Chapter Four

The probability

Dif:1 The random experiment

It is an experiment which we know in advance all its possible outcomes but we cannot predict which of these outcomes will occure when the experiment. carried on

Dif:2 The Sample space

It is the set of all possible outcomes of the random experiment. and denoted by S.

Example.1

In the following exp. write down the sample space:

i) Tossing a coin one time.

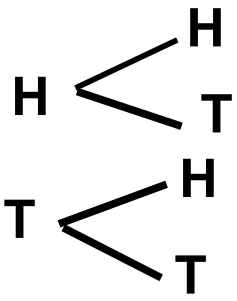
ii) Tossing a coin two times.

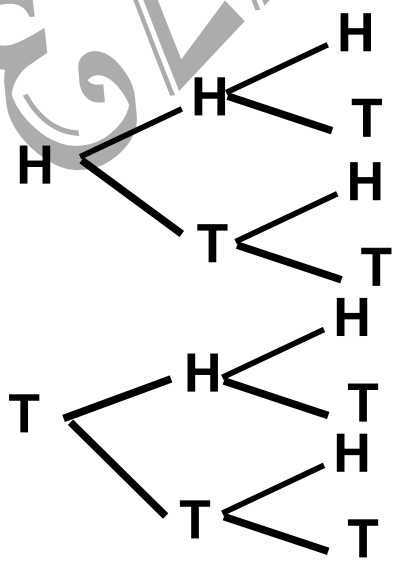
iii) Tossing a coin three times.

Solution

i) S = { (H,T) } , n(s) = 2^1 = 2

ii) S = { (H,H) , (H,T) , (T,T) , (T,H) } , n(s) = 2^2 = 4





iii) S = { (H,H,H) , (H,H,T) , (H,T,H) , (H,T,T),

(T,H,H) , (T,H,T) , (T,T,H) , (T,T,T) } , n(s) = 2^3 = 8

Example.2

In the following exp. write down the sample space:

i) Tossing a die one time.

ii) Tossing a die two times.

Solution

i) S = { 1 , 2 , 3 , 4 , 5 , 6 } , n(s) = $2^1 = 2$

ii)

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

, n(s) = $6^2 = 36$

Types of events

1) The certain event

It is the event which must occure when we do the experiment

2) The impossible event ϕ

It is the event which is impossile to occure

3) The simple event

It is the set which contain one element

4) mutually exclusive

They cannot occure simultaneously

Example.3

In the following exp. write down the sample space:

i) Tossing a coin then a die one time.

Solution

i) S = { (H,1) , (H,2) , (H,3) , (H,4) , (H,5) , (H,6) ,
(T,1) , (T,2) , (T,3) , (T,4) , (T,5) , (T,6) }

Example.4

In the following exp. write down the sample space:
Selection of two cards one after the other with replacement from a box containing 4 cards numbered from 1 to 4

Solution **" with replacement"**

S = { (1,1) , (1,2) , (1,3) , (1,4) , (2,1) , (2,2) , (2,3) , (2,4) ,
(3,1) , (3,2) , (3,3) , (3,4) , (4,1) , (4,2) , (4,3) , (4,4) }

" without replacement"

S = { (1,2) , (1,3) , (1,4) , (2,1) , (2,3) , (2,4) ,
(3,1) , (3,2) , (3,4) , (4,1) , (4,2) , (4,3) }

Axioms of probability

(1) $P(A) = \dfrac{\text{The numbers of elements of A}}{\text{The number of elements of S}} \Rightarrow P(A) = \dfrac{n(A)}{n(S)}$

(2) $0 \le P(A) \le 1$

(3) $P(\varnothing) = 0$, $P(S) = 1$

Rule. 1

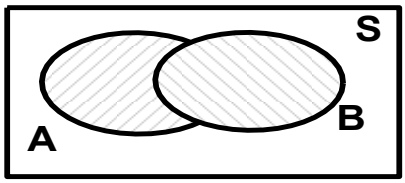
If A , B are two mutually exclusive events then $P(A \cap B) = 0$

Rule. 2

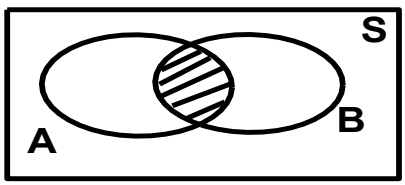
If $A \subset B$, then $P(A \cap B) = P(A)$, and $P(A \cup B) = P(B)$

Operations on events

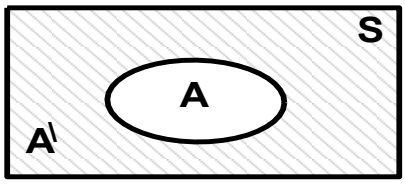
(1) Occ. of A or B
Occ. of one of them at least
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$



