Unit One

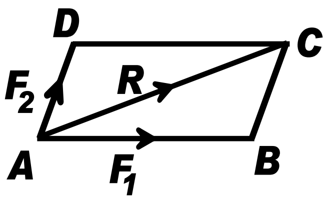
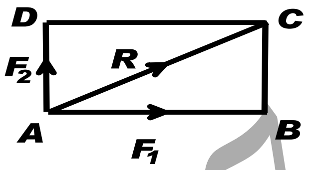
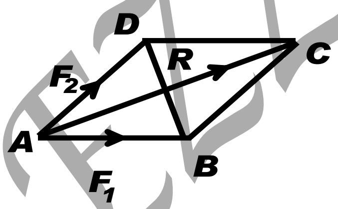
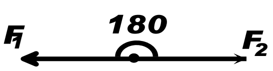
Force : is defined as the effect of a natural body upon another one.

The effect of any force depends on the following factors:

First: magnitude of a force. **Secondly: Direction of a force**

Thirdly: Point of action of the force and its line of action

The resultant of two forces meeting at a point analytically

Case	*Drawing*	*Resultant*	*Direction*
Measure of the angle between $\overline{F_1}$, $\overline{F_2}$ *is* α		$R=\sqrt{F_1^{\,2}+F_2^{\,2}+2F_1F_2\cos\alpha}$	$\tan\theta=\dfrac{F_2\sin\alpha}{F_1+F_2\cos\alpha}$
$\overrightarrow{F}_1$ *and* $\overrightarrow{F}_2$ *perpendicular*		$R=\sqrt{F_1^{\,2}+F_2^{\,2}}$	$\tan\theta=\dfrac{F_2}{F_1}$
$\overrightarrow{F}_1$ *and* $\overrightarrow{F}_2$ *are equal in magnitude*		$R=2F\cos\dfrac{\alpha}{2}$	$\overrightarrow{R}$ *bisects the angle between the two forces*
$\overrightarrow{F}_1$ *and* $\overrightarrow{F}_2$ *in the same direction*		$R=F_1+F_2$	$\overrightarrow{R}$ *in the same direction of the two forces*
$\overrightarrow{F}_1$ *and* $\overrightarrow{F}_2$ *in the opposite directions*		$R=\left\lvert F_1-F_2\right\rvert$	$\overrightarrow{R}$ *with the direction of greatest force*

Example:1 The forces of magnitude 8 , $8\sqrt{3}$ newton act at a point include an angle of measure 150^{o} Find their resultant in magnitude **and direction**

Solution

$$F_1 = 8 \quad , \quad F_2 = 8\sqrt{3} \quad , \quad \alpha = 150^\circ$$

$$\because R = \sqrt{F_1{}^2 + F_2{}^2 + 2F_1F_2\cos\alpha}$$

$$R = \sqrt{(8)^2 + (8\sqrt{3})^2 + 2\times 8\times 8\sqrt{3}\times\cos 150^\circ} = \sqrt{64} = 8 \ newton$$

$$\tan\theta = \frac{F_2\sin\alpha}{F_1 + F_2\cos\alpha} = \frac{8\sin 150^\circ}{8 + 8\sqrt{3}\cos 150^\circ} = \frac{\sqrt{3}}{3}$$

$$m(\angle\theta) = 30^\circ$$

the resultant is 8 N. and make an angle of measure 30^{o} with the first force

Example 2 Two forces of magnitude 5 , $5\sqrt{2}$ newton act at a point include an angle of measure 45° Find their resultant in magnitude and direction

Solution

$$F_1 = 5 \quad , \quad F_2 = 5\sqrt{2} \quad , \quad \alpha = 45^\circ$$

$$\because R = \sqrt{F_1{}^2 + F_2{}^2 + 2F_1F_2\cos\alpha}$$

$$R = \sqrt{(5)^2 + (5\sqrt{2})^2 + 2\times 5\times 5\sqrt{2}\times\cos 45^\circ} = \sqrt{125} = 5\sqrt{5} \ newton$$

$$\tan\theta = \frac{F_2\sin\alpha}{F_1 + F_2\cos\alpha} = \frac{5\sin 45^\circ}{5 + 5\sqrt{2}\cos 45^\circ} = \frac{1}{2} \qquad \Rightarrow \therefore m(\angle\theta) = 26^\circ 35^\backslash$$

the resultant is $5\sqrt{5}$N. and make an angle of measure $26^\circ 35^\backslash$ with the first force

Example :3 Two forces of magnitude 10 , $10\sqrt{3}$ newton act at a point their resultant is 10 newton find the measure of the angle between the two forces and direction

Solution

$$F_1 = 10 \qquad , \qquad F_2 = 10\sqrt{3} \qquad , \qquad R = 10$$

$$\cos\alpha = \frac{R^2 - F_1{}^2 - F_2{}^2}{2F_1F_2} = \frac{(10)^2 - (10)^2 - (10\sqrt{3})^2}{2\times10\times10\sqrt{3}}$$

$$\cos\alpha = \frac{-\sqrt{3}}{2} \qquad \therefore \Rightarrow m(\angle\alpha) = 150^\circ$$

Example: 4 Two forces of magnitude F , $\sqrt{3}F$ newton act at a point their resultant is $2F$ newton find the measure of the angle between the two forces and direction

Solution

$$F_1 = F \qquad , \qquad F_2 = \sqrt{3}F \qquad , \qquad R = 2F$$

$$R^2 = F_1{}^2 + F_2{}^2 + 2F_1F_2\cos\alpha$$

$$(2F)^2 = (F)^2 + (\sqrt{3}F)^2 + 2\times F\times\sqrt{3}F\times\cos\alpha$$

$$4F^2 = F^2 + 3F^2 + 2\sqrt{3}F^2\cos\alpha \qquad \Rightarrow 0 = 2\sqrt{3}F^2\cos\alpha$$

$$\cos\alpha = 0 \qquad \Rightarrow m(\angle\alpha) = 90^\circ$$

Exampl: 5 Two forces of magnitude 4 ,F newton act at a point the measure of the angle between them 135° If the direction of their resultant inclend to the force F by angle of measure 45° find the value of F

Solution

$F_1 = 4 \quad , \quad F_2 = F \quad , \quad \alpha = 135^\circ \quad , \quad \theta = 45^\circ$

$$\tan\theta = \frac{F_2 \sin\alpha}{F_1 + F_2 \cos\alpha} \quad \Rightarrow \tan 45^\circ = \frac{4\sin 135^\circ}{F + 4\cos 135^\circ}$$

$$\Rightarrow 1 = \frac{4 \times \frac{\sqrt{2}}{2}}{F + 4 \times \frac{-\sqrt{2}}{2}} \quad \Rightarrow F - 2\sqrt{2} = 2\sqrt{2} \quad \Rightarrow F = 4\sqrt{2}$$

Example: 6 Two forces one of them twice the other and having a resultant. If the larger force is dubeled and the smaller force increased by 4 gm.wt. then the resultant will be in the same direction. find the two forces

