***Unit (1)***

***Lesson(1)***

***Solving quadratic equations in one variable***

***:*** Solving the quadratic equations in one variable algebraically:

(1) by factorization (2) by the general formula

 where a is coefficient of , b is coefficient of and c is the absolute term.

**Second**: Solving the quadratic equation in one variable graphically:

 Case (1) Case(2) Case (3)

 – axis at two points – axis at one point intersect – axis

 Remark

 case of the interval is not given , then we can graph the function by finding the

 vertex of the curve which is ( , and then we find some points to the right

 of it , and the same number of points to the left of lt.

 ***Exercises*** (1) on Solving quadratic equation in one variable

 1 :

(1)

(3) = 0

 Solution

(1)= 0

(2) ……………………………………………………………………………………………………………………………….

(3) ……………………………………………………………………………………………………………………………….

(4) ……………………………………………………………………………………………………………………………….

2the solution set of the following equations by using the general formula :

(1) + 7 = 0<< knowing that = 1.4>>

(2) 3 – 65 = 0 << Approximate the result to the nearest tenth >>

 Solution

(1) a = 1 , b = - 6 , c = 71.4

(2) ……………………………………………………………………………………………………………………………….

 3 the solution set of the following equations graphically :

(1) 3 + 2 = 0 ( Draw in the interval [ - 1 , 4 ] )

(2) + 1 = 0

 4 the value of a and the value of the other root if :

(1) = - 1 is one of the roots of the equation :

(2) =7 is one of the roots of the equation :

 Solution

 (1)+ a (-1) – 7=0

 1- a -7 = 0 a = 1 – 7 = - 6 a = - 6

 (2) ……………………………………………………………………………………………………………………………….

Lesson 2 An introduction in Complex numbers

The imaginary number '' i ''

 The imaginary number '' i '' is defined as the number whose square is – 1 i. e. = - 1

 '' i ''

|  |  |  |  |
| --- | --- | --- | --- |
|  = i | = - 1  | = - i |  = 1  |
|  = i |  = - i |  = - i |  = 1  |
|  = i |  = - 1 |  = - i |  = 1 |

 The Complex number

The Complex number is the number that can be written in the form : a + bi , where a

 and b are two real numbers and = - 1

* a is called the real part.
* bi is called the imaginary part.

Set of Complex numbers

 The Set of Complex number C is defined as : C= { a + bi : a }

 Equality of two Complex numbers

 Two Complex numbers are equal if and only if( ) the two real parts are equal and the

 two imaginary parts are equal.

i.e. if : (a + bi) and (c + di) are two Complex numbers and if : a = c , b = d , then :

 a +bi = c + di and vice versa : if : a + bi = c + di , then : a = c , b = d

The two Conjugate numbers

 The two numbers : a + bi and a – bi are called Conjugate numbers.

 Note : Take care that the Complex number and its Conjugate differ only

in the sign of their imaginary parts.

 Exercises 2 on an introduction in Complex numbers

 1 Simplify each of the following :

 (1) (2) (3) (- 4 i) (- 6 i) (4) (

 2 Find the result of each of the following in the simplest form :

 (1) (3 + 2 i) + (2 – 5 i) (2) (12 -5 i) – (7 – 9 i)

 (3) (2 + 3 i) (3 – 4 i) (4) (4 – 3 i) (4 + 3 i)

 3 each of the following in the form ( a + b i )

 (1)

 (4)

 4 Solve each of the following equations :

 (1)

 5

 6 equations:

 (1)

 7

 8

Lesson 3 Determining the types of roots of a quadratic equation

Discriminate

 \*The expression : is called the discriminant of the quadratic equation

 because it is used to determine the types of roots of the quadratic equation as follows:

|  |  |  |
| --- | --- | --- |
| Discriminate  | The types of the two roots | A sketch for the function related to the equation |
| Is positive () >0 | Two different real roots  |  o o   |
| Is equal to zero – 4 ac = 0 | Two equal real roots  |  o o  |
| is negative | Two Complex and non Real roots |  |

 Remark

(1) = 0 are rational

numbers the discriminate is a perfect square ,then the roots are real rational numbers.

(2)the discriminant of the quadratic equation isn't positive , then the two roots of the

 quadratic equation are complex numbers and conjugate.

