

UNIT (2)

Fluid Mechanics

Chapter (4)

Hydrostatic (fluids at rest)

Fluids: materials which can flow and can't take a fixed shape, but take the shape of the container (liquids and gases).

- **Gases** are compressible (take the volume and shape of the container). They fill any volume they occupy (diffuse).
- **Liquids** are incompressible (have a fixed volume and take the shape of its container.).

Density: the mass per unit volume.

$$\rho = \frac{m}{V_{ol}} \quad \text{kg/m}^3$$

- **The density is a basic property of matter as it depends on:**
 - 1) Type of material.
 - 2) Temperature.
- **The density varies from one substance to another due to:**
 - 1) Atomic weights (the density increases as atomic weight increases).
 - 2) Inter-atomic or intermolecular spaces (the density increases as intermolecular spaces decrease).
- **The density of a substance change by changing temperature** (because the volume of a certain mass of any substance change by changing temperature).
- **The density of a gas varies according to the pressure applied on it** (so density is not a physical property for gases).
- Bodies of less density float over more dense liquids.
- The density of fresh water at 4° C is 1000 kg/m³ or 1g/cm³ or 1kg/liter.
- $1 \text{ g/cm}^3 = 1000 \text{ kg/m}^3$.

Relative density (specific weight):

The ratio between the density of any material and the density of water at the same temperature.

$$R D_{\text{substance}} = \frac{\rho_{\text{substance}}}{\rho_{\text{water}}} \quad \text{at the same temperature}$$

$$RD_{\text{substance}} = \frac{m_{\text{substance}} / V_{\text{ol}}}{m_{\text{water}} / V_{\text{ol}}} \Rightarrow RD_{\text{substance}} = \frac{m_{\text{substance}}}{m_{\text{water}}}$$

Or; the ratio between the mass of any volume of a substance to the mass of the same volume of water at the same temperature.

- The relative density has no units (because it is a ratio between 2 similar quantities).
- $\rho_{\text{substance}} = (RD_{\text{substance}}) (\rho_{\text{water}}) = (RD_{\text{substance}}) (1000 \text{ kg/m}^3)$

What is meant by:

1. The density of oil = 600 kg/m³.

It means that the mass per unit volume of oil = 600 kg.

2. The relative density of aluminum = 2.7

It means that the ratio between the density of aluminum to that of water at the same temperature = 2.7

Applications on density

1) How well the car battery is charged:

By measuring the density of the electrolyte (dilute sulfuric acid) in the car battery.

- **When battery is discharged:** the acid reacts with lead forming lead sulphate on the lead plates of the battery, so the density of the electrolyte is low due to the consumption of the acid.
- **When battery is charged:** sulphate groups liberate from the lead plates and return to the electrolyte, so the density of the electrolyte increases.

2) Clinical analysis:

a. Measuring blood density:

- a) Higher density indicates higher concentrations of red blood cells.
 - b) Lower density (lower concentration of red blood cells) indicates anemia.
- The normal blood density is (1040 kg/m³ - 1060 kg/m³).

b. Measuring urine density:

Some diseases increase the concentrations of salts in the urine, so the density increases.

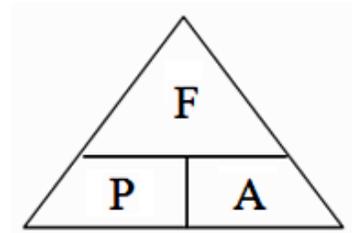
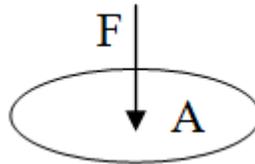
- The normal urine density is 1020 kg/m³.

Pressure at a point

The average force which acts normally on unit area surrounding that point.

- If the force is affecting \perp on the area:

$$P = \frac{F_{\perp}}{A}$$



- **Units of pressure:**

(N/m²) or (kg m⁻¹ sec⁻²) or (Pascal) or (J/m³)

- **Factors affecting pressure:**

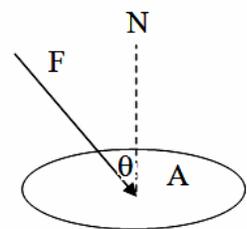
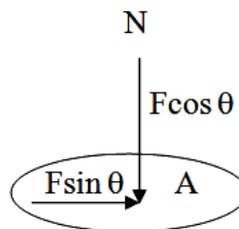
1) **Force:** the pressure is directly proportional with the force at constant area
($P \propto F$).

2) **Area:** the pressure is inversely proportional with the area at constant force.

$$P \propto \frac{1}{A}$$

- If the force is inclined with an angle (θ) with the normal to the area:

$$P = \frac{F \cos \theta}{A}$$



What is meant by:

The pressure at a point = 70 N/m²

The average force which acts normally on unit area surrounding that point = 70 N.

