**Unit 3 Similarity**

**Lesson 1 Similarity of polygons**

**Definition:**

**(Having the same number of sides) are said to be similar**

**If the following two conditions satisfied together:**

**1**

**2**

**1**

**,**

**2**

**Remark 1**

**On writing the polygons, write them according to the order of their corresponding**

**Vertices to make it easy to deduce the equal angles in measure and write**

**The proportion of corresponding side lengths.**

**:**

**A B C D**

**1**

**2**

**2**

**3**

**If each one of two polygons is similar to a third polygon, then the two polygon are similar**

**,**

**4**

**All regular polygons that have the same number of sides are similar.**

* **All equilateral triangles are similar.**
* **All squares are similar.**
* **All regular pentagons are similar, and so on.**

**5**

**1 cm.**

**D E C**

**\*We can calculate the golden ratio as follows let the length of**

**The golden rectangle ABCD = cm. and its width = 1 cm., and**

**Draw the square AOBC The rectangle ABCD the rectangle EOBC**

**a = 1 , b = -1 , c = -1 A cm. O B**

**1.618**

**Ratio between the length of the rectangle and its width = 1.618 which is called the golden**

**Ratio [Note] All golden rectangles are similar.**

**Exercise 1**

**1**

**cm.**

**15 cm.**

**C G**

**(1)  D \* H \***

**8 cm.**

**12 cm.** 4.8 cm.

**(2) E 6 cm. F**

**A B**

**= scale factor**

**2**

**= 5 cm. Y 5 cm. X**

**6 cm.**

**C B**

**3 cm.**

**3 :**

**4.8 cm.**

**(1)**

**(2)**

**C 6 cm. B Z 8 cm. Y**

**4 : D 4 cm. C**

**4.8 cm.**

**2.5 cm.**

**M**

**.**

**= 2.5 cm.**

**B 8 cm. A**

**5perimeter**

**:**

**(1) (2)**

**6**

**10 cm.**

**7**

**The perimeter of the second is 200 cm.ind the length of the second rectangle and its area**

**et the length of two dimensions of the second rectangle be and cm.**

**he two rectangles are similar = = 40 cm.**

**, = = 60 cm.Area of second rectangle = 40 x 60 = 2400**

**H.W. Page 79 No. (2), Page 80 No.(6),Page 81 No. (8), Page 82(14) and Page 83 No. (19)**

**Lesson 2**

**(A. A. )**

**: Z**

**C**

**,**

**x**

**\***

**\***

**x**

**1 right – angled triangles are similar if the measure of an acute angle in One**

**of them is equal to the measure of an acute angle in the other triangle.**

**2 :**

* **, is equal to the measure of**

**an angle of the base in the other triangle.**

**3**

**[1]**

**a line is drawn parallel to one side of a triangle and intersects the other two sides or**

**The lines containing them, them the resulting triangle is similar to the original triangle.**

**: D E**

**\***

**x**

**A A**

**Ex D C x B A**

**\***

**\***

**\***

**x**

**x**

**x**

**C B E D C B**

**\***

**\***

**and intersects**

**[2]**

**-**

**Into two triangles which are similar to each other and to the original triangle.**

**: A**

**and**

**: C D B**

**Remarks on the previous figure:**

**1**

**is a mean proportional between**

**2**

**is a mean proportional between**

**3**

**is a mean proportional between**

**4**

**Exercise 2 On Similarity of triangles**

**1 A**

**M**

**Proof**

**x**

**x**

**C B**

**2 : A**

**Intersecting H D**

**C O B**

**Proof**

**(1) (2)**

**from (1) and (2)**

**3 A**

**Proof**

**C D B**

**4**

**16 cm.**

**: X**

**Z Y**

**12 cm.**

**Proof**

**5 = {E} , where Elies outside the Circle,**

**AB = 4 cm. , DC = 7 cm. and BE = 6 cm.**

**Prove that:**

**Proof E 6 cm. B 4 cm. A**

**D**

**C**

**7 cm.