Universal gravitational law

The forces of attraction between two bodies are directly proportional to the product of their masses and indirectly proportional with the square of distance between their centers

Let m_1 , m_2 are the masses of two bodies and the distance between their centers s: Then

$$F \alpha \frac{m_1 \times m_2}{s^2} \qquad \Rightarrow \therefore F = G \times \frac{m_1 \times m_2}{s^2}$$

M measured in kg. , S in meter , f in newton

Example (1) Calculate the gravitational force between two bodies their masses are 10 kg, 50 kg and distance between their centers 0.5 m. known that the universal gravitational constant equals 6.67×10^{-11} newton .m^2/kg^2

Solution

$$F = G \times \frac{m_1 \times m_2}{s^2} \Rightarrow \therefore F = 6.67 \times 10^{-11} \times \frac{10 \times 50}{(0.5)^2} = 1.334 \times 10^{-8}$$

Example (2) Calculate the gravitational force between two planets the mass of the first 2×10^{21}ton, mass of the other 4×10^{25} ton and the distance between their centers 2×10^{6} km

Solution

$$F = G \times \frac{m_1 \times m_2}{s^2}$$

$$F = 6.67 \times 10^{-11} \times \frac{2 \times 10^{24} \times 4 \times 10^{28}}{(2 \times 10^{9})^2} = 1.334 \times 10^{-8}$$

$$F = 6.67 \times 10^{-11} \times \frac{2 \times 10^{52}}{4 \times 10^{18}} = 6.67 \times 10^{-11} \times 2 \times 10^{34}$$

$$F = 1.334 \times 10^{24} \ \textit{newton}$$

Example (3) A piece of iron is put at a distance of 50 cm from another piece of Nickel of mass 25 kg then the gravitational force between them became 6×10^{-8} N. What is the mass of the piece of iron approximated to the nearest integer number ?

Solution

$$F = G\times\frac{m_1\times m_2}{s^2}$$

$$6\times10^{-8} = 6.67\times10^{-11}\times\frac{m_1\times25}{(0.5)^2}$$

$$m_1 = \frac{0.25\times6\times10^{-8}}{6.67\times10^{-11}\times25} = 9 \text{ kg.}$$

Example (4) A satellite of mass 1500 kg revolves at a height of 540 km from the Earth's surface whose mass is 6×10^{24} kg and radius is 6360 km. Find the Earth's gravitational force on the moon known that the universal gravitational constant equals 6.67×10^{-11} newton.m^2/kg^2

Solution

$$F = G\times\frac{m_1\times m_2}{s^2}$$

$$F = 6.67\times10^{-11}\times\frac{1500\times6\times10^{24}}{(540000+6360000)^2} = 1260807 \ \textit{newton}$$

Example (5) If the attraction force between the Earth and the Moon is 3011×10^{24} newton and the mass of the Earth 6×10^{24} kg, the mass of the Moon 7×10^{22} kg. Find the distance between their centers.

Solution

$$F = G\times\frac{m_1\times m_2}{s^2}$$

$$3011\times10^{4} = 6.67\times10^{-11}\times\frac{7\times10^{22}\times6\times10^{24}}{S^2}$$

$$S = \sqrt{\frac{7\times10^{22}\times6\times10^{24}\times6.67\times10^{-11}}{3011\times10^{4}}} = 3\times10^{6} \ \textit{meters}$$

Example (6) Calculate the mass of the Earth in kg if known that its radius length 6360 km and gravitational constant is 6.67×10^{-11} and the Earth's acceleration due to gravity is 9.8 m/s^2

Solution

Let a body of mass m_1 put on the surface of the Earth and m_2 is the mass of the Earth

The forces of attraction between the body and the Earth is the weight of the body itself = $m_1 \times g$

$$F = G \times \frac{m_1 \times m_2}{s^2}$$

$$m_1 \times g = 6.67 \times 10^{-11} \times \frac{m_1 \times m_2}{(6360000)^2}$$

$$9.8 = 6.67 \times 10^{-11} \times \frac{m_1 \times m_2}{4.045 \times 10^{24}} \Rightarrow \therefore m_2 = 5.9 \times 10^{24} kg$$

Comparing the accelerations due to gravities on the surfaces of two planets:

If g_1 , g_2 the acceleration due to gravity for each planet, m_1 , m_2 their mass in kg, r_1 , r_2 their radii in meters respectively, then from the previous

it is possible to deduce the following relation: $\frac{g_1}{g_2} = \frac{m_1}{m_2} \times \frac{r_2^2}{r_1^2}$

If the mass of the Earth is 81 times the mass of the moon and their diameter equal 12756 km, 3476 km respectively. If the acceleration due to gravity on the Earth equals 9.8 m / sec^2 what is the acceleration due to gravity on the moon's surface?

