mathematics Applications

(1) A building worker dropped a piece of concrete from a high scaffold :

(1) What is the velocity of the concrete after half second ?

(2) What is the distance covered by the concrete within this time ?

((4.9 m./sec., 1.225m.))

(2) A stone is projected in a well with velocity 4 m./sec.It reached the reached the bottom of the well after 2 seconds. Find the depth of the well and the stone velocity when it collides with bottom of the well.

((27.3 m. , 23.6 m/sec.))

(3)A body is projected vertically upwards with velocity 49 m./sec . find the time needed to reach maximum height and the distance which it covered .

((5 seconds , 122.5 metres.))

(4) A particle is projected vertically upwards with velocity 14 m./sec. from a point at a height 350 metres from the ground surface . Find the time taken for the body reaches ground surface . **((10 seconds))**

(5) From the top of a tower , a particle is projected vertically upwards with velocity 24.5 m./sec. It reached the ground surface after 8 seconds. Find:

(1) The height of the tower .

(2) The maximum height to which the particle reach above ground surface.

(3) The total distance covered by the particle within this time interval .

((117.6m. , 148.225m. , 178.85 m.))

(6) A body is projected vertically upwards with velocity 14m./ sec. from a point at height 350 metres, from the ground surface. Find :

(1) Time taken by the body to reach the ground surface.

(2) The total distance covered by the body till it reaches ground surface.

((10 second , 370 metres))

(7) A body fell down from a height 22.5 metres on a sandy ground to imbedded on it a distance 25 cm. Calculate each of :

(1) The velocity of the body at the ground surface.

(2) The acceleration that the body moved with inside the ground .

((21m./sec. downwards , - 882 m./sec^2.))

(8) Two balls , the mass of the first = 5.2 kg. and the mass of the second is 0.25 kg. the two balls are put such that the distance between their centers is 50 cm. Calculate the gravitational force between them given that the universal gravitational constant = 6.67×10^{-11} newton.m^2./kg^2.

((3.4684×10^{-10} newton))

(9) Two identical balls the mass of each of them = 6.8 kg. and the distance between their centers is 21.8 cm. What is the gravitational force between them ? **((6.49×10^{-8} newton))**

(10) A and B are two events in a sample space of a random experiment , if P (A\`) = $\frac{3}{5}$, P (A U B) = 0.45 , find P (B) in the following cases :

(1) A and B are two mutually exclusive events .

(2) $A \subset B$ (3) P (A-B) = 0.2 **((0.2 , 0.75 , 0.55))**

(11) If A and B are two events of the sample space (S) of a random experiment , P (A) = $\frac{1}{2}$, P (B) = x and P ($A \cup B$\`) = $\frac{1}{3}$

(1) Find the value of x in each of the following cases :

(i) A and B are mutually exclusive events . (ii) $A \subset B$

(2) If $x = \frac{1}{4}$, find the value of : P ($A \cap B$) **(($\frac{1}{6}$, $\frac{2}{3}$, $\frac{1}{12}$))**

(12) If A and B are two events of the sample space (S) of a random experiment. P (B) = $\frac{4}{5}$ P (A) , P (A – B) = 0.24 , P ($B \cap A$\`) = 0.15

Find : P (A) , P (B) , P ($A \cup B$) , P (A\` $\cup$ B\`)

((0.45 , 0.36 , 0.6 , 0.79))

(13) (S) is the sample space of a random experiment where S = { A , B , C }

, if $\frac{P(A')}{P(A)} = \frac{7}{3}$, 2 P (B) = 3 P (B`) Find : $\frac{P(C')}{P(C)}$ **((9))**

(14) If (S) = { A , B , C , D } is the sample space of a random experiment ,

Find : P (A) , P (B) given that : P (A) = 3 P (B) , P (C) = P (D) = $\frac{7}{18}$

(($\frac{1}{6}$, $\frac{1}{18}$))

(15) If (S) = { A, B, C } is the sample space of a random experiment ,

and 20 P (A) = 15 P (B) = 12 P (C) .

Find : P (A) , P (B) , P (C) **(($\frac{1}{4}$, $\frac{1}{3}$, $\frac{5}{12}$))**

(16) If A and B are two events of the space(S)of a random experiment

, and p (A) = $\frac{1}{3}$ P (A) , P(B) = $\frac{1}{2}$, P (A` ∪ B`) = $\frac{5}{8}$, Find :

(1) The probability of occurrence of one of the two events at least .

(2) The probability of occurrence of one of the two events at most .