**

**6**

**D A**

**Proof**

**F Y E**

**(1) C x B**

**(2)**

**7 A**

**E**

**, F**

**(1) C D B**

**(2) Area of the rectangle AEDF =**

**H.W. Page 94 Nos. (10), (13), Page 95 No.(15) and Page 96 Nos.(24),(26)**

**Lesson 3 Follow: similarity of triangles**

**Theorem [1]: S.S.S. Similarity theorem**

**the side lengths of two triangles are in proportion, then the two triangles are similar.**

**A**

**Const. D**

**and intersects Y X**

**ProofF E C B**

**"Corollary"**

**"Construction"**

**"given"**

**"S.S.S. Congruency theorem"**

**"Proved"**

**Theorem [2]:**

**An angle of one triangle is congruent to an angle of another triangle and lengths of**

**The sides including those angles are in proportion and then the triangle are Similar.**

**GivenD A**

**.**

**O H**

**C B**

**"Corollary"**

**"Given", "Construction" =**

**"S.A.S. Congruency theorem" (2)**

**3 On theorem (1) and theorem (2)**

**1 :**

**A**

**= {F} , AB = 6 cm. , BC = 12 cm. AC = 8 cm.**

**8 cm.**

**, FC = 3 cm. , BD = 4.5 cm. , DF = 6 cm. Prove that:**

**9 cm.**

**E**

**C B**

**4.5 cm.**

**6 cm.**

**D**

**2 : A**

**7.5 cm.**

**ABC is a triangle in which: AB = 6 cm. , BC = 9 cm. , D**

**4 cm.**

**5 cm.**

**AC = 7.5 cm. , D is a point outside the triangle ABC where:**

**C**

**B**

**9 cm.**

**6 cm.**

**DB = 4 cm. , DA = 5 cm. Prove that**

**Proof**

**3 : A**

**ABCD is a quadrilateral , E where: B**

**Prove that**

**D C**

**Proof**

**4 :**

**3 cm.**

**A**

**ABC, in which AB =8 cm. , AC = 6 cm. , D,**

**6 cm.**

**Where AD = 3 cm. , E , where EC = 2 cm. D**

**8 cm.**

**Prove that: AED ~ ABC**

**Proof**

**C**

**B**

**2 cm.**

**5:**

**7.5 cm.**

**C**

**A**

**= {E}, AE = 7.5 cm., EC = 12 cm.**

**10 cm.**

**, BE= 9 cm. , ED = 10 cm. , AB = 6 cm.**

**12 cm.**

**Prove that: ABE ~ DCE, then find the length of:**

**D**

**B**

**Proof**

**9 cm.**

**6 cm.**

**6 ABC is a triangle, AB = 8 cm. , AC = 10 cm. ,BC= 12 cm. E, where AE = 2 cm. ,**

**10 cm.**

**A**

**Where BD = 4 cm. Prove that:**

**( 1 ) ~**

**( 2 )**

**4 cm.**

**2 cm**

**D**

**C**

**Proof**

**12 cm.**

**B**

**8 cm.**

**7 :**

**A**

**6 cm.**

**, DC = 5 cm. ,**

**8 cm.**

**C 5 cm. D 4 cm. B**

**is a tangent segment for the circle passing through the**

**Vertices of**

**Proof**

**is common angle**

**cm.**

**= is tangent segment for the circle passing through**

**TheVertices of**

**8 : D 16 cm. A**

**24 cm.**

**24 cm.**

**= {E} ,**

**36 cm.**

**AB = CD = 24 cm. , BC = 54 cm. , AD = 16 cm. , AC = 36 cm.**

**C**

**54 cm.**

**B**

**A**

**9 :**

**4 cm.**

**C**

**= 5 cm.**

**10 cm.**

**D**

**B**

**, is a tangent segment to**

**8 cm.**

**The circle at A, AD = 10 cm.**

**5 cm.**

**10 ABC is a right – angled triangle at A, D where**

**D**

**B**

**: (1)**

**(2)**

**C**

**A**

**11 where**

**D**

**A**

**X**

**C**

**B**

**Lesson 4 The relation between the areas of two similar polygons**

**First The ratio between the areas of the surfaces of two similar triangles**

**Theorem [3]**

**The ratio between the areas of the surfaces of two similar triangles equals the square of the**

**Ratio between the lengths of any two corresponding sides of the two triangles.**

**Given**

**D**

**A**

**Const. Draw**

**O M H**

**C**

**B**

**and (1)**

**:**

**(2)**

**(3)**

**:**

**(Q.E.D.)**

**Notice that:**

**I.e. The ratio between the areas of the surface of two similar triangles equals the square of the**

**ratio between the lengths of any two corresponding altitudes of the two triangles.