Exercises 3 on determining the types of roots of

 a quadratic equation

1 Determine the type of the two roots of each of the following equations :

(1)

 2 + 2 = 0 are

complex and not real , then use the general formula to find those two roots.

 3hen find

 the value = 0

4

 (1) = 0 are real and different

 (2) + 16 = 0 are complex and not real.

5

 + m = 0 ( has no real roots )

6 that the two roots of

 the equation :

7 the interval to which a belongs that makes the two roots of the

 equation : (a + 2) .

8

9

 (

 Lesson 4 Relation between the two roots of the second

 Degree equation and the coefficients of its terms

 we know that the two roots of the quadratic equation : are :

Let one of the two roots be and the other be Then :

1

i.e.

2=

 i.e.

 Remarks

 = 0

 1

 i.e.

the product of the two roots = the absolute term.

 2

 i.e. one of the two roots of the equation is the additive inverse of the other.

3

i.e. One of the two roots of the equation is the multiplicative invers of the

 other.

4 is double

 the additive inverse of the other root

 Exercises 4 On the relation between the two roots of the second

 Degree equation and the coefficients of its terms

1 Without solving the equation find the sum and the product of the two

 roots of the following equations :

 (1) 4 + 2) =0

2 – c = 0 is

 ,find the value of c , then solve the equation in the set of complex number.

3 , find

 the value of b , then solve the equation in the set of the complex number.

 Solution

4

 :

 (1) = 0

 (2) = 0

 Solution

5 if :

 (1) 2 , 5 are the two roots of the equation :

 (2)

 Solution

6

(1) One of the roots of the equation : – 3 = 0 is the additive

 invers of the other roots.

(2) One of the roots of the equation : is the

 multiplicative invers of the other.

 Solution

7 if one of the two roots of the equation :

 is double the other root.

 Solution

8 , if one of the two roots of the equation :

 is double the additive inverse of the other.

 Solution

Lesson 5 Forming the quadratic equation whose two

 roots are known

 be the two roots of the quadratic equation:+ c = 0

 By multiplying the two sides by , the equation becomes in

 the form :

By substituting in (1) , we get the quadratic

 equation whose roots are (2)

 i.e. – (the sum of the two roots) + product of the two roots = 0

 by factorizing , we get another form of the last equation : =0

 Remember the following identities

 1

 2 (

 3

 4

 5

 6

Exercise 5 On forming the quadratic equation whose

 its two roots are known

1

(1) – 2 , 4 (2) – 5 i , 5 i (3) 3 – 2 i(4)

 Solution

 (1)sum of two roots = product of two roots =

-( the sum of the two roots)+ the product of the two roots=0

 (2) sum of two roots = product of two roots =

 (3) sum of two roots = product of two roots =

 (4) sum of two roots = product of two roots =

 2 are the two roots of the equation:

 the numerical value of each of the following expressions

(1) (2)

(3) (2)

 Solution

 3

 find the equation whose roots are :

 Solution

4 are the two roots of the equation :

 form the quadratic equation whose roots are :

 Solution

5

 Solution

 Let the two roots of the given equation are : = 7and = - 9

and the two roots of the required equation are :

6

 Solution

7

 ,

 Solution

8 = 0,equals

 twice the product of the two roots of the equation : = 0, then find

 Solution

 Let the two roots of the first equationare: ,

 let the two roots of the second equation are :

 (by squaring both sides)

Lesson 6 sign of a function

Investigating the sign of a function

 the values of the function*f* are as follows :

* Positive , i.e. = 0

|  |  |
| --- | --- |
| + + + ++ + + + + + + + + + + + C  O We notice that :  |  O  C We notice that :  |

 From the previous , we deduce that

is the same sign of c R

 Second (linear function)

The following figures represent the two functions :

|  |  |
| --- | --- |
|  + c ( b is positive ) + + +. + O We notice that the sign of the function : * is the same as the sign of b(positive) at
* is opposite to the sign of b(negative) at
* equals Zero at
 |  ( b is negative ) ++ + O We notice that the sign of the function : * is the same as the sign of b (negative) at
* is opposite to the sign of b (positive)

at * equals Zero at
 |

From the previous ,we deduce that

 To find the sign of the linear function + c , b 0 , we put

1

 2 -

3 is opposite to the is the same as the

We can show that on coefficient of coefficient of

the number line as follows : 0

Third The sign of the second degree function ( quadratic function )