(2) Occ. of A and B together
$P(A \cap B) = P(A) + P(B) - P(A \cup B)$



(3) Non occ. of A
$P(A') = 1 - P(A)$

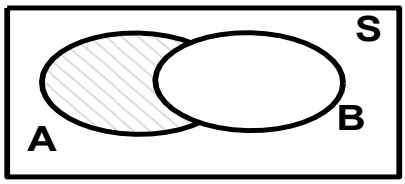


(4) Occ. of A and non B

Occ. of A only

$P(A-B) = P(A \cap B^{\setminus})$

$= P(A) - P(A \cap B)$



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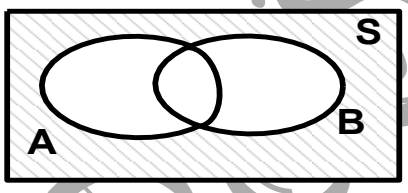
(5) Non occ. of neither A nor B

$P(A^{\setminus} \cap B^{\setminus}) = P(A \cup B)^{\setminus}$

$= 1 - P(A \cup B)$



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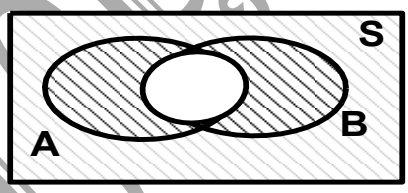
(6) Non occ. of A and B together

at most one of them

$P(A^{\setminus} \cup B^{\setminus}) = P(A \cap B)^{\setminus}$

$= 1 - P(A \cap B)$



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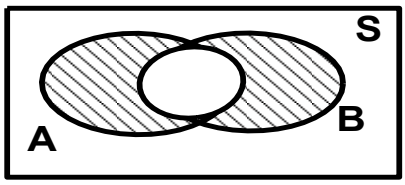
(7) Occ. of only one of A or B

$P[(A-B) \cup (B-A)] = p(A) + P(B) - 2P(A \cap B)$



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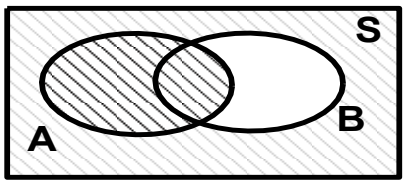
(8) Occ. of A or non occ. of B

$P(A \cup B^{\setminus}) = 1 - P(B-A)$



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(9) $P(A^{\setminus}-B^{\setminus}) = P(B-A) = P(B) - P(A \cap B)$

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(10) $P(A-B^{\setminus}) = P(A \cap B)$

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(11) $P(A^{\setminus}-B) = P(A^{\setminus} \cap B^{\setminus}) = 1 - P(A \cup B)$

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(12) $P(A-B)^{\setminus} = 1 - P(A-B)$

Example.1

One card is drawn randomly from a box containing 9 cards numbered from 1 to 9. describe the sample space of this exp. and then calculate the peob. of the following events:

A: The event of the occ. of an odd number on the selected card
B: The event of the occ. of a prime number on the selected card
C: The event of the occ. of A or B
D: The event of the occ. of A and B
E: The event of the occ. of only A
F: The event of non occ. of neither A nor B

Solution:

$S = \{ 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9 \}$

$A = \{ 1 , 3 , 5 , 7 , 9 \}$ $\quad P(A) = \frac{5}{9}$

$B = \{ 2 , 3 , 5 , 7 \}$ $\quad P(B) = \frac{4}{9}$

$C = A \cup B = \{ 1 , 2 , 3 , 5 , 7 , 9 \}$ $\quad P(A \cup B) = \frac{6}{9} = \frac{2}{3}$

$D = A \cap B = \{ 3 , 5 \}$ $\quad P(A \cap B) = \frac{3}{9} = \frac{1}{3}$

$E = A - B = \{ 1 , 9 \}$ $\quad P(A \cup B) = \frac{2}{9}$

$F = (A \cup B)' = \{ 4 , 6 , 8 \}$ $\quad P((A \cup B)') = \frac{3}{9} = \frac{1}{3}$

Example.2

A die is tossed twice, and the upper face is observed in each time, Find the probability of each of the following events:

A: The event of the occ. of two numbers whose sum is 8.
B: The event of the occ. of two numbers whose difference is 3.
C: The event of the occ. of two numbers whose sum divisable by 3.
D: The event of the occ. of two numbers whose sum divisable by 5.

E: The event of the occ. of two numbers whose sum divisable by 3 and 5.

F: The event of the occ. of two numbers whose sum divisable by 3 or 5.