(3) The probability of occurrence of the event B only.

(4) The probability of occurrence of only one of the two events .

(($\frac{7}{8}$, $\frac{5}{8}$, $\frac{1}{8}$, $\frac{1}{2}$))

(17) A and B are two events in a sample space for a random experiment , if $P(A) = \frac{2}{3} P(B)$, the probability of the occurrence of one of them at most equals 0.75 , the probability of the occurrence of one of them at least equals 0.6

Find the probability of the following events :

(1) the occurrence of both of them all together .

(2) the occurrence of only one of them .

(3) the occurrence of B or the non- occurrence of A .

((0.25 , 0.35 , 0.91))

Algebra

1- Find the geometric means of the sequence : (4 , ….. , ….. , ….. , ….. , …. , 2916)

Solution :- 12 , 36 , 108 , 324 , 972 or -12 , 36 , -108 , 324 , -972

2- If 6a , 3b, 2c, 2 d are positive quantities in an arithmetic sequence, prove that

b c > 2a d

3- Find the sum of the geometric sequence in which :

a = 3, r =2, n = 8 .

Solution :- $S_n = \frac{a\,(1-r^n)}{1-r}$ **the sum formula of the geometric sequence**

$S_8 = \frac{3\,(1-2^8)}{1-2}$ **by substituting : a = 3, r = 2 and n = 8**

$S_8 = 3 \times 255 = 765$ **by simplifying**

4- Find the sum of the geometric series : 1 + 3 + 9 + ….. + 6561

Solution :- $S_n = \frac{a-l\,r}{1-r}$ **the sum formula of the geometric sequence**

$S = \frac{1-6561\times3}{1-3}$ **by substituting : a = 1, r = 3 and l = 6561**

$S = \frac{19682}{2} = 9841$ **by simplifying**

5- Find $\sum_{r=5}^{12} 3\,(2)^{r-1}$

Solution :- $T_5 = a = 3(2)^{5-1} = 48$, r = 2 , n = 12 – 5 + 1 = 8

$S_n = \frac{a\,(1-r^n)}{1-r}$ **the sum formula of the geometric sequence**

$S_8 = \frac{48\,(1-2^8)}{1-2}$ **by substituting : a = 48, r = 2 , n = 8**

$S_8 = 48 \times 255 = 12240$ **by simplifying**

6- Find the sum for each of the following two geometric series if found :

a) $\frac{81}{8} + \frac{27}{4} + \frac{9}{2} + \ldots\ldots\ldots$ b) $\frac{2}{3} + \frac{5}{6} + \frac{25}{24} + \ldots\ldots\ldots$

Solution :-

a) **Find the common ratio of the geometric sequence** : $r = \frac{27}{4} \div \frac{81}{8} = \frac{27}{4} \times \frac{8}{81} = \frac{2}{3}$

$\because -1 < \frac{2}{3} < 1$ $\therefore$ **The series has a sum**

$\because a = \frac{81}{8}\ , r = \frac{2}{3}$ **by substituting in the sum formula** $S_\infty = \frac{a}{1-r}$

$$\therefore S_\infty = \frac{\frac{18}{8}}{1-\frac{2}{3}} = \frac{\frac{18}{8}}{\frac{1}{3}} = \frac{243}{8}$$

b) **Find the common ratio of the geometric sequence** : $r = \frac{5}{6} \div \frac{2}{3} = \frac{5}{6} \times \frac{3}{2} = \frac{5}{4}$

$\because \frac{5}{4} > 1$ $\therefore$ The series is divergent and has no sum .

7- Put 0.432 in the form of a common fraction .

Solution :-

First : using the sum of an infinite geometric series

$0.\overline{432} = 0.432 + 0.000432 + 0.000000432 + \ldots\ldots\ldots$

Sum formula of the geometric sequence : $S_\infty = \frac{a}{1-r}$

let $a = \frac{432}{1000}$, $r = \frac{1}{1000}$ then $S_\infty = \frac{\frac{432}{1000}}{1-\frac{1}{1000}}$

$$S_\infty = \frac{432}{1000} \times \frac{1000}{999} = \frac{16}{37}$$

8- How many ways can khaled have a meal out of three meals (Liver – chicken – fish) and a drink out of three drinks (orange – lemon – Mango) ?

Solution :- Number of ways to choose a meal = 3 ways and the number of ways to choose a drink = 3 ways.

The total number of ways to choose = 3 × 3 = 9 ways .