**

**1 The ratio between the Perimeters of two similar triangles equals the ratio between any two**

**Corresponding sides.**

**2 The ratio between the areas of two triangles having the same base equals the ratio between**

**Their heights.**

**3 The ratio between the areas of two triangles having the same height equals the ratio between**

**Their bases lengths.**

**The ratio of the areas of the surfaces of two similar triangles equals the square of the ratio of**

**A**

**The Lengths of any two corresponding medians of the two triangles.**

**D**

**F M E C L B**

**Second The ratio between the areas of the surfaces of two similar polygons**

**Any two similar polygons can be divided into the same number of triangles, each is similar to its**

**Corresponding one.**

**A**

**:**

**and from two corresponding vertices say C and**

**Then each polygon**

**Will be divided into three triangles such that:**

**= (n–2) triangles**

**Theorem [4]**

**Ratio betweenthe areas of the surfaces of two similar polygons equals the Square**

**Of the ratio between the lengths of any two corresponding sides of the polygons.**

**Const.**

**are divided into the same number of the triangles each is similar to**

**its corresponding one "fact"**

**"from similar polygons"**

**Proportion properties:**

**Remember that:**

**ratio between the perimeters of two similar polygons equals the ratio**

**between The lengths of two corresponding sides in them.**

**Exercise 4 On the relation between the areas of two similar polygons**

**1 Complete:**

**(1)= …………**

**(2) = ….. cm.**

**2 , , then write the value of each of**

**The following: and**

**3**

**. find the area of the greater polygon.**

**4**

**A**

**C B**

**A**

**5 :**

**:**

**B C D**

**6:**

**,**

**(1) (2)**

**7 – angled at B, and drawn on**

**Y B X**

**the two similar triangles XBA, YCB such that X and Y lie outside**

**: the ratio between the area of the two triangles XBA**

**C**

**D A**

**and YCB equals the ratio between the lengths of the two projection**

**Of**

**8ABC is a right – angled triangle at B. are drawn,**

**Y**

**X**

**B**

**C A**

**Z**

**Adding (1) and (2)**

**Lesson 5 Application of Similarity in the circle**

**Well known problem:**

**A**

**C A**

**:**

**B**

**Of a circle intersecting at the point**

**D**

**C**

**D**

**B**

**B D**

**B D**

**Then:**

**Lie on a circle.**

**:**

**,**

**Lie on the same circle.**

**When a secant segment and a tangent segment are drawn to a circle from**

**an external point, the product of the lengths of the secant segment and its external**

**Segment is equal to the square of the length of the tangent segment.**

**:**

**D C A**

**Intersects it at C and D**

**B**

**isa tangent to the circle**

**Which passes through the points**

**Exercise 5 On application of similarity in the circle**

**2 cm.**

**D A**

**1 :**

**3 cm.**

**9 cm.**

**B C**

**A**

**2:**

**5 cm.**

**C**

**B**

**3 cm.**

**4 cm.**

**D**

**3:**

**15 cm.**

**.**

**9 cm.**

**4:**

**2 cm. 6 cm.**

**8 cm.**

**:**

**(1) (2)**

**5**

**.**

**6 :**

**(1)**

**(2)**

**6 cm.**

**(3)**

**4 cm. 5 cm.**

**7**

**6 cm.**

**(1)**

**8 cm.**

**(2)**

**is right – angled at B (1)**

**(because m (<D) + m(<FEC) =)**

**(2) From (1) and (2)**

**= 4.5 cm.