Solution:

$A = \{ (2,6) , (3,5) , (4,4) , (6,2) , (5,3) \}$ $\quad P(A) = \frac{5}{36}$

$B = \{ (1,4) , (2,5) , (3,6) , (4,1) , (5,2) , (6,3) \}$ $\quad P(B) = \frac{6}{36}$

$C = \{ (1,2) , (1,5) , (2,1) , (2,4) , (3,3) , (3,6) , (4,2) , (4,5) , (5,1) , (5,4) , (6,3) , (6,6) \}$ $\quad P(C) = \frac{12}{36} = \frac{1}{3}$

$D = \{ (1,4) , (2,3) , (3,2) , (4,1) , (4,6) , (5,5) , (6,4) \}$ $\quad P(C) = \frac{7}{36}$

$E = E \cap C = \varnothing$ $\quad P(E) = P(\varnothing) = 0$

$F = C \cup D \rightarrow P(F) = P(C \cup D) = P(C) + P(D) - P(C \cap D)$

$$= \frac{1}{3} + \frac{7}{36} - 0 = \frac{19}{36}$$

Example.3

A coin is tossed three consecutive times, and the upper face is observed in each time, find the probability of each of the following events:

A: The event of the occ. of at least one head

B: The event of the occ. of at most one head

C: The event of the occ. of exactly one head

D: The event of the occ. of two consecutive heads

Solution:

$S = \{ (H,H,H) , (H,H,T) , (H,T,H) , (H,T,T) , (T,H,H) , (T,H,T) , (T,T,H) , (T,T,T) \}$

$A = \{ (H,H,H) , (H,H,T) , (H,T,H) , (H,T,T) , (T,H,H) , (T,H,T) , (T,T,H) \}$

$P(A) = \frac{7}{8}$

$B = \{ (H,T,T) , (T,H,T) , (T,T,H) , (T,T,T) \}$ $\quad P(B) = \frac{4}{8} = \frac{1}{2}$

$C = \{ (T,H,T) , (T,T,H) , (H,T,T) \}$ $\quad P(B) = \frac{3}{8}$

$D = \{ (H,H,H) , (H,H,T) , (T,H,H) \}$ $\quad P(B) = \frac{3}{8}$

Example.4

A box containing 3 balls one red, one white and one black, two balls are selected at random one after the other with replacement.
calculate the probability of:

i: The two selected balles have the same color.
ii: At least one red ball.
iii: At most one red ball.

Solution:

S = { (b,b) , (b,w) , (b,r) , (r,w) , (r,b) , (r,r) , (w,r) , (w,b) , (w,w) }

i: A = { (b,b) , (r,r) , (w,w) } $P(B) = \frac{3}{9} = \frac{1}{3}$

ii: B = { (b,r) , (r,w) , (r,b) , (r,r) , (w,r) } $P(B) = \frac{5}{9}$

iii: C = { (b,b) , (b,w) , (b,r) , (r,w) , (r,b) (w,r) , (w,b) , (w,w)} $P(B) = \frac{8}{9}$

Example.5

The prob. that Mohamed succeeds in math's exam is 0.75, and the prob. that he succeeds in physics is 0.65, and the prob. that he succeeds in both of them is 0.45, Find

i: The prob. that he succeeds in at least one of them.
ii: The prob. that he fails in both.
iii: The prob. that he succeeds only in maths.
iv: The prob. that he succeeds in at most one of them.

Solution:

let P(maths) = P(A) = 0.75 , P(physics) = P(B) = 0.65
P(both) = P(A∩B) = 0.45

i)P(A∪B) = P(A) + P(B) - P(A∩B) = 0.75 + 0.65 - 0.45 = 0.95

ii) $P(A \cup B)^{\setminus}$ = 1 - P(A∪B) = 1 - 0.95 = 0.05

iii) P(A - B) = P(A) - P(A∩B) = 0.75 - 0.45 = 0.3

iv) $P(A \cap B)^{\setminus}$ = 1 - P(A∩B) = 1 - 0.45 = 0.55

Example.6

If A, B are two events in S, P(A) = 0.7 , P(B) = 0.6 , P(A∩B) = 0.4,
Find: P(A∪B) , P(A∩B$^{\backslash}$) , P(A$^{\backslash}$∩B$^{\backslash}$)

Solution:

P(A∪B) = P(A) + P(B) - P(A∩B) =0.7 + 0.6 - 0.4 = 0.9

P(A∩B$^{\backslash}$) = P(A) - P(A∩B) = 0.7 - 0.4 = 0.3

P(A$^{\backslash}$∩B$^{\backslash}$) = P(A∪B)$^{\backslash}$ = 1 - P(A∪B) = 1 - 0.9 = 0.1

Example.7

If A, B are two events in S, P(A) = 0.6 , P(B$^{\backslash}$) = 0.6 , P(A$^{\backslash}$∪B$^{\backslash}$) = 0.7,
Find: P(A-B) , P(A$^{\backslash}$∩B$^{\backslash}$) , P(A$^{\backslash}$∩B)

Solution:

P(B) = 1 - P(B$^{\backslash}$) = 1 - 0.6 = 0.4 and,

P(A$^{\backslash}$∪B$^{\backslash}$) =P(A∩B)$^{\backslash}$ = 1 - P(A∩B) = 0.7
⇒ P(A∩B) = 1 - 0.7 = 0.3