G R F:

1. **Needles, nails and knives have sharp edges.**

Because the pressure is inversely proportional with the area.

2. **Big load cars that move in deserts have large area tires.**

Because the pressure is inversely proportional with the area,

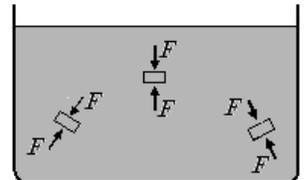
Pressure at a point inside a liquid

If you push a piece of foam under water and let it go, it will rise and float. Why?

Because water pushes the immersed foam with an upward force due to the pressure difference across the piece of foam.

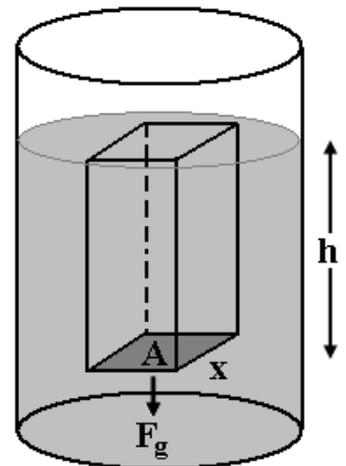
Notes:

- 1- At any point inside a liquid, the pressure acts in any direction.
- 2- The direction of the force on any surface is normal that surface.
- 3- The pressure on the surface of a body is equal to the pressure on the liquid that has the same volume and shape.
- 4- For a certain size of a liquid, there is an equilibrium between two forces:
 - a) The weight of the liquid.
 - b) The force due to the pressure of the liquid around it.



Calculating the pressure (P) at a point inside a liquid:

- Imagine a horizontal plate (**x**) of area (**A**) at depth (**h**) inside a liquid of density (**ρ**).
- This plate acts as the base of a column of the liquid.
- The force acting on the plate (**x**) is the weight of the column of the liquid whose height (**h**) and cross section area (**A**).
- The volume of this column ($V_{ol} = A h$).
- The mass of this column ($m = \rho V_{ol} = A h \rho$).
- The weight of this column ($F_g = m g = \rho V_{ol} g = A h \rho g$).
- The force resulting from the liquid pressure balances with the weight of the column of the liquid.



$$\begin{aligned} \mathbf{F} &= \mathbf{F}_g \\ &= \mathbf{A h \rho g} \end{aligned}$$

The pressure due to the liquid from under the plate (**x**) (acting upwards).

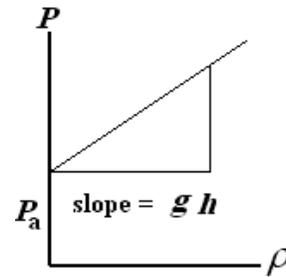
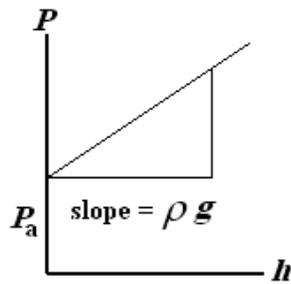
$$\begin{aligned} \mathbf{P} &= \mathbf{F / A = A h \rho g / A} \\ \mathbf{P} &= \mathbf{\rho g h} \end{aligned}$$

The pressure at a point inside a liquid:

The weight of a column of the liquid around that point, its cross section area 1 m^3 and its length equals the depth of that point from the liquid surface.

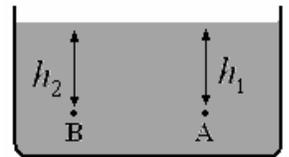
Factors affecting the pressure at a point inside a liquid:

- 1- The depth of that point: directly.
- 2- The density of the liquid: directly.



Notes:

- ❖ All points that lie on the same horizontal level inside a liquid have the same pressure.



$$P_A = P_a + \rho g h_1$$

$$P_B = P_a + \rho g h_2$$

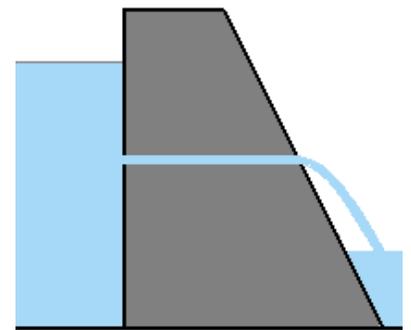
$$\because h_1 = h_2 \Rightarrow \therefore P_A = P_B$$

- ❖ The liquid rises to the same height (level) in the connecting vessels regardless of the geometrical shape.



- The average sea level is constant for all connected seas and oceans.