 + c, a 0 ,

We have to obtain the discriminant of the equation: + c = zero

Threecases:

1

|  |  |
| --- | --- |
| +++ ++ + |  + + + + |

* []
* ][
* {}
* And we illustrate this on the same as opposite to the same as

the opposite number line. the sign of a the sign of a the sign of a

 0 0

 2

the equation and this the sign of the function as follows:

|  |  |
| --- | --- |
| + + + + + + + +O + |  |

3 = 0

|  |  |
| --- | --- |
| + ++ + + ++ ++ ++ +O  |  O  |

We can illustrate this on is the same as is the same as

the opposite number line. 0

Exercise 6 On sign of a function

1rules

(1)

 (4)

Solution

 (1) (2) positive for all R

 (3) \*

(4) \*

 (5)

* At

2 the following functions which are defined

 by the following rules , Then represent your answer on the number line :

(1) (2)

 (3) (4)

 Solution

(1) 4

(where 0

(2)

(3)

 (4)

3 in the interval [- 3 , 4].

 From the graph , determine the sign of *f* in that interval.

 4 + 4 in [- 3 , 5]. From the graph

 , determine the sign of*f* in that interval.

H.W. Page 41 Nos. (1) [6 , 7] , (2) [5 , 11]

 Page 42 Nos. (9) , (10) and Page 44 No. (18).

Lesson 7

 the following steps:

 1 We write the quadratic function related to the inequality.

 2 We study the sign of the quadratic function.

 3 We determine the intervals, which satisfy the inequality.

xample :

 Solution

1Write the quadratic function related to the inequality as follows :

 2 3

 2 We study the sign of *f* by putting: + 6 = 0

 + + + + + - - - - - + + + + +

= 0 = 3 0 0

3 We determine the intervals, which satisfy : (positive)

 Notice that : 2 3

 From the previous example :

 is ]2 , 3[

H.W. Page 49 Nos. (1) [4 , 5 , 9 and 15] (2) [6 and 13] and(3)

Exercise 7 On quadratic inequalities in one variable

1

(1)

(4)

 Solution

 (1) -4 2

– [- 4 , 2] + + + + + - - - - - + + + + +

 – [- 4 , 2] 0 0

 (2)

 (3)

 (4)

 (5)

2

(1) (2)

 Solution

 (1)

 (2)

Unit 2 Trigonometry

Lesson 1 Directed angle

Directed angle

 are the two sides of an angle whose vertex is ''O '', then :

 1 ( represents the directed B

 angle

O A

2 ( represents the directed B

 angle

 , and terminal side is O A

 i.e. The directed angle

.

 Remark

 ( , So: AOB directed angle BOA directed angle

 Positive and negative measures of a directed angle

1 The measure of the directed angle AOB is positive if the direction of the

 rotation from the initial side to the terminal side is anticlockwise

2 The measure of the directed angle AOB is negative if the directed of the

 rotation from the initial side to the terminal side is the clockwise

Remarks

1Each directed angle has two measures , one is positive and the other is negative such

 that the sum of the absolute values of the two measures equals

 i.e.

2\* the positive measure of the directed angle = , then the negative

 measure of the same directed angle =

\*\* the negative measure of the directed angle = , then the positive

 measure of the same angle =

 The Standard position of the directed angle

 A directed angle is in the standard position if the following two conditions are satisfied:

1 rectangular co- ordinate

system. B.

 2 initial side (lies on the positive direction O A

 of the – axis.

 Angle position in the orthogonal Co- ordinate plane

 measure is lies in one of the quadrants :

|  |  |
| --- | --- |
|  B O  |  B O A  |
|  O A  B |  O A  B |

 Remark

" quadrantal angle" i.e. are quadrantal angles.

Several directed angles in the standard position are said to be equivalent

 they have one common terminal side.