1) P(A-B) = P(A) - P(A∩B) = 0.6 - 0.3 = 0.3

P(A∪B) = P(A) + P(B) - P(A∩B) =0.6 + 0.4 - 0.3 = 0.7

2) P(A$^{\backslash}$∩B$^{\backslash}$) = 1 - P(A∪B) = 1 - 0.7 = 0.3

3) P(A$^{\backslash}$∩B$^{\backslash}$) = P(A∪B)$^{\backslash}$ = 1 - P(A∪B) = 1 - 0.9 = 0.1

Example.8

If A, B are two events in S, P(A) = 0.7 , P(A-B) = 0.3 , P(B$^{\backslash}$) = 0.4,
Find: P(A∪B) , P(A$^{\backslash}$∩B$^{\backslash}$) , P(A$^{\backslash}$∩B)

Solution:

P(B) = 1 - P(B$^{\backslash}$) = 1 - 0.4 = 0.6

P(A-B) = P(A) - P(A∩B)
0.3 = 0.7 - P(A∩B) ⇒ P(A∩B) = 0.7 - 0.3 = 0.4

P(A∪B) = P(A) + P(B) - P(A∩B) = 0.7 + 0.6 - 0.4 = 0.9

$P(A^{\setminus} \cap B^{\setminus}) = P(A \cup B)^{\setminus} = 1 - P(A \cup B) = 1 - 0.9 = 0.1$

$P(A^{\setminus} \cap B) = P(B) - P(A \cap B) = 0.6 - 0.4 = 0.2$

Example.9

A class contains 50 students, 25 of them study chemistry, 20 of them study history and 15 study both. A student is chosen at random. find the pro. that

i) The student studies chemistry or history

ii) The student does not study either of these subjects

iii) The student studies only history

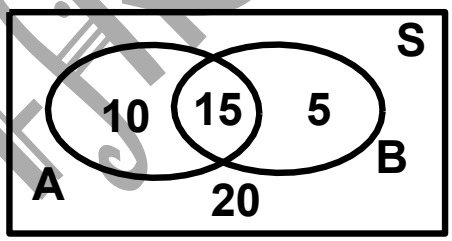
Solution:

chemistry = A $\Rightarrow P(A) = \frac{25}{50} = \frac{1}{2}$

history = B $\Rightarrow P(B) = \frac{20}{50} = \frac{2}{5}$

Both = $A \cap B \Rightarrow P(A \cap B) = \frac{15}{50} = \frac{3}{10}$

another solution



i) The student studies chemistry **or** history

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{2} + \frac{2}{5} - \frac{3}{10} = \frac{3}{5}$

ii) The student does **not** study either of these subjects

$P(A^{\setminus} \cap B^{\setminus}) = 1 - P(A \cup B) = 1 - \frac{3}{5} = \frac{2}{5}$

iii) The student studies **only** history

$P(B-A) = P(B) - P(A \cap B) = \frac{2}{5} - \frac{3}{10} = \frac{1}{10}$

Example.10 **Two students try to solve a problem if the prob. that 1st student only solve it is 0.3 and the prob. that the 2nd solve it is o.6 and the prob. that the both of them solve it is 0.4. calculate the prob. that the problem is solvrd by only one of them**

Solution:

$P(A \cap B) = 0.4$, $P(B) = 0.6$

$P(A-B) = P(A) - P(A \cap B) \quad \therefore \Rightarrow 0.3 = P(A) - 0.4 \ \therefore \Rightarrow P(A) = 0.7$

$P(A-B) \cup P(B-A) = P(A) + P(B) - 2P(A \cap B)$

$= 0.7 + 0.6 - 2 \times 0.4 = 0.5$

Example.11

Three players A, B, C associated in a swimming race, if the prob. that A wins equals double the prob. that B wins, and the prob. that B wins equals the prob. that C wins. Find the prob. that B or C wins

Solution:

$P(B) = P(C) = x \quad , P(A) = 2x$

$x + x + 2x = 1 \quad \therefore\Rightarrow \quad 4x = 1 \quad \therefore\Rightarrow \quad x = \frac{1}{4}$

$P(A) = 2\times\frac{1}{4} = \frac{1}{2}$ and $P(B) = P(C) = x = \frac{1}{4}$

$P(B\cup C) = P(B) + P(C) - P(B\cap C) = \frac{1}{4}+\frac{1}{4}-0 = \frac{1}{2}$

Example.12 **A die is designed such that the prob. of occurrence of the numbers 1,2,3,4,5 are equal, and the prob. of occ. of the number 6 is double the prob. of occ. of the number one, if this die is tossed one time. Calculate the prob. of occ. of an even number.**

Solution:

$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$

$x + x + x + x + x + 2x = 1 \quad \therefore\Rightarrow 7x = 1 \quad \Rightarrow x = \frac{1}{7}$