- ❖ **The base of a dam is thicker than its top.**
To withstand the increasing pressure with increasing depth.



The atmospheric pressure

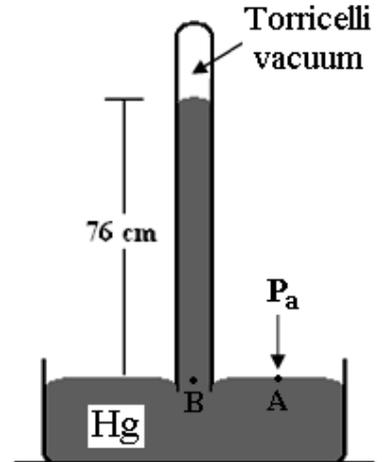
The weight of air column over unit area of earth surface at sea level

Mercury Barometer:

Use: measures the atmospheric pressure.

Structure:

- 1 m long glass tube filled completely with mercury (**Hg**).
- A tank of **Hg** in which the glass tube is turned upside down.



Observation:

- The level of **Hg** went down to a certain level (≈ 0.76 m from the surface of **Hg** in the tank).
- A void is formed above the **Hg** column (**Torricelli vacuum**).

Calculating the atmospheric pressure:

Taking two points (**A** & **B**) in one horizontal plane (**A** is outside the tube at the surface of **Hg** in the tank, while **B** is inside the tube).

The pressure at point **B** = The pressure at point **A**

$$P_B = P_A$$

But $P_A = P_a$ (atmospheric pressure) and

$P_B = \rho_{Hg} g h$ + the pressure of Torricelli vacuum = $\rho_{Hg} g h + 0$

$$P_B = \rho_{Hg} g h$$

$$P_a = \rho_{Hg} g h$$

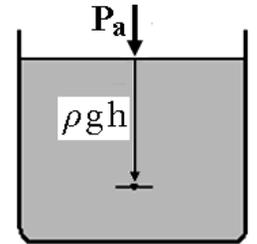
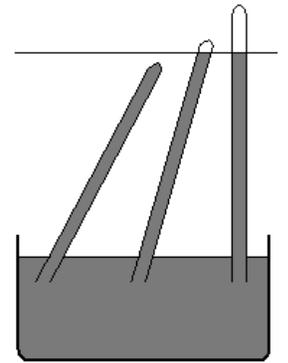
- ❖ The atmospheric pressure is equivalent to the weight of a column of **Hg** whose height is **0.76** m and cross section area 1 m^2 at 0°C .
- ❖ At **STP** (standard temperature and pressure)

$$t = 0^\circ\text{C}, \quad \rho_{Hg} = 13595 \text{ kg / m}^3 \quad \text{and} \quad g = 9.8 \text{ m / s}^2.$$

$$P_a = \rho_{Hg} g h = 13594 \text{ kg / m}^3 \times 9.8 \text{ m / s}^2 \times 0.76 \text{ m} = 1.013 \times 10^5 \text{ N / m}^2$$

Notes:

- 1- The height of **Hg** column in the tube is constant whether the tube is upright or inclined.
- 2- Torricelli vacuum doesn't appear if the length of the tube is less than 76 cm.
- 3- The atmospheric pressure decreases with increasing height from sea level (because the length of air column decreases).
- 4- When the free surface of the liquid is subjected to atmospheric pressure (**P_a**), then the total pressure at a point inside a liquid.



$$P = P_a + \rho g h$$

Units for measuring the atmospheric pressure:

1- Pascal (N / m ²).	P_a = 1.013 x 10⁵ Pascal (N / m²).
2- cm Hg.	P_a = 76 cm Hg.
3- mm Hg.	P_a = 760 mm Hg.
4- Torr = mm Hg.	P_a = 760 torr.
5- Bar = 10 ⁵ Pascal.	P_a = 1.013 bar.

Using the mercury barometer to measure the height of a mountain:

- ❖ The difference between the atmospheric pressure at sea level and the top of the mountain is (**ΔP**).
- ❖ The height of the mountain is (**h**).
- ❖ The density of air is (**ρ_{air}**) and the density of **Hg** is (**ρ_{Hg}**).
- ❖ The difference between the two levels of **Hg** in the barometer at sea level and the top of the mountain is (**Δh**).

$$(\Delta P)_{Hg} = (\Delta P)_{air}$$

$$\rho_{Hg} g (\Delta h) = \rho_{air} g h$$

$$\rho_{Hg} (\Delta h) = \rho_{air} h$$

Balance of liquids in a U-shaped tube:

- ❖ When a liquid is put in a U-shaped tube, the level of the liquid in the two branches is the same.
- ❖ When another liquid is added in one branch of the tube (both liquids don't mix).