 Exercise 1 On directed angles

 1 Determinethe smallest positive measure for each of the following angles whose

measures are follows ,then determine the quadrant in which each angles lies:

 (1) –(2) (3) - (4)

(5) (6) – (7) (8) –

Solution

(1)

 (2)

 (3)

 (4)

 (5)

 (6)

 (7)

 (8)

2

 negative measure having common terminal side for each of the following

 angles : (1) (2) – (3) –

Solution

 (1)

 (2)

Lesson2systems of measuring angle

 (Degree measure – Radian measure)

Definition

of radius length r subtends an arc of length , Then M r

Remark

r

 The unit of measurement of the radian angle :

Definition of the radian angle B

rr =

M r A

 The relation between the radian measure and the degree measure :

M

 Assuming that: m(< AMB)= in degrees , in radians , length of B A

Exercise 2 On degree measure and radian measure of an angle

1 measures

 are as follows : (1) (2) – (3) (4)

Solution

 (1)

 (2)

 (3)

 (4)

2 angle

 that subtend an arc of length () in a circle of radius (r) in each of the following cases:

 (1) = 12 cm. , r = 10 cm. (2) cm. , r = 6 cm.

Solution

 (1) = =

 (2)

3is drawn

Subtend an arc of length ( Cases:

 (1) = 22.5 cm. (2) = 38.35 cm.

(3) = 24.325 cm. (4) = 43.92 cm.

 Solution

 (1) 6.4 cm. (2)

 (3) (4)

4 to the nearest one decimal place of a centimeter the length of an arc in a circle

 of radius length( r) subtending a central angle of measure() in each of the following :

 (1) r = 12.5 cm. , (2) r = 7.5 cm. ,

Solution

 (1) = 20 cm.

 (2)

5 an arc length of 12 cm. subtended

Solution

6 the measure of a central angle in a circle equals subtending an arc of length

cm.

Solution

7 their measures is .

the measures of the two angles in radian and in degrees.

Solution

8

 M

 B A

Solution

 9 are two

 C

AB = 12 cm. M. A

 B

Solution

H.W. Page 64 Nos. (3) , (4) , Page 65 Nos. (10) , (13) , (15) and page 66 No.(18)

Lesson 3 Trigonometric Functions

 The unit Circle

 (0 , 1)

 A circle of center at the origin point and of 1

 radius equals the unit of the length is called (- 1 , 0) O (1 , 0)

 a unit circle.

 (0 , - 1)

 Remarks:

1at two points of coordinates (1 , 0) , (- 1 , 0) and

 intersects the *y* – axis at two points of coordinates (0 , 1) , (0 , - 1)

2the unit circle , then

 \*= 1 from Pythagoras' theorem. \*

 The basic Trigonometric functions and their reciprocals

 we draw the directed angle AOB in the standard position B

 and its terminal side intersects the unit circle

 at the point B () and if m ( , then O A

1

 (1) – coordinate of the point B i.e.

 (2) – coordinate of the point B i.e.

(3)

 i.e.

 Notice that : )

2 trigonometric function for the angle of measure are:

 (1) secant of the angle (sec) =

i.e.

 (2) cosecant of the angle (csc) =

i.e.

(3) cotangent of the angle =

i.e.

Signs of trigonometric functions (Astc)

* We can summarize signs of the trigonometric functions in the following table :

 QuadrantSign of Sign of Sign of

cos , sec Sin , csc tan , cotsin , The All

 First ]0 , [ + + + csc (+ve) are (+ve)

 Second ] + tan ,cotcos , sec

 Third ] + (+ve)(+ve)

 Forth ] [ +

The trigonometric functions of some special angles

|  |  |  |
| --- | --- | --- |
| The measure of  | The coordinates of the point of the intersection of the terminal side with the unit circle  | The values of the trigonometric functionsSin cos tan  |
|  |  (1 , 0 )  |  0 1 0  |
|  |  ( 0 , 1 )  |  1 0 undefined |
|  |  (- 1 , 0 )  |  0 - 1 0  |
|  |  (0 , - 1 )  |  -1 0 undefined |
|  |  (  |  |
|  |  (  |  |
|  |  ( ) |  1 |

Exercise 3 On trigonometric functions

 1

 (1) (2) (3) (4)

Solution

(1)= + = lies in is positive

(2) is negative

(3)

(4)

2 the terminal side of the directed angle whose measure is in the standard position

 intersects the unit circle at the point A (- :

 (1) the quadrant in which the angle lies.