Solution

first Let: $F_1 = F \quad , \quad F_2 = 2F \Rightarrow \tan\theta_1 = \frac{2F\sin\alpha}{F + 2F\cos\alpha}$

Second $\quad F_1 = F + 4 \quad , \quad F_2 = 4F \Rightarrow \tan\theta_2 = \frac{4F\sin\alpha}{(F+4) + 4F\cos\alpha}$

$$\tan\theta = \frac{F_2 \sin\alpha}{F_1 + F_2 \cos\alpha} \quad \Rightarrow \tan 45^\circ = \frac{4\sin 135^\circ}{F + 4\cos 135^\circ}$$

$\because$ *The direction is constant* $\Rightarrow \tan\theta_1 = \tan\theta_2$

$$\therefore \frac{2\cancel{F\sin\alpha}}{F + 2F\cos\alpha} = \frac{4\cancel{F\sin\alpha}}{(F+4) + 4F\cos\alpha}$$

$$\therefore \quad \frac{1}{F + 2F\cos\alpha} = \frac{2}{(F+4) + 4F\cos\alpha}$$

$$F + 4 + \cancel{4F\cos\alpha} = 2F + \cancel{4F\cos\alpha}$$

$\therefore F = 4 \qquad \therefore$ *The two forces are 4 and 8*

Example: 7 Two forces of magnitude 7 , 14 newton act at a point their resultant perpendicular to the first force. find the measure of the angle between the two forces and resultant

Solution

$F_1 = 7$, $F_2 = 14$, $\theta = 90^\circ$

$$\tan\theta = \frac{F_2 \sin\alpha}{F_1 + F_2 \cos\alpha} \Rightarrow \tan 90^\circ = \frac{14\sin\alpha}{7 + 14\cos\alpha} = \frac{1}{0}$$

$$\therefore 7 + 14\cos\alpha = 0 \quad \Rightarrow \cos\alpha = \frac{-1}{2} \quad \Rightarrow m(\angle\alpha) = 120^\circ$$

$$R = \sqrt{F_1^{\ 2} + F_2^{\ 2} + 2F_1F_2 \cos\alpha}$$

$$R = \sqrt{(7)^2 + (14)^2 + 2\times7\times14\times\cos 120^\circ} = \sqrt{147} = 7\sqrt{3} \ newton$$

Example: 8 Two forces 6 and F *meet at a point and the angle between them* 120°*. If the line of resultant perpendicular to the first force find the value of* F

Solution

Solution $F_1 = 6$, $F_2 = F$ $\alpha = 120^\circ$, $\theta = 90^\circ$

$$\tan\theta = \frac{F_2 \sin\alpha}{F_1 + F_2 \cos\alpha} \quad \therefore \quad \tan 90^\circ = \frac{6\sin 120^\circ}{6 + F\cos 120^\circ} = \frac{1}{0}$$

$$\therefore 6 + F\cos 120^\circ = 0 \quad \Rightarrow F \times \frac{-1}{2} = -6 \quad \Rightarrow F = 12 \ \ N.$$

Special cases

(1) If $\overrightarrow{F}_1, \overrightarrow{F}_2$ *are equal in magnitude* $\overrightarrow{F_1} = \overrightarrow{F}_2 = F$ *and thier lines of actions not on the same S.line then*

$$\boxed{\mathcal{R} = 2F\cos\frac{\alpha}{2} \qquad \theta = \frac{\alpha}{2}}$$

Remark: $\alpha \rightarrow$ *is the angle between* F_1 , F_2

$\theta \rightarrow$ *is the angle between* R , F_1

2. If $\vec{F}_1, \vec{F}_2$ are perpendicular : then

$$\mathcal{R} = \sqrt{F_1^{\,2} + F_2^{\,2}} \qquad , \qquad \tan\theta = \frac{F_2}{F_1}$$

3. If $\vec{F}_1, \vec{F}_2$ are in the same direction : then

$\mathcal{R} = F_1 + F_2$ and the direction of the resultant is the same of the two forces (the resultant is maximum)

4. If $\vec{F}_1, \vec{F}_2$ are in opposite directions : then

$\mathcal{R} = |F_1 - F_2|$ and the direction of the resultant is the same of the greatest forces (the resultant is minimum)

EX .9 Two perpendicular forces of magnitude 30 , 40 gm.wt. find its resultant perfectelly

Solution

$$F_1 = 30 \qquad , \qquad F_2 = 40 \qquad , \qquad \alpha = 90^\circ$$

$$R = \sqrt{F_1^{\,2} + F_2^{\,2}}$$

$$R = \sqrt{(30)^2 + (40)^2} = \sqrt{2500} = 50 \text{ gm.wt}$$

$$\tan\theta = \frac{F_2}{F_1} = \frac{40}{30} = \frac{4}{3}$$

$$m(\angle\theta) = 53^\circ 8'$$

EX .10 *Two forces of magnitude 10 , 20 gm.wt act at a point thier line of action on the same st.line find its resultant if : (1) the two forces in the same direction (2) the two forces in opposite directions*

Solution $F_1 = 10$, $F_2 = 20$

(1) the two forces in the same direction

$R = F_1 + F_2 = 10 + 20 = 30$

and the direction of the resultant is the same of two forces

(2) the two forces in opposite directions

$\mathcal{R} = |F_1 - F_2| = \mathcal{R} = |10 - 20| = 10$

and the direction of the resultant is the same of greatest forces

EX .11 *Two forces of magnitude 10 , 15 act at a point, thier line of action on the same st.line find (1) the maximum resultant and its direction (2) the minimum resultant and its direction*

Solution $F_1 = 10$, $F_2 = 15$

(1) the maximum resultant

$R = F_1 + F_2 = 10 + 15 = 25 \quad gm.wt$

and the direction of the resultant is the same of two forces

(2) the minimum resultant

$\mathcal{R} = |F_1 - F_2| = \mathcal{R} = |10 - 15| = 5 \quad gm.wt$

and the direction of the resultant is the same of greatest forces

12 *Two forces of magnitude 10 , 10 $gm.wt$ act at a point, and the angle between them 120° find thier resultant and direction*

Solution $F_1 = 10$, $F_2 = 10$

$$R = 2F\cos\frac{\alpha}{2} = 2\times 10\cos 60^\circ = \frac{1}{2}\ \ gm.wt$$

and the direction of the resultant bisects the angle between the two forces

13 *Three forces of magnitude 5 , 10 , $4\sqrt{7}$ $gm.wt$ act at a point, and the angle between the first and second 60° find the max and min. value of the resultant of forces*

Solution

First find the resultant of first and second forces

$$R = \sqrt{F_1^{\,2} + F_2^{\,2} + 2F_1F_1\cos\alpha} = \sqrt{(5)^2 + (10)^2 + 2\times 5\times 10\cos 60^\circ} = \sqrt{175} = 5\sqrt{7}$$

second : find the resultant of two forces $4\sqrt{7}$ *and* $5\sqrt{7}$

The max. limit = $R = F_1 + F_2 = 4\sqrt{7} + 5\sqrt{7} = 9\sqrt{7}$

The min. limit = $R = |F_1 - F|_2 = \left|4\sqrt{7} - 5\sqrt{7}\right| = \sqrt{7}$