The pressure at point (A)

$$P_A = P_a + \rho_1 g h_1$$

The pressure at point (B)

$$P_B = P_a + \rho_2 g h_2$$

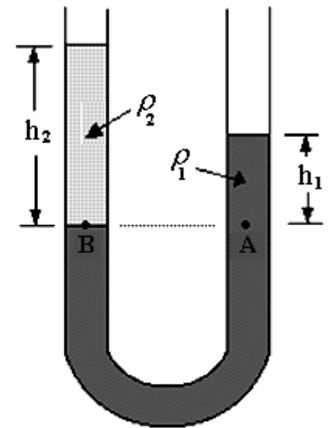
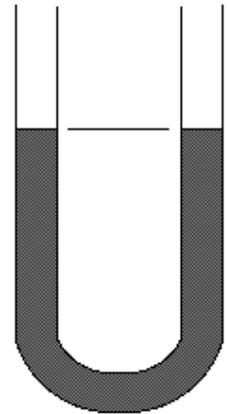
As point (A) and point (B) are in the same level in the same liquid.

$$P_A = P_B$$

$$P_a + \rho_1 g h_1 = P_a + \rho_2 g h_2$$

$$\rho_1 h_1 = \rho_2 h_2$$

$$\frac{\rho_2}{\rho_1} = \frac{h_1}{h_2}$$



- ❖ If the 1st liquid is water, then the ratio (ρ_2 / ρ_1) represents the relative density of the 2nd liquid.

The manometer

Use:

- 1- Measures the pressure of a gas enclosed in a reservoir.
- 2- Measures the difference between the pressure of an enclosed gas and the atmospheric pressure.

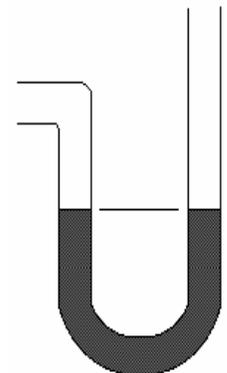
Structure:

A U-shaped tube containing an amount of liquid of known density.

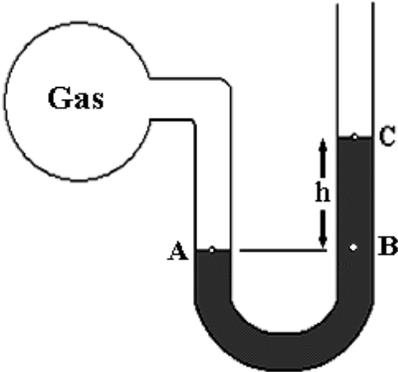
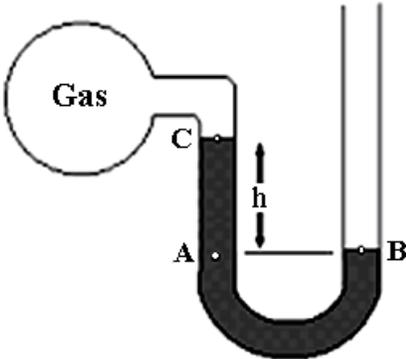
- ❖ One end of the tube is connected to a gas reservoir.

How to use it:

- ❖ When the manometer is not connected to the gas reservoir, the level of the liquid in the two branches is the same.
- ❖ When one end of the manometer is connected to the gas reservoir, then the level of the liquid may rise in one branch and go down in the other.



- ❖ The pressure at point **B** = The pressure at point **A** (at the same level).

When $P > P_a$	When $P < P_a$
<p>The level of the liquid in the free branch is higher than the branch connected to the reservoir by (h).</p> <p style="text-align: center;">P (gas pressure) = $P_A = P_B$</p> <p style="text-align: center;">$P = P_a + \rho g h$</p> <p style="text-align: center;">$\Delta P = P - P_a = \rho g h$</p>	<p>The level of the liquid in the free branch is lower than the branch connected to the reservoir by (h).</p> <p style="text-align: center;">P (gas pressure) = P_C</p> <p style="text-align: center;">$P = P_a - \rho g h$</p> <p style="text-align: center;">$\Delta P = P_a - P = \rho g h$</p>
	

- ❖ Water is used in the manometer when measuring small gas pressure (because the density of water is small, so the difference between the 2 levels of water in the 2 branches will be great and the pressure is measured accurately).

APPLICATIONS TO PRESSURE

1) MEASURING BLOOD PRESSURE:

- ❖ Blood is a viscous liquid pumped through a network of arteries and veins by the muscular effect of the heart.

Types of blood pressure:

- a. **Systolic pressure:** the maximum blood pressure (120 torr) when the cardiac (heart) muscle contracts to push the blood from the left ventricle through the aorta onto the arteries.
- b. **Diastolic pressure:** the minimum blood pressure (80 torr) when the cardiac (heart) muscle relaxes.