 (2)

Solution

3 is the measure of the directed angle in the standard position and B is the

 intersection point of its terminal side with the unit circle , then find all trigonometric

 functions of the angle in each of the following cases:

 (1) B ( (2) B ( where

Solution

4 the value of each of :

 (1) (2)

Solution

 (1) =

 (2) =

5

(1) (2)

Solution

6

(1)

 (2)

 Solution

9, then find the value of which satisfies the following equations:

 (1)

 (2)

Solution

8 the unit

Solution

9

 Solution

H.W. Page 70 No. (4) , Page 71 No. (10) Page 72 No. (14)

 Lesson 4 Related angles

 Definition of the related angles

are two angles the difference between their measures or the sum of their

 measures equals a whole number of right angles.

 The relation between trigonometric functions of related angles

 are (+ive)

 (

 1Relation between trigonometric functions of related angles of measures- )

2Relation between trigonometric functions of related angles of measures+

3Relation between trigonometric functions of related angles of measures-)

4Relation between trigonometric functions of related angles of measures+)

5Relation between trigonometric functions of related angles of measures -)

6 Relation between trigonometric functions of related angles of measure + )

7 Relation between trigonometric functions of related angles of measures - )

 Remark

 H.W. Page 87 Nos. (1) , (2) Page 89 No. (7) and Page 9o Nos. (14) , (19)

Exercise 4 On related angles

 1

 (1) (2)

 Solution

2

 (1)

(2)

Solution

3= - 1

Solution

4 the terminal side of an angle of measure in its standard position intersects the

 unit circle at the point (

 (1)

 Solution

 (1) =

 (2) =

 (3) =

 (4) =

 (5) =

 (6)=

5 is the measure of a positive acute angle in the standard position

and its terminal side intersects the unit circle at the point ( ,

Solution

 = B ( )

 = = 0

6 one of the values of , where which satisfies each of the following

 (1)

 (3)

 Solution

 (1)

 (2)

 (3)

 (4)

7

Solution

8

Solution

9 = 1 where , find the value of , then find the value of :

 Solution

Lesson 5 Graphing trigonometric functions

First

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | 0.5 | 0.87 | 1 | 0.87 | 0.5 |  0 |  |  |  |  |  |  |

 1

 0.8

 0.6

 0.4

 0.2

 O

From the previous , we can deduce that :

Properties of the sine function in the form :

 1

 2\* maximum value of the function is 1 and it happens when +

\*he minimum value of the function is – 1 and it happens when = + 2n where nZ

 \*

3

Second

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |

 .

 .

 .

 .

From the previous , we can deduce that :

 Properties of the cosine function in the form :

1

2\*and it happens when = +2n

 \* minimum value of the function is – 1 and it happens when

 \* =[-1,1]

3

 Note : Each of the two functions : is periodic , its period

 is and its range [ - a , a ] where a is positive.

 Exercise 5 On Graph trigonometric functions

 1

 (1) is …………….

 (2) is ………….

 (3) is………….

 (4) is…………

 (5) is………..

 (6) is………

 (7) is a periodic function and its period equals………

2 the maximum and minimum values , Then calculate the range of each

of the following functions :

(1) (2) (3) (4)

Solution

|  |  |  |  |
| --- | --- | --- | --- |
|  | Maximum value | Minimum value | range |
| (1) |  1  |  |  [  |
| (2) |  |  |  [ |
| (3) |  3 |  |  [ |
| (4) |  |  |  [ |

3

 (1)

 (2)

Lesson6 Finding the measure of an angle given the value

 of one of its trigonometric ratios

 \*we know thatis known

\*here is another form used to find the value of

 which means that equals the value of the measure of the angle whose sine is *y*

 1i.e. :

2 i.e. equals the measure of the positive acute angle whose

 cosine =

 3 i.e. equals the measure of the positive acute angle whose

 tan = 1 then

Exercise 6 On finding the measure of an angle given

 the value of one of its trigonometric ratios

1 :

 (1)

 (4)

 Solution

 (1)

 (4)

 (6) , - 0.5206 (negative)

2

(1) (2)

 Solution

 (1)

 (2)

3

 (1) B(2) B (3) B

Solution

4 = :

 (1) Calculate the measure of the angle to the nearest second.

 (2)

Solution 1 3

 (1) O

 (2)

 5

Solution

6

 Solution

7

 Solution

 O

 = 5 - 4