14 *Two forces of magnitude F , $\sqrt{2}F$ act at a point, and the angle between them its tangent =-1 , thier resultant = 4 N.*
Find : (1) the norm of F
(2) measure of the angle between the resultant and the first force

Solution $F_1 = F$, $F_2 = \sqrt{2}F$ $\tan\alpha = -1 \Rightarrow \alpha = 135^\circ$

$$R^2 = F_1^{\,2} + F_2^{\,2} + 2F_1F_2\cos\alpha = (F)^2 + (\sqrt{2}F)^2 + 2\times F\times\sqrt{2}F\cos 135^\circ$$

$$16 = F^2 + 2F^2 + 2\cancel{\sqrt{2}}F^2\times\frac{-1}{\cancel{\sqrt{2}}} = F^2 \qquad \Rightarrow \boxed{F = 4}$$

$$\tan\theta = \frac{F_2\sin\alpha}{F_1 + F_2\cos\alpha} = \frac{4\sqrt{2}\sin 135^\circ}{4 + 4\sqrt{2}\cos 135^\circ} = \frac{4}{0} \Rightarrow m(\angle\theta) = 90^\circ$$

15 *The ratio between magnitude of two forces is 1 : $\sqrt{2}$, and the line of action of the resultant inclined the greatest force by 45° Find the measure of the angle between the two forces then find the value of each force, given that thier resultant = $3\sqrt{2}$*

Solution

Let the two forces F *and* $F\sqrt{2}$

$\because \theta = 45^\circ \qquad \therefore \Rightarrow \tan\theta = \dfrac{F_2 \sin\alpha}{F_1 + F_2 \cos\alpha}$

$\therefore \Rightarrow \tan 45^\circ = \dfrac{F \sin\alpha}{\sqrt{2}F + F\cos\alpha} \qquad \therefore \Rightarrow 1 = \dfrac{F \sin\alpha}{\sqrt{2}F + F\cos\alpha}$

$\therefore F\sin\alpha = \sqrt{2}F + F\cos\alpha \qquad \therefore \Rightarrow \sin\alpha = \sqrt{2} + \cos\alpha$

$\therefore \sin\alpha - \cos\alpha = \sqrt{2} \xrightarrow{\text{by squaring}} \therefore \underbrace{\sin^2\alpha + \cos^2\alpha}_{=1} - \underbrace{2\sin\alpha\cos\alpha}_{=\sin 2\alpha} = 2$

$\therefore \sin 2\alpha = -1 \qquad \therefore \Rightarrow m(2\alpha) = 270^\circ \qquad \therefore \Rightarrow m(\alpha) = 135^\circ$

$R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\alpha$

$(2\sqrt{3})^2 = F^2 + 2F^2 + 2 \times F \times \sqrt{2}F \cos 135^\circ$

$18 = 3F^2 - 2F^2$

$F^2 = 18 \qquad \therefore \Rightarrow F = 3\sqrt{2}$

$\therefore$ *The two forces are* $3\sqrt{2}$ *and* $6\sqrt{2}$

16 *If the angle between two forces is right then thier resultant will be* $\sqrt{10}$ N. *and if the measure of the angle between them is 60° then thier resultant will be* $\sqrt{13}$ N. *find the value of these forces*

Solution *In the first case* $\alpha = 90^\circ$

$R^2 = F_1^2 + F_2^2 \qquad \therefore \Rightarrow 10 = F_1^2 + F_2^2 \rightarrow (1)$

In the second case $\alpha = 60^\circ$

$R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\alpha \qquad \therefore \Rightarrow 13 = \underbrace{F_1^2 + F_2^2}_{=10 \text{ "from 1"}} + 2F_1F_2\cos 60^\circ$

$\therefore 13 = 10 + \cancel{2}F_1F_2 \times \dfrac{1}{\cancel{2}} \qquad \therefore \Rightarrow F_1F_2 = 3 \rightarrow (2)$

from(1) & (2) $F_1 = 3$ *or* 1 & $F_2 = 1$ *or* 3

17 *Two prependicular forces one of them $\frac{3}{4}$ the other, and thier resultant $=20N$. find the two forces then find the angle between them if the resultant became $4\sqrt{13}$*

Solution *Let the two forces* $F_1 = 3F\,,\ F_2 = 4F$

$\because$ *Two forces are prependicular*

$R^2 = F_1^2 + F_2^2 \qquad \therefore \Rightarrow 400 = 9F^2 + 16F^2$

$\therefore \Rightarrow 400 = 25F^2 \ \therefore \Rightarrow F = 4$

$F_1 = 3\times 4 = 12N\,.\ ,\ F_2 = 4\times 4 = 16N\,.$

To find the angle :

$R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\alpha \qquad \therefore \Rightarrow (4\sqrt{13})^2 = (12)^2 + (16)^2 + 2\times 12\times 16\cos\alpha$

$208 = 400 + 384\cos\alpha \qquad \therefore \Rightarrow m(\angle\alpha) = 120^\circ$

18 *F_1 , F_2 are two forces act at a point. their resultant is R gm.wt when the angle between them is 120 and their resultant becomes $R\sqrt{3}$ gm.wt., when F_2 becomes in the opposite direction. prove that $F_1 = F_2$ and the resultant in the first case is perp. to the resultant in the second case.*

Solution

$R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\alpha$

in cses 1. $\therefore \Rightarrow R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos 120^\circ$

$\therefore \Rightarrow R^2 = F_1^2 + F_2^2 + F_1F_2 \xrightarrow{\times 3}$

$\therefore \Rightarrow 3R^2 = 3F_1^2 + 3F_2^2 + 3F_1F_2 \longrightarrow (1)$

in cses 1. $\therefore \Rightarrow R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos 60^\circ$

$\therefore \Rightarrow 3R^2 = F_1^2 + F_2^2 + F_1F_2 \longrightarrow (2)$

from (1) & (2) by subtraction

$2F_1^2 + 2F_2^2 - 4F_1F_2 = 0 \xrightarrow{\div 2} F_1^2 + F_2^2 - 2F_1F_2 = 0$

$\therefore (F_1 - F_2)^2 = 0 \qquad \Rightarrow F_1 - F_2 = 0 \qquad \Rightarrow F_1 = F_2$

$\therefore$ *the two forces are equal* $\quad \therefore$ *the resultant bisects the angle between them in the two cases*

Homework

[1] Complete :

(1) The action of a force on a body determined by ....

(2) The resultant force of two forces $\overline{F_1}$, $\overline{F_2}$ ewuals ....

(3) The maximum falue of the resultant of two forces 4 , 6 newton meeting at a point equals ....

(4) The minimum falue of the resultant of two forces 4 , 6 newton meeting at a point equals ....

