Math

GEOMETRY

Senior 1

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FIRST TERM 2018

Unit 3 Similarity

Lesson 1 Similarity of polygons

Definition:

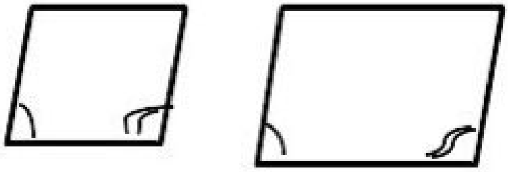
Tow polygons $M_1$and M_2(Having the same number of sides) are said to be similar

If the following two conditions satisfied together:

1. Their corresponding angles are congruent.
2. The length of their corresponding sides are proportional.

In this case , we shall write :the polygon $M_1 \sim$ the polygonM_2, that means the polygon $M_1$is similar to the polygon M_2

In the opposite figure if :



1. $m\angle A = m\angle X$, $m\angle B = m\angle Y$, $m\angle C = m\angle Z$, $m\angle D = m\angle L$
2. $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZL} = \frac{DA}{LX}$

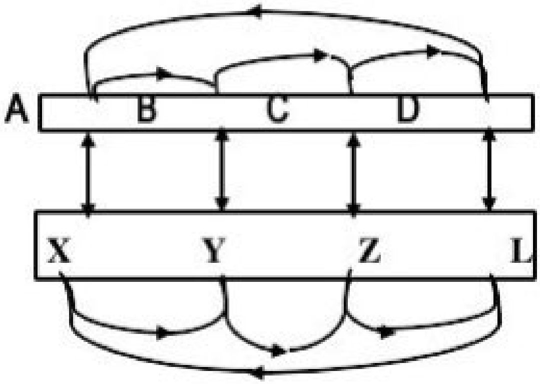
Then , the polygon ABCD$\sim$the polygon XYZL

Remark 1

On writing the polygons, write them according to the order of their corresponding Vertices to make it easy to deduce the equal angles in measure and write The proportion of corresponding side lengths.

For exampal :

If the polygon ABCD$\sim$the polygon XYZL , then:



1. $m\angle A = m\angle X$, $m\angle B = m\angle Y$ $m\angle C = m\angle Z$, $m\angle D = m\angle L$
2. $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZL} = \frac{DA}{LX}$

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Remark 2

If the two polygons are congruent,then they are similary,but it'snot necessary that similar poygons are congrunt.

- In the case of congrunt polygons , the ratio between the lengths of the corresponding sides similarity ratioequals one.

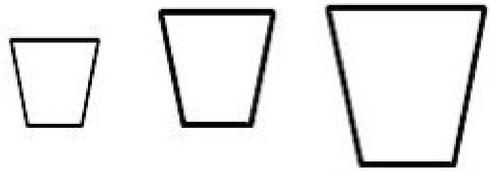
Remark3

If each one of two polygons is similar to a third polygon, then the two polygon are similar

i.e. If polygon M_1~polygon M_3

,polygon M_2~polygon $M_3 M_1 M_2 M_3$

$\therefore$ polygon M_1~polygon M_2



Remark4

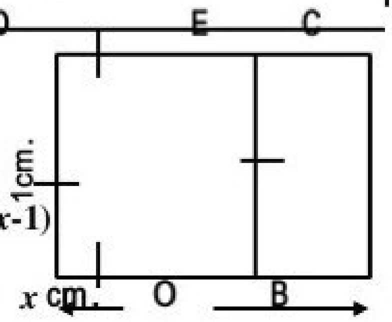
All regular polygons that have the same number of sides are similar.

- All equilateral triangles are similar.
- All squares are similar.
- All regular pentagons are similar, and so on.

Remark5

i.e. The ratio between the permeters of two similar polygons= theratio between the lengths of two corresponding sides of them.

The golden rectangle



*We can calculate the golden ratio as follows let the length of The golden rectangle ABCD = x cm. and its width = 1 cm., and Draw the square AOBC $\therefore$ The rectangle ABCD ~ the rectangle EOBC$(x-1)$

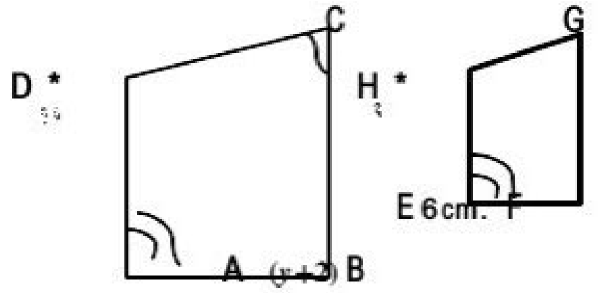
$\therefore \frac{AB}{EO}=\frac{BC}{OB} \therefore \frac{x}{1}=\frac{1}{x-1} \therefore x^2-x-1=0 \therefore$ a = 1 , b = -1 , c = -1

$\because x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \therefore x=\frac{1\pm\sqrt{1-4\times1\times-1}}{2} \therefore \frac{1-\sqrt{5}}{2}$ "refused " $\therefore x=\frac{1+\sqrt{5}}{2} \therefore x\simeq 1.618$

$\therefore$ TheRatio between the length of the rectangle and its width = 1.618 which is called the golden Ratio [Note] All golden rectangles are similar.

Exercise 1 On similarty of polygons

1 In the opposite figure :



polygon ABCD~polygon EFGH

(1) Find the scale factor of similarty of polygon ABCD to polygon EFGH

(2) Find the value of : x and y

Proof

$\because$ polygon ABCD~polygon EFGH $\therefore \frac{AB}{EF}=\frac{BC}{FG}=\frac{CD}{GH}=\frac{DA}{HE}=$ scale factor

$\therefore \frac{y+2}{6}=\frac{BC}{FG}=\frac{15}{x}=\frac{12}{8}$ $\therefore$ scale factor $=\frac{12}{8}=\frac{3}{2}$ first req.

$\therefore x=\frac{15\times 8}{12}=10$, $y+2=\frac{6\times 12}{8}=9$, $y=9-2=7$ second req.

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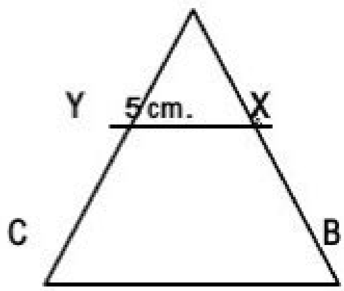
2 In the opposite figure :



$\Delta ABC \sim \Delta AXY$ Prove that : $XY // \overline{BC}$

and if : $AX = 6$ cm. , $XB = 3$ cm. and $YX = 5$ cm.

Find : The length of $\overline{BC}$

Proof

3 In the opposite figure :

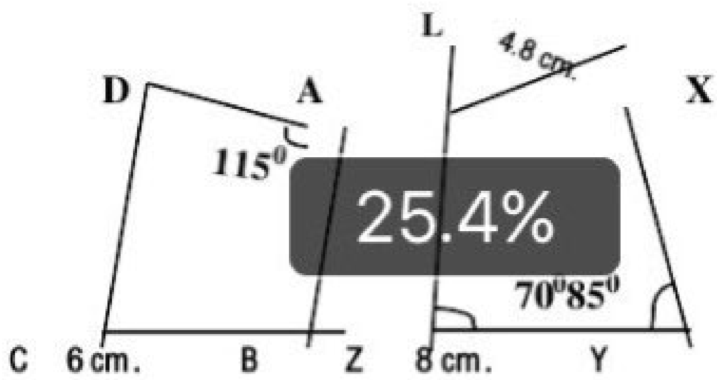
Polygon ABCD~Polygon XYZL

(1) Calculate :mXLZ , length of $\overline{AD}$

(2) If the perimeter of the polygon

ABCD= 19.5 cm.

Find :the perimeter of polygon XYZL



Proof

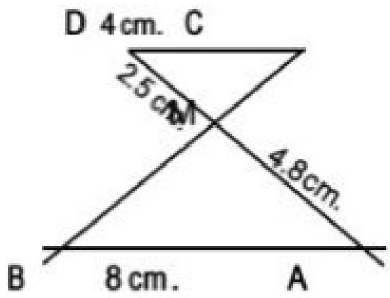
4 In the opposite figure :

$\Delta MAB \sim \Delta MCD$

Prove that :the figure ABCD is a cyclic quadrilatral.

And if :AB= 8 cm. , CD= 4 cm. , MA= 4.8 cm. MD = 2.5 cm.

Find :the length of $\overline{BC}$



Proof

5 The dimensions of arectangl are 10 cm. and 6 cm. Find the perimeter and the area of another rectangle similar to it if :

(1) The scale factor equals 3 (2) The scale factor equals 0.4

Proof

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6 The length of a golden rectangular box is 16.2 cm. Calculate the width of the box to the nearest centimetres.

Proof

$\because$ length/width= 1.618/1 $\therefore$ 16.2/width= 1.618/1 $\therefore$ width$\simeq$ 10 cm.

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7 Two similar rectangles , the dimensions of the first are 8 cm. and 12 cm. and The perimeter of the second is 200 cm.Find the length of the second rectangle and its area

Proof

Let the length of two dimensions of the second rectangle be x cm. and y cm.

$\because$ The two rectangles are similar $\therefore \frac{8}{x} = \frac{12}{y} = \frac{40}{200} \therefore X = \frac{8\times200}{40}$ = 40 cm.

$y = \frac{12\times200}{40}$ = 60 cm. $\therefore$ Area of second rectangle = 40 x 60 = 2400cm^2

H.W. Page 79 No. (2), Page 80 No.(6),Page 81 No. (8), Page 82(14) and Page 83 No. (19)

Lesson 2 Similarty of triangles

Postuate (A.A. Similarity postulate)

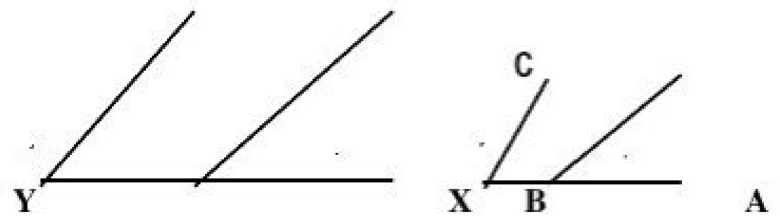
If two angles of one triangle are congrunt to their two corresponding anglesof another triangle , then the two triangles are similar.

In the opposite figure :

If : $\angle A \equiv \angle X$

, $\angle B \equiv \angle Y$

then : $\Delta ABC \sim \Delta XYZ$



Remarks

1 Two right – angled triangles are similar if the measure of an acute angle in One of them is equal to the measure of an acute angle in the other triangle.

2 Tow isosceles triangles are similar if :

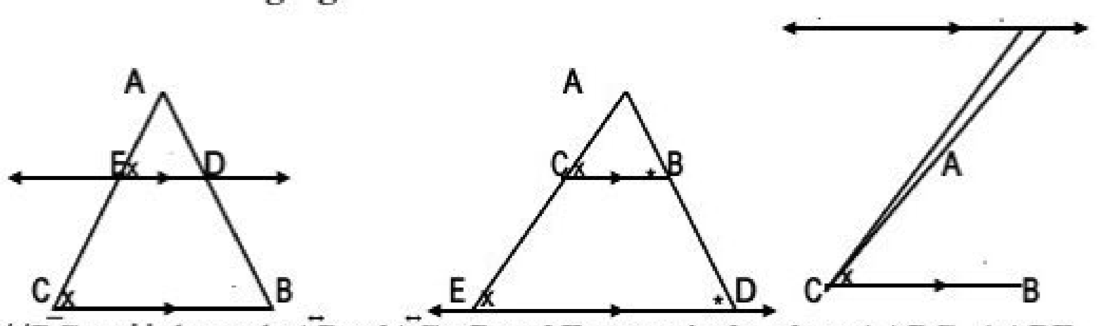
- The measure of one angle of the base in one triangle, is equal to the measure of an angle of the base in the other triangle.
- Their vertices angles are equal in measure.