$A = \text{Even numbers} = \{ 2 , 4 , 6 \} \quad P(A) = \frac{1}{7}$

Example.13 **If A,B are two mutually exclusive events, P(A - B) = o.7 $P(A\cup B) = 0.9$ Find : $P(B^{\setminus})$, $P(A^{\setminus}\cup B^{\setminus})$, $P(A^{\setminus}\cap B^{\setminus})$**

Solution:

$P(A\cap B) = 0$

$P(A - B) = P(A) - P(A\cap B) \quad \therefore\Rightarrow \quad 0.7 = P(A) - 0 \quad \therefore\Rightarrow \quad P(A) = 0.7$

1] $P(B^{\setminus}) = 0.9 - 0.7 = 0.2 \quad \therefore\Rightarrow \quad P(B^{\setminus}) = 1 - 0.2 \quad \therefore\Rightarrow \quad P(B^{\setminus}) = 0.7$

2] $P(A^{\setminus}\cup B^{\setminus}) = 1 - P(A\cap B) = 1 - 0 = 1$

3] $P(A^{\setminus}\cap B^{\setminus}) = 1 - P(A\cup B) = 1 - 0.9 = 0.1$

Example.14

If A, B are two events in S, P(A) = 0.5 , P(B) = 0.4 , Find $P(A' \cap B')$ in each of the following cases:

i. $P(A' \cup B') = 0.8$

ii. A , B are mutually exclusive events

iii. $B \subset A$

Solution:

i. $P(A' \cup B') = 1 - P(A \cap B)$

$0.8 = 1 - P(A \cap B)$

$P(A \cap B) = 0.2$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.4 - 0.2 = 0.7$

$P(A' \cap B') = 1 - (A \cup B) = 1 - 0.7 = 0.3$

ii. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.4 = 0.9$

$P(A' \cap B') = 1 - (A \cup B) = 1 - 0.9 = 0.1$

iii. $B \subset A \quad \therefore \Rightarrow \quad P(A \cup B) = P(A) = 0.5$

$P(A' \cap B') = 1 - (A \cup B) = 1 - 0.5 = 0.5$

Example.15 **If A,B are two events in S, $A \subset B$, given that P(A) = 0.5 and the prob. that only B occurrence = 0.2**

Find the prob. that B does not occurred

Solution:

$\because A \subset B \quad \therefore \Rightarrow P(A \cap B) = P(A)$ and $P(A \cup B) = P(B)$

$\because$ the prob. that only B occurrence = 0.2 $\quad \therefore \Rightarrow P(B-A) = 0.2$

$P(B-A) = P(A) - P(B)$

$0.2 = P(B) - 0.5 \quad \therefore \Rightarrow P(B) = o.2 + 0.5 = 0.7$

the prob. that B does not occurred = $P(B')$

$P(B') = 1 - P(B) = 1 - 0.7 = 0.3$

Example.16 If A,B are two events in S, $\frac{P(A)}{P(A^{\setminus})} = \frac{3}{2}$, and P(B) = 0.7,

P(A-B) = 0.3 Find $P(A^{\setminus} \cap B^{\setminus})$, $P(A-B^{\setminus})$

Solution:

$2P(A) = 3\,P(A^{\setminus})$ $\quad 2P(A) = 3[1-P(A)] = 3 - 3\,P(A)$

$5P(A) = 3 \quad P(A) = 3/5 = 0.6$ and $P(B) = 0.7$

$P(A-B) = P(A) - P(A \cap B)$

$0.3 = 0.6 - P(A \cap B) \quad \therefore \Rightarrow P(A \cap B) = 0.3$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = O.6 + 0.7 - 0.4 = 0.9$

$P(A^{\setminus} \cap B^{\setminus}) = 1 = P(A \cup B) = 1 - 0.9 = 0.1$

$P(A-B^{\setminus}) = P(A \cap B) = 0.3$

Example.17 If A,B are two events, in S $P(A) = 2P(A^{\setminus})$, and P(B) = 0.5 ,

$P(A \cap B) = 0.25$, find $P(A \cup B)$, $P(A \cap B^{\setminus})$, $P(A-B^{\setminus})$, $P(A^{\setminus}-B^{\setminus})$

Solution:

$P(A) = 1 - P(A^{\setminus}) \quad \therefore \Rightarrow 2P(A^{\setminus}) = 1 - P(A^{\setminus})$

$3P(A^{\setminus}) = 1 \quad \therefore \Rightarrow P(A^{\setminus}) = 1/3$ and $P(A) = 2/3$

i) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = (2/3) + (1/2) - (1/4) = 11/12$

ii) $P(A \cap B^{\setminus}) = P(A) - P(A \cap B) = (2/3) - (1/4) = 5/12$

iii) $P(A-B^{\setminus}) = P(A^{\setminus}) - P(A \cap B^{\setminus}) = (2/3) - (5/12) = 1/4$

iv) $P(A^{\setminus}-B^{\setminus}) = P(B-A) = P(B) - P(A \cap B) = 0.5 - 0.25 = 0.25$