(5) 2 , 3 newton two forces if the angle between them 60 , then the resultant of two forces = ...

[2] Choose

(1) The resultant of two forces 3 , 5 newton and the angle between them 60° = ... (2 , 6 , 7 , 8) newton

(2) Two forces of magnitude 3 , 4 newton and their resultant = 5 newton , then the measure of the angle between them = ... (30° , 45° , 60° , 90°)

(3) Two forces are equal in magnitude of 6 newton for each and their resultant = 6 newton , then the measure of the angle between them = (30° , 60° , 120° , 150°)

(4) Two forces of magnitude 3 , F newton , the measure of the angle between them = 120 , if their resultant is perpendicular to the first force , then the value of F = ...

(1.5 , 2 , $2\sqrt{3}$, 6) newton

[3] Answer the following questions :

(1) Two forces of magnitude 5 , 10 newton act at a partical and included between them angle of 120 , Find their resultant and the measure of angle between the resultant and the first force

(2) Two forces of magnitude 3 ,$3\sqrt{2}$ kg.wt act at a partical and included between them angle of 120 , Find their resultant and its direction

(3) Two forces of magnitude 15 , 8 kg.wt act at a partical and their resultant = 13 kg.wt , Find measure of the angle between these two forces

(4) Two forces of magnitude 8 , F newton act at a partical the measure of the angle the angle betwwen them 120° , if their resultant = $F\sqrt{3}$ newton Find falue of F

(5) Two forces of magnitude 4 , F newton act at a partical the measure of the angle betwwen them 135° , if their resultant inclined to the force F by angle of measure 45°. Find value of F

(6) Two forces of magnitude 4 , F newton act at a partical the measure of the angle the angle betwwen them 120° , if their resultant perpendicular to the first force, Find falue of F

(7) Two forces of magnitude F , $F\sqrt{3}$ newton act at a partical if their resultant = 2F newton Find the measure of the angle between thes two forces.

(8) Two forces of magnitude 12 , 15 newton act at a partical the angle betwwen them of cosine $\frac{-4}{5}$. Find their resultant

(9) Two equal forces of magnitude F kg.wt. and included an angle of measure 120. If the two forces are doubled and the angle between them became 60 . the resultant will increase by 11 kg.wt. than the first case Find the value of F

(10) Two forces of magnitude 12 , F kg.wt. act at a point, and the first act in east direction , the other act at 60 in the direction south of west. Find the value of F and the resultant of two forces if the line of action of the resultant act at 30 in the direction south of east

(11) F_1,F_2 are two forces act at a point and include angle of measure 120° between them , and their resultant = $\sqrt{19}$ newton. If the angle between them became 60° , Then the resultant became 7 newton Find each of F_1 , F_2

(12) Two forces of magnitude F , 2F kg.wt. act at a point, If the second force is doubled and the first force increased by 15 kg.wt. Then the direction of the resultant not change Find the value of F

Creative thinking:

23 Two forces of equal magnitude meeting at a point and the magnitude of their resultant equals 12 kg.wt. If the direction of one of them is reversed, then the magnitude of the resultant becomes 6 kg.wt. Find the magnitude of each force.

24 Two forces of magnitudes K , F and the magnitude of their resultant equals 2K, if the measure of the angle between them equals θ, If the measure of the angle changes and become $(180^\circ - \theta)$ then the magnitude of their resultant will decreased to its half. Find the ratio between K , F.

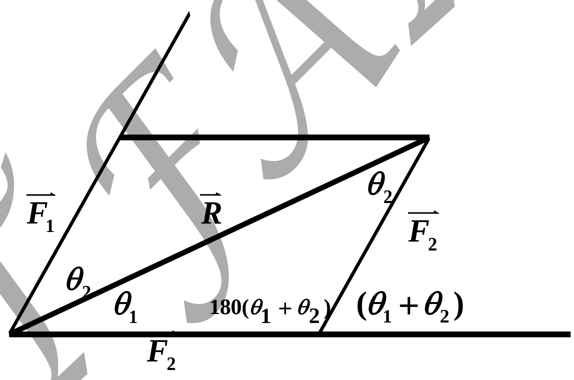
25 F , 2F are two forces act on a particle and enclose between them an angle of measure α. The magnitude of their resultant equals $\sqrt{5}$ F (m + 1) and if the measure of the angle between them becomes $(90^\circ - \alpha)$, then the magnitude of the resultant will be $\sqrt{5}$ F (m - 1).

Prove that $\tan \alpha = \frac{m - 2}{m + 2}$

Lesson (2) Resolution of a force into two components

It is the inverse operation of the resultant which find the two forces and with given the resultant R. So we can perform this operation by infinite number of ways, becaus we can draw an infinite number of parallelograms which have one diagonal. But if the direction of the two forces F , F are given. Then the resultant is unique.

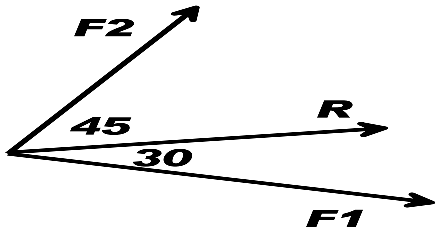
From the sine law $\boxed{\dfrac{F_1}{\sin\theta_2} = \dfrac{F_2}{\sin\theta_1} = \dfrac{R}{\sin(\theta_1+\theta_2)}}$



$\boxed{F_1 = \dfrac{R\sin\theta_2}{\sin(\theta_1+\theta_2)}}$ $\boxed{F_2 = \dfrac{R\sin\theta_1}{\sin(\theta_1+\theta_2)}}$

EX .1 Resolve a force 100N. in two directions. The first inclines by 30° to the force and the second by 45° in the other directionof force.

Solution : R=100 $\theta_1 = 30^\circ$ $\theta_2 = 45^\circ$



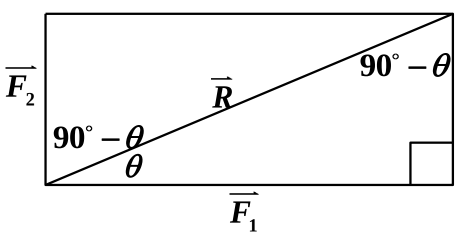
$$F_1 = \frac{R\sin\theta_2}{\sin(\theta_1+\theta_2)} = \frac{100\sin 45^\circ}{\sin 75^\circ} = 73.2N$$

$$F_2 = \frac{R\sin\theta_2}{\sin(\theta_1+\theta_2)} = \frac{100\sin 30^\circ}{\sin 75^\circ} = 51.8N$$

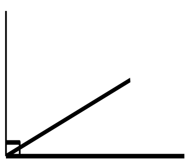
Resolution of a force into two perp.directions

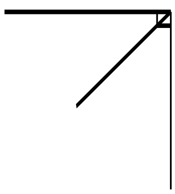
$$\theta_1+\theta_2 = 90^\circ \Rightarrow \frac{F_1}{\sin(90^\circ-\theta)} = \frac{F_2}{\sin\theta} = \frac{R}{\sin 90^\circ}$$