3 Any two equilatral triangles are similar.

Corollary [1]

If a line is drawn parallel to one side of a triangle and intersects the other two sides or The lines containing them, them the resulting triangle is similar to the original triangle.

In each of the following figure :



If: $\overleftrightarrow{DE} // \overline{BC}$ and intersects $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ at D and E respectively , then: $\Delta ABC \sim \Delta ADE$

Corollary [2]

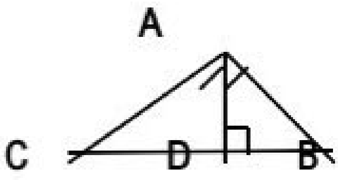
In any right - angled triangle , the altitude to the hypotenuse separtes the triangle

Into two triangles which are similar to each other and to the original triangle.

In the opposite figure :

If : Δ ABC is a right-angled triangle at A and $\overline{AD} \perp \overline{BC}$

,Then : Δ DBA $\sim \Delta$ DAC $\sim \Delta$ ABC



Remarks on the previous figure:

1 From similarity of $\Delta\Delta$ DBA and ABC , we get : $\frac{DB}{AB} = \frac{BA}{BC}$

$\therefore (AC)^2 = DB \times BC$ i.e. AB is a mean proportional between DB and BC

2 From similarity of $\Delta\Delta$ DAC and ABC , we get : $\frac{DC}{AC} = \frac{AC}{BC}$

$\therefore (AC)^2 = DC \times BC$ i.e. AC is a mean proportional between DC and BC

3 From similarity of $\Delta\Delta$ DBA and DAC , we get : $\frac{DA}{DC} = \frac{DB}{DA}$

$\therefore (DA)^2 = DB \times DC$ i.e. DA is a mean proportional between DB and DC

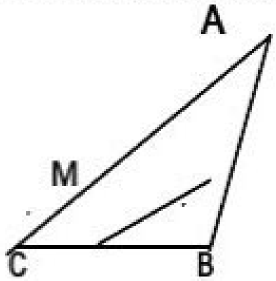
4 From similarity of $\Delta\Delta$ DBA and ABC , we get : $\frac{AB}{CB} = \frac{AD}{CA}$

$\therefore AD \times CB = AB \times CA$

Exercise 2 On Similarity of triangles

1 In Δ ABC , $AC > AB$, $M \in \overline{AC}$ where : m $\angle$ ABM = m$\angle$ C

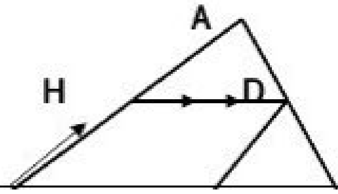
Prove that : $(AB)^2 = AM \times AC$



Proof

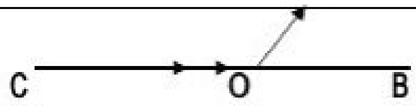
2 In the opposite figure :

ABC is a triangle , $D \in \overline{AB}$, draw $\overrightarrow{DH} // \overline{BC}$ Intersecting



$\overline{AC}$ at H , $\overline{DO}//\overline{AC}$ intersecting $\overline{BC}$at O

Prove that : Δ ADH~Δ DBO

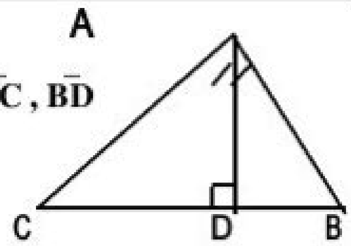


Proof

$\because$ $\overline{DH}//\overline{BC}$ $\therefore$ Δ ADH~Δ ABC (1) $\because$ $\overline{DO}//\overline{AC}$ $\therefore$ Δ DBO~Δ ABC (2)

from (1) and (2) $\therefore$ Δ ADH~ Δ DBO

3 ABC is a right-angled triangle at A , $\overrightarrow{AD}\perp\overline{BC}$to intersec it at D

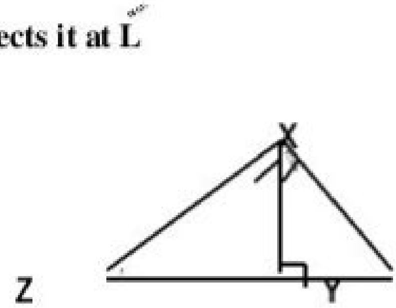


Prove that : $\overline{DA}$is a mean proportional between$\overline{DC}$, $\overline{BD}$

Proof

4 XYZ is a right-angled triangle at X , draw $\overrightarrow{XL}\perp\overline{YZ}$and intersects it at L

prove that : $\dfrac{(XY)^2}{(XZ)^2}=\dfrac{YL}{LZ}$



If XY= 12 cm. and XZ= 16 cm.

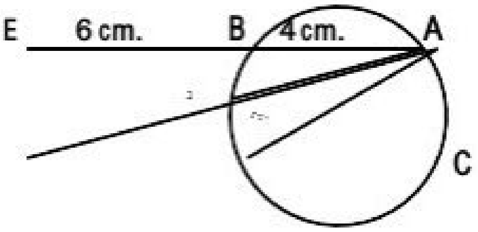
Calculate the length of each of : $\overline{YL}$,$\overline{XL}$

Proof

5 $\overline{AB}$and $\overline{DC}$are two chords in a circle , $\overrightarrow{AB}\cap\overrightarrow{CD}$= {E} , where E lies outside the Circle,

AB = 4 cm. , DC= 7 cm. and BE = 6 cm.

Prove that: $\Delta ADE \sim \Delta\ CBE$, then find the length of: $\overline{CE}$



Proof

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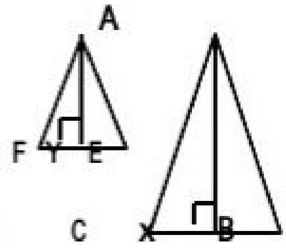
6 ABC and DEF are two similar triangles , $\overrightarrow{AX} \perp \overrightarrow{BC}$ to intersect it at X , $\overrightarrow{DY} \perp \overrightarrow{EF}$ to intersect it at Y. prove that : $BX \times YF = CX \times YE$



Proof $\because\ \Delta\ ABC \sim \Delta\ DEF\ m\angle B = m\angle E,\ m\angle C = m\angle F$

In $\Delta\Delta$ ABX, DEY $\therefore\ m\angle B = m\angle E$,

$m\angle BXA = m\angle EYD = 90^0 \therefore\ \Delta\ ABX \sim \Delta DEY \therefore \frac{BX}{EY} = \frac{AX}{DY}$? (1)

In $\Delta\Delta$ AXC , DYF : $\because\ m\angle C = m\angle F\ ,\ m\angle AXC = m\angle DYF = 90^0$

$\therefore\ \Delta\ AXC \sim \Delta\ DYF \therefore \frac{AX}{DY} = \frac{XC}{YF}$? (2)

From 1,2 : $\therefore \frac{BX}{EY} = \frac{XC}{YF} \therefore BX \times YF = XC \times YE$

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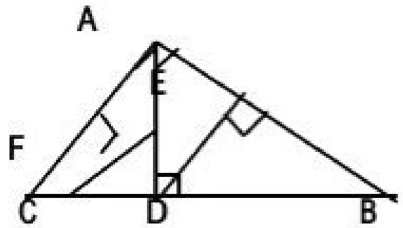
7 In the opposite figure :

ABC is a right-angled triangle at A

, $\overline{AD} \perp \overline{BC}$, $\overline{DE} \perp \overline{AB}$, $\overline{DF} \perp \overline{AC}$ Prove that :

(1) $\Delta ADE \sim \Delta\ CDF$

(2) Area of the rectangle AEDF = $\sqrt{AE \times EB \times AF \times FC}$



H.W. Page 94 Nos. (10), (13), Page 95 No.(15) and Page 96 Nos.(24),(26)

Lesson 3 Follow: similarity of triangles

Theorem [1]: S.S.S. Similarity theorem

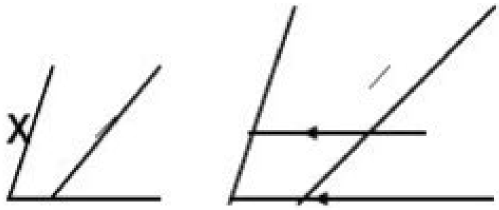
If the side lengths of two triangles are in proportion, then the two triangles are similar.

Given In $\Delta\Delta$ ABC , DEF : $\frac{AB}{DE}=\frac{BC}{EF}=\frac{CA}{FD}$

R.T.P. Δ ABC $\sim\Delta$ DEF A

Const. Take $X\in \overline{AB}$ where AX = DE Draw D

$\overline{XY}//\overline{BC}$ and intersects $\overline{AC}$ at Y Y



Proof $\because \overline{XY}//\overline{BC}$ F E C B

$\therefore \Delta ABC \sim \Delta AXY$ "Corollary≪1≫" $\therefore \frac{AB}{AX}=\frac{BC}{XY}=\frac{CA}{YA}$

$\because AX = DE$ "Construction"

$\therefore \frac{AB}{DE}=\frac{BC}{XY}=\frac{CA}{YA}$? 1 $\therefore \frac{AB}{DE}=\frac{BC}{EF}=\frac{CA}{FD}$ "given" ? (2)

From 1 and 2 we deduce that : XY = EF, YE = FD

and $\Delta AXY \equiv \Delta DEF$ "S.S.S. Congruency theorem" $\therefore \Delta DEF \sim \Delta AXY$

$\because \Delta ABC \sim \Delta AXY$ "Proved" $\therefore \Delta ABC \sim \Delta DEF$

Theorem [2]: S.A.S. Similarity theorem

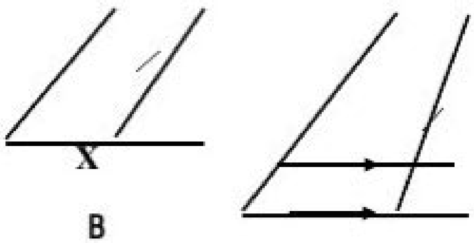
If An angle of one triangle is congruent to an angle of another triangle and lengths of The sides including those angles are in proportion and then the triangle are Similar.

Given $\angle A \equiv \angle D$ and $\frac{AB}{DH}=\frac{AC}{DO}$ D A

R.T.P. $\Delta ABC \sim \Delta DHO$

Const. Let $X\in \overline{AB}$ such that AX = DHO H Y

and draw $\overline{XY}//\overline{BC}$ and intersects $\overline{AC}$ at Y C B



Proof $\because \overline{XY}//\overline{BC} \therefore \Delta ABC \sim \Delta AXY$ "Corollary" ? (1) $\therefore \frac{AB}{AX}=\frac{AC}{AY}$

$\because \frac{AB}{DH}=\frac{AC}{DO}$ "Given", AX = DH "Construction" $\therefore \frac{AB}{AX}=\frac{AC}{DO} \therefore AY=DO$

$\therefore \Delta AXY \equiv \Delta DHO$ "S.A.S. Congruency theorem" $\therefore \Delta AXY \sim \Delta DHO$? (2)

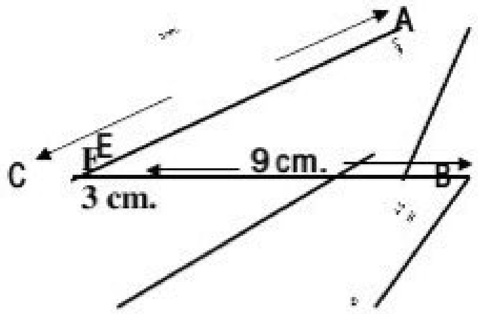
From 1 and 2 we get : $\Delta ABC \sim \Delta DHO$ Q.E.D.

Exercise (3) On theorem (1) and theorem (2)

1 **In the opposite figure :**

$\overline{BC} \cap \overline{DE} = \{F\}$, AB = 6 cm. , BC = 12 cm. AC = 8 cm.