9- a) Find $\dfrac{\lfloor\!\underline{10}}{\lfloor\!\underline{8}}$ b) If $\lfloor\!\underline{n}$ = 120 find the value of n

<u>Solution :-</u>

a) $\dfrac{\lfloor\!\underline{10}}{\lfloor\!\underline{8}} = \dfrac{10\times\ 9\ \lfloor\!\underline{8}}{\lfloor\!\underline{8}} = 10 \times 9 = 90$

b) $n = 5 \times 4 \times 3 \times 2 \times 1$ $\therefore \lfloor\!\underline{n} = \lfloor\!\underline{5}$ then $n = 5$

10- Find the value for reach of the following :

a) ${}^{7}P_{4}$ b) ${}^{4}P_{4}$ c) ${}^{4}P_{3}$

<u>Solution :-</u>

a- ${}^{7}P_{4} = 7 \times 6 \times 5 \times 4 = 840$

b- ${}^{4}P_{4} = 4 \times 3 \times 2 \times 1 = 24$

c- ${}^{4}P_{3} = 4 \times 3 \times 2 = 24$. What do you notice in the two phrases b and c ?

11- If ${}^{7}P_{r}$ = 840, find the value of $\lfloor\!\underline{r-4}$

<u>Solution :-</u>

Start by dividing the number 840 by 7 , then divide the quotient by 6, then divide the resulted quotient by 5 and so on till you reach number 1

$\therefore$ Number $840 = 7 \times 6 \times 5 \times 4 = {}^{7}P_{4}$

$\therefore {}^{7}P_{r} = {}^{7}P_{4}$ $\therefore r = 4$ $\therefore \lfloor\!\underline{r-4} = \lfloor\!\underline{0} = 1$

12-Find the value of each of the following :

a) ${}^{7}C_{5}$ b) ${}^{7}C_{2}$ (what do you notice) ?

<u>Solution :-</u>

a- ${}^{7}C_{5} = \dfrac{7P5}{\lfloor\!\underline{5}} = \dfrac{7\times6\times5\times4\times3}{5\times4\times3\times2\times1} = 21$ b- ${}^{7}C_{2} = \dfrac{7\times6}{2\times1} = 21$

13- If : $^{28}C_r = {}^{28}C_{2r-47}$

Solution :-

$\because \; {}^{28}C_r = {}^{28}C_{2r-47}$

Either : $r = 2r - 47$ i.e.: $r = 47$

It is greater than the value of n, so it is refused.

Or : $r + 2r - 47 = 28$ $\therefore 3r = 75$ $\therefore r = 25$

Calculus

1- If y = (x^2 – 3 x + 1)5 , find $\frac{dy}{dx}$

Solution :-

let z = x^2 – 3 x + 1 $\quad\therefore$ y = z^5

it is clear that y is differentiable with respect to z (polynomial at z) and $\frac{dy}{dz} = 5z^4$

and also z is differentiable with respect to x (polynomial at x) and $\frac{dz}{dx} = 2x - 3$,

By applying the chain rule $\therefore \frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx} = 5z^2 \times (2x - 3)$

By substituting z

$$\therefore \frac{dy}{dx} = 5\,(x^2 - 3x + 1)^4 \times (2x - 3)$$

2- Find $\frac{dy}{dx}$ if

a) $y = (6x^3 + 3x + 1)^{10}$ $\qquad$ b) $y = \left(\frac{x-1}{x+1}\right)^5$

Solution :-

$Y = (6x^3 + 3x + 1)^{10}$

$\therefore \frac{dy}{dx} = 10\,(6x^3 + 3x + 1)^9 \times \frac{d}{dx}(6x^3 + 3x + 1)$

$= 10\,(18x^2 + 3)(6x^3 + 3x + 1)^9$

$= 30\,(6x^2 + 1)(6x^3 + 3x + 1)^9$

$Y = \left(\frac{x-1}{x+1}\right)^5$ $\qquad \therefore \frac{dy}{dx} = 5\left(\frac{x-1}{x+1}\right)^4 \times \frac{(x+1)\times 1-(x-1)\times 1}{(x+1)2}$

$= 5\left(\frac{x-1}{x+1}\right)^4 \times \frac{x+1-x+1}{(x+1)2}$

$= \frac{10}{(x+1)2} \times \left(\frac{x-1}{x+1}\right)^4 = \frac{10\,(x-1)4}{(x+1)6}$

3- y = $\sqrt[3]{z}$, z = x^2 – 3x + 2, find $\frac{dy}{dx}$

Solution :-

$\because$ y = $z^{\frac{1}{3}}$ $\qquad$ $\frac{dy}{dz} = \frac{1}{3} z^{-\frac{2}{3}}$

$\because$ z = x^2 – 3x + 2 $\qquad$ $\frac{dy}{dz}$ = 2x -3

a) $\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx} = \frac{1}{3} z^{-\frac{2}{3}}$ (2x – 3) $\qquad$ $\therefore \frac{dy}{dx} = \frac{1}{3}$ (2x – 3) (x^2 – 3x + 2) $^{-\frac{2}{3}}$

4- Find the points which lie on the curve of y = x^3 – 4x + 3 at which the tangent makes a positive angle of measure 135° with the positive direction of x axis .