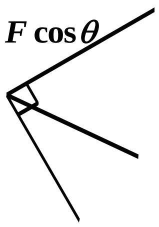
$$\Rightarrow \frac{F_1}{\cos\theta} = \frac{F_2}{\sin\theta} = \frac{R}{1}$$



$\boxed{F_1 = R\cos\theta}$ $\boxed{F_2 = R\sin\theta}$



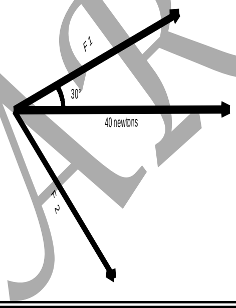




Study Carefully these questions

1 *Resolve a horizontal force 40 newtons into two perp. directions one of them inclines by 30 with the horizzontal upwords*

Solution



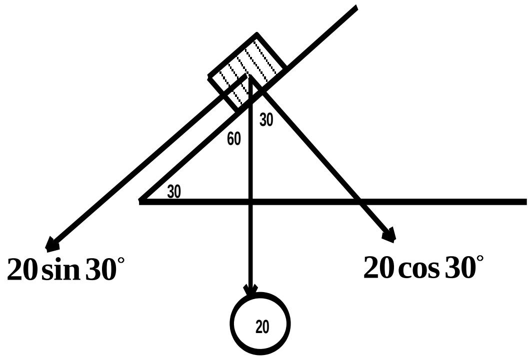
$$F_1 40\cos 30 = 20\sqrt{3}\ \mathcal{N}$$
$$F_2 40\sin 30 = 20\ \mathcal{N}$$

2 *A body of weight 20 newtons is placed on an inclined plane of inclination 30° to the horizontal. Find the components of the weight in the direction of the plane and the normal to it*

Solution

$$\because F_1 = R\cos\theta = 20\cos 30^\circ = 10\sqrt{3}\ \textit{newtons}$$

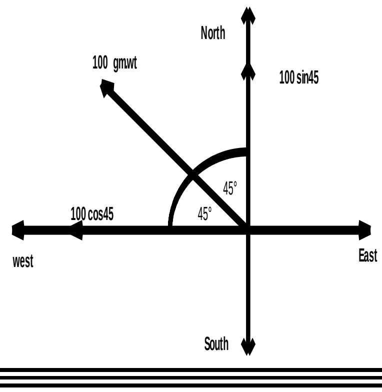
$$\because F_2 = R\sin\theta = 20\sin 30^\circ = 10\ \textit{newtons}$$



3 *A Force of magnitude 100 gm.wt acts in the directions north west. find its components due to north and west.*

Solution

$$\because F_1 = R\cos\theta = 100\cos 45^\circ = 50\sqrt{2}\ \textit{newtons}$$
$$\because F_2 = R\sin\theta = 100\sin 45^\circ = 50\sqrt{2}\ \textit{newtons}$$



Homework

[1] Complete :

(1) A force of magnitude 6 newton act in the north direction resolvrd into two perpendicular components, then its component in the east direction = ... newton

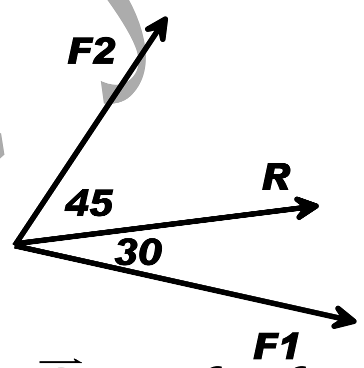
(2) A force of magnitude $4\sqrt{2}$ newton act in the east direction resolved into two perpendicular components then its component in the south of east = ... newton

(3) In the opposite figure : If the force $\overrightarrow{\mathcal{R}}$ resolved into two components $\overrightarrow{\mathcal{F}_1}$, $\overrightarrow{\mathcal{F}_2}$,

If $\left\|\overrightarrow{\mathcal{R}}\right\|$ = 12 newton

Then $\mathcal{F}_1$= ...... newton

$\mathcal{F}_2$ = ... newton

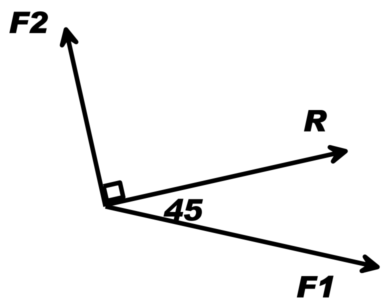


(4) In the opposite figure : If the force $\overrightarrow{\mathcal{R}}$ resolved into two components $\overrightarrow{\mathcal{F}_1}$, $\overrightarrow{\mathcal{F}_2}$,

If $\left\|\overrightarrow{\mathcal{R}}\right\|$ = 18 newton

Then $\mathcal{F}_1$ = ... newton

$\mathcal{F}_2$ = ... newton

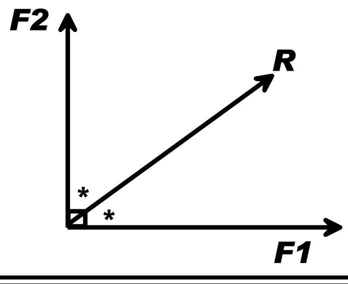


(5) In the opposite figure : If the force $\overrightarrow{\mathcal{R}}$ resolved into two perpendicular components $\overrightarrow{\mathcal{F}_1}$, $\overrightarrow{\mathcal{F}_2}$,

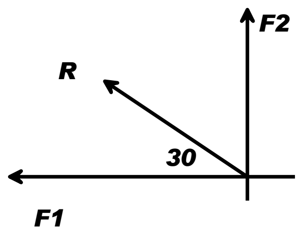
If $\left\|\overrightarrow{\mathcal{R}}\right\|$ = $6\sqrt{2}$ kg.wt.

Then $\left\|\overrightarrow{\mathcal{F}_1}\right\|$= ...... kg.wt.

$\left\|\overrightarrow{\mathcal{F}_2}\right\|$ = ... kg.wt.



(6) In the opposite figure : The force $12\sqrt{2}$ act in direction 30° south of west then:
the magnitude of its component in weast direction = ...
the magnitude of its component in south direction = ...



[2] Answer the following questions :

(1) The magnitude of a force is 600 gm.wt. act at a point Find the two components in two opposite directions make two angles of measure 30° , 45°

(2) A force of magnitude 120 newton act in north of east direction Find the two components in east direction and north direction

(3) Resolve the force of magnitude 160 gm.wt into two perpendicular directions one of them inclined to the horizontal by angle of measure 30° upwards

(4) A force of magnitude 18 newton act in south. find the two two components in two directions 60 east of south and the other in 30 west of south

(5) A body of weight 42 newton is placed on a smooth inclined plane by 60 with the horizontal. Find the two components of the weight in the direction of the line of the gratest slope and the direction perpendicular to it

Creative thinking:

12 An inclined plane of length 130 cm and height 50 cm, a rigid body of weight 390 gm wt. is placed on it. Find the two components of the weight in the direction of the line of the greatest slope of the plane and the direction normal to it.