, FC = 3 cm. , BD = 4.5 cm. , DF = 6 cm. Prove that:



1 Δ **ABC~**Δ **DBF** **2**Δ **EFC is isosceles**

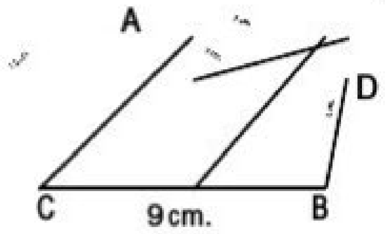
Proof

2 **In the opposite figure :**

ABC is a triangle in which: AB = 6 cm. , BC = 9 cm. ,

AC = 7.5 cm. , D is a point outside the triangle ABC where:

DB = 4 cm. , DA = 5 cm. Prove that



1 Δ **ABC ~** Δ **DBA** **2**$\overrightarrow{BA}$ **bisects** $\angle$ **DBC**

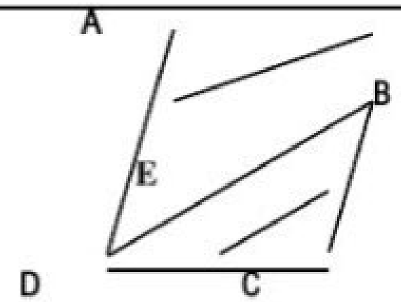
Proof

3 In the opposite figure :

ABCD is a quadrilateral , $E \in \overline{BD}$ where:

$\frac{AB}{DA}=\frac{CE}{BC}$, $\frac{BD}{DA}=\frac{EB}{BC}$ Prove that

1 $\overline{AD} // \overline{BC}$ 2 $\overline{AB} // \overline{CE}$



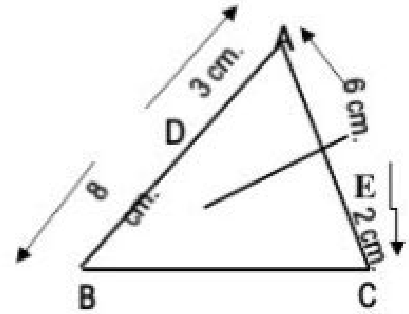
Proof

4 In the opposite figure :

ΔABC, in which AB = 8 cm. , AC = 6 cm. , $D \in \overline{AB}$,

Where AD = 3 cm. , $E \in \overline{AC}$, where EC = 2 cm.

Prove that: $\Delta AED \sim \Delta ABC$



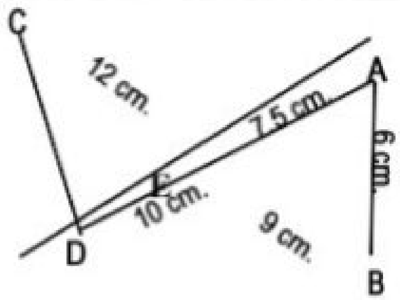
Proof

5 In the opposite figure:

$\overline{AD} \cap \overline{BC} = \{E\}$, AE = 7.5 cm., EC = 12 cm.

, BE = 9 cm. , ED = 10 cm. , AB = 6 cm.

Prove that: $\Delta ABE \sim \Delta DCE$, then find the length of: $\overline{CD}$



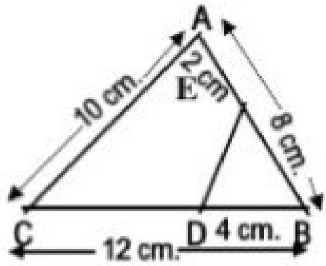
Proof

6 ABC is a triangle, AB = 8 cm. , AC = 10 cm. ,BC= 12 cm. E∈ $\overline{AB}$, where AE = 2 cm.,

D∈ $\overline{BC}$ Where BD = 4 cm. Prove that:

(1) $\triangle$BDE ~$\triangle$ BAC and deduce the length of $\overline{DE}$

(2) The figure ACDE is a cyclic quadrilateral.



Proof

7 In the opposite figure :

ABC is a triangle in which D∈ $\overline{BC}$ where BD= 4 cm.

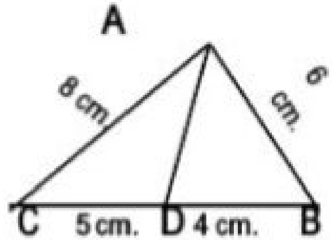
, DC = 5 cm. , If AB= 6 cm. , AC= 8 cm.

1Prove that : Δ ABC~Δ DBA

2Find the length : $\overline{AD}$

3Prove that: $\overline{AB}$ is a tangent segment for the circle passing through the

Vertices of Δ ADC



Proof

$\because \frac{AB}{BD}=\frac{6}{4}=\frac{3}{2}, \frac{BC}{BA}=\frac{9}{6}=\frac{3}{2}$

In $\Delta\Delta$ ABC , DBA : $\frac{AB}{BD}=\frac{BC}{BA}=\frac{3}{2}$ $\angle$ B is common angle $\therefore$ Δ ABC~Δ DBA

$\therefore \frac{AB}{DB}=\frac{AC}{AD} \therefore \frac{6}{4}=\frac{8}{AD}$ $\therefore$ AD= $5\frac{1}{3}$ cm.

$\therefore$ m$\angle$ BAD = m($\angle$ C)$\therefore$ $\overline{AB}$ is tangent segment for the circle passing through

TheVertices of Δ ADC

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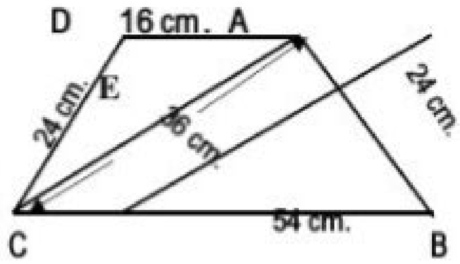
8 In the opposite figure :



ABCD is a quadrilateral in which : $\overline{BD} \cap \overline{AC} = \{E\}$,

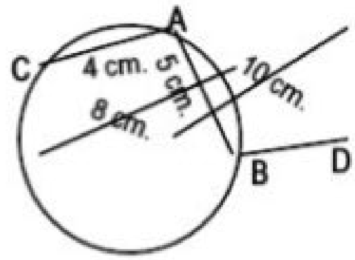
AB = CD = 24 cm. , BC = 54 cm. , AD = 16 cm. , AC = 36 cm.

Prove that : $\Delta BAC \sim \Delta ADC$

Then prove that $\overrightarrow{CA}$ bisects $\angle BCD$

Proof

9 In the opposite figure :



ABC is an inscribed triangle in a circle where AB = 5 cm.

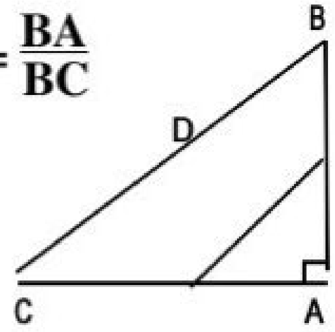
,BC= 8 cm. , CA= 4 cm. , $\overline{AD}$ is a tangent segment to

The circle at A, AD = 10 cm.

Prove that : $\Delta DAB \sim \Delta BCA$, then find the length of $\overline{BD}$

Proof

10 ABC is a right − angled triangle at A, $D \in \overline{BC}$ where $\frac{DB}{AB} = \frac{BA}{BC}$



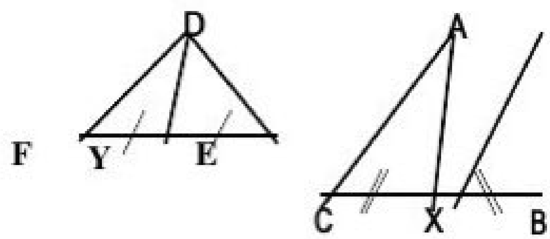
Prove that: (1) Δ ABC$\sim\Delta$ DBA

(2) $\overline{AD} \perp \overline{BC}$

Proof

11 If Δ ABC$\sim\Delta$ DEF and X is a midpoint of $\overline{BC}$, Y is midpoint of $\overline{EF}$ where $\overline{BC}$, $\overline{EF}$ two Corresponding sides in the two triangles.

Prove that : Δ ABX$\sim\Delta$ DEY



Proof

Lesson ④ The relation between the areas of two similar polygons

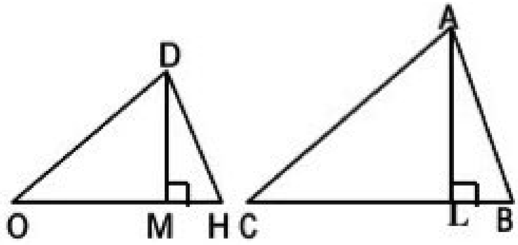
First The ratio between the areas of the surfaces of two similar triangles

Theorem [3]

The ratio between the areas of the surfaces of two similar triangles equals the square of the Ratio between the lengths of any two corresponding sides of the two triangles.

Given $\Delta\ ABC \sim \Delta\ DHO$

R.T.P. $\dfrac{\text{The area of } \Delta\ ABC}{\text{The area of } \Delta\ DHO} = \dfrac{AB^2}{DH} = \dfrac{BC^2}{HO} = \dfrac{AC^2}{DO}$



Const. Draw $\overrightarrow{AL} \perp \overleftrightarrow{BC}$ such that $\overrightarrow{AL} \cap \overleftrightarrow{BC} = \{L\}$

and $\overrightarrow{DM} \perp \overleftrightarrow{HO}$ such that $\overrightarrow{DM} \cap \overleftrightarrow{HO} = \{M\}$

Proof $\because \Delta\ ABC \sim \Delta\ DHO \therefore m\angle B = m(\angle H)$ and $\dfrac{AB}{DH} = \dfrac{BC}{HO} = \dfrac{CA}{OD}$ (1)

In the two right-angled triangles ABL and DHM :

$\because m\angle B = m\angle H \therefore \Delta\ ABL \sim \Delta\ DHM \therefore \dfrac{AB}{DH} = \dfrac{AL}{DM}$ (2)

$\because \dfrac{\text{The area of } \Delta\ ABC}{\text{The area of } \Delta\ DHO} = \dfrac{\frac{1}{2} BC \times AL}{\frac{1}{2} HO \times DM} = \dfrac{BC}{HO} \times \dfrac{AL}{DM}$

(3)

From 1 , 2 and 3 We get:

$$\frac{\text{The area of } \Delta\ ABC}{\text{The area of } \Delta\ DHO} = \frac{BC}{HO} \times \frac{BC}{HO} = \frac{BC^2}{HO} = \frac{AB^2}{DH} = \frac{CA^2}{OD} \text{ (Q.E.D.)}$$

Notice that:

$$\frac{a\ \Delta\ ABC}{a\ \Delta\ DHO} = \frac{AB^2}{DH}$$

$\because \dfrac{AB}{DH} = \dfrac{AL}{DM}$ $\qquad \therefore \dfrac{a\ \Delta\ ABC}{a\ \Delta\ DHO} = \dfrac{AL^2}{DM}$

I.e. The ratio between the areas of the surface of two similar triangles equals the square of the ratio between the lengths of any two corresponding altitudes of the two triangles.

Remarks

1 The ratio between the Perimeters of two similar triangles equals the ratio between any two Corresponding sides.

2 The ratio between the areas of two triangles having the same base equals the ratio between Their heights.

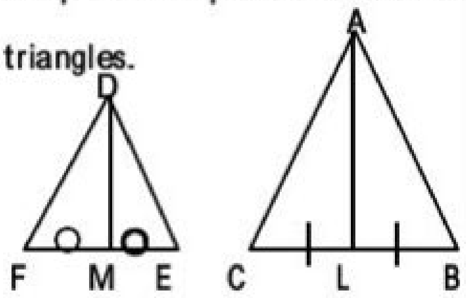
3 The ratio between the areas of two triangles having the same height equals the ratio between Their bases lengths.

Important remark

The ratio of the areas of the surfaces of two similar triangles equals the square of the ratio of The Lengths of any two corresponding medians of the two triangles.

