Algebra First Stage (2nd sec)

Answer all the fallowing questions:

1) Prove that:

i) $$\frac{\sqrt[3]{\sqrt[4]{125}} \times \sqrt[3]{275} \times \sqrt[4]{(11)^5}}{\sqrt[4]{125} \times (605)^{\frac{1}{6}} \times \sqrt[4]{11}} = 11$$

ii) $$\frac{3 \times (8)^{n-\frac{1}{3}} + (4)^{n+1} \times \left(\sqrt{2}\right)^{2n+4}}{\left(\frac{1}{8}\right)^{-\frac{1}{3}-n} - (16)^{n+\frac{1}{2}} \times \left(\frac{1}{2}\right)^{n+4}} = 10$$

2) If $y^{\frac{3}{4}} = 2x^{\frac{5}{3}} = 64$. Find the value of $5x+2y$

3)

Find the solution set of each of the following:-

i) $5^{x(x-5)} = \dfrac{1}{\left(\sqrt{5}\right)^{12}}$

ii) $5^{x+1} + 5^{x} = \dfrac{6}{125}$

iii) $4^{x+y} = 128$, $5^{x-2y-3} = 1$

iv) $2y + 3\sqrt{y} - 119 = 0$

4)

If $f(x) = 3^x$

i) prove that: $\dfrac{f(x+5)+f(x+3)}{f(x+3)+f(x+1)} = f(2)$

ii) Find the value of x where:

$f(x+2)+f(x-2)=246$

5)

Graph $f(x)=2^x$ where $x \in [-3,3]$,

From the graph find:

The range & Discuss its monotony.

Also from the graph find:-

$f(1.6)$, $f(-0.5)$ and the value of $\sqrt[5]{8}$.

6)

If $f(x)=a^{x+b}$, $g(x)=\left(\frac{1}{2}\right)^x + 8$ and each of

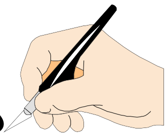
$(0,128), (-7,1) \in f$. Find:

i) The value of a,b.

ii) The point of intersection of f & g.

iii) The range of g.

The answers



1) i)

$$\text{L.H.S} = \frac{\sqrt[3]{\sqrt[4]{125}} \times \sqrt[3]{275} \times \sqrt[4]{11^5}}{\sqrt[4]{125} \times (605)^{\frac{1}{6}} \times \sqrt[4]{11}}$$

$$= \frac{\sqrt[3]{\sqrt[4]{5^3}} \times \sqrt[3]{(5^2 \times 11)} \times \sqrt[4]{11^5}}{\sqrt[4]{5^3} \times (5 \times 11^2)^{\frac{1}{6}} \times \sqrt[4]{11}}$$

$$= \frac{\sqrt[3]{5^{\frac{3}{4}}} \times (5^2 \times 11)^{\frac{1}{3}} \times 11^{\frac{5}{4}}}{5^{\frac{3}{4}} \times (5 \times 11^2)^{\frac{1}{6}} \times 11^{\frac{1}{4}}}$$

$$= \frac{5^{\frac{3}{4\times 3}} \times 5^{\frac{2}{3}} \times 11^{\frac{1}{3}} \times 11^{\frac{5}{4}}}{5^{\frac{3}{4}} \times 5^{\frac{1}{6}} \times 11^{\frac{2}{6}} \times 11^{\frac{1}{4}}}$$

$$= \frac{5^{\frac{1}{4}+\frac{2}{3}} \times 11^{\frac{1}{3}+\frac{5}{4}}}{5^{\frac{3}{4}+\frac{1}{6}} \times 11^{\frac{1}{3}+\frac{1}{4}}} = \frac{5^{\frac{3+8}{12}} \times 11^{\frac{4+15}{12}}}{5^{\frac{9+2}{12}} \times 11^{\frac{4+3}{12}}}$$

$$= \frac{\cancel{5^{\frac{11}{12}}} \times 11^{\frac{19}{12}}}{\cancel{5^{\frac{11}{12}}} \times 11^{\frac{7}{12}}} = 11^{\frac{19}{12}-\frac{7}{12}} = 11 = \text{R.H.S}$$

ii) $$\text{L.H.S} = \frac{3 \times (8)^{n-\frac{1}{3}} + (4)^{n+1} \times \left(\sqrt{2}\right)^{2n+4}}{\left(\frac{1}{8}\right)^{-\frac{1}{3}-n} - (16)^{n+\frac{1}{2}} \times \left(\frac{1}{2}\right)^{n+4}}$$

$$= \frac{3 \times (2^3)^{n-\frac{1}{3}} + (2^2)^{n+1} \times \left(2^{\frac{1}{2}}\right)^{2n+4}}{\left(2^{-3}\right)^{-\frac{1}{3}-n} - (2^4)^{n+\frac{1}{2}} \times \left(2^{-1}\right)^{n+4}}$$

$$= \frac{3 \times 2^{3n-1} + 2^{2n+2} \times 2^{n+2}}{2^{1+3n} - 2^{4n+2} \times 2^{-n-4}}$$

