Leyan Series IN Mathematics

Final Revision

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The slope of the straight line

(1) The equation of the straight line which is parallel to X-axis and pass through the point (- 2 , 3) .

The equation of the straight line : ∵ $y = m\,x + c$,,, the straight line parallel to X – axis then $m = 0$

$y = c$ from the point (- 2 , 3) ,, $c = 3$,, the equation is ∴ $y = 3$

(2) If $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{CD} = \frac{2}{3}$ then the slope of $\overleftrightarrow{AB} = \frac{2}{3}$

∵ $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$,,, ∴The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{CD}$

(3) If $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{CD} = \frac{2}{3}$ then the slope of $\overleftrightarrow{AB} = -\frac{3}{2}$

∵$\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$,,, ∴The slope of $\overleftrightarrow{AB}$ × the slope of $\overleftrightarrow{CD}$ = - 1

(4) The slope of the straight line whose equation 2 x – 3 y + 5 = 0 is ……….

∵The slope = $-\frac{\textit{The coefficient of x}}{\textit{The coefficient of y}} = -\frac{2}{-3} = \frac{2}{3}$

(5) If the two straight lines 2 x + b y + 3 = 0 and 3 x – y + 2 = 0 are perpendicular then b = ……..

∵The two straight lines Perpendicular ∴ $-\frac{2}{b} \times -\frac{3}{-1} = -1$,,, $-\frac{6}{b} = -1$,, $b = 6$

(6) If the two straight lines K x – 2 y + 3 = 0 , 6 x + 3 y – 5 = 0 parallel , then K = …….

∵The two straight lines parallel ∴ $\frac{-K}{-2} = -\frac{6}{3}$,,, $K = -4$

(7) The slope of the perpendicular line to the line passes through the two points (2 , 6) ,(- 4 , 1) = …..

∵The slope = $\frac{\textit{difference of y co-ordinates}}{\textit{difference of x co-ordinates}} = \frac{6-1}{2-(-1)} = \frac{5}{3}$

∴The slope of the perpendicular line = $\frac{-3}{5}$

(8) The equation of the straight line whose slope = 1 and passes through the origin point is …….

The equation is $y = m\,x + c$ and the point (0 , 0) satisfies it and $m = 1$ then $y = x$ or $y - x = 0$

(9) The slope of the straight line which is parallel to the straight line which passes through the two points (3 , 1) , (5 , - 1) = …………

The slope = $\frac{\textit{difference of y co-ordinates}}{\textit{difference of x co-ordinates}} = \frac{1-(-1)}{3-5} = \frac{2}{-2} = -1$

(10) The equation of the straight line which passes through the origin point and perpendicular to the straight line y = 2 x is ……. ∵The slope of the straight line = $\frac{2}{1}$ then ∴ the slope of perpendicular = $-\frac{1}{2}$

∴The equation $y = m\,x$,,, $y = -\frac{1}{2}x$ then $2y + x = 0$

(11) If the straight line y = x sin 30° + c : passes through (4 , 6) then c = ….. ,∵ 6 = 4 sin30° + c

$\therefore c = 6 - 2 = 4$

(12) The straight line passes through the two points (1 , y) , (3 , 4) , its slope is tan 45° , then y = ……

The slope = $\frac{difference\ of\ y\ co-ordinates}{difference\ of\ x\ co-ordinates} = \frac{y-4}{1-3} = \tan 45°$,,, $\frac{y-4}{-2} = 1$,,, y – 4 = - 2 , y = 2

(13) If the two equations of the two straight lines L_1 , L_2 respectively are 2 x – 3 y + a = 0 ,

3 x + b y – 6 = 0 , Find (1) the value of b when $L_1 // L_2$ (2) the value of b when $L_1 \perp L_2$

(3) if the point (1 , 3) lies on L_1 , then find the value of a .

(1) when $L_1 // L_2$ then $m_1 = m_2$

$$\frac{-2}{-3} = \frac{-3}{b}$$

2 b = - 9 ,, b = 4.5

(2) when $L_1 \perp L_2$ then $m_1 \times m_2 = -1$

$$\frac{-2}{-3} \times \frac{-3}{b} = -1$$

b = 2

(3) (1 , 3) $\in L_1$

2(1) – 3(3) + a = 0

2 – 9 + a =0 , a = 7

(14) Find the equation of the straight line passes through the two points (2 , 3) , (- 3 , 2)

The slope = $\frac{difference\ of\ y\ co-ordinates}{difference\ of\ x\ co-ordinates} = \frac{3-2}{2-(-3)} = \frac{1}{5}$

The equation when the straight line $\in$ (2 , 3)

y = m x + c

$y = \frac{1}{5}x + c$ when it satisfies (2 , 3)

$3 = = \frac{2}{5} + c$,,, $c = \frac{13}{5}$

$\therefore$The equation : $y = \frac{1}{5}x + \frac{13}{5}$

Another solution

The equation when the straight line $\in$ (- 3 , 2)

y = m x + c

$y = = \frac{1}{5}x + c$ when it satisfies (- 3 , 2)

$2 = = \frac{-3}{5} + c$,,, $c = \frac{13}{5}$

$\therefore$ The equation : $y = \frac{1}{5}x + \frac{13}{5}$

(15) $\overline{AB}$ is a diameter of circle M if B (8 , 11) , M (5 , 7) , then Find (1) the coordinates of A .