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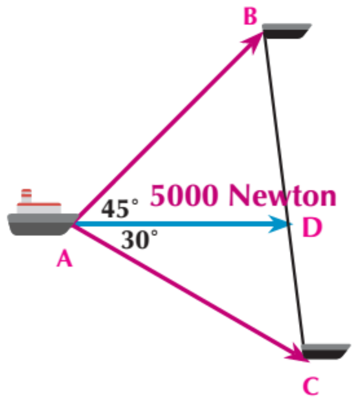
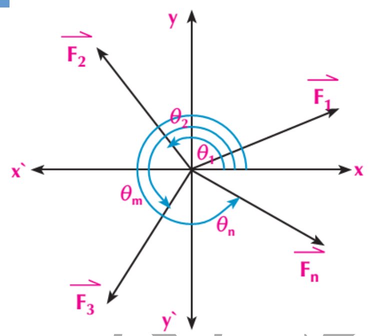
13 A cruiser is pulled by two ships B and C using two strings hanged to a point A on the cruiser ,the angle between the two strings equals 75° , if the angle between one of the strings and $\overrightarrow{AD}$ equals 45° and the resultant of the forces used to pull the cruiser equals 5000 Newton and acts on $\overrightarrow{AD}$ Find the tension in the two strands.



The resultant of coplanar forces meeting at a point

$$\overrightarrow{R} = \left(\sum_{r=1}^{n} F_r \cos\theta_r\right)\overrightarrow{i} + \left(\sum_{r=1}^{n} F_r \sin\theta_r\right)\overrightarrow{j}$$



1 *Four coplanar forces act at a point, the 1st of magnitude 8N and acts towards east, the 2nd of magnitude 4N and acts in direction 60° north of eastn , the 3rd of magnitude 10N and acts in direction 60° north of west, and the 4th of magnitude $6\sqrt{3}$N and acts in direction 30° south of west, find the magnitude and direction of their resultant.*

Solution

$\because X = 8 + 4\cos 60^\circ - 10\cos 60^\circ - 6\sqrt{3}\cos 30^\circ =$

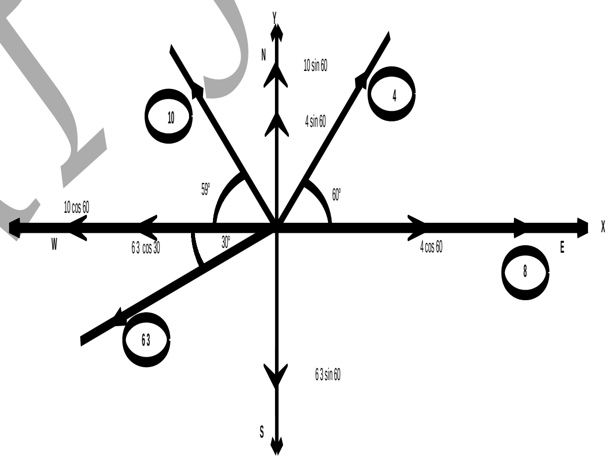
$\therefore X = 8 + 2 - 5 - 9 = -4\ \hat{i}$

$\because Y = 4\sin 60^\circ + 10\sin 60^\circ - 6\sqrt{3}\sin 60^\circ$

$\therefore Y = 2\sqrt{3} + 5\sqrt{3} - 3\sqrt{3} = 2F\ \sin 60^\circ = 4\sqrt{3}\ \hat{j}$

$\overrightarrow{R} = x\hat{i} + y\hat{j} \qquad \Rightarrow \therefore \overrightarrow{R} = -4\hat{i} + 4\sqrt{3}\hat{j}$

$R = \sqrt{x^2 + y^2} = \sqrt{(-4)^2 + \left(4\sqrt{3}\right)^2} = 8N$



the direction of the resultant

$\tan\alpha = \dfrac{y}{x} = \dfrac{4\sqrt{3}}{-4} = -\sqrt{3}$ *(forget negative)*

$\alpha = 60^\circ \quad or \quad \alpha = 120^\circ$

Another Solution by using polar system

Forces are $(8, 0^\circ)$, $(4, 60^\circ)$, $(10, 120^\circ)$, $(6\sqrt{3}, 210^\circ)$

$X = 8\cos 0^\circ + 4\cos 60^\circ + 10\cos 120^\circ + 6\sqrt{3}\cos 210^\circ$

$X = -4\hat{i}$

$Y = 8\sin 0^\circ + 4\sin 60^\circ + 10\sin 120^\circ + 6\sqrt{3}\sin 210^\circ$

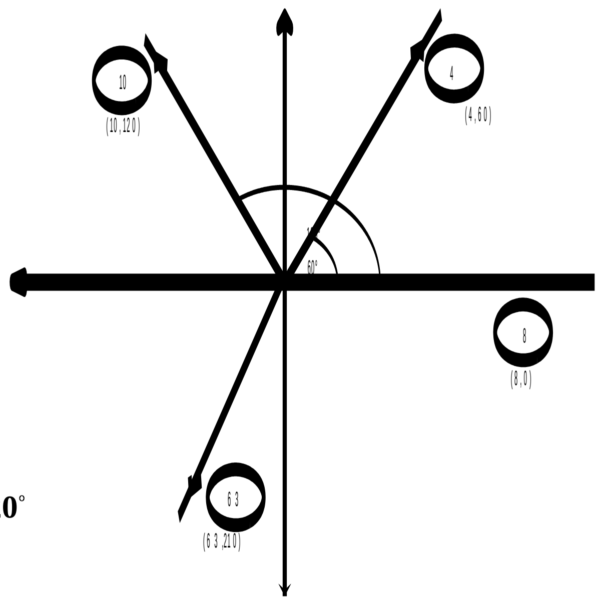
$Y = 4\sqrt{3}\hat{j}$

$\mathcal{R} = -4\hat{i} + 4\sqrt{3}\hat{j}$

$R = \sqrt{(-4)^2 + (4\sqrt{3})^2} \qquad \Rightarrow \therefore R = 8N$

$\tan\theta = \dfrac{y}{x} = \dfrac{4\sqrt{3}}{-4} = -\sqrt{3} \qquad \Rightarrow \therefore \alpha = 60^\circ \ \ or \ \ \alpha = 120^\circ$

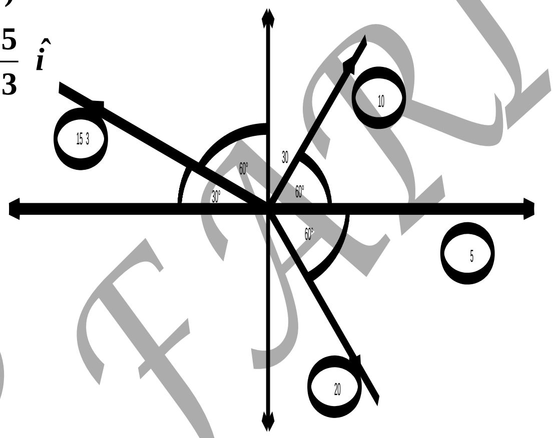
with the positive direction of X-axis



2 *The forces 5 , 10 , $15\sqrt{3}$, 20 newtons act at a partical, the measure of the angle between the first two forces is 60° , between the second and the third force is 90° , between the third and the fourth is 150° , find the resultant and its direction*