2) MEASURING THE AIR PRESSURE INSIDE A CAR TIRE:

- When a tire is well inflated (under high pressure), the area of contact with the road is small (low friction force with the road).
- An underinflated tire (under low pressure), the area of contact with the road is large (the friction force with the road increases) so the tire is heated.
- ❖ Air pressure inside a car tire can be measured by the pressure gauge.

Pascal's Principle

Consider a liquid in a container provided with a piston at its top.

- The pressure immediately underneath the piston (on the liquid surface).

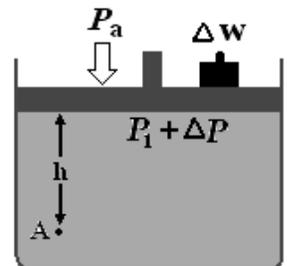
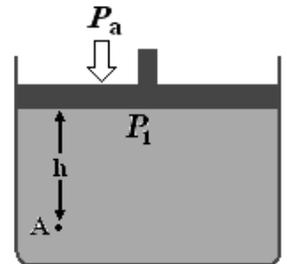
P_1 = the atmospheric pressure + the pressure due to the weight of the piston.

- The pressure at point (A) at depth (h) inside the liquid.

$$P_A = P_1 + \rho g h$$

- Placing an additional weight on the piston, the pressure on the piston increases by ΔP .
- The piston doesn't move inside (because the liquid is incompressible).
- The pressure underneath the piston must increase in turn by ΔP .
- The pressure at point (A) increases by ΔP .

$$P_A = P_1 + \rho g h + \Delta P$$



Conclusion:

The increase in the pressure is transmitted in full to all parts of the liquid.

Pascal's Principle:

When a pressure is applied on a liquid enclosed in a container, the pressure is transmitted in full to all parts of the liquid as well as the walls of the container.

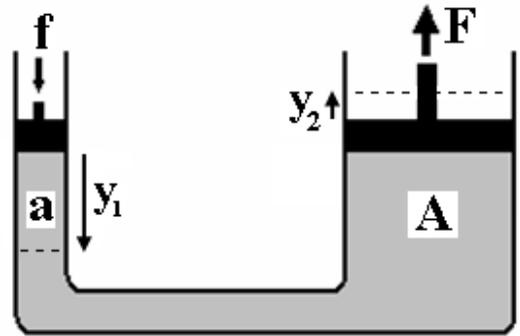
- ❖ Pascal's Principle is not applied on gases (because the gases are compressible, then a part of the applied pressure is consumed in compressing the gas and the pressure is not transmitted in full).

APPLICATIONS TO PASCAL'S PRINCIPLE

The hydraulic press

Structure:

- 1- Small cylinder whose cross section area is (**a**).
- 2- Large cylinder whose cross section area is (**A**).
- 3- Each cylinder is provided with a piston.
- 4- The 2 cylinders are connected together with a tube and filled with a liquid.



Idea of work: Pascal's Principle

When a pressure is applied on the small piston, this pressure is transmitted in full to the large piston.

- When a force (**f**) is applied on the small piston, then
- The pressure applied by the small piston ($P = \frac{f}{a}$) and this pressure is transmitted in full to the large piston.
- The pressure affecting the large piston ($P = \frac{F}{A}$)
- $\therefore P = \frac{f}{a} = \frac{F}{A}$
- $\therefore F = \frac{A}{a} f$ or $\frac{F}{f} = \frac{A}{a}$
- ❖ If a small force is applied on the small piston, a larger force is generated from the large piston.

Function: amplifying force (gaining a larger force from a small one).

Mechanical advantage:

$$\eta = \frac{F}{f} = \frac{A}{a}$$

The ratio between the force generated by the large piston to the force applied on the small piston. **or**

The ratio between the area of the large piston to the area of the small piston.

What is meant by:

The mechanical advantage of a hydraulic press = 40.

It means that the ratio between the force generated by the large piston to the force applied on the small piston = **40**.

$$\text{❖ } \because \frac{F}{f} = \frac{A}{a} \quad \Rightarrow \quad \frac{F}{f} = \frac{\pi R^2}{\pi r^2} \quad \Rightarrow \quad \therefore \frac{F}{f} = \frac{R^2}{r^2}$$

❖ According to the law of conservation of energy:

The work done on the small piston = the work gained from the large piston.