**

**Unit 4 The triangle proportionality theorems**

**Lesson 1 Parallel lines and proportional parts**

**Theorem [1]**

**"Similarity postulate"**

**: (1)**

**(2)**

**From (1) and (2) we get :**

**From the properties of the proportion**

**, say intersecting at respectively, as shown in the figures, then:=**

**,**

**, we can deduce that :**

**:**

**then:**

**Exercise 1 On parallel lines and proportional parts**

**1:**

**6 cm.**

**,**

**3 cm. 4 cm.**

**2**

**9 cm.**

**15 cm.**

**3**

**8.4 cm.**

**14 cm.**

**21 cm.**

**5.6 cm.**

**4**

**6 cm.**

**10 cm.**

**7.8 cm.**

**13 cm.**

**5, where**

**=**

**6,**

**7and ,**

**3**

**4**

**(1)**

**4 3**

**H.W. Page 148 No. (6) and Page 150 Nos. (16), (17) and (21)**

**Lesson 2 Talis' Theorem**

**Theorem [2]**

**are two transversals**

**, then**

**are two transversals to them**

**Similarly:**

**: (exchange the means) (1)**

**Similarly (exchange the means) (2)**

**:**

**in the previous figure , notice that :**

**1**

**If , then**

**2:**

**.**

**:**

**,**

**Exercise 2 On Talis' theorem**

**1:**

**,**

**4.8 cm. 3.6 cm.**

**1.6 cm. 2.4 cm.**

**,**

**2:**

**5 cm. 4 cm.**

**7.5 cm. 6 cm.**

**3:**

**3 cm. 7 cm.**

**6 cm.**

**4.8 cm.**

**4:**

**15 cm.**

**6 cm.**

**9 cm.**

**18 cm.**

**5**

**Lesson 3 Angle bisectors and proportional parts**

**Theorem [3]**

**Bisectors of the interior or exterior angle of a triangle at any vertex divides the opposite**

**base of the triangle internally or externally into two parts, the ratio of their lengths is equal to**

**the ratio of the lengths of the other two sides of the triangle.**

**3**

**2**

**\***

**2**

**1**

**4**

**3**

**1**

**\***

**\***

**\***

**4**

**and intersect**

**,**

**,**

**, :**

**1:**

**\* \* \***

**then:**

**2 :**

**\* \* \* \* \*\***

**3**

**:**

**\* \***

**\* \***

**\* \***

**\* \***

**:**

**Internally and intersects at D then: AD=**

**is a triangle, internally, = {D}**

**\*\***

**x**

**x**

**= (inscribed angles subtended by**

**=**

**=AB X AC – AD X DE= AB XAC – BD XDC AD=**

**Notice that:**

**\* \***

**:**

**then:**

**H.W. Page 172 No. (7) and Page 174 No. (20)**

**Exercise 3 On angle bisectors and proportional parts**

**1**

**4 cm.**

**2.4cm.**

**\* \***

**6 cm.**

**2**

**8 cm.**

**\* \***

**6 cm.**

**7 cm.**

**3**

**\* \***

**4 cm. 5 cm.**

**4 right – angled triangle at B , draw**

**24 cm.**

**5**

**Of the triangle at A and intersects at E**

**.**

**(1)**

**6 cm.**

**(2)**

**3 cm.**

**\* \***

**7 cm.**

**(First req.)**

**= 14cm.**

**(second req.)**

**Lesson 4 Follow: bisectors and proportional parts**

**.**

**:**

**, , at A**

**.**

**:**

**\* \***

**x x**

**Exercise 4 On converse of theorem (3)**

**.**

**1**

**6 cm.**

**.**

**6 cm. 4 cm.**

**2**

**4 cm.**

**.**

**6 cm.**

**6 cm.**

**(1)**

**9 cm.**

**(2)**

**3**

**.**

**12 cm.**

**18 cm.**

**4 :**

**,**

**5**

**and intersects the circle M at B, C and**

**the circle N at D, E respectively.**

**H.W. P. 182 No. (3) &P. 183 Nos. (10), (12) P. 185 Nos.(22) ,(23)**

**Lesson 5 Applications of proportionality in the circle**

**:**

**Note 1**

**.**

**:**

* **.**
* **.**
* **.**

**Note 2**

**Notice that:**

**:**

**:**

**,**

**Note 3**

**:**

**.**

**.**

**Well Known problem:**

.

**,**

**[ ]( )**

**[ ]**

**,**

**[ ]**

**Exercise 5**

**1**

**(1)**

**(2)**

**(3)**

**(4)**

**2**

**4**

**4 = 8 cm.**

**= 64**

**5**

**6**

**.**

**8 cm. 4 cm.**