If Δ ABC~Δ DEF , L is midpoint of $\overline{BC}$, M is midpoint of $\overline{EF}$

Then $\dfrac{a(\Delta ABC)}{a(\Delta DEF)} = \dfrac{AL^2}{DM}$



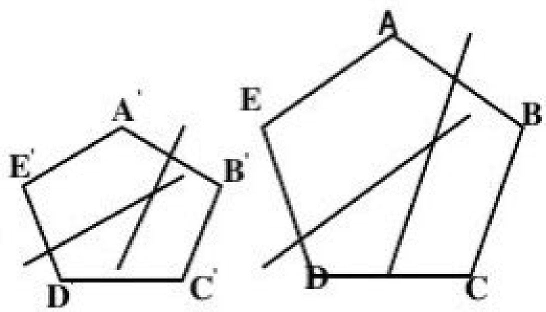
Second The ratio between the areas of the surfaces of two similar polygons

Fact

Any two similar polygons can be divided into the same number of triangles, each is similar to its Corresponding one.

In the opposite figure :

If the two polygons ABCDE and A'B'C'D'E' are similar and from two corresponding vertices say C and C' we draw $\overline{CA}$, $\overline{CE}$, $\overline{C'A'}$ and $\overline{C'E'}$ Then each polygon Will be divided into three triangles such that:



$\Delta ABC \sim \Delta A'B'C'$, $\Delta ACE \sim \Delta A'C'E'$ and $\Delta ECD \sim \Delta E'C'D'$

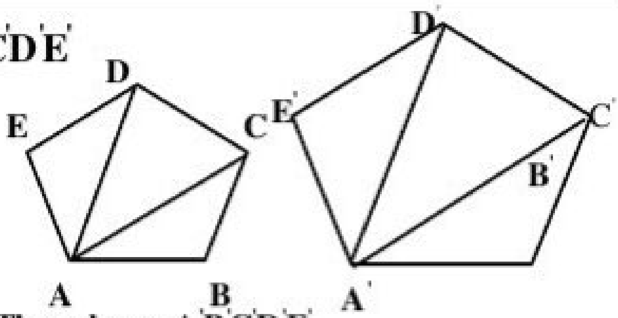
Important remark

- If the number of sides of a polygon is n sides , then the number of triangles that the polygon is divided by drawing the diagonals from One of its vertices = (n−2) triangles

Theorem [4]

The Ratio between the areas of the surfaces of two similar polygons equals the Square Of the ratio between the lengths of any two corresponding sides of the polygons.

Given The polygon ABCDE~ the polygon A'B'C'D'E'



R.T.P. $\frac{a\text{ the polygon }ABCDE}{a(\text{the polygon }A'B'C'D'E')} = \left(\frac{AB}{A'B'}\right)^2$

Const. From A , A' draw $\overline{AC}$, $\overline{AD}$, $\overline{A'C'}$, $\overline{A'D'}$

Proof ∵ The polygon ABCD~The polygon A'B'C'D'E'

∴ They are divided into the same number of the triangles each is similar to its corresponding one "fact"

$$\therefore \frac{a\,\Delta ABC}{a\,\Delta A'B'C'} = \left(\frac{BC}{B'C'}\right)^2 , \frac{a\,\Delta ACD}{a(\Delta A'C'D')} = \left(\frac{CD}{C'D'}\right)^2 , \frac{a\,\Delta ADE}{a\,\Delta A'D'E'} = \left(\frac{DE}{D'E'}\right)^2$$

$$\because \frac{BC}{B'C'} = \frac{CD}{C'D'} = \frac{DE}{D'E'} = \frac{AB}{A'B'} \text{ "from similar polygons"}$$

$$\therefore \frac{a\,\Delta ABC}{a\,\Delta A'B'C'} = \frac{a\,\Delta ACD}{a\,\Delta A'C'D'} = \frac{a\,\Delta ADE}{a\,\Delta A'D'E'} = \left(\frac{AB}{A'B'}\right)^2$$

From Proportion properties: $\frac{a\,\Delta ABC + a\,\Delta ACD + a\,\Delta AD}{a\,\Delta A'B'c' + a\,\Delta A'C'D' + a(\Delta A'D'E')} = \left(\frac{AB}{A'B'}\right)^2$

$$\therefore \frac{a(\text{the polygon }ABCDE)}{a\text{ the polygon }A'B'C'D'E'} = \left(\frac{AB}{A'B'}\right)^2$$

Remember that:

The ratio between the perimeters of two similar polygons equals the ratio between The lengths of two corresponding sides in them.

Exercise (4) On the relation between the areas of two similar polygons

(1) Complete:

(1) If $\Delta\ ABC \sim \Delta\ XYZ$, $AB = 3XY$, then $\frac{a\ \Delta\ XYZ}{a\ \Delta\ ABC} =$ ....

(2) If $\Delta\ ABC \sim \Delta\ DEF$ $a\ \Delta\ ABC = 9\ a\ \Delta\ DEF$ and $DE = 4$ cm. , then $AB =$ .... cm.

---

(2) If the polygon $ABCD \sim$ the polygon $A'B'C'D'$, $\frac{AB}{A'B'} = \frac{1}{3}$, then write the value of each of

The following: $\frac{a \text{ the polygon } ABCD}{a \text{ the polygon } A'B'C'D'}$ and $\frac{\text{Perimeter of } ABCD}{\text{Perimeter of } A'B'C'D'}$

Proof

---

(3) If the lengths of two corresponding sides in two similar polygons are 12 cm. ,6 cm.

and the area of the smaller polygon= 135cm.^2 ,then. find the area of the greater polygon.

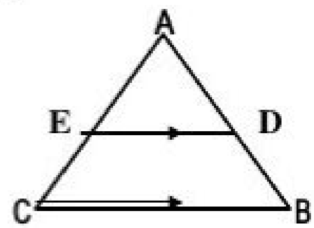
Proof

---

(4) ABC is a triangle , $D \in \overline{AB}$ where $AD = 2\ BD$, $E \in \overline{AC}$ where $\overline{DE} \parallel \overline{BC}$

If the area of $\Delta\ ADE = 60 \text{ cm.}^2$ Find the area of the trapezium DBCE

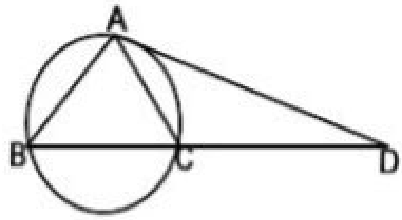
Proof



5 In the opposite figure:

$\overline{AD}$ is a tangent segment to the circumcircle of Δ ABC

, 2 AB = 3 AC Find: $\dfrac{a\,\Delta\,ACD}{a\,\Delta\,ACB}$

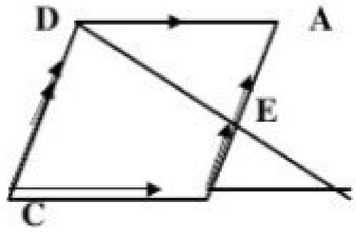


Proof |

6 In the opposite figure:

ABCD is a parallelogram , E $\in \overline{AB}$ where

$\dfrac{AE}{EB} = \dfrac{3}{2}$, $\overrightarrow{DE} \cap \overrightarrow{CB} = \{f\}$

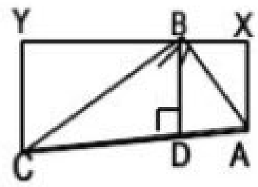


(1) Prove that: Δ DCF ~ Δ EAD (2) Find: $\dfrac{a\,\Delta\,DCF}{a\,\Delta\,EAD}$

Proof |

7 ABC is a right − angled at B, and drawn on $\overline{AB}$ and $\overline{BC}$ the two similar triangles XBA, YCB such that X and Y lie outside ΔABC

Prove that: the ratio between the area of the two triangles XBA and YCB equals the ratio between the lengths of the two projection Of $\overline{AB}$ and $\overline{BC}$ on $\overleftrightarrow{AC}$ respectively.



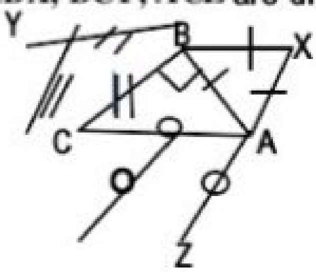
Proof |

8 ABC is a right − angled triangle at B. The equilateral triangles ABX, BCY, ACZ are drawn,

Prove that: a Δ ABX + a Δ BCY = a Δ ACZ

Proof: | $\because \Delta\Delta\Delta$ ABX , BCY , ACZ are equilatral triangles

$\therefore \Delta$ ABX ~ Δ BCY ~ Δ ACZ



$\frac{a(\Delta ABX)}{aACZ}=\frac{AB^2}{AC}=\frac{AB^2}{AC^2}1$, $\frac{a\Delta BCY}{a\Delta ACZ}=\frac{BC^2}{AC}=\frac{BC^2}{AC^2}$ D(2)Adding (1) and (2)

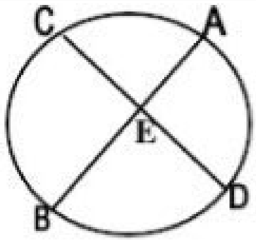
Lesson (5) Application of Similarity in the circle

Well known problem:

If the lines containing the two chords:

$\overline{AB}$, $\overline{CD}$ Of a circle intersecting at the point E

Then: EA×EB= EC×ED



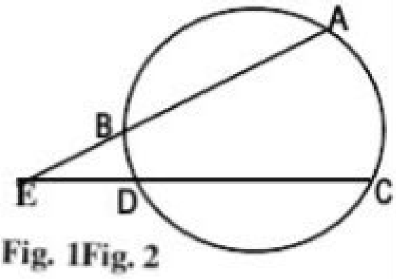


Fig. 1Fig. 2

Convers of the well Known Problem

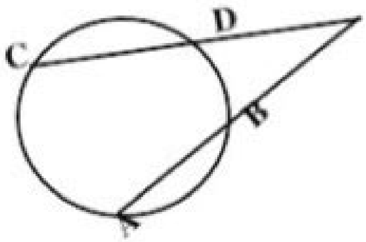
If the two lines containing the two segments $\overline{AB}$and $\overline{CD}$intersect at E (A,B,C,D,and E are distinct points) and EA×EB= EC×ED , then the points:A , B , C and DLie on a circle.

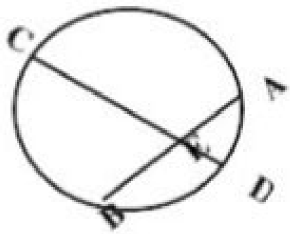
In the opposite figure:

If:EA×EB= EC×ED ,

then the points:A , B , C and D

Lie on the same circle.





Corollary [1]

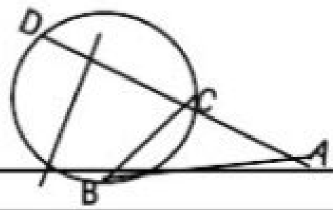
When a secant segment and a tangent segment are drawn to a circle from an external point, the product of the lengths of the secant segment and its external

Segment is equal to the square of the length of the tangent segment.

In the opposite figure:

If $\overline{AB}$ is tangent to the circle,$\overrightarrow{DA}$Intersects it at C and D

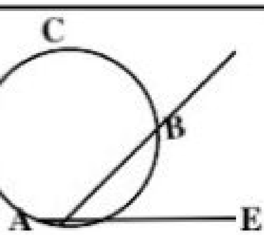
Then AB^2= AC×AD



Corollary [2]

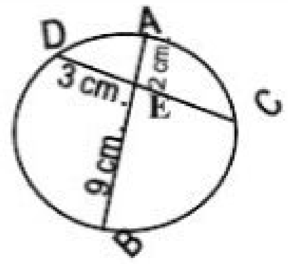
If: $(EA)^2$= EB×EC , Then $\overline{EA}$ isa tangent to the circle

Which passes through the points A , B and C



Exercise (5) On application of similarity in the circle

◇ 1 In the opposite figure:

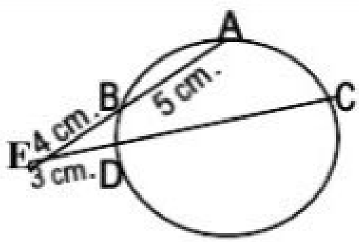


If $\overline{AB} \cap \overline{CD} = \{E\}$, AE = 2 cm. , ED = 3 cm. , EB = 9 cm.