Solution

Forces are $(5,0^\circ),\ (10,60^\circ),\ (15\sqrt{3},150^\circ),\ (20,300^\circ)$



$\because X = 5\cos 0^\circ + 10\cos 60^\circ + 15\sqrt{3}\cos 150^\circ + 20\cos 300^\circ = -\dfrac{5}{3}\ \hat{i}$

$\because Y = 5\sin 0^\circ + 10\sin 60^\circ + 15\sqrt{3}\sin 150^\circ + 20\sin 300^\circ$

$= 3F \times \dfrac{\sqrt{3}}{2} - 4F \times \dfrac{1}{2} - 3F\ \dfrac{\sqrt{3}}{2} = \dfrac{5\sqrt{3}}{2}\ \hat{j}$

$\overrightarrow{R} = x\hat{i} + y\hat{j} \qquad \Rightarrow \therefore \overrightarrow{R} = -\dfrac{5}{2}\hat{i} + \dfrac{5\sqrt{3}}{2}\hat{j}$

$R = \sqrt{x^2 + y^2} = \sqrt{\left(-\dfrac{5}{2}\right)^2 + \left(-\dfrac{5\sqrt{3}}{2}\right)^2} = 5$ *newtons*

the direction of the resultant

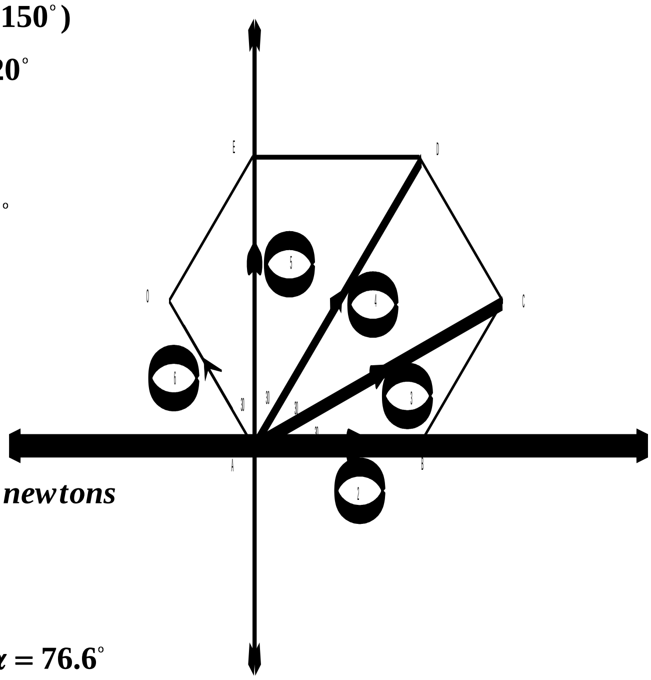
$tan\alpha = \dfrac{y}{x} = \dfrac{-\dfrac{5\sqrt{3}}{2}}{\dfrac{5}{2}} = -\sqrt{3} \qquad \Rightarrow \therefore \alpha = 60^\circ$ *or* 120°

---

3 *ABCDEO is regular hexagon forces of magnitude 2, 3, 4, 5, 6 newtons act at $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$, $\overrightarrow{AE}$, $\overrightarrow{AO}$ respectively, find their resultant*

Solution

Forces are $(2,0^\circ),\ (3,30^\circ),\ (4,60^\circ),\ (5,90^\circ),\ (6,150^\circ)$



$\because X = 2\cos 0^\circ + 3\cos 30^\circ + 4\cos 60^\circ + 5\cos 90^\circ + 6\cos 120^\circ$

$= \dfrac{2+3\sqrt{3}}{2}\ \hat{i}$

$\because Y = 2\sin 0^\circ + 3\sin 30^\circ + 4\sin 60^\circ + 5\sin 90^\circ + 6\sin 120^\circ$

$= \dfrac{13+10\sqrt{3}}{2}\ \hat{j}$

$\overrightarrow{R} = x\hat{i} + y\hat{j} \qquad \Rightarrow \therefore \overrightarrow{R} = \dfrac{2+3\sqrt{3}}{2}\ \hat{i} + \dfrac{13+10\sqrt{3}}{2}\ \hat{j}$

$R = \sqrt{x^2 + y^2} = \sqrt{\left(\dfrac{2+3\sqrt{3}}{2}\right)^2 + \left(\dfrac{13+10\sqrt{3}}{2}\right)^2} = 15.58$ *newtons*

the direction of the resultant

$tan\alpha = \dfrac{y}{x} = \left(\dfrac{13+10\sqrt{3}}{2}\right) \div \left(\dfrac{2+3\sqrt{3}}{2}\right) = 4.2 \qquad \Rightarrow \therefore \alpha = 76.6^\circ$

$\boxed{4}$ *ABCD is a rectangle in which* $AB = 7cm$, $BC = 3cm$, $O \in \overline{AB}$, *where* $AO = 3cm$, *four forces of magnitude* $2,\ 5,\ 3,\ 6\sqrt{2}$, *kg.wt act at the point O, in the direction of* $\overrightarrow{OB}$, $\overrightarrow{OC}$, $\overrightarrow{BC}$, $\overrightarrow{OD}$ *respectively, find the magnitude of the resultant of these forces and prove that it is parallel to* $\overrightarrow{BC}$

Solution

$\because X = 2 + 5\cos\theta - 6\sqrt{2}\cos 45^\circ$

$\therefore X = 2 + 5(\frac{4}{5}) - 6\sqrt{2} \times \frac{\sqrt{2}}{2} = 0\ \hat{i}$

$\because Y = 5\sin\theta^\circ + 3 + 6\sqrt{2}\sin 45^\circ$

$= 5 \times \frac{3}{5} + 3 + 6\sqrt{2} \times \frac{\sqrt{2}}{2} = 12\ \hat{j}$

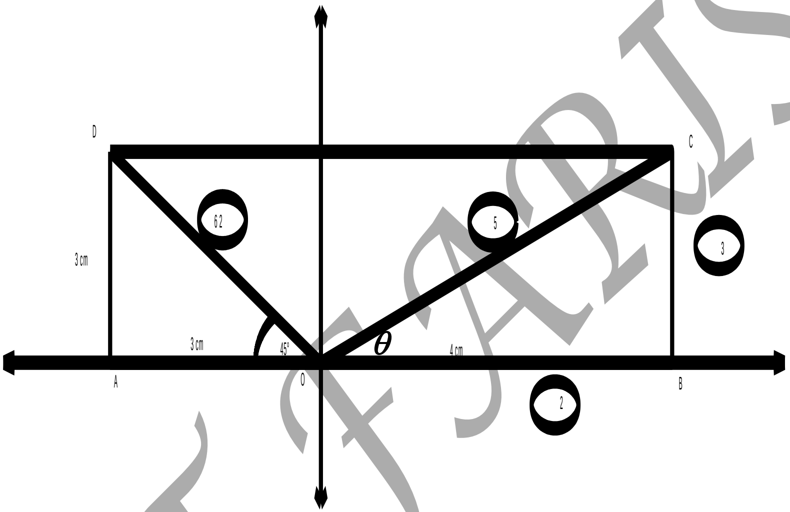
$\overrightarrow{R} = x\hat{i} + y\hat{j} \qquad \Rightarrow \therefore \overrightarrow{R} = 0\hat{i} + 12\hat{j}$