Home work

1. A student wants to buy a bag. It is possible to choose from three types. Each one has two sizes and the color of the bag is either black or brown. Represent the sample space by a tree diagram.
2. In an experiment of tossing a coin once, then a die is rolled, observing the upper faces.
   a. Write down the sample space of this experiment, then determine the following events.
   - A: appearance of a head and an odd number.
   - B: appearance of a tail and an even number
   - C: appearance of a prime number > 2
   - D: appearance of a number divisible by 3
3. A die is rolled two consecutive times, the number on the upper face is observed in each time Determine each of the following events:
   - A: The appearance of two equal numbers
   - B: The appearance of two numbers their sum equals 9
   - C: The appearance of two numbers their sum equals 13
   - D: The appearance of number 3 one time at least.
4. From the set of numbers {1, 2, 3, 4} we need to form a two different digit number. Represent the sample space in a tree diagram, and then determine the following events:
   - A: The event "the unit digit is an odd number ".
   - B: The event "the tens digit is an odd number".
   - C The event "the two digits are odd numbers".
   - D The event "the unit digit or the tens digit is an odd number.
5. A bag contains 20 identical cards numbered from 1 to 20, If a card is selected randomly and the number written on it is recorded. Write the following events:
   A the event "the recorded number is even and greater than 10
   B the event "the recorded number is a factor of 12"
   C the event "the recorded number is odd and divisible by 3
   D the event "the recorded number is a multiple of the two numbers 2, 5
   E the event "the recorded number is prime"
   F the event "the recorded number satisfying the inequality $5x - 3 \leqslant 17$
6. Two cards are drawn one after the other from a set of 8 identical cards numbered from 1 to 8 and the drawn card must returned before drawn another card. What is the number of the elements in the sample space? and if:
   A: is the event "the number in the second draw is three times the number in the first draw"
   B: is the event "the sum of the two numbers is more than 13"
   Write the events A, B. Are there two mutually exclusive events? Explain that.

7 In the experiment of tossing a coin three consecutive times and observing the sequence of heads and tails .represent the sample space with tree diagram, then determine the following events:

A the event "appearance of two tails at least" B the event "appearance of two tails at most"

C the event " appearance of a head in the first toss"

A the event " non-appearance of a head in the three tosses"

8 In an experiment of tossing a coin once, then a die is rolled, observing the upper faces Represent the sample space of this experiment by a probability tree diagram, and then determine the following events:

A appearance of a Tail and an even number"

B appearance of a head and an odd number"

C non-occurrence of A or non- occurrence of B"

D occurrence of the event A only

E occurrence of the event A and occurrence of the event B

Choose the correct answer from those given :

9 If a regular die is rolled once, then the probability of the appearance of an odd number less than 5 in the upper face equals:

a $\frac{2}{5}$ b $\frac{1}{2}$ c $\frac{1}{3}$ d $\frac{1}{6}$

10 If a regular die is rolled twice, then the probability of the appearance of an even number in the first roll and a prime number in the second roll equals:

a $\frac{1}{3}$ b $\frac{1}{6}$ c $\frac{1}{9}$ d $\frac{1}{4}$

11 If a ball is drawn randomly from a box contained 3 white balls, 5 red balls and 7 green balls, then the probability that the selected ball is white or green equals:

a $\frac{1}{5}$ b $\frac{2}{3}$ c $\frac{7}{15}$ d $\frac{1}{2}$

12 A card is drawn from a set of 9 identical cards numbered from 1 to 9. What is the probability that the drawn card carrying a divisor of (factor of) 9 or an odd number equals:

a $\frac{1}{3}$ **b** $\frac{7}{9}$ **c** $\frac{1}{2}$ **d** $\frac{5}{9}$

13 If A , B are two events in a sample space of a random experiment $B \subset A$,
$P(A) = 2P(B) = 0.6$ then $P(A - B)$ equals:

a 0.6 **b** 0.3 **c** 0.4 **d** 0.2

14 A uniform die, the numbers 8, 9, 10, 11, 12, 13 written in its faces. If the die is rolled once, observing the number appearing on its upper face

a Find the probability of each of the following events:

- A: "appearance of an odd number."
- B "appearance of a prime number."
- C: "appearance of an even number."
- D "appearance of a number great than 12."
- E: "appearance of a number consists of two digits."
- F "appearance of a number consists of only one digit."

b Calculate: $P(A \cup C)$, $P(E \cup F)$, $P(B \cap D)$.

15 If is a sample space of a random experiment, where $S = \{A , B, C, D\}$, find:
$P(A)$, $P(B)$, given that $P(A) = 3\,P(B)$, $P(C) = P(D) = \frac{7}{18}$

16 If A, B are two mutually exclusive events, S is a sample space of its random experiment, If $P(A \cup B) = 0.6$, $P(A - B) = 0.25$ find, $P(A)$, $P(B)$.