$$(\text{Work})_{\text{in}} = (\text{Work})_{\text{out}}$$

$$f y_1 = F y_2$$

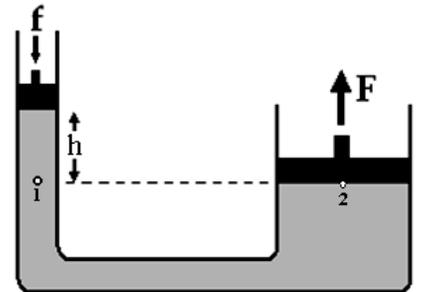
$$\therefore \frac{F}{f} = \frac{y_1}{y_2}$$

❖ The mechanical advantage can be expressed by the ratio

$$(\eta = \frac{F}{f} = \frac{y_1}{y_2})$$

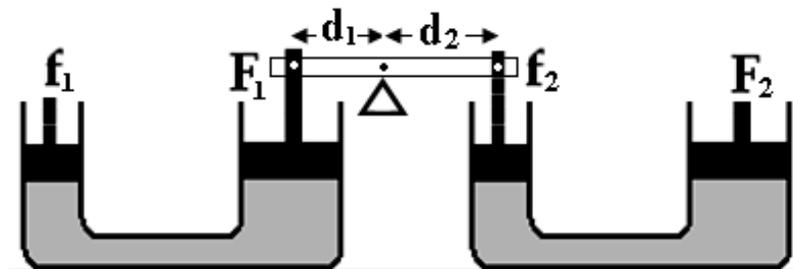
❖ When the 2 piston are not in the same level

$$P_1 = P_2 \quad \Rightarrow \quad \frac{f}{a} + \rho gh = \frac{F}{A}$$



❖ When a lever is connected between 2 hydraulic presses.

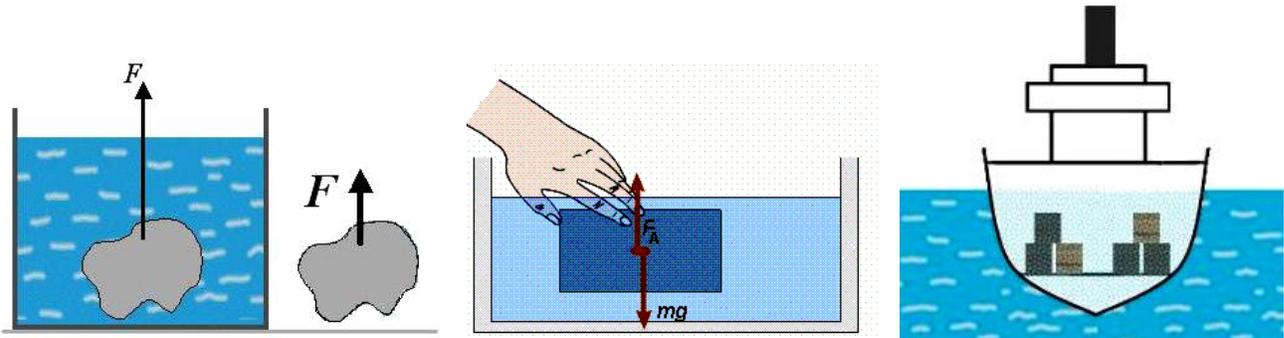
$$F_1 d_1 = f_2 d_2$$



Archimedes' Principle

Related observation:

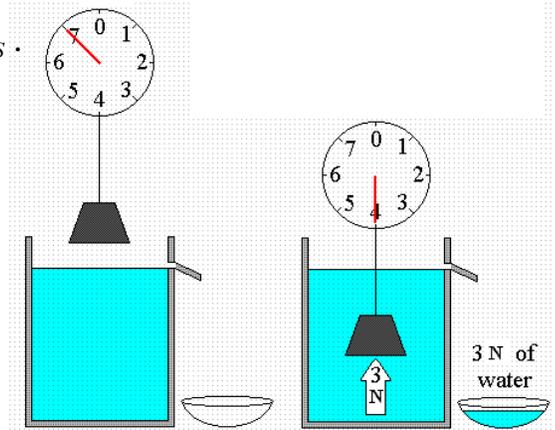
- 1- An object can be easily lifted in water than air.
- 2- When a piece of foam is immersed in water, it is pushed upwards and floats on water surface.
- 3- An iron nail sinks in water while a large steel ship floats.
- 4- Balloons filled with helium rise up.



Experiment:

- 1- Weigh a body in air using a spring balance $(F_g)_s$.
- 2- Weigh the body in water in a displacement cylinder $(F_g)_s^{\setminus}$ and collect the displaced water.
- 3- Calculate the difference between the two weights $(F_g)_s - (F_g)_s^{\setminus}$.
- 4- Weigh the displaced water $(F_g)_w$.

Observation: $(F_g)_w = (F_g)_s - (F_g)_s^{\setminus}$.



Conclusion:

When a body is immersed in a fluid (liquid & gas), the fluid pushes on the body by an upward force (buoyancy or buoyant force).