Calculate the length of : $\overline{CE}$

Proof

◇ 2 In the opposite figure:

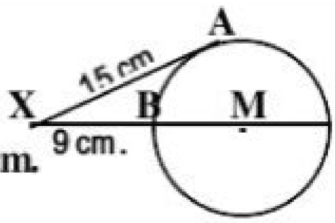


AB = 5 cm. , BE = 4 cm. , DE = 3 cm.

Calculate the length of : $\overline{CD}$

Proof

◇ 3 In the opposite figure:



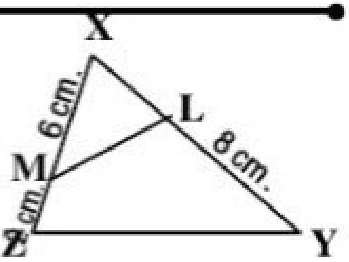
$\overline{XA}$ is a tangent to the circle M at A

Where XA = 15 cm. , if XB = 9 cm.

Calculate the length of the radius of the circle.

Proof

◇ 4 In the opposite figure:



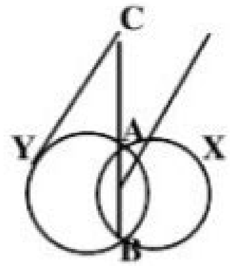
L ∈ $\overline{XY}$ where XL = 4 cm. YL = 8 cm., M ∈ $\overline{XZ}$

where XM = 6 cm. , ZM = 2 cm. Prove that:

(1) $\Delta XLM \sim \Delta XZY$ (2) LYZM is a cyclic quadrilateral

Proof

5 Two circles are intersected at A and B , $C \in \overleftrightarrow{AB}$ and $C \notin \overline{AB}$,
from C the two tangent segments $\overline{CX}$ and $\overline{CY}$ are drawn
to touch the circle at X and Y respectively.



Prove that : CX = CY

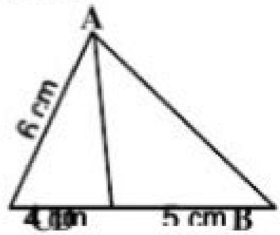
Proof

6 ABC is a triangle. $D \in \overline{BC}$ where BD = 5 cm. and DC = 4 cm. If AC = 6 cm. Prove that :

(1) $\overline{AC}$ is a tangent segment to the circle passing through the points A , B and D

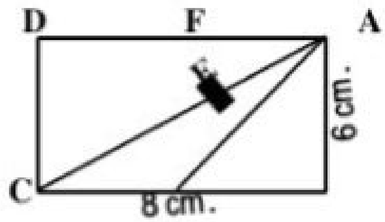
(2) $\Delta ACD \sim \Delta BCA$

(3) Area of Δ ABD : Area of Δ ABC = 5 : 9



Proof

7 ABCD is a rectange in which AB = 6 cm. and BC = 8 cm. ,
$\overrightarrow{BE} \perp \overline{AC}$ and intersects and $\overline{AC}$ at E and $\overline{AD}$ at F



(1) Prove that : $AB^2 = AF \times AD$

(2) Find the length of : $\overline{AF}$

Proof $\because \Delta$ ABC is right − angled at B $\overline{BE} \perp \overline{AC}$ $\therefore$ $AB^2 = AE \times AC$ (1)

$\because$ Figure FECD is a cyclic quadrilateral.(because m (<D) + m(<FEC) = 180^0)

$\therefore$ $AF \times AD = AE \times AC$ (2) From (1) and (2) $\therefore$ $AB^2 = AF \times AD$ $\therefore$ $6^2 = AF \times 8$

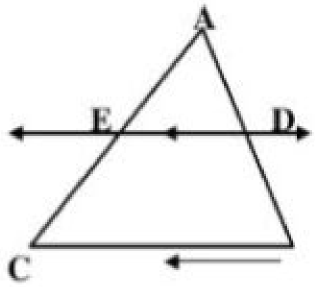
$\therefore$ AF = 36 ÷ 8 = 4.5 cm.

Unit 4 The triangle proportionality theorems

Lesson 1 Parallel lines and proportional parts

Theorem [1]

If a line is darwn parallel to one side of a triangle and intersects the other two sides , Then it divides them into segments whose lengths are proportional.



Given ABC is a tringle , $\overleftrightarrow{DE} // \overline{BC}$

R.T.P. $\frac{AD}{DB} = \frac{AE}{EC}$

Proof $\because \overleftrightarrow{DE} // \overline{BC} \therefore \Delta\, ABC \sim \Delta\, ADE$ "Similarity postulate"

, then: $\frac{AB}{AD} = \frac{AC}{AE}$ (1)

$\because D \in \overline{AB}$, $E \in \overline{AC}$ $\therefore AB = AD + DB$, $AC = AE + EC$ (2)

From (1) and (2) we get : $\frac{AD+DB}{AD} = \frac{AE+EC}{AE}$, then : $\frac{AD}{AD} + \frac{DB}{AD} = \frac{AE}{AE} + \frac{EC}{AE}$

$\therefore 1 + \frac{DB}{AD} = 1 + \frac{EC}{AE} \therefore \frac{DB}{AD} = \frac{EC}{AE}$ From the properties of the proportion $\frac{AD}{DB} = \frac{AE}{EC}$

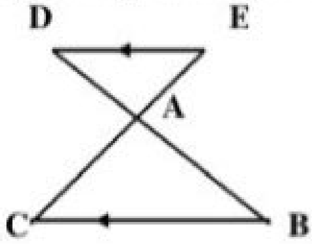
Corollary

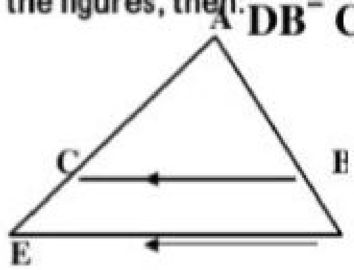
If a straight line is drawn outside the triangle ABC parallel to one side of the triangle , say $\overline{BC}$ intersecting $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ at D and E respectively, as shown in the figures, then: $\frac{AB}{DB} = \frac{AC}{CE}$

,From the properties of the proportion

, we can deduce that :

$\frac{AD}{AB} = \frac{AE}{AC}$, $\frac{AD}{BD} = \frac{AE}{CE}$

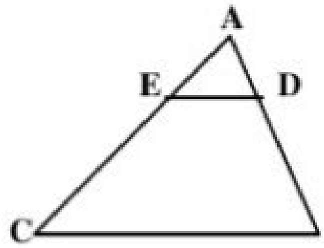




Converse of thearem [1]

If a straight line intersects two sides of a triangle and divides them into segments whose lengths are proportional , then it is parallel to the third side of the triangle.

In the opposite figure:



ABC is a triangle, $\overleftrightarrow{DE}$ intersects $\overleftrightarrow{AB}$ at D, $\overleftrightarrow{AC}$ at E

and $\frac{AD}{DB}=\frac{AE}{EC}$ then: $\frac{AD+DB}{AD}=\frac{AE+EC}{AE}$

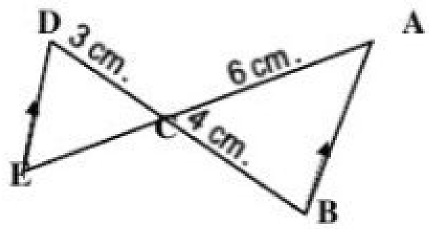
because: $\frac{\text{antecedent+consequent}}{\text{antecedent}}=\frac{\text{antecedent+consequent}}{\text{antecedent}}$

$\therefore \frac{AB}{AD}=\frac{AC}{AE} \because \angle A$ is common $\therefore \Delta ABC \sim \Delta ADC$

$\therefore \angle B \equiv \angle ADE$ and they are corresponding angles $\therefore \overleftrightarrow{DE} // \overleftrightarrow{BC}$

Exercise 1 On parallel lines and proportional parts

1 In the opposite figure:



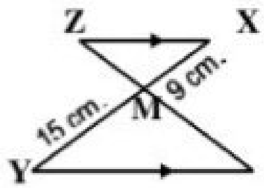
$\overline{AB} // \overline{DE}$, $\overline{AE} \cap \overline{BD} = \{C\}$,

AC = 6 cm., BC = 4 cm. and CD = 3 cm.

Find the length of : $\overline{AE}$

Proof

2 $\overline{XY} \cap \overline{ZL} = \{M\}$, where $\overline{XZ} // \overline{LY}$, if XM = 9 cm., YM = 15 cm.

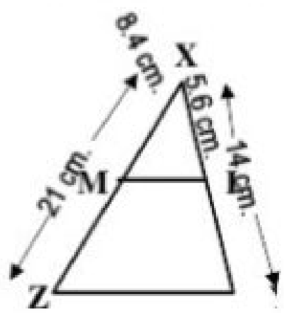


and ZL = 36 cm. Find the length of : $\overline{ZM}$

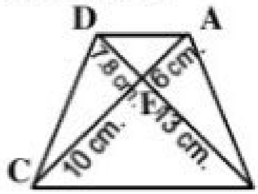
Proof

3 XYZ is a triangle in which XY = 14 cm., XZ = 21 cm. L $\in \overline{XY}$, where XL = 5.6 cm. and M $\in \overline{XZ}$ where XM = 8.4 cm. Prove that: $\overline{LM} // \overline{YZ}$

Proof



4 ABCD is a quadrilatral , its diagonals are intersected at E If AE= 6 cm. , BE= 13 cm. , EC= 10 cm. and ED= 7.8 cm. Prove that: ABCD is atrapezium.

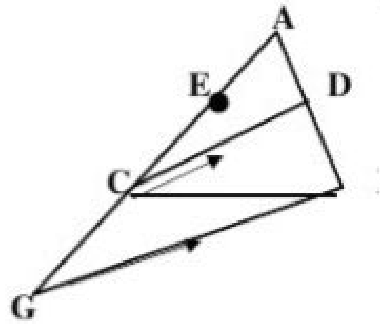


Proof

5 ABC is a triangle , D∈ $\overline{AB}$, E∈ $\overline{AC}$, where $\frac{AD}{DB}=\frac{AE}{EC}$

draw $\overline{DC}$ then from B draw $\overrightarrow{BG}//\overline{DC}$ and intersect $\overrightarrow{AC}$ at G

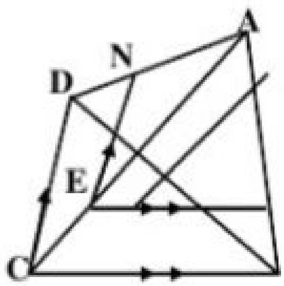
Prove that: $AC^2 = AE \cdot AG$



Proof

6 ABCD is aquadrilateral , E∈ $\overline{AC}$ draw $\overrightarrow{EF}//\overline{CB}$ to intersect $\overline{AB}$ at F,

draw $\overrightarrow{EN}//\overline{CD}$ to intersect $\overline{AD}$ at N Prove that : $\overline{FN}//\overline{BD}$

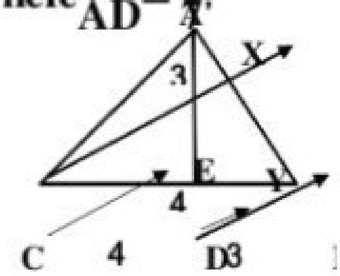


Proof

7 ABC is a triangle , D∈ $\overline{BC}$, where $\frac{BD}{DC}=\frac{3}{4}$ and E∈ $\overline{AD}$, where $\frac{AE}{AD}=\frac{3}{7}$,

$\overrightarrow{CE}$ is drawn to

intersect $\overline{AB}$ at X $\overline{DY}//\overline{CX}$ and intersects $\overline{AB}$ at Y Prove that: AX= BY



Proof In Δ ADY : ∵ $\overline{EX}//\overline{DY}$ ∴ $\frac{AE}{ED}=\frac{AX}{XY}=\frac{3}{4}$ (1)