17 If A, B is a sample space of a random experiment, and $P(A) = \frac{1}{3}$, $P(B) = \frac{3}{8}$, $P(A \cap B) = \frac{1}{4}$ find:

a $P(A')$ **b** $P(A \cup B)$ **c** $P(A - B)$ **d** $P(A' \cap B')$

18 If A, B are two events, of a sample space of a random experiment, where: $P(A) = 0.4$,
$P(B') = 3P(B)$, $P(A \cap B) = 0.2$ find the probability of:

a Occurrence of A only. **b** Occurrence of A or B
c Occurrence of A and non-Occurrence of B.

19 A box contains colored identical balls: 4 red, 6 blue, and 5 yellow. A ball is selected at random from the box, find the probability that the drawn ball is:

a Red. **b** Blue or yellow. **c** Not blue. **d** Not red and not yellow.

20 One card is selected at random from 30 identical cards numbered from 1 to 30. Find the probability that the selected card is carring a number:

a Divisible by 3 **b** divisible by 5
c Divisible by 3 and 5 **d** divisible by 3 or 5

21 Three distinct coins are tossed once. Observing the upper faces, find the probability of the following events:

- A: appearance of a head or two heads.
- B: appearance of at least one head.
- C: appearance of a head at most.
- D: appearance of at two consecutive tails at least.

22 A die is rolled two consecutive times, the number on the upper face is observed in each time. Find the probability of each of the following events:

- Appearance of the number 4 in the first roll.
- Appearance of two numbers, their sum equals 8
- Appearance of two numbers, their sum is less than or equal 5

23 **Join with sport :** A random sample consists of 60 persons in a survey, it is found that 40 of them encourage Al Hilal club, 28 of encourage El negma club and 8 of them don't encourage any of them. A person is chosen at random from the sample. Find the probability that the chosen person encourages:

a At least one of the two clubs. **b** Both clubs.
c Al Hilal club only. **d** Only one of the two clubs.

24 In an experiment of tossing a coin once, then a die is rolled once , observing the upper faces. If A is the event of the appearance of a head and a prime number, B is the event of the appearance of an even number. Find the probability of the occurrence of each of the two events, and then calculate the probability of the following events:

a The occurrence of one of the events at least
b The occurrence of the two events together
c The occurrence of only the event B
d The occurrence of only one of the two events

25 a card is selected randomly from 50 identical cards numbered from 1 to 50, if the number written on it is recorded. Find the probability that the number written on the selected card is:

a A multiple of number 7 **b** A perfect square number
c A multiple of number 7 and a perfect square number
d Not a perfect square number and not a multiple of 7

26 If A , B are two events, in a sample space of a random experiment, where: $P(B) = \frac{4}{5} P(A)$, $P(A - B) = 0.24$ $P(B \cap A') = 0.15$ then find: $P(A)$, $P(B)$, $P(A \cup B)$, $P(A' \cup B')$

27 Tarek wrote 75 letters on the typewriter ,he found that 60% of them are without mistakes and Zead wrote 25 letters on the typewriter ,he found that 80% of them are without mistakes. If a letter is selected randomly from all letters written by both Tarek and Zead ,then find the probability that the selected letter is:

a Without mistakes. **b** Written by Zead.
c Written by Zead without mistakes. **d** Written by Tarek with mistakes.

28 If A , B are two events, in a sample space of a random experiment, where: $P(A) = 0.6$, $P(B) = 0.8$, $P(A' \cup B') = 0.5$ then find $P(A' \cap B)$

General Exercise on (unit four)

Choose the correct answer from those given:

In an experiment of rolling a die twice then:

1. The probability of the appearance of number 5 in the first roll and number 6 in the second roll is:

   a $\frac{1}{24}$ b $\frac{1}{30}$ c $\frac{1}{36}$ d $\frac{1}{6}$

2. The probability of the appearance of number 5 in one of the two rolls and the number 6 in the other roll is:

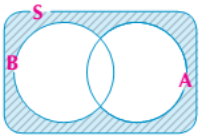
   a $\frac{1}{12}$ b $\frac{1}{6}$ c $\frac{5}{36}$ d $\frac{1}{18}$

3. The probability of the appearance of two equal numbers in the two rolls

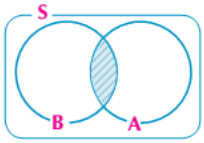
   a $\frac{1}{5}$ b $\frac{1}{36}$ c $\frac{1}{6}$ d $\frac{1}{18}$

If A and B are two events in a sample space of a random experiment, choose the event that represents the shaded parts in the opposite venn diagram :