Archimedes' Principle:

A body partially or fully immersed in a fluid (liquid & gas) is pushed upwards by a force equal to the weight of the displaced fluid by the body.

Theoretical derivation:

Imagine a cylinder of a liquid of volume (V_{ol}), cross section area (A) and height (h).

❖ **The forces acting on this cylinder:**

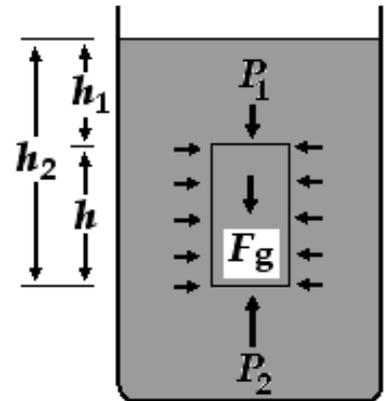
a. **Horizontal forces:** cancel each other out (because each two opposite forces are equal in magnitude).

b. **Vertical forces:** which are:

1. The weight of that cylinder downwards [$(F_g)_L = mg = \rho_L V_{ol} g$].
2. Buoyant force: results from the difference of the pressure on the upper and lower surfaces of the cylinder.

$$\begin{aligned}
 F_b &= (\Delta P) A = (P_2 - P_1) A \\
 &= (\rho_L g h_2 - \rho_L g h_1) A \\
 &= \rho_L g A (h_2 - h_1) \\
 &= \rho_L g A h \\
 F_b &= \rho_L g V_{ol}
 \end{aligned}$$

$$\therefore F_b = (F_g)_L$$



❖ Equilibrium requires that (F_b) works upwards.

❖ F_b = the weight of the displaced liquid.

The relation between the weight of a body in air and its weight when immersed in a liquid.

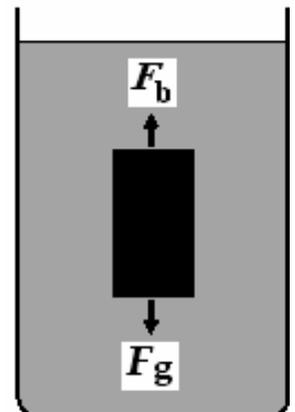
❖ Substituting the virtual cylinder by a solid cylinder of the same shape and volume whose density is (ρ_s).

❖ The buoyant force which the liquid exerts on the solid cylinder remains the same (F_b) acting upwards.

❖ The weight of the solid cylinder (F_g)_s.

❖ The resultant force on the immerse solid cylinder

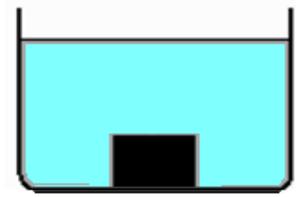
$$\begin{aligned}
 F &= (F_g)_s \downarrow = F_b \uparrow - (F_g)_s \downarrow \\
 &= \rho_L g V_{ol} - \rho_s g V_{ol} \\
 &= (\rho_L - \rho_s) g V_{ol}
 \end{aligned}$$



Floating cases

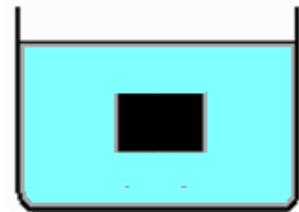
1- When $\rho_s > \rho_L \Rightarrow (F_g)_s > F_b$

- The net force on the body acting downwards.
- The body sinks to the bottom.
- The apparent weight $(F_g)_s^{\downarrow} = (F_g)_s \downarrow - F_b \uparrow$



2- When $\rho_s = \rho_L \Rightarrow (F_g)_s = F_b$

- The net force = 0
- The body is hanging (suspended in the liquid).
- The apparent weight = 0.



3- When $\rho_s < \rho_L \Rightarrow (F_g)_s < F_b$

- The net force on the body acting upwards.
- The body moves up to float on the liquid surface.

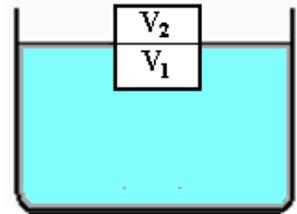
❖ When the body settles on the liquid surface.