Solution

Let the mass of the moon is (m) kg ,then the mass of the Earth equals (81 m)

$r_1 = 6378$ km , $r_2 = 1738$ km , $g_1 = 9.8$ m / sec^2 , $g_2 = ?$

$\because \frac{g_1}{g_2} = \frac{m_1}{m_2} \times \frac{r_2^2}{r_1^2}$ $\quad \therefore \frac{9.8}{r_2} = \frac{81\ m}{m} \times (\frac{1738}{6378})^2$

Simplifying: $\quad \therefore g\ moon \simeq 1.63$ m / sec^2

Homework

1. Two identical balls each of mass 6.8 kg and the distance between their centers equals 21.8 cm. What is the gravitational force between them?

2. Calculate the gravitational force between two bodies of masses 10 kg , 15 kg and the distance between them is 2 meters.

3. A satellite of mass 2000 kg revolves at a height 440 km from the Earth's surface whose mass equals 6×10^{24} kg . Find the gravitational force of the Earth on the satellite knowing that the Earth's radius equals 6360 km.

4. If the Earth's acceleration due to gravity (g) is 10 m / sec^2 the Earth's radius equals 6.36×10^6 m. Calculate the Earth's mass.

5. Calculate the gravitational force between the Sun and the Earth if you know that the Earth moves in an elliptical orbit around the sun, the Earth's mass equals 6×10^{24} kg the sun's mass equals 9×10^{29} kg and the distance between their centers equals 1.5×10^{11} m.

6. If you know that the Earth's mass equals 5.97×10^{24} and its radius 6.34×10^6 m, the mass of the moon equals 7.36×10^{22} kg, find the radius of the moon if the gravity on the Earth's surface equals six times the gravity on the moon's surface.

7. If you know that the Earth's mass equals 6.06×10^{24} and its radius is 6.36×10^6 , find the intensity of the Earth's gravitational field.

8. A planet whose mass equals 3 times the Earth's mass, and its diameter equals 3 times the Earth's diameter. Calculate the ratio between the acceleration due to gravity on this planet and that on the Earth.

9. Find the universal gravitational force between two planets the mass of the first equals 2×10^{21} ton and the mass of the second equals 4×10^{25} ton and the distance between their centers equals 2×10^6 km.

10. A piece of iron is put at a distance of 50 cm from another piece of Nickel of mass 25 kg then the gravitational force between them became $10^{-8} \times 6$ N. What is the mass of the piece of iron approximated to the nearest integer number?

11. If a body of math m\ kg on a height s meter from the surface of the earth whose radius is r meter and mass m kg .find the value of the gravitational force that act on the body.

12. **Join with space:** an international space station with weight on the surface of the earth 421997.6 newton .find its weight when it became in an external orbit on a height 350 km from the surface of the earth known that the Earth's mass equals 6.37×10^3 and its radius is 5.6×10^{24} . (Hint : Force in Newton = Mass in kg × gravitational force of the earth 9.8 m/sec^2)

General Exercises (Unit two)

Complete the following:

1. If $\overrightarrow{V_A} = 7\ \overrightarrow{i}$, $\overrightarrow{V_B} = -3\ \overrightarrow{i}$ then $\overrightarrow{V_{AB}}$ = ...
2. If $\overrightarrow{V_{CD}} = 70\ \overrightarrow{n}$, $\overrightarrow{V_c} = 50\ \overrightarrow{n}$ then $\overrightarrow{V_D}$ = ...
3. If two cars A, B are moving with two velocities 65 km/h , 75 km/h ,then
   a. V_{AB} = ... if they are moving in the same direction.
   b. V_{AB} = ... if they are moving in opposite directions.
4. A car started its motion from rest with a uniform acceleration 20 cm/sec^2 for 10 seconds.
   a. The final velocity of the car = ... m/sec.
   b. The distance covered during this interval = ... m.
5. A body started its motion from rest with a uniform acceleration 72 km/hr and deceleration 2m/sec^2.
   a. The time required for the body to stop = ... sec.
   b. The distance covered during this interval = ... m.
6. A car used its brakes to stop in 10 seconds after covering a distance of 25 m.
   a. The acceleration of the body during using the brakes = ... m/sec^2.
   b. The velocity of the body at the beginning of using the brakes = ... m/sec.
7. body has fallen from the top of a vertical tower to reach the ground after 5 seconds:
   a. The velocity of the body when it reached the ground = ... m/sec.
   b. The height of the tower = ... m.
8. A body is projected vertically upwards from a point on the Earth's surface to return to it after 4 seconds:
   a. The velocity of projecting the body = ... m/sec.
   b. The maximum height reached by the body = ... m.
9. From the top of a tower of 20 m height a body is projected upwards with velocity 7m/sec:
   a. The velocity of reaching the ground = ... m/sec.
   b. The time interval to reach the ground = ... seconds.
10. A planet of mass equals 3 times the Earth's mass and its diameter equals 3 times the Earth's diameter, then the ratio between the acceleration due to gravity on this planet to the acceleration due to gravity on the Earth ..... : .....
11. A body moves in a straight line a distance of 100 m with velocity 5 m/sec. Then it moves with a velocity of 8 m/sec. in the same direction for 10 seconds. Find the average velocity during the whole trip.

12 Two bodies A, B move in a straight line in the direction $\overrightarrow{B A}$ with two velocities 100 m/min, 120 km/hr. If the distance between them equals 30 km. find where and when they will meet?

13 A car (A) moving on a straight road has measured the relative velocity of another car (B) coming from the opposite direction to find it 130 km/hr. and when the car (A) doubled its velocity and measure of car B again , it found that it has become 180 km/hr. Find the actual velocities of the two cars.

14 A controlling speed car (Radar) moves on the highway (desert road) with a speed of 30 km/hr , it watched a truck coming from the opposite direction which seemed to be moving with velocity 110 km/h. Find the actual velocity of the truck.

15 A body has moved in a straight line with a velocity 7 m/sec.and with acceleration 4 m/sec^2. Find its speed and the distance cut by it in 6 sec.

16 A body started its motion from rest with uniform acceleration 20 km/sec^2 when its velocity became 8 m/sec it moved with deceleration until it stopped after 112 seconds from the start of the motion. Calculate the deceleration and the total distance covered by the body.

17 A body started its motion from rest to cover 150 m and when its velocity became 54 km/hr the acceleration stopped and it moved with its acquired velocity a distance of 300 m, then it moved with a uniform deceleration of value $\frac{3}{2}$ m/sec^2 till it stopped. Calculate the average velocity during the whole trip.

18 A body moves in a straight line with uniform acceleration to cover 52 meters in the first four seconds, then it cover a distance of 92 meters in the next four seconds. Calculate the acceleration of motion, the initial velocity and the distance covered during the first 10 seconds of its motion.

19 If $\overrightarrow{r}$ is the position vector of a body that moves in a straight line from point "O" which is determined by the relation $\overrightarrow{r} = (3n^2 - 3)\ \overrightarrow{n}$ find the displacement vector after 4 seconds from the starting of the motion.

20 A body has fallen from a high S above the Earth's surface to cover a distance of 34.3 m at the last second of its motion . Find:

a The velocity of the body when it reaches the Earth's surface.

b The height from which the body has fallen.

21 A body is projected vertically upwards with a velocity of 14 m/sec from a point at height 350 meters above the Earth's surface. Find:

a The time taken by the body to reach the Earth's surface.

b The total distance covered by the body to reach the Earth's surface.

22 A sphere of iron is put at a distance of 40 cm from another sphere of Nikcel of mass 50 kg such that the gravitational force between them equals 12×10^{-8} N, then what is the mass of the sphere of iron if you know that the universal gravitational constant equals 6.67×10^{-11} newton . m^2/kg^2