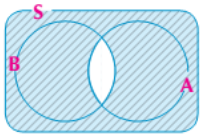
4. a $S - (A \cup B)$ b $A' \cup B'$
   c $S - (A' \cup B')$ d $(A \cap B)'$



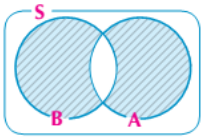
5. a $A \cap B$ b $A \cup B'$
   c $(A \cup B)'$ d $A' \cup B'$



6. a $(A \cup B) - (A \cap B)$ b $S - (A \cap B)'$
   c $S - (A \cup B)'$ d $S - (A \cap B)$



7. a $S - (A \cap B)$ b $A' \cap B'$
   c $(A - B) \cup (B - A)$ d $S - (A \cup B)'$



8. Form two digit number from the digits of the number 4321 , and represents the sample space using the probability tree diagram ,then write the sample space and the following events:
   A: event" the set of prime numbers"
   B: event" the set of numbers that divisible by 3"
   C: event" the set of numbers that divisible by 3 and 5"
   D: event" the set of numbers that its unit digit is twice of its tens digit"

9. In the experiment of rolling a die once and observing its upper face .Find the probability that the appearance number is:
   a A prime number b A factor of 6 c An odd number and divisible by 3

10 If A, B are two events in a sample space S of a random experiment.

Complete each of the following to get true statement:

a If A, B are two mutually exclusive events:

- $A \cap B =$ ..... - $A - B =$ ..... - $B - A =$ .....

b If $B \subset A$ then:

- $(A \cap B) =$ ..... - $A \cup B =$ ..... - $B - A =$ .....

c If $P(A) = 0.3$, $P(B) = 0.5$, $P(A \cap B) = 0.1$ then:

- $P(A \cup B) =$ ..... - $P(B - A) =$ ..... - $P(A' \cap B') =$ .....

d If $A \cap B = \phi$, $P(A') = 0.7$, $P(B') = 0.4$ then:

- $P(A \cup B) =$ ..... - $P(A \cap B) =$ ..... - $P(A - B) =$ .....

11 If A, B are two events in a sample space S of a random experiment, $A \subset B$, $P(A) = \frac{1}{2}$, and probability of occurrence of B only = 0.2. Calculate the probability of non-occurrence of B.

12 If A, B are two events in a sample space S of a random experiment , $P(A) = \frac{1}{2}$, , $P(B) = x$ and $P(A \cup B)' = \frac{1}{3}$

a Find the value of x in each of the following:

- If A, B are two mutually exclusive events. - $A \subset B$.

b If $x = \frac{1}{4}$ Find the value of $P(A \cap B)$.

13 If A, B are two events, of a sample space S of a random experiment, write a symbolical expression of each of the following events, representing them by Venn -diagrams:

a Non occurrence of A

b Occurrence of A or B.

c Occurrence of B only.

d Occurrence of A or non-occurrence of B.

e Non occurrence of both of them together.

f Occurrence of only one of them.

14 A coin is tossed three consecutive times, observing the upper faces, represent the sample space by a tree diagram. Find the probability of the following events:

a A: appearance of only two heads

b B: appearance of at most two heads

c C: appearance of at most one tail

d D: appearance of same outcomes in the three tosses

15 A die is tossed two consecutive times, the number on the upper face is observed in each time.

a Draw a tree diagram that represents the sample space, explaining the following events on it.

- A: appearance of two numbers, their sum is an odd number which is greater than 6
- B: appearance of two numbers , one of them is 2 and their sum $\leqslant 5$
- C: appearance of two equal numbers

b Are A, B and C two by two mutually exclusive events?

c Calculate the probability of each of the following: $P(A \cup B)$, $P(B \cap C)$, $P(A \cup C)'$, $P(B - C)$.

16 Five identical cards numbered from 2 to 6, two cards are drawn randomly one after the other with replacement and observing the sequence of the numbers on the selected cards to form all possible two digit numbers. Find the probability that:

a its unit digit is a prime number.

b its tens digit is an odd number.

c its unit digit is a prime number or its tens digit is an odd number.

Newspaper:

17 In a sample of 50 persons, it is found that 27 of them read newspaper (A), 24 of them read newspaper (B) and 9 persons read both newspapers. A person is chosen at random from this sample. Find the probability that the chosen person read:

a Newspaper A only.

b At least one of the two newspapers.

Tourism:

18 In one of the sound and light shows at the pyramids, 200 people attend from various nationalities, the opposite table shows their data. If one of them was chosen at random by their access cards to give him the souvenir prize, then: Find the probability that the chosen person is :

	Arab	European	American	Total
Man	32	47	15	94
Woman	23	63	20	106
Total	55	110	35	200

a An European man.

b An American Woman.

c A Woman.

d With Arab or European nationality.

19 If $S = \{A, B, C\}$, is a sample space of a random experiment and, $20P(A) = 15P(B) = 12P(C)$ Find: $P(A)$, $P(B)$, $P(C)$