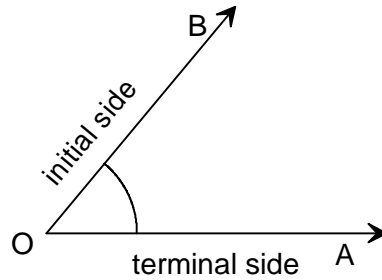
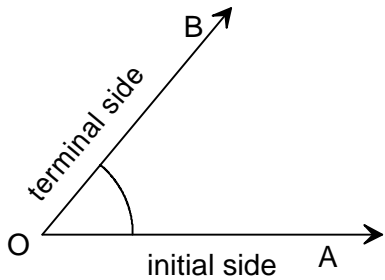


Trigonometry

Unit (3): Measures of angles

Directed angle: is an ordered pair of two rays called the sides with a common Starting point called "vertex"

Remark: The first projection of the ordered pair is the "initial side" of the angle.
The second projection of the ordered pair is the "terminal side"



Measure of directed angle:

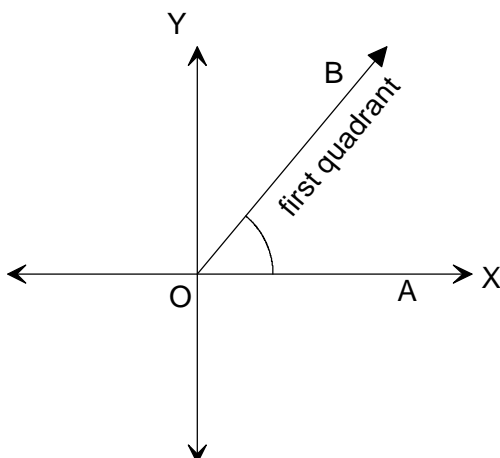
1. *Positive measure:* the directed angle $\angle AOB$ is the ordered pair $(\overrightarrow{OA}, \overrightarrow{OB})$ where \overrightarrow{OA} is the initial side and \overrightarrow{OB} is the terminal side and the direction of rotation is anti-clockwise
2. *Negative measure:* the directed angle $\angle BOA$ is the ordered pair $(\overrightarrow{OB}, \overrightarrow{OA})$ where \overrightarrow{OB} is the initial side and \overrightarrow{OA} is the terminal side and the direction of rotation is clockwise

Angles in the standard position:

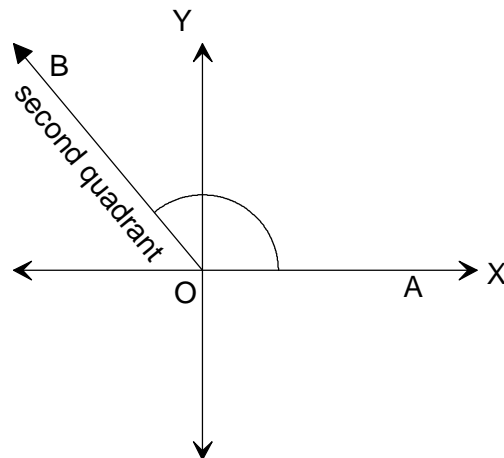
An angle is said to be in "the standard position", If:

1. Its vertex is the origin of a rectangular coordinate system.
2. its initial side coincide with \overrightarrow{OX} "the positive direction of x-axis"

If $\angle AOB$ in the standard position and $m(\angle AOB) = \theta$ so:

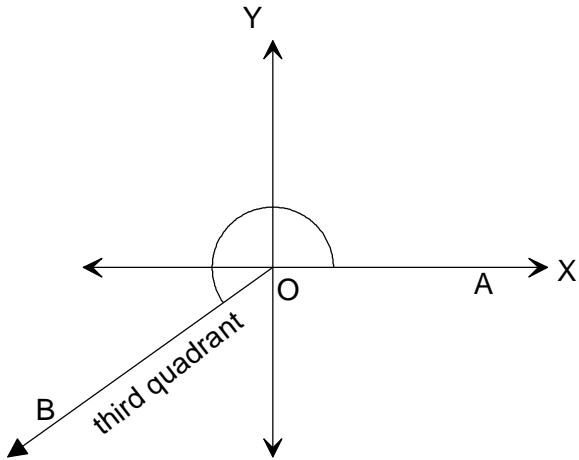


$$\theta \in] 0^\circ, 90^\circ[$$

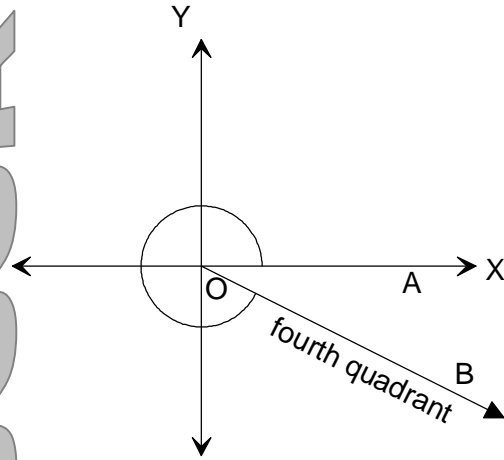


$$\theta \in] 90^\circ, 180^\circ[$$

(2)



$$\theta \in] 180^\circ, 270^\circ[$$



$$\theta \in] 270^\circ, 360^\circ[$$

Notes:

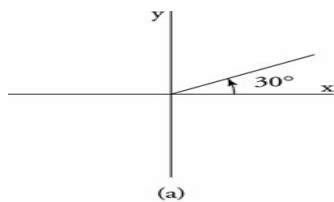
- The measure of the directed angle $\angle AOB \neq \angle BOA$
- Each directed angle in the standard position, has two measures (+ve and -ve)
- The numerical sum of the two measures equals 360°
- If x° represents the +ve measure for a directed angle, then the -ve measure of this angle will be $x^\circ - 360^\circ$. and if $(-x^\circ)$ is the -ve measure of this angle, then the +ve measure will be $-x^\circ + 360^\circ$

Ex (1): Determine the quadrant in which each of the following angles lies. Then find the other measure of the angle: 70° , 160° , -60° , -310°

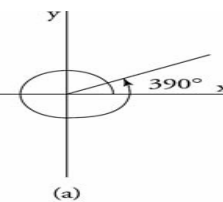
Solution

- 1) 70° lies in the 1st quad. And the other measure is -290°
- 2) 160° lies in the 2nd quad. And the other measure is -200°
- 3) -60° lies in the 4th quad. And the other measure is 300°
- 4) -310° lies in the 2nd quad. And the other measure is 50°

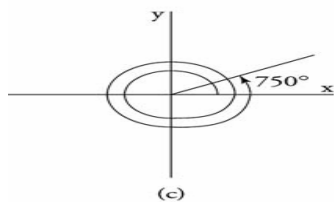
Equivalent angles (co-terminal angles): angles having the same terminal ray



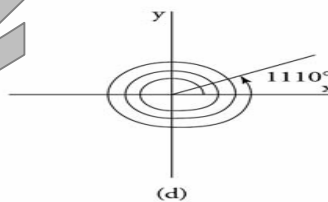
(a)



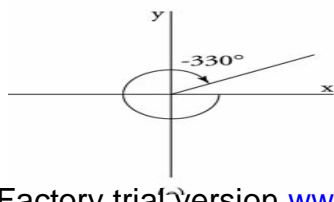
(a)



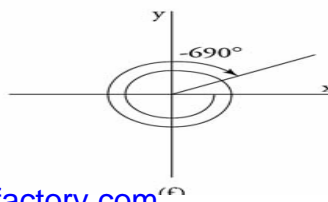
(c)



(d)



(e)



(f)

(3)

Note: if θ is the measure of a directed angle in its standard position, then the angles of measures $(\theta \pm n \times 360^\circ)$ where $n \in \mathbb{C}$, will have the same terminal side and called equivalent (co-terminal) angles.

Ex (2): Determine the quadrant in which each of the following angles lies
 530° , 1670° , -740° , -3530°

Solution

- 1) 530° Equivalent to 170° so it lies in the 2nd quad.
 - 2) 1670° Equivalent to 230° so it lies in the 3rd quad.
 - 3) -740° Equivalent to 340° so it lies in the 4th quad.
 - 4) -3530° Equivalent to 70° so it lies in the 1st quad.
-

Exercise (1)

- 1) Complete the following statements correctly :
 - a) The smallest +ve measure of the angle whose measure is (-490°) is and it lies in the quadrant
 - b) The angle of measure (405°) can be equivalent to the angle
 - c) The angle of measure (-135°) lies in the quad.
 - d) The quadrant in which the angle $89^\circ 59'$ lies is the quad.
 - e) The - ve measure of the angle $275^\circ 19'$ is
 - f) The + ve measure of the angle $-130^\circ 54' 12''$ is
- 2) Determine the - ve measure of the following angles:
 - a) 72°
 - b) 200.5°
 - c) 108°
- 3) Determine the + ve measure of the following angles:
 - a) -216°
 - b) $-54^\circ 54'$
 - c) $-310^\circ 45' 30''$
- 4) Find two angles one of them is of + ve measure and the other is of - ve measure and they are equivalent to the given angles:
 - a) 135°
 - b) $179^\circ 26'$
 - c) -230°
 - d) $-590^\circ 18'$

5) In questions (2 \rightarrow 4) determine the quadrant in which each angle lies