$$= \frac{3 \times 2^{3n-1} + 2^{3n+4}}{2^{1+3n} - 2^{3n-2}} = \frac{2^{3n-1}\left(3 + 2^5\right)}{2^{3n-2}\left(2^3 - 1\right)}$$

$$= \frac{2^{3n-1}(35)}{2^{3n-2}(7)} = 2^{\cancel{3n}-1-\cancel{3n}+2} \times 5$$

$$= 2 \times 5 = 10 = \text{R.H.S}$$

2)

$$y^{\frac{3}{4}} = 2x^{\frac{5}{3}} = 64$$

$y^{\frac{3}{4}} = 64$

$\Rightarrow y^{\frac{3}{4}} = \left(2^6\right)$

$\left(y^{\frac{3}{4}}\right)^{\frac{4}{3}} = \left(2^6\right)^{\frac{4}{3}}$

$\therefore\ y = 2^8 \Rightarrow \boxed{y = 256}$

& $2x^{\frac{5}{3}} = 64$

$x^{\frac{5}{3}} = 32 = 2^5$

$\left(x^{\frac{5}{3}}\right)^{\frac{3}{5}} = \left(2^5\right)^{\frac{3}{5}}$

$\boxed{\therefore x = 8}$

$$\therefore 5x + 2y = 40 + 512 = 552.$$

3)

(i) $5^{x(x-5)} = \dfrac{1}{\left(\sqrt{5}\right)^{12}}$

$= \dfrac{1}{\left(5^{\frac{1}{2}}\right)^{12}}$

$= \dfrac{1}{5^6} = 5^{-6}$

$\therefore x(x-5) = -6$

$\therefore x^2 - 5x + 6 = 0$

$\therefore (x-2)(x-3) = 0$

$\therefore\ x = 2 \text{ or } x = 3$

$\therefore\ S.S = \{2,3\}$

ii) $5^{x+1} + 5^x = \dfrac{6}{125}$

$5^x(5+1) = \dfrac{6}{125}$

$5^x \times \cancel{6} = \dfrac{\cancel{6}}{125}$

$5^x = \dfrac{1}{5^3} = 5^{-3}$

$\therefore\ x = -3$

$\therefore S.S = \{-3\}$

iii) $4^{x+y} = 128$

$\left(2^2\right)^{x+y} = 2^7$

$(2)^{2x+2y} = 2^7$

$\boxed{2x + 2y = 7}$ (1)

$, 5^{x-2y-3} = 1$

$\therefore x - 2y - 3 = 0$

$\therefore \boxed{x - 2y = 3}$ (2)

$(1)+(2) \Longrightarrow 3x = 10$

$\therefore\ \boxed{x = 10/3}$ . . . . (3)

From (3) in (1) we get $\boxed{y = 1/6}$

$$S.S = \left\{\left(\frac{10}{3}, \frac{1}{6}\right)\right\}$$

iv) $2y+3\sqrt{y}-119=0$

$2\left(\sqrt{y}\right)^2+3\sqrt{y}-119=0$

$\left(2\sqrt{y}+17\right)\left(\sqrt{y}-7\right)=0$

$2\sqrt{y}+17=0 \Rightarrow \sqrt{y}=\frac{-17}{2}\notin R^{+}\cup\{0\}$

$\therefore \sqrt{y}=\frac{-17}{2}$ (is refused)

OR $\sqrt{y}-7=0 \Rightarrow \sqrt{y}=7 \Rightarrow y=49.$

$\therefore$ S.S={49}

4)

i) L.H.S

$=\frac{f(x+5)+f(x+3)}{f(x+3)+f(x+1)}$

$=\frac{3^{x+5}+3^{x+3}}{3^{x+3}+3^{x+1}}=\frac{3^{x+3}\cancel{\left(3^2+1\right)}}{3^{x+1}\cancel{\left(3^2+1\right)}}$

$=3^{x+3-x-1}=3^2=f(2)=$ R.H.S

ii) $f(x+2)+f(x-2)=246$

$\therefore 3^{x+2}+3^{x-2}=246$

$\therefore 3^{x-2}\left(3^4+1\right)=246$

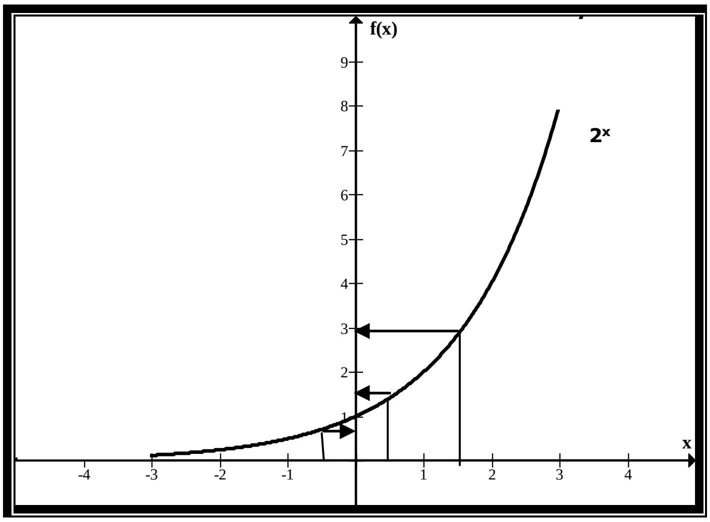
$\therefore 3^{x-2}\times 82=246$

$\therefore 3^{x-2}=\frac{246}{82}=3$

$\therefore x-2=1 \Rightarrow x=3$

5)