(2) The length of the radius of the circle (3) The equation of the perpendicular straight line to $\overline{AB}$ from the point B .

(1)The coordinates of A if M is a center of the circle then M is the midpoint of diameter $\overline{AB}$, A(x , y)

$(5, 7) = \left(\frac{x+8}{2}, \frac{y+11}{2}\right)$ Then x + 8 = 10 ,,, x = 2 and y +11 = 14 ,,, y = 3 A(2 , 3)

(2) The length of the radius = The distance $\overline{AM}$ = The distance $\overline{AM}$ = $\frac{The\ distance\ \overline{AB}}{2}$

The distance $\overline{AM} = \sqrt{(2-5)^2 + (3-7)^2} = 5\ units$

(3) The slope of $\overleftrightarrow{AB} = \frac{difference\ of\ y\ co-ordinates}{difference\ of\ x\ co-ordinates} = \frac{3-11}{2-8} = \frac{-8}{-6} = \frac{4}{3}$, the slope of perpendicular = $\frac{-3}{4}$

The equation y = m x + c satisfying at B (8 , 11) then $y = \frac{-3}{4}x + c$ at (8 , 11)

$11 = \frac{-3}{4}(8) + c$ then c = 17 ,,,,,, $y = \frac{-3}{4}x + 17$

(16) A straight line , its slope = $\frac{1}{2}$, intersects a positive part of y – axis of length two units , find

(1) the equation of this straight line . (2) its intersection point with y – axis .

$\because$The equation $y = m x + c$, where m is a slope and c is the intersect part of y – axis .

$\therefore y = \frac{1}{2} x + 2$ by direct substitution , (2) the point of intersection with y– axis at x = 0 , y = 2 , (0 , 2)

(17) Find the equation of the straight line which passes through the point (1 , 6) and the midpoint of $\overline{AB}$ where A (1 , -2) , B (3 , - 4) .

The midpoint of $\overline{AB}$ = $\left(\frac{1+3}{2} , \frac{-2-4}{2} \right)$ = (2 , - 3)

The slope = $\frac{6-(-3)}{1-2} = \frac{9}{-1} = -9$

y = mx + c at (1 , 6)

y = - 9 x + c ,,,,, 6 = - 9 (1) + c

c = 15 ,,, y = -9 x + 15

Another solution

,,,,,

y = m x + c at (2 , - 3)

y = -9 x + c ,,,, - 3 = -9 (2) + c

c = 15 ,,, y = -9 x + 15

(18) If the straight line L_1 passes through the Two points (3 , 1) , (2 , k) , and the straight line L_2 makes with the positive direction with x – axis an angle of measure 45 Find the value of K if :

The slope of L_1 = $\frac{difference\ of\ y\ co-ordinates}{difference\ of\ x\ co-ordinates} = \frac{1-K}{3-2} = \frac{1-K}{1} = 1 - K = m_1$

The slope of L_2 = Tan θ = Tan 45° = 1 = m_2

(1) $L_1 \parallel L_2$ Then $m_1 = m_2$

1 – K = 1 ,, K = 0

(2) $L_1 \perp L_2$ then $m_1 \times m_2 = -1$

(1 – k) (1) = - 1 ,, - K = -2 , K = 2

(19) Find the value of K if the points A (5 , - 5) , B (- 1 , K) , C (15 , 15) are the vertices of right angled triangle at B .

$\because$ ABC is a right angled triangle at B $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$

The slope of $\overleftrightarrow{AB}$ $\times$ The slope of $\overleftrightarrow{BC}$ = -1

The slope of $\overleftrightarrow{AB}$ = $\frac{K-(-5)}{-1-5} = \frac{K+5}{-6}$, The slope of $\overleftrightarrow{BC}$ = $\frac{K-15}{-1-15} = \frac{K-15}{-16}$

$\frac{K+5}{-6} \times \frac{K-15}{-16} = -1$,,,,, k^2 - 10 k – 75 = - 96 ,,, k^2 - 10 k + 21 = 0

(K – 3) (K – 7) = 0 then k – 3 = 0 ,,, k = 3 or k – 7 = 0 ,,, k = 7

(20) If the points A (0 , 1) , B (a , 3) , and C (2 , 5) are collinear find the value of a .

The slope of $\overleftrightarrow{AB}$ = The slope of $\overleftrightarrow{AC}$

$\frac{3-1}{a-0} = \frac{5-1}{2-0}$,,,, $\frac{2}{a} = \frac{4}{2}$,,,,, $2a = 2$

a = 1