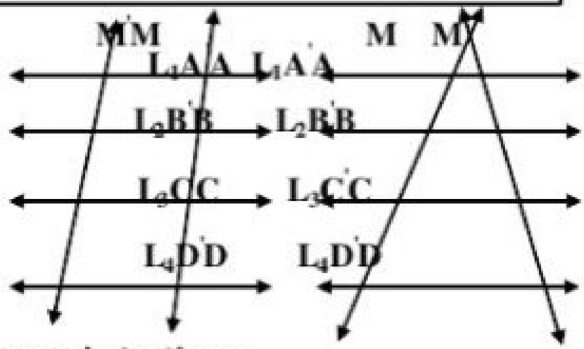
In Δ BCX: ∵ $\overline{DY}//\overline{CX}$ ∴ $\frac{BD}{DC}=\frac{BY}{XY}=\frac{3}{4}$ 2 From 1,2: ∴ $\frac{AX}{XY}=\frac{BY}{XY}$ ∴ AX= BY

H.W. Page 148 No. (6) and Page 150 Nos. (16) , (17) and (21)

Lesson 2 Talis' Theorem

Theorem [2]

Given several coplanar parallel lines and two transversals , then the lengths of the corresponding segments on the transversals are proportional

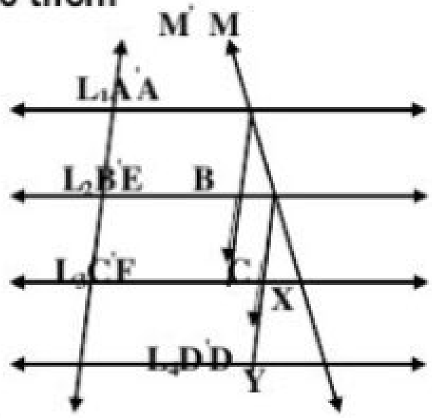


In the opposite two figures:

If $L_1//L_2//L_3//L_4$ and M , M' are two transversals

, then $\frac{AB}{A'B'}=\frac{BC}{B'C'}=\frac{CD}{C'D'}=\frac{AC}{A'C'}$

In the following the Proof of the theorem



Given $L_1//L_2//L_3//L_4$ and M , M' are two transversals to them

R.T.P. $AB : BC : CD = A'B' : B'C' : C'D'$

Const.draw $\overrightarrow{AF}//M'$ and intersects L_2 at E

L_3 at F , $\overrightarrow{BY}//M'$ and intersects L_3 at X , L_4 at Y

Proof $\because \overline{AA'}//\overline{EB'}$, $\overline{AE}//\overline{A'B'}$

$\therefore AEB'A'$ is a parallelogram , then $AE = A'B'$

Similarly: $EF = B'C'$, $BX = B'C'$, $XY = C'D'$ In ΔACF : $\because \overline{BE}//\overline{CF}$

$\therefore \frac{AB}{BC}=\frac{AE}{EF}$,Then: $\frac{AB}{BC}=\frac{A'B'}{B'C'}$, $\frac{AB}{A'B'}=\frac{BC}{B'C'}$ (exchange the means) (1)

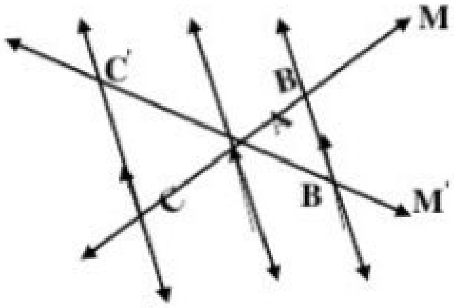
Similarly ΔBDY : $\therefore \frac{BC}{CD}=\frac{B'C'}{C'D'}$, $\frac{BC}{B'C'}=\frac{CD}{C'D'}$ (exchange the means) (2)

From 1 , 2 we get : $\frac{AB}{A'B'}=\frac{BC}{B'C'}=\frac{CD}{C'D'}$ $\therefore AB:BC:CD = A'B':B'C':C'D'$

In the previous figure , notice that :

$$\frac{AC}{CD}=\frac{A'C'}{C'D'} , \frac{AC}{CB}=\frac{A'C'}{C'B'} , \frac{BD}{DA}=\frac{B'D'}{D'A'}$$

TWO Special Cases



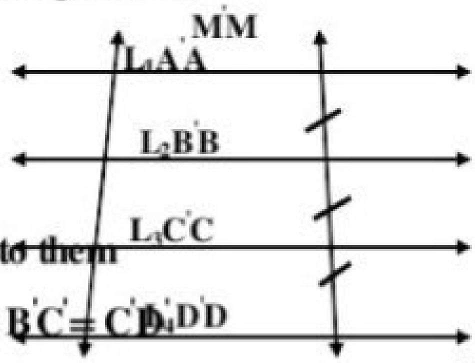
1 If the two lines M and M' intersect at the point A

and $\overleftrightarrow{BB'} \parallel \overleftrightarrow{CC'}$, then $\frac{AB}{AC} = \frac{AB'}{AC'}$ and conversely

If $\frac{AB}{AC} = \frac{AB'}{AC'}$, then $\overleftrightarrow{BB'} // \overleftrightarrow{CC'}$

2 Talis' Special theorem:

If the lengths of segments on the transversals are equal , then the lengths of the segments on any other transversal will be also equal.



In the opposite figure:

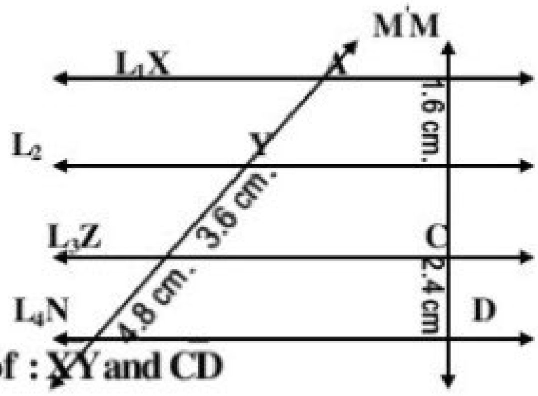
If $L_1 // L_2 // L_3 // L_4$,

M and M' are two transversals to them

and if AB = BC = CD , then A'B' = B'C' = C'D'

Exercise 2 On Talis' theorem

In the opposite figure:



$L_1 // L_2 // L_3 // L_4$

M , M' are two transversals ,

If AB = 1.6 cm. , BC = 2.4 cm. ,

YZ = 3.6 cm. , ZN = 4.8 cm.

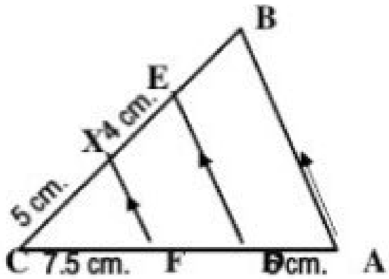
Calculate the length of each of : XY and CD

Proof

2 In the opposite figure:

If : $\overline{AB} // \overline{DE} // \overline{FX}$, AD = 6 cm. , EX = 4 cm. , FC = 7.5 cm.

CX = 5 cm. Find the length of each of : $\overline{DF}$, $\overline{BE}$

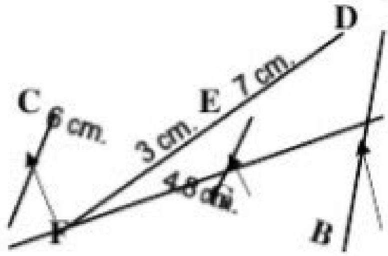


Proof

3 In the opposite figure:

$\overline{AD} // \overline{BE} // \overline{Fc}$, $\overline{AC} \cap \overline{DF} = \{G\}$, DE = 7 cm. EG = 3 cm. ,

GC = 6 cm. , BG = 4.8 cm. Find the length of each of : $\overline{GF}$, $\overline{AG}$

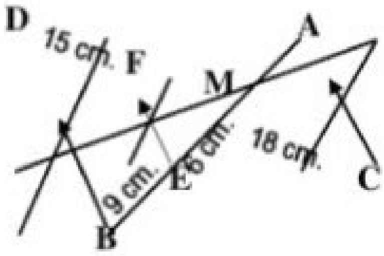


Proof

4 In the opposite fifure:

$\overline{AB} \cap \overline{CD} = \{ M \}$ E ∈ $\overline{MB}$, F ∈ $\overline{MD}$ and $\overline{AC} // \overline{FE} // \overline{DB}$

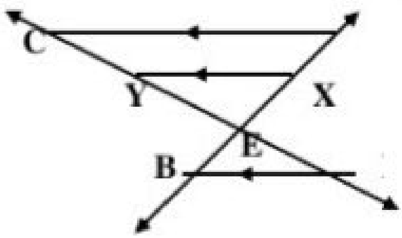
Find : 1 The length of $\overline{AF}$ 2 The length of $\overline{AM}$



Proof

5) $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{ E \}$, X ∈ $\overline{AB}$, Y ∈ $\overline{CD}$ and $\overline{XY} // \overline{BD} // \overline{AC}$

Prove that : AX × ED = CY × EB



Proof

Lesson 3 Angle bisectors and proportional parts

Theorem [3]

The Bisectors of the interior or exterior angle of a triangle at any vertex divides the opposite base of the triangle internally or externally into two parts, the ratio of their lengths is equal to the ratio of the lengths of the other two sides of the triangle.

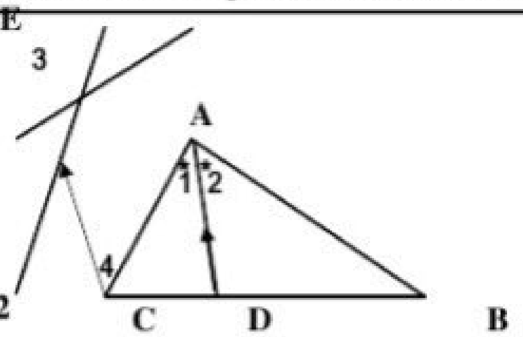


Figure 2

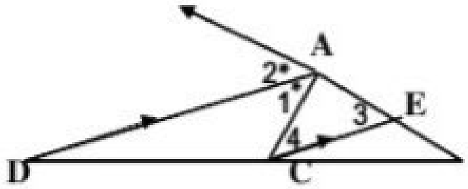


Figure 1

Given ABC is a triangle, $\overrightarrow{AD}$ bisects $\angle$ BAC internally in fig.1 and externally in fig.2

$$\text{R.T.P.}\ \frac{BD}{DC}=\frac{AB}{AC}$$

Const. Draw $\overrightarrow{CE} // \overrightarrow{AD}$ and intersect $\overrightarrow{BA}$ at E

Proof $\because \overrightarrow{AD}$ bisects $\angle$ BAC $\therefore \angle 1 \equiv \angle 2$

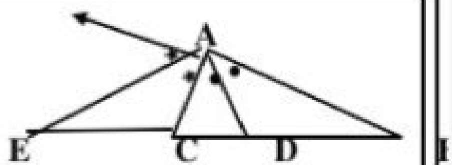
, $\because \overrightarrow{CE} // \overrightarrow{AD} \therefore \angle 1 \equiv \angle 4$ alternate angles, $\angle 3 \equiv \angle 2$ (corresponding angles)

, $\because \angle 1 \equiv \angle 2$ $\therefore \angle 3 \equiv \angle 4$ $\therefore \overline{AE} \equiv \overline{AC}$? 1

, $\because \overline{CE} // \overline{AD} \therefore \frac{BD}{DC}=\frac{AB}{AE}$? 2 from 1 and 2 : $\frac{BD}{DC}=\frac{AB}{AC}$ (Q.E.D.)