Solution :-

$\because$ y = x^3 – 4x + 3 $\qquad$ $\therefore \frac{dy}{dx} = 3x^2$ - 4

$\therefore$ the tangent makes an angle of measure135°with the positive direction of x– axis

$\therefore$ the slope of the tangent = tan 135 = -1

$\therefore \frac{dy}{dx} = 3x^2$ - 4 = -1 $\qquad$ $\therefore$ $3x^2 = 3$ $\qquad$ $\therefore$ x = $\pm$ 1

When x = -1 $\qquad$ $\therefore$y = $(-1)^3$ – 4 (-1) + 3 = 6

, when x = 1 $\qquad$ $\therefore$ y = 1 – 4 + 3 = 0

$\therefore$ the points are (-1 , 6) , (1 , 0)

5- Find the slope of the normal on the curve of y = tan (π - $\frac{2}{3}$ x) at point (π , $\sqrt{3}$)

Solution :-

$\because$ y = tan (π - $\frac{2}{3}$ x) $\qquad$ $\therefore \frac{dy}{dx} = -\frac{2}{3} \sec^2 (\pi - \frac{2}{3} x)$

the slope of the tangent to the curve at point (π , $\sqrt{3}$) = $-\frac{2}{3} \sec^2 (\pi - \frac{2\pi}{3})$

$= -\frac{2}{3} \sec^2 \frac{\pi}{3} = \frac{-2}{3} \times 4 = \frac{-8}{3}$

The slope of the normail at point (π , $\sqrt{3}$) = $\frac{3}{8}$

6- Find the two equations of the tangent and normal to the curve of $y = 2x^3 - 4x^2 + 3$ at the point lying on the curve and whise abscissa = 2

Solution :-

$\because y = 2x^3 - 4x^2 + 3$

$\therefore$ when x = 2 $\quad\therefore y = 2(2)^3 - 4(2)^2 + 3 = 3$

$\therefore$ point (2,3) lies on the curve

$\because \frac{dy}{dx} = 6x^2 - 8x \quad \therefore \left[\frac{dy}{dx}\right]_{(2,3)} = 6(2)^2 - 8(2) = 8$

the slope of the tangent = 8 and its and its equation is

$y - 3 = 8(x-2)$ $\quad$ i.e $y - 8x + 13 = 0$

the slope of the normal = $\frac{-1}{8}$ and its equation is

$y - 3 = \frac{-1}{8}(x-2)$ $\quad$ i.e. $8y + x - 26 = 0$

7- Check the correctness for each of the following :

a) $\int x^7 \, dx = \frac{1}{8} x^8 + C$ $\qquad$ b) $\int \frac{x}{\sqrt{1+x^2}} \, dx = \sqrt{1+x^2} + C$

Solution :-

a- $\because \frac{d}{dx}\left(\frac{1}{8} x^8 + C\right) = x^7$ $\qquad \therefore \int x^7 \, dx = \frac{1}{8} x^8 + C$

b- $\frac{d}{dx}(\sqrt{1+x^2} + C = \frac{2x}{2\sqrt{1+x^2}} = \frac{x}{\sqrt{1+x^2}}$

$\therefore) \int \frac{x}{\sqrt{1+x^2}} \, dx = \sqrt{1+x^2} + C$

8- Find : a) $\int (4x + 3x^2)\,dx$ **b) $\int \frac{(x^2+2)2}{x^2}\,dx$**

Solution :-

a- $\int (4x + 3x^2)\,dx$

$= \int 4x\,dx + 3x^2\,dx$

$= 4 \int x\,dx + 3 \int x^2\,dx$

$= \frac{4}{2} x^2 + 3 \times \frac{1}{3} x^3 + C$

$= 2x^2 + x^3 + C$

b- $\int \frac{(x^2+2)2}{x^2}\,dx$

$= \int \frac{x^4+4x^2+4}{x^2}\,dx$

$= \int x^2\,dx + \int 4\,dx + \int 4x^{-2}\,dx$

$= \frac{1}{3} x^3 + 4x - 4x^{-1} + C$

9- Find the following integrations :