The weight of the body = the buoyant force on the immersed part
the weight of the displaced liquid by the immersed part.

$$(F_g)_s = F_b$$

$$\rho_s g V_{oi} = \rho_L g (V_{oi})_1$$

$$\frac{\rho_s}{\rho_L} = \frac{(V_{oi})_1}{V_{oi}}$$

**Notes:**

- 1- When different bodies of equal volumes are immersed in the same liquid, the loss in their weights is the same.

$$\text{The loss in weight} = F_b = (F_g)_s - (F_g)_s^{\downarrow} = \rho_L g V_{oi}$$

- 2- When a body floats on two different liquids separately, the buoyant force is the same.
- $F_{b1} = F_{b2} = (F_g)_s$

- ❖ When a ship travels from fresh water to salt water, the buoyant force on the ship doesn't change, but the immersed part of the ship decreases.

$$F_{b1} = \rho_{L1} g (V_{oi})_1$$

$$F_{b2} = \rho_{L2} g (V_{oi})_2$$

$$\therefore F_{b1} = F_{b2} = (F_g) \Rightarrow \rho_{L1} g (V_{oi})_1 = \rho_{L2} g (V_{oi})_2$$

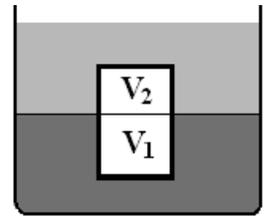
$$\frac{(V_{oi})_1}{(V_{oi})_2} = \frac{\rho_{L2}}{\rho_{L1}}$$

- 3- When a body is suspended in two immiscible liquids.

$$(F_g)_s = F_{b1} + F_{b2}$$

$$\rho_s g V_{ol} = \rho_{L1} g (V_{ol})_1 + \rho_{L2} g (V_{ol})_2$$

$$\rho_s V_{ol} = \rho_{L1} (V_{ol})_1 + \rho_{L2} (V_{ol})_2$$

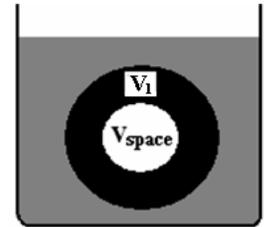


- 4- When a hollow body is suspended in a liquid.

$$F_b = (F_g)_s$$

$$\rho_L g V_{ol} = \rho_s g (V_{ol})_1$$

$$\rho_L V_{ol} = \rho_s [V_{ol} - (V_{ol})_{space}]$$

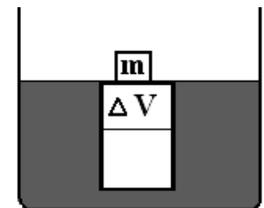


- 5- When an extra mass is added on a floating body to be completely immersed

$$m g = \Delta F_b$$

$$m g = \rho_L g \Delta V_{ol}$$

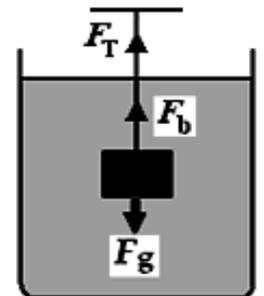
$$m = \rho_L \Delta V_{ol}$$



- 6- When a body is hung with a thread and immersed in a liquid.

$$(F_g)_s = F_T + F_b$$

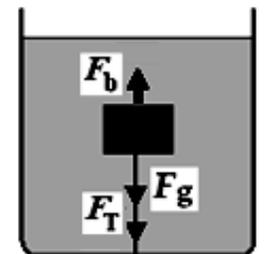
$$F_T = (F_g)_s - F_b$$



- 7- When a body is tied with a thread to the bottom of the container.

$$F_b = F_T + (F_g)_s$$

$$F_T = F_b - (F_g)_s$$



- 8- The buoyant force of air on a balloon.

$$F_b = \rho_{air} g V_{ol}$$

- 9- The rising force on a balloon = the buoyant force – the weight of the balloon.

$$F = F_b \uparrow - F_g \downarrow$$

$$= F_b \uparrow - [(F_g)_{gas} + (F_g)_{acc}]$$

$$= \rho_{air} g V_{ol} - [\rho_{gas} g V_{ol} + (mg)_{acc}]$$



The acceleration of the balloon $a = \frac{F}{M} = \frac{F}{\rho_{gas} V_{ol} + m_{acc}}$

Applications of buoyancy

1) Hydrotherapy technique:

The patients who are not able to lift their limbs because of a disease in the muscles or joints are treated by immersing their bodies in water, their bodies become nearly weightless, so the force required to move the limbs is reduced and the therapeutic exercises become possible.

2) Weightlessness experiments:

They involve immersion in containers filled with a liquid whose concentration is adjusted so that the buoyant force cancels out the weight.

3) Controlling the floating and submerging:

- a. A submarine floats when its tanks are filled with air.
- b. It submerges when its tanks are filled with water.

❖ Fish and whales fill their air sacs with air to enable them to float and empty them from air when they go under.

4) Diving:

- a. At shallow depths: the diver breathes compressed air to equate the pressure.
- b. At larger depth: the diver adjusts the pressure in the diving suit to control the buoyant force.

Mr / Essam