$R = \sqrt{x^2 + y^2} = \sqrt{(0)^2 + (12)^2} = 12$ *kg.wt*

the direction of the resultant

$tan\alpha = \frac{y}{x} = \frac{12}{0} = -\sqrt{3} \qquad \Rightarrow \therefore \alpha = 90^\circ$

$\therefore R \parallel \overrightarrow{BC}$



$\boxed{5}$ *The forces F, 8, k, 5,* $8\sqrt{3}$ *act at a particle due east, 60 north of east, north, west, and south. If the resultant is 4N, due 60 north of the east. Find the value of F and k.*

Solution <u>**first**</u> : $F \to E \quad , \ 8 \to 60\ N\ of\ E$

$k \to N \ , \qquad 5 \to W \ , \qquad 8\sqrt{3} \to S$

Forces are $(F, 0^\circ),\ (8, 60^\circ),\ (k, 90^\circ),\ (5, 180^\circ),\ (8\sqrt{3}, 270^\circ)$

$\because X = F\cos 0^\circ + 8\cos 60^\circ + k\cos 90^\circ + 5\cos 180^\circ + 8\sqrt{3}\cos 270^\circ$

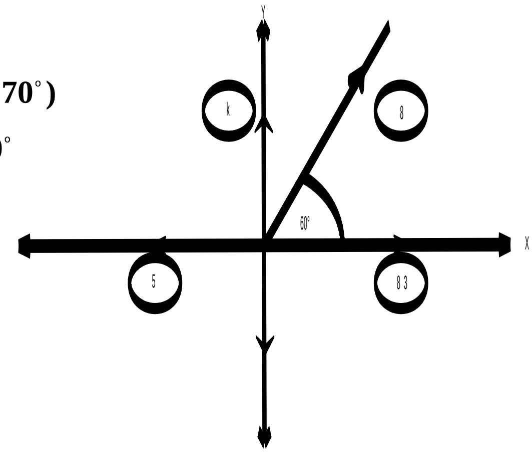
$= (F - 1)\ \hat{i}$

$\because Y = F\sin 0^\circ + 8\sin 60^\circ + k\sin 90^\circ + 5\sin 180^\circ + 8\sqrt{3}\sin 270^\circ$

$= (k - 4\sqrt{3})\ \hat{j}$

$\overrightarrow{R} = x\hat{i} + y\hat{j}$

$\Rightarrow \therefore \overrightarrow{R} = (F - 1)\ \hat{i} + (k - 4\sqrt{3})\ \hat{j} \to (1)$



<u>**Second**</u> :

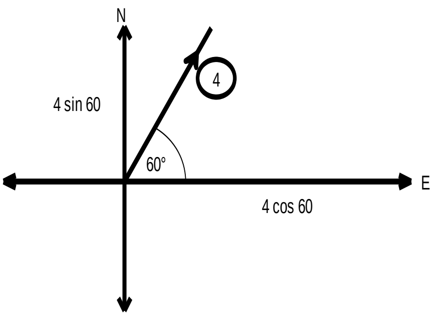
$R = 4 \to 60^\circ\ N\ of\ E$

$X = 4\cos 60 i$

$Y = 4\sin 60 j$

$R = 2i + 2\sqrt{3}j \to (2)$

from (1) & (2) *we get* $F = 3, \quad k = 6\sqrt{3}$



Homework

Complete the following:

1. If the forces $\overrightarrow{F_1} = 2\,\overrightarrow{i}$, $\overrightarrow{F_2} = \overrightarrow{i} - 2\,\overrightarrow{j}$, $\overrightarrow{F_3} = 6\,\overrightarrow{j}$ then:
   the magnitude of the resultant of the forces = ... and its direction = ...

2. If the forces $\overrightarrow{F_1} = 2\,\overrightarrow{i} - 2\,\overrightarrow{j}$, $\overrightarrow{F_2} = 4\,\overrightarrow{i} - 8\,\overrightarrow{j}$, $\overrightarrow{R} = 2a\,\overrightarrow{i} - 3b\,\overrightarrow{j}$
   then: a = ... , b = ...

3. If $\overrightarrow{F_1} = 3\,\overrightarrow{i} - 2\,\overrightarrow{j}$, $\overrightarrow{F_2} = a\,\overrightarrow{i} - \overrightarrow{j}$, $\overrightarrow{F_3} = 4\,\overrightarrow{i} - b\,\overrightarrow{j}$, $\overrightarrow{R} = 6\,\overrightarrow{i} - 4\,\overrightarrow{j}$
   then: a = ... , b = ...

4. Find the magnitude and the direction of resultant of the forces shown in each of the following figures:

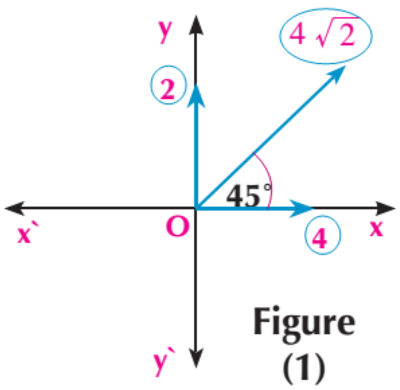


Figure (1)

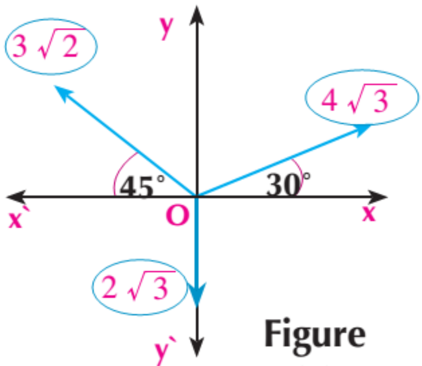


Figure (2)

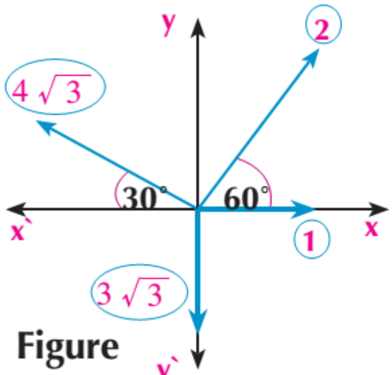
