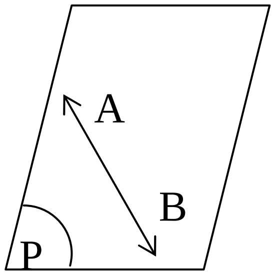
Lines and Planes

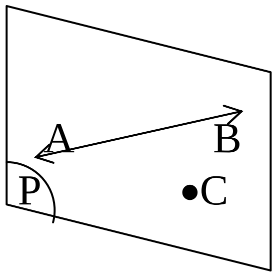


The Plane : is the surface where the line joining any two points
(The wall surface , the faces of a cube , ……)

Notice:
1) The plane has no limit and it has no thickness
2) It divides the space in two parts

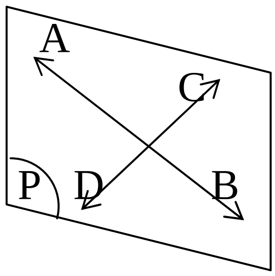
Determining a plane in the space :

The plane is determining by one of the following



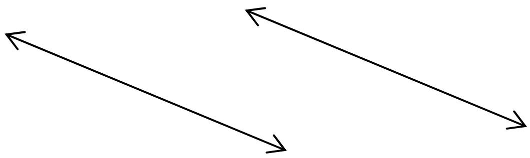
1) Three distinct non – collinear points

2) line and a point does not belong to it



3) Two intersecting Lines

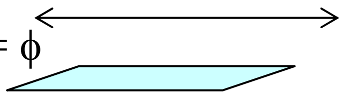
4) Any two parallel lines

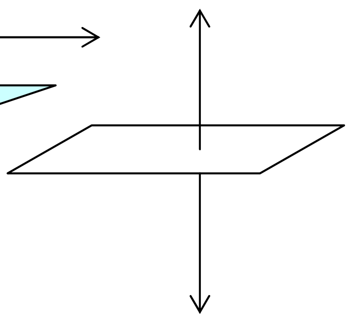


Relative positions of a line w.r.t a plane

Let L be a line and P be a plane

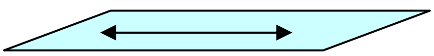
1) L // plane P $L \cap P = \phi$





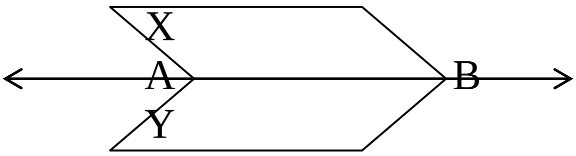
2) L intersects the plane in one point

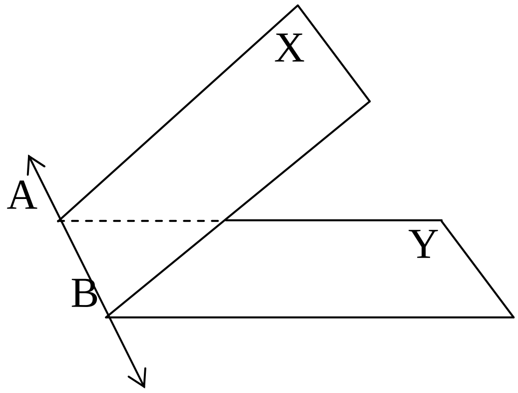
3) L lies completely in the plane $L \subset P$