Important Remarks

1 In the opposite figure:

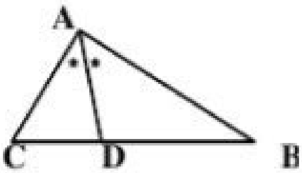


If $\overrightarrow{AD}$, $\overrightarrow{AE}$ are the bisectors of the angle A and the exterior angle of Δ ABC at A respectively, then: $\frac{BD}{DC}=\frac{AB}{AC}, \frac{BE}{EC}=\frac{AB}{AC} \therefore \frac{BD}{DC}=\frac{BE}{EC}$

$\because$ The base $\overline{BC}$ is divided internally at D , externally at E by the same ratio AB :AC and

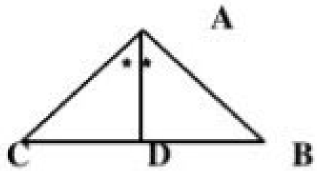
We notice that :the two bisectors $\overrightarrow{AD}$ and $\overrightarrow{AE}$ are perpendicular. i.e. : $m\angle DAE = 90^0$

2 If $\overrightarrow{AD}$ bisects $\angle BAC$ and intersects $\overline{BC}$ at D, then D taks one of the following:



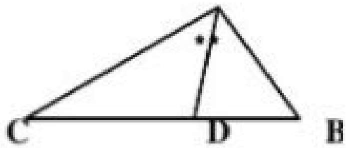
If AB $> AC$, *then* $BD > DC$

i.e. D is nearer to C than to B



If AB $=$ AC, then BD $=$ DC

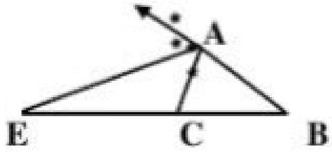
i.e. D is equidistant from each of B and C



If AB $< AC$, *then* $BD < DC$

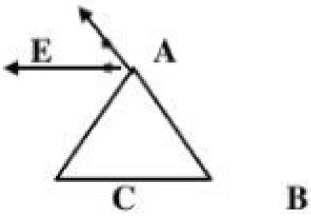
i.e. D is nearer to B than to C

3 If $\overrightarrow{AE}$ bisects the exterior angle of ΔABC at A, where $E \notin \overline{BC}$, then E takes one of the following:

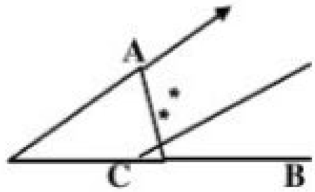


If AB $> AC$, *then* $BE > EC$

i.e. $E \in \overrightarrow{BC}$



If $AB = AC$, *then* $\overrightarrow{AE} // \overrightarrow{BC}$



If AB $< AC$, *then* $BE < EC$

i.e $E \in \overrightarrow{CB}$

Find the length of the interior and the exterior bisectors of an angle of a triangle

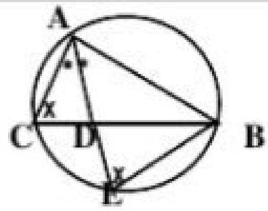
A well-known Problem:

If $\overrightarrow{AD}$ bisects $\angle A$ in ΔABC Internally and intersects $\overline{BC}$ at D then: $AD = \sqrt{AB \times AC - BD \times DC}$

Given ABC is a triangle, $\overrightarrow{AD}$ bisects $\angle BAC$ internally, $\overrightarrow{AD} \cap \overline{BC} = \{D\}$

R.T.P. $AD = \sqrt{AB \times AC - BD \times DC}$

Const. Draw a circle passes through the vertices of ΔABC and intersects $\overrightarrow{AD}$ at E, draw $\overline{BE}$



Proof $\because m\angle CAD = m\angle EAB$ given, $m\angle E = m(\angle C)$ (inscribed angles subtended by $\widehat{AB}$)

$\therefore \Delta ACD \sim \Delta AEB$, then: $\frac{AC}{AE} = \frac{AD}{AB}$ $\therefore AD \times AE = AB \times AC$ $\therefore AD \times AD + DE = AB \times AC$

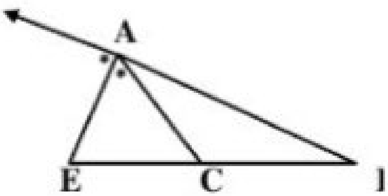
$\therefore (AD)^2 = AB \times AC - AD \times DE$ $\therefore (AD)^2 = AB \times AC - BD \times DC$ $\therefore AD = \sqrt{AB \times AC - BD \times DC}$

Notice that:

In the opposite figure:

If $\overrightarrow{AE}$ bisects $\angle BAC$ externally and intersects $\overrightarrow{BC}$ at E,

then: $AE = \sqrt{BE \times EC - AB \times AC}$

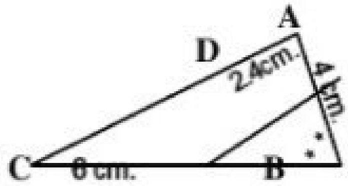


H.W. Page 172 No. (7) and Page 174 No. (20)

Exercise (3) On angle bisectors and proportional parts

(1) ABC is a triangle in which AB= 4 cm. , BC= 6 cm. , draw $\overrightarrow{BD}$ bisects $\angle$ ABC and inersects $\overline{AC}$ at D , if AD= 2.4 cm.

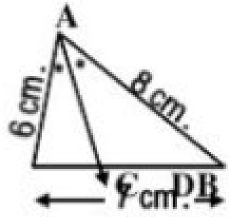
Find the length of : $\overline{AC}$



Proof

(2) ABC is a triangle in which AB= 8 cm. , AC= 6 cm. , BC= 7 cm.

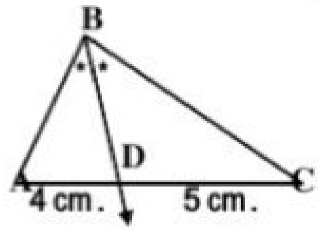
$\overrightarrow{AD}$ bisects $\angle$ BAC and intersects $\overline{BC}$ at D Find the length of: $\overline{DB}$, $\overline{DC}$



Proof

(3) ABC is a triangle , its Perimeter is 27 cm. , $\overrightarrow{BD}$ bisects $\angle$ B and intersects $\overline{AC}$ at D If AD= 4 cm. and CD= 5 cm. ,

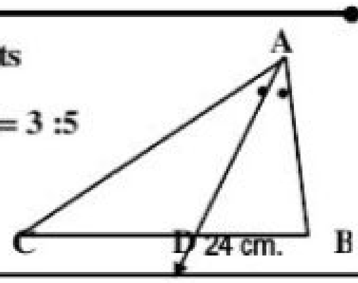
find the length of : $\overline{AB}$, $\overline{BC}$ and $\overline{BD}$



Proof

(4) ABC is right – angled triangle at B , draw $\overrightarrow{AD}$ bisects $\angle$ A and intersects $\overline{BC}$ at D If the length of $\overline{BD}$ equals 24 cm. ,BA :AC= 3 :5

Find the Perimeter of Δ ABC

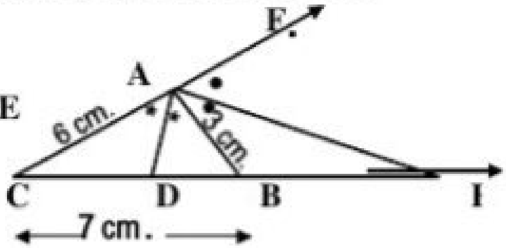


Proof

5 ABC is a triangle in which AB = 3 cm. , BC = 7 cm. , CA = 6 cm. , $\overrightarrow{AD}$ bisects $\angle$ A and intersects $\overline{BC}$ at D , $\overrightarrow{AE}$ bisects the exterior angle Of the triangle at A and intersects $\overrightarrow{CB}$ at E

(1) Prove that : $\overline{AB}$ is a median in the triangle ACE

(2) Find the ratio of the area of Δ ADE to the area of Δ ACE



Proof $\because$ $\overrightarrow{AE}$ bisects $\angle$ BAF $\therefore \frac{CE}{EB} = \frac{AC}{AB} = \frac{6}{3} = 2$

$\therefore$ CE = 2EB $\therefore$ CB = BE $\therefore$ B is midpoint of $\overline{CE}$ $\therefore$ $\overline{AB}$ is a median of Δ ACE (First req.)

$\because$ $\overrightarrow{AD}$ bisects $\angle$ BAC $\therefore \frac{BD}{DC} = \frac{BA}{AC} = \frac{3}{6} = \frac{1}{2} \therefore \frac{BD+DC}{DC} = \frac{3}{2} \therefore \frac{BC}{DC} = \frac{3}{2} \therefore \frac{7}{DC} = \frac{3}{2}$

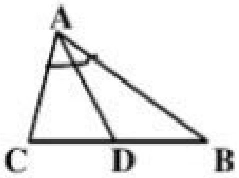
$\therefore$ DC = 14/3 BD = 7 - 14/3 = 7/3, BE = 7cm. , ED = 7/3 + 7 = 28/3, EC = 14cm.

$\therefore$ The area of (ΔADE)/The area of(ΔACE) = ED/CE = 28/3/14 = 2/3 (second req.)

Lesson 4 Follow: bisectors and proportional parts

Converse of theorem[3]

In the opposite two figures:



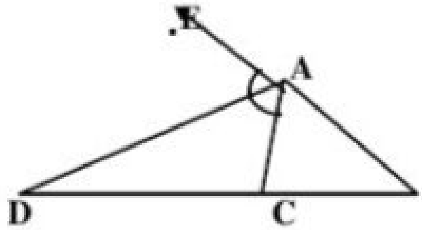


Figure 1 Figure 2

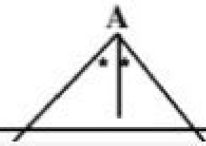
If D $\in$ $\overline{BC}$ figure 1 such that: If D $\in$ $\overrightarrow{BC}$, D $\notin$ $\overline{BC}$ figure 2 such that : $\frac{BD}{DC} = \frac{BA}{AC}$

$\frac{BD}{DC} = \frac{BA}{AC}$, then $\overrightarrow{AD}$ bisects $\angle$ BAC, then $\overrightarrow{AD}$ bisects the exterior angle of ΔABC at A

Fact

The bisectors of angles of a triangle are concurrent.

In the opposite figure:



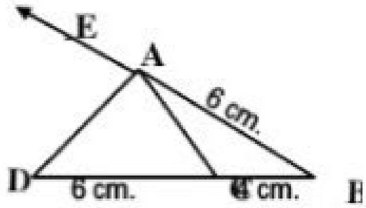
$\overrightarrow{AM}$, $\overrightarrow{BM}$ and $\overrightarrow{CM}$ are concurrent at the point M

Exercise 4 On converse of theorem (3)

1. ABC is a triangle , in which AB = 6 cm. , BC = 4 cm. ,

CA = 3.6 cm. , D ∈ $\overrightarrow{BC}$ such that CD = 6 cm. ,

Prove that : $\overrightarrow{AD}$ bisects the exterior angle of △ ABC at A.



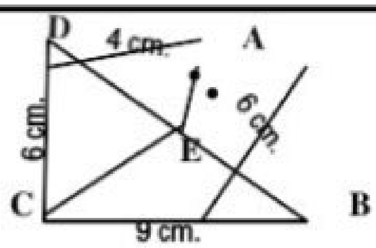
Proof

2. ABCD is a quadrilateral in which AB = 6 cm. , BC = 9 cm. ,

CD = 6 cm. , AD = 4 cm. , $\overrightarrow{AE}$ bisects ∠ A , intersects $\overline{BD}$ at E.

(1) Find the value of the ratio BE/ED

(2) Prove that : $\overrightarrow{CE}$ bisects ∠ BCD

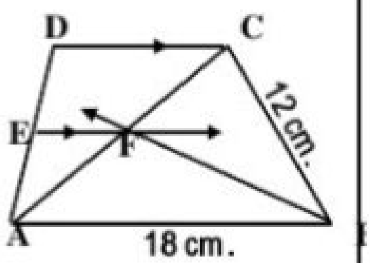


Proof

3. ABCD is a quadrilateral in which AB = 18 cm. , BC = 12 cm.

E ∈ $\overline{AD}$, where 2 AE = 3 ED , draw $\overrightarrow{EF}$ // $\overline{DC}$ and intersects $\overline{AC}$ at F.