a) $\int (x \sin x)\,dx$ b) $\int (4\cos x + \frac{1}{\cos^2 x} + 1)\,dx$

Solution :-

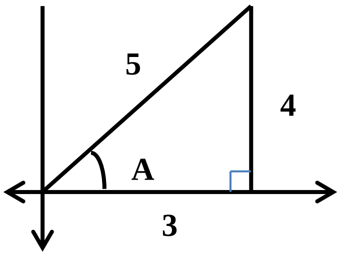
a- $\int (x \sin x)\,dx = \frac{1}{2} x^2 + \cos x + C$

b- $\int (4\cos x + \frac{1}{\cos^2 x} + 1)\,dx = \int (4\cos x + \sec^2 x + 1)\,dx$

$= 4\sin x + \tan x + x + C$

1- If you know sin A = $\frac{4}{5}$ where 0° < A < 90° , find the value for each of the following without using the calculator :

a) sin 2A   b) cos 2A   c) tan 2A



Solution :-

$\because \sin A = \frac{4}{5}$ $\because$ A lies in the first quadrant

$\therefore \cos A = \frac{3}{5}$ (positive because A is an acute angle)

a- sin2A $= 2 \sin A \cos A = 2 \times \frac{4}{5} \times \frac{3}{5} = \frac{24}{25}$

b- cos 2C $= 1 - 2 \sin^2 A = 1 - 2 \times \frac{16}{25} = \frac{-7}{25}$

(you can use the other froms of the cosine rule of the double – angle)

c- $\tan 2A = \frac{\sin 2A}{\cos 2A} = \frac{24}{25} \div \left(\frac{-7}{25}\right) = \frac{-24}{7}$

2- Find the value for each of the following , without using the calculator , :

a) 2 sin 15° cos 15   b) $2\cos^2 22° 30' - 1$

Solution :-

a- $2 \sin 15° \cos 15° = \sin 2 \times 15° = \sin 30° = \frac{1}{2}$

b- $2 \cos^2 22° 30' - 1 = \cos (2 \times 22° 30') = \cos 45° = \frac{1}{\sqrt{2}}$

3- Prove the correctness of the identity : csc 2 x + cot 2x = cot x , then use the previous identity to find the value of cot 15° .

Solution :-

The left side $= \frac{1}{\sin 2x} + \frac{\cos 2x}{\sin 2x} = \frac{1+\cos 2x}{\sin 2x}$

$= \frac{1+(2\cos^2 x - 1)}{2 \sin x \cos x} = \frac{2\cos^2 x}{2 \sin x \cos x} = \frac{\cos x}{2 \sin x}$ = cot x (the right side)

By putting x = 15°   in the identity : csc 2 x + cot 2 x = cot x

$\therefore \cot 15° = \csc 30° + \cot 30° = 2 + \sqrt{3}$

4- Find the surface area of the triangle whose side lengths are 6 , 8 and 10 centimetres using Heron' formula .

Solution :-

$\because$ 2P = 6 + 8 + 10 = 24 cm P = 12 cm

P – a = 12 – 6 = 6 cm , p – b = 12 – 8 = 4 cm , p – c = 12 – 10 = 2 cm

$\therefore$ Area of $\Delta = \sqrt{p\,(p-a)(p-a)(p-c)}$

$= \sqrt{12 \times 6 \times 4 \times 2} =$ 24 cm^2

5- Find the area of the following triangles.

a) A triangle of side lengths 7 , 9 and 12 centimeters .

b) A triangle of two-side lengths 24 and 40 cm and the included angle of measurement 30° .

c) Is the triangle whose side lengths are 12 , 14 and 30 existed ? find its area if possible .

Solution :-

a- $\because$ 2 P = 28 cm $\therefore$ p = 14 cm, p – a = 7 cm , p – b = 5 cm, p-c = 2 cm .

By applying Heron's formula :

Area of the triangle = $\sqrt{14 \times 7 \times 5 \times 2} = 14\sqrt{5}$ cm^2

b- Area of the triangle = $\frac{1}{2} \times 24 \times 40 \times \sin 30$

$= \frac{1}{2} \times 24 \times 40 \times \frac{1}{2} = 240$ cm^2

c- 2 P = 56 cm $\therefore$ P = 28 cm , P < a side

$\therefore$ there is no triangle to find its area .