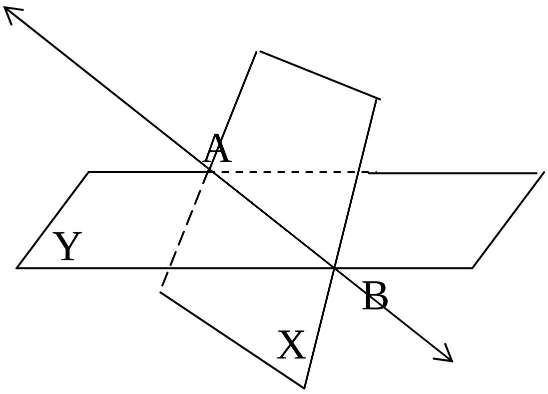


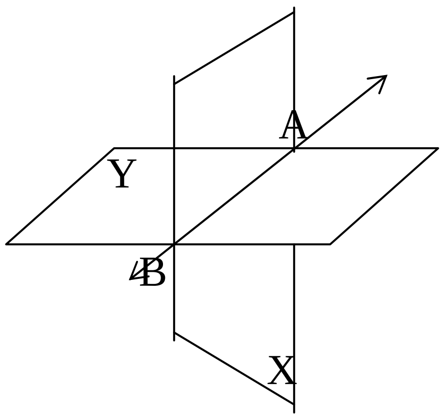
Relative position of two planes

1) the two planes intersect

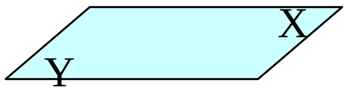


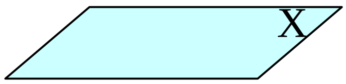






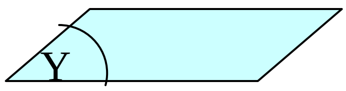
2) The two plane coincide :





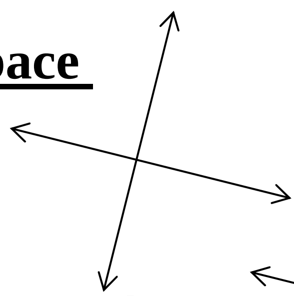
3) The two plane are parallel

$X \cap Y = \phi$

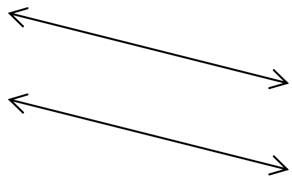


The relation between two lines in space

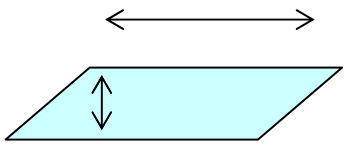
1) The two lines intersect



2) The two lines are parallel (the two lines lie in one plane)

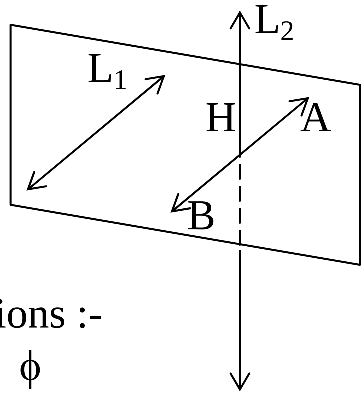


3) The two are SKEW (non-coplanar)



The angle between two Skew Lines

$L_1 \subset X$, $L_2 \subset X$, $L_2 \cap X = \{H\}$. to determine the angle between them
Draw $\overleftrightarrow{AB}$ passing through H such that $\overleftrightarrow{AB} // L_2$
The angle included
between L_2 and $\overleftrightarrow{AB}$ is
that included between L_1 and L_2



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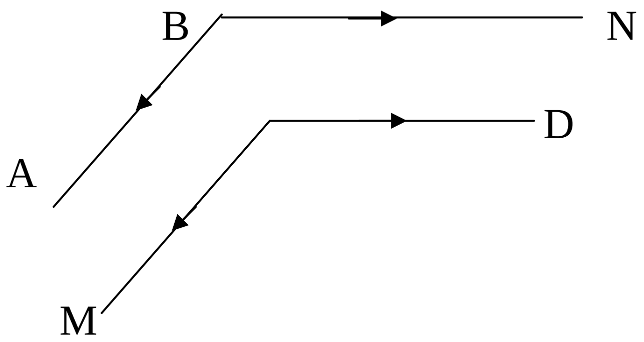
N.B : to prove that $L_1 // L_2$, prove each of the conditions :-
i) L_1 and L_2 are lying in one plane ii) $L_1 \cap L_2 = \phi$

Through two given Skew lines , can pass two parallel planes
$\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are skew lines
Draw $\overleftrightarrow{BN} // \overleftrightarrow{CD}$ & $\overleftrightarrow{CM} // \overleftrightarrow{BA}$
Plane ABN // plane MCD



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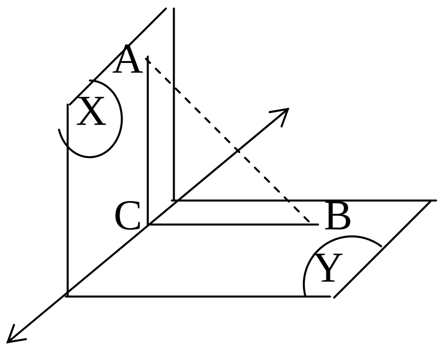
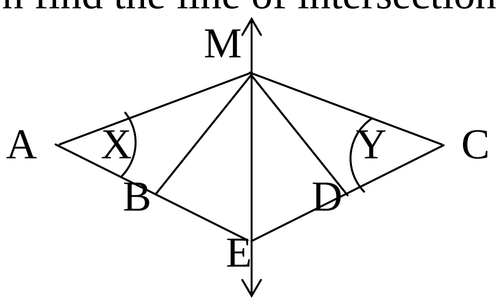
Examples

1) In the figure . Complete
a) plane ABC $\cap$ X =
b) plane ABC $\cap$ Y =
c) plane ABC $\cap$ L =
d) plane ABC $\cap$ X $\cap$ Y =

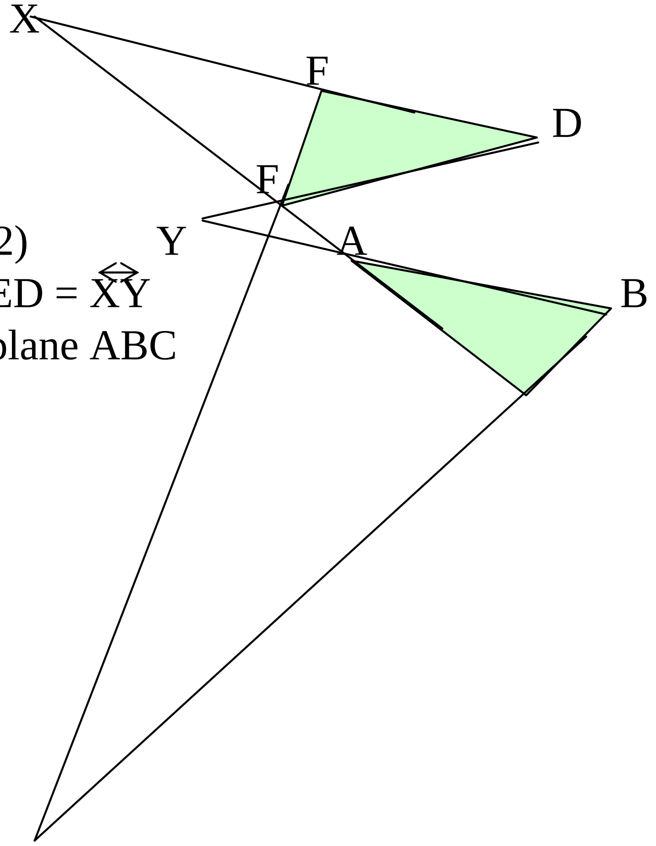


2) If A, B , C and D are four coplanar points , such that $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{E\}$
M is a point not belonging to their plane then find the line of intersection of the two planes MAB and MDC



E $\in$ plane X , E $\in$ plane Y
E$\in$ X $\cap$ Y , similarly M $\in$ X $\cap$ Y
$\therefore X \cap Y = \overleftrightarrow{ME}$

3) A,B and C are three non-collinear points lying in one plane
D,E and F are three non-collinear points lying in another plane
If $\overleftrightarrow{AC} \cap \overleftrightarrow{DF} = \{x\}$, $\overleftrightarrow{AB} \cap \overleftrightarrow{DE} = \{y\}$ and $\overleftrightarrow{BC} \cap \overleftrightarrow{EF} = \{z\}$ .Prove that $z \in \overleftrightarrow{XY}$



$X \in \overleftrightarrow{FD}$ $X \in$ plane FED
$X \in \overleftrightarrow{AC}$ $X \in$ plane ABC
$\therefore$ $X \in$ plane ABC $\cap$ EFD …..(1)
Similarly $Y \in$ plane EFD $\cap$ ABC …(2)
From (1) & (2) plane ABC $\cap$ plane FED = $\overleftrightarrow{XY}$
$Z \in \overleftrightarrow{EF}$ $Z \in$ plane EFD & $Z \in$ plane ABC
$\therefore Z \in \overleftrightarrow{XY}$

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Geometrical fact :
If each of two lines is parallel to the third then they are themselves parallel .

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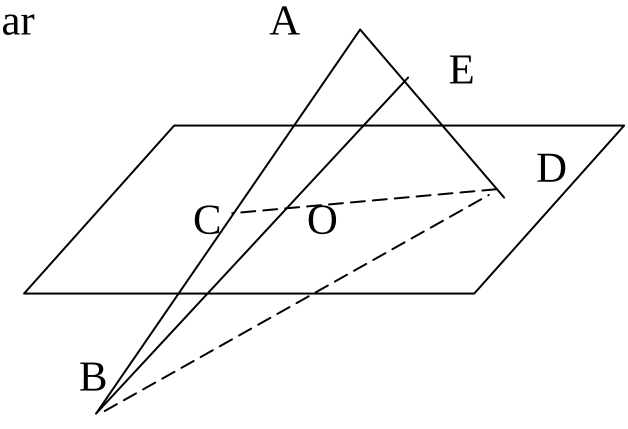
EX: $\overleftrightarrow{AB} \cap$ plane X = {C} . $\overline{AD}$ is drawn such that $\overline{AD} \cap$ plane X = {D}
E $\in \overline{AD}$ and $\overline{EB}$ intersect the plane X in O .
prove that C , O and D are collinear



A,B and D non collinear
then A , B and D form a plane Y
C$\in$ AB , AB $\subset$ plane Y
C$\in$plane Y , D $\in$ X , D$\in$ Y
Plane X $\cap$ plane Y = $\overleftrightarrow{CD}$
O$\in\overleftrightarrow{EB}$, $\overleftrightarrow{EB}$ $\subset$ plane Y
O$\in$ plane Y , O $\in$ plane X
O lies on the line of intersection of plane X and plane Y
O $\in \overleftrightarrow{CD}$
O , C and D collinear

Exercise

Which of the following statements are true and which are false?

1) There is one and only one plane through three points in space

2) There is an infinite number of planes passing through two distinct points

3) There is one and only one plane passing through a point

4) There is an infinite number of planes passing through two intersecting lines

5) There is an infinite number of planes passing through two skew lines

6) There is one and only one plane passing through two parallel lines

7) There is one and only one plane passing through three non-collinear points

8) If a line $L \subset$ plane X , a point $A \in L$. So $A \in X$

9) Any two lines form a plane

10) Any two skew lines form a plane

11) Any two planes can intersected in 3 non – collinear points

12) Any two lines each of which is // to the third are //

13) Any two lines of which is $\perp$ to the third are //

14) If L_1 , L_2 and L_3 are three lines in space , $L_1 // L_2$, $L_1 \perp L_3$
Then L_1 , L_2 and L_3 are coplanar

15) All vertical lines in space are //

16) All horizontal planes are //

17) All vertical planes are //

18) If two lines do not intersected so they are //

19) If two planes have a line and a point outside it in common . then these two planes are intersecting

20) The planes containing two skew lines can be //

21) The planes containing two skew lines must be intersected

22) The sides of any quadrilateral are coplanar

23) If $\overleftrightarrow{AB} \cap$ plane X $\neq \phi$ so A and B are in opposite sides of the plane X

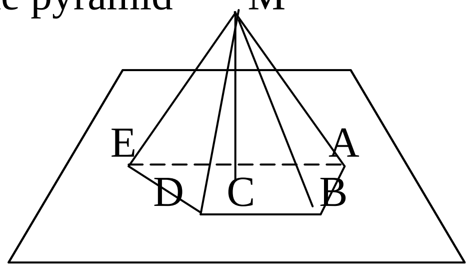
24) if A and B are in the same side of plane X , then AB $\cap$ plane X = ϕ

The Pyramid

In The figure

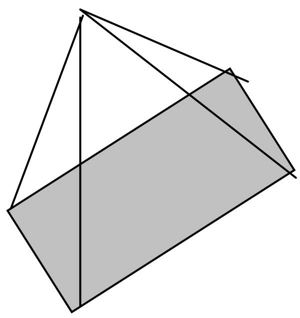
ABCDE is a polygon $\subset$ plane X and M $\not\in$ plane X

The union of all segments $\overline{MA}$, $\overline{MB}$, …. Is called the pyramid



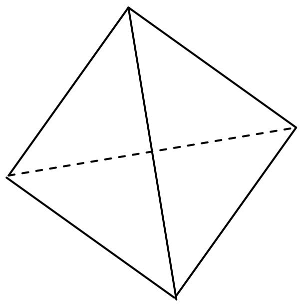
The type of pyramid according to the type of its base

The right pyramid. Whose its base is a regular polygon
And its lateral edges are equal



The regular pyramid (triangular)

All faces are equilateral triangles



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Parallelism of a lines and a plane

Theorem (1)

If a line is parallel to a plane , then it is parallel to a line of intersection of this plane with the plane containing the given line .

Given : $\overleftrightarrow{AB}$ // plane Y and $\overleftrightarrow{AB} \subset$ X , plane X $\cap$ plane Y = $\overleftrightarrow{CD}$

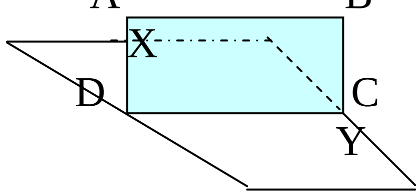
R.T.P : $\overleftrightarrow{CD}$ // $\overleftrightarrow{AB}$

Proof: $\overleftrightarrow{AB}$ // Y so $\overleftrightarrow{AB} \cap$ plane Y = ϕ , $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \phi$ … (1)

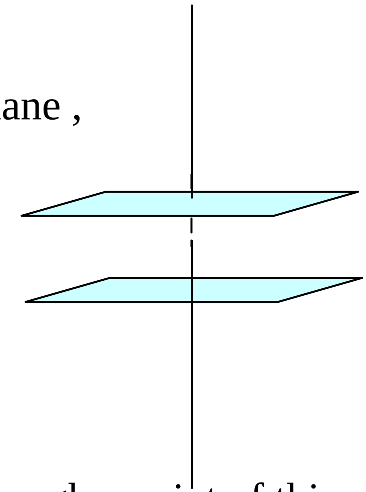
$\overleftrightarrow{AB} \subset$ plane X , $\overleftrightarrow{DC} \subset$ plane Y

Then $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are coplanar….(2)

Then $\overleftrightarrow{AB}$// $\overleftrightarrow{CD}$



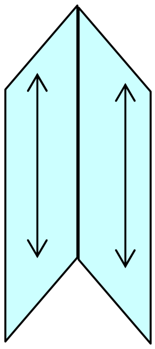
Fact
If a line outside a plane is parallel to a line in the plane ,
then it is parallel to the plane



Corollaries
1) If a line intersects one of two parallel planes
then it intersects the other

2) If a line is // to a plane , then any line passing through a point of this plane
and // to the given line lies in the plane

3) If a plane intersects two parallel plane , then the lines of intersection are //



4) If two intersecting planes pass through two // lines
Then their line of intersection is // to those two lines

5) If a line is // to each of intersecting planes so it // to
It // to their line of intersection