Prove that : $\overrightarrow{BF}$ bisects ∠ ABC



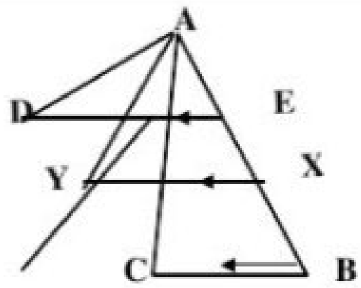
Proof

4. In the opposite figure:

$\overline{ED}$ // $\overline{XY}$ // $\overline{BC}$,

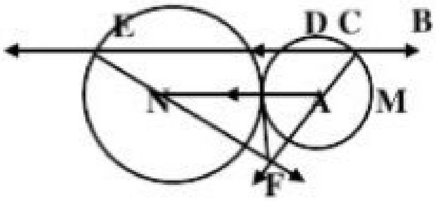
and AD × BX = AC × EX ,

Prove that : $\overrightarrow{AY}$ bisects ∠ CAD



Proof

5 Two circles M and N touch externally at A, a straight line is drawn parallel to $\overline{MN}$ and intersects the circle M at B, C and the circle N at D, E respectively. If $\overrightarrow{BM} \cap \overrightarrow{EN} = \{F\}$,



Prove that : $\overrightarrow{AF}$ bisects $\angle$ MFN

Proof In Δ BFE : $\because \overline{MN} // \overline{BE}$ $\therefore \frac{BM}{MF} = \frac{EN}{FN}$, $\because$ BM = MA , EN = AN $\therefore \frac{MA}{MF} = \frac{AN}{FN}$ $\therefore \frac{MA}{AN} = \frac{MF}{FN}$

$\therefore \overrightarrow{FA}$ bisects $\angle$ MFN H.W. P. 182 No. (3) & P. 183 Nos. (10), (12) P. 185 Nos. (22), (23)

Lesson 5 Applications of proportionality in the circle

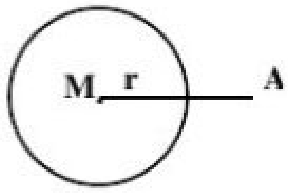
Power of a point with respect to a circle

Definition:

Power of the point A with respect to the circle M in which, the length of its radius r is the real number $P_M(A)$ where : $P_M A = (AM)^2 - r^2$

Note 1

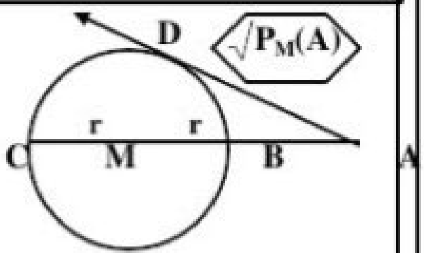
we can expect the position of point A with respect to the circle M if :



- $P_M A > 0$ *then A lies outside the circle.*
- $P_M A = 0$ then A lies on the circle.
- $P_M A < 0$ *then A lies insid the circle.*

Note 2

If point A lies outside the circle M



Then : $P_M A = AM^2 - r^2$

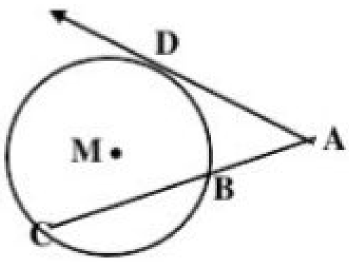
$= AM - r AM + r$

$= AB \times AC = AD^2$

$\therefore$ length of the tangent drawn from A to circle M $= \sqrt{P_M(A)}$

Notice that:

In the opposite figure :



If point A lies outside the circle, $\overline{AC}$ intersects the circle at B, C

Then $P_M(A) = AB \times AC$

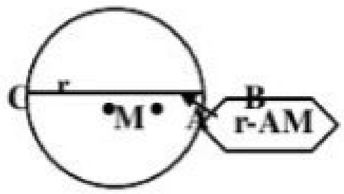
And this can be concluded from the previous not, where :

$P_M A = (AD)^2$ where $\overrightarrow{AD}$ is tangent to the circle M at D

, ∵ $(AD)^2 = AB \times AC$ ∴ $P_M A = AB \times AC$

Note (3)

If point A lies inside the circle M , then :



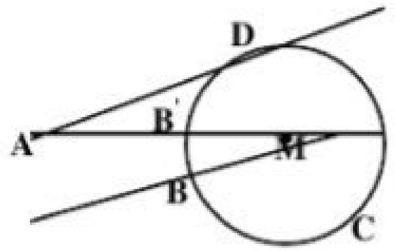
$P_M A = AM^2 - r^2$

$= AM-rAM+r$

$= -r-AMAM+r = -AB \times AC$

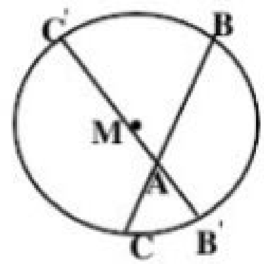
Summary of the previous as follow

If A lies outside circle M , then:



$P_M A = AB \times AC = AB' \times AC' = (AD)^2$

If A lies inside circle M , then:



$P_M = -AB \times AC = -AB' \times AC'$

Importnat Note :

The set of points which have the same power with respect to two distinct circles is called the principle axis of the two circles.

If : $P_M A = P_N A$ then A lies on the principle axis of the two circles M and N.

For example

If $P_M A = P_N A$, $P_M B = P_N B$

Then $\overleftrightarrow{AB}$ is the principle axis of the two circles M and N

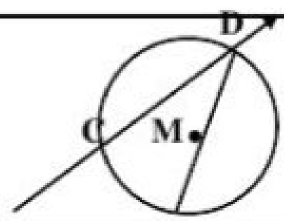
Secant , tangent and measures of angles

Well Known problem:

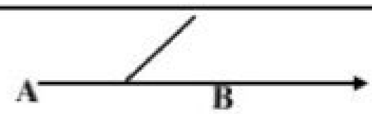
The measure of an angle formed by a secant and a tangent or two tangents drawn from a point outside a circle is equal to half the positive difference of the measures of the intercepted arcs.

❖ First case:Intersection of a sectant and a tangent to a circle

Given $\overleftrightarrow{AB}$ is a tangent to the circle at B , $\overrightarrow{AD} \cap$ the circle M= {C, D}



R.T.P. $m\angle A=\frac{1}{2}[m\widehat{BD}-m(\widehat{BC})]$



Const. Draw $\overline{BC}$, $\overline{BD}$

Proof: ∵ $\angle$ BCD is an exterior angle of Δ ABC

∴ $m\angle BCD = m\angle A + m\angle ABC$ ∴ $m\angle A = m\angle BCD - m\angle ABC$

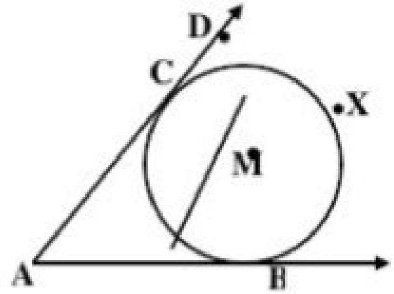
, ∵ $\angle$ BCD is an inscribed angle ∴ $m\angle BCD=\frac{1}{2}m(\widehat{BD})$

, ∵ $\angle$ ABC is a tangency angle ∴ $m\angle ABC=\frac{1}{2}m\widehat{BC}$

∴ $m\angle A=\frac{1}{2}m\widehat{BD}-\frac{1}{2}m\widehat{BC}=\frac{1}{2}[m\widehat{BD}-m\widehat{BC}]$ (Q.E.D.)

❖ Second case :Intersection of two tangents to a circle

Given $\overrightarrow{AB}$, $\overrightarrow{AC}$ two tangents to the circle M at B and C



R.T.P. $m\angle A=\frac{1}{2}[m\widehat{BXC}-m(\widehat{BC})]$

Const. Draw $\overline{BC}$

Proof: ∵ $\angle$ BCD is an exterior angle of Δ ABC

∴ $m\angle BCD = m\angle A + m\angle B$ ∴ $m\angle A = m\angle BCD - m(\angle B)$

, ∵ $\angle$ BCD is a tangency angle ∴ $m\angle BCD=\frac{1}{2}m(\widehat{BXC})$

∵ $\angle$ B is a tangency angle ∴ $m\angle B=\frac{1}{2}m(\widehat{BC})$

∴ $m\angle A=\frac{1}{2}m\widehat{BXC}-\frac{1}{2}m\widehat{BC}=\frac{1}{2}[m\widehat{BXC}-m\widehat{BC}]$ (Q.E.D.)

H.W. Page 195 No. 3 , Page 196 No. 10and Page 197 No. 12

Exercise 5 On applications of proportionality in the circle

◇ Find the power of the given point with respect to the circle M which its radius length is r :

(1) The point A where AM= 12 cm. and r= 9 cm.

(2) The point B where BM= 8 cm. and r= 15 cm.

(3) The point C where CM= 7 cm. and r= 7 cm.

(4) The point D where $DM=\sqrt{17}$ cm. and r= 4 cm.

Proof

Determint the position of each of the following points with respect to the circle M, of radius length 10 cm., then calculate the distance between each point from the centre of the circle: 1 $P_M A = -36$ 2 $P_M B = 96$ 3 $P_M C = zero$

Proof(1) $\because P_M A = -36 < 0$ $\therefore$ *A lies inside the circle.*

, $\because P_M A = (AM)^2 - r^2$ $\therefore -36 = AM^2 - 100 \therefore AM^2 = 64 \therefore AM = 8$ cm.

2 $\because P_M B = 96 > 0$ $\therefore$ *B lies outside the circle.*

, $P_M B = BM^2 - r^2$ $\therefore 96 = BM^2 - 100 \therefore BM^2 = 196 \therefore BM = 13$ cm.

3 $P_M C = 0$ $\therefore$ C lies on the circle $\therefore MC = r = 10$ cm.

4 If the distance between a point and the centre of a circle equals 25 cm., and the power of this point with respect to the circle equals 400

Find the radius length of this circle.

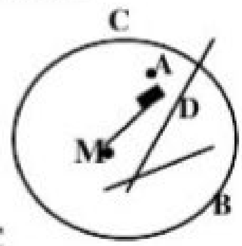
Proof

4 If a point A is outside the circle M, $\overline{AD}$ is a tangent to the circle at D where AD = 8 cm.

Find the power of point A with respect to circle M.

Proof $\because \overline{AD}$ is a tangent to the circle at D $\therefore AD = \sqrt{P_M(A)} \therefore P_M A = (AD)^2 = (8)^2 = 64$

5 The radius length of circle M equals 31 cm. The point A lies at 23 cm. distance from its centre. Draw the chord $\overline{BC}$ where $A \in \overline{BC}$, AB= 3 AC. Calaculate:



1 The of the chord $\overline{BC}$

2 The distance between the chord $\overline{BC}$ and the centre of the circle.

Proof $\because P_M A = AM^2 - r^2 = 23^2 - 31^2 = -432$, $\because P_M A = -AB \times AC$

$\therefore -432 = -AB \times AC \therefore 432 = AB \times AC$, $\because AB = 3\,AC \therefore 432 = 3\,AC \times AC$

$AC^2 = 144 \therefore AC = 12$ cm. $\therefore AB = 3 \times 12 = 36$ cm. $\therefore BC = 36 + 12 = 48$ cm.

Let the distance between the chord $\overline{BC}$ and the centre of the circle is $\overline{MD}$

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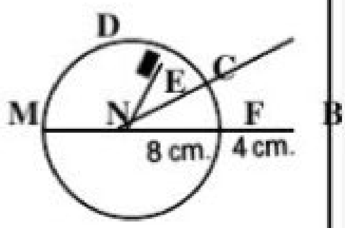
, where : $\overline{MD} \perp \overline{BC}$ ∵ D is the midpoint of $\overline{BC}$,

$\because P_M D = DM^2 - r^2 = -BD \times DC$ $\therefore MD^2 - 31^2 = -24 \times 24$

$\therefore (MD)^2 = 961 - 576 = 385$ $\therefore MD = \sqrt{385} \simeq 19.6$ cm.

6 The radius length of circle N equals 8 cm. The point B lies at 12 cm. distance from its centre , draw a straight line passes through the point B and intersects the circle at C and D where CB = CD

Calculate the length of the chord $\overline{CD}$ and its distance from the point N.



Proof