x	-3	-2	-1	0	1	2	3
F(x)	1/8	1/4	1/2	1	2	4	8



The range = $[\frac{1}{8}, 8]$
f is increasing on its domain
To find f(1.6)
at x=1.6 draw a vertical line till it intersect the graph find the opposite value of y
i.e $f(1.6) \approx 3$
To find $f(-\frac{1}{2})$ from the graph
at $x=-\frac{1}{2}$ then $y \approx 0.7$
To find $\sqrt[5]{8}=\sqrt[5]{2^3}=2^{\frac{3}{5}}=2^{0.6}$
from the graph at x=0.6
then $y \approx 1.5 \Rightarrow \sqrt[5]{8} \approx 1.5$

6)

i) $\because (0,128) \in f \Rightarrow f(0) = 128$

$\Rightarrow a^b = 128 \quad (1)$

$\because (-7,1) \in f \Rightarrow f(-7) = 1$

$\Rightarrow a^{-7+b} = 1 \qquad (2)$

$(1) \div (2)$ we get $\dfrac{a^b}{a^{-7+b}} = \dfrac{128}{1}$

$\Rightarrow a^{b+7-b} = 128 \Rightarrow a^7 = 2^7 \Rightarrow \boxed{a=2}$ (3)

From (3) in (1) we get $2^b = 2^7 \Rightarrow \boxed{b=7}$.

ii) $f(x) = 2^{x+7}$, $g(x) = \left(\dfrac{1}{2}\right)^x + 8$

To find point of intersection of f,g
you must solve f(x)=g(x)

$\therefore 2^{x+7} = \left(\dfrac{1}{2}\right)^x + 8 \Rightarrow 2^7 \times 2^x = \dfrac{1}{2^x} + 8$

By multiplying both sides by 2^x

$\therefore 2^7 \times 2^x \times 2^x = \dfrac{1}{\cancel{2^x}} \times \cancel{2^x} + 8 \times 2^x$

$\therefore 2^7 \times 2^{2x} = 1 + 8 \times 2^x$

$\therefore 128 \times 2^{2x} - 8 \times 2^x - 1 = 0$

$\therefore \left(8 \times 2^x - 1\right)\left(16 \times 2^x + 1\right) = 0$

$\therefore$ Either $8 \times 2^x - 1 = 0 \Rightarrow 2^x = \dfrac{1}{8} \Rightarrow \boxed{x=-3}$

Or $16 \times 2^x + 1 = 0 \Rightarrow 2^x = \dfrac{-1}{16} \notin R^+$ (refused)

At $x=-3 \Rightarrow f(-3) = 2^4 = 16$

The point of intersection of f,g is (-3,16)

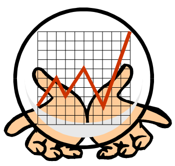
iii) The range of exponential function $\in R^+$

$\therefore \left(\dfrac{1}{2}\right)^x > 0 \Rightarrow \left(\dfrac{1}{2}\right)^x + 8 > 8$

$g(x) > 8 \Rightarrow$ The range of g is $]8, \infty[$

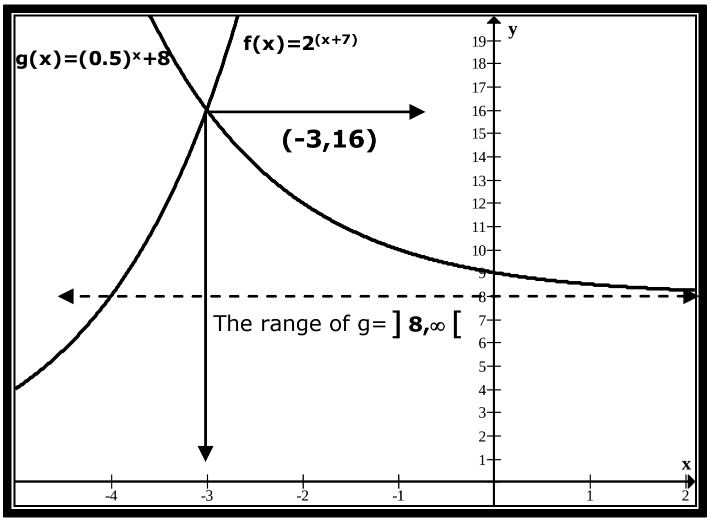
Another method

for ii & iii "graphically"



Graph f & g on the same rectangular co-ordinate system

x	. .	-4	-3	-1	0
2^{x+7}	. .	8	16	64	128
$\left(\frac{1}{2}\right)^x + 8$	. .	24	16	10	9



From the graph:-

ii) The point of intersection is (-3,16)

iii) The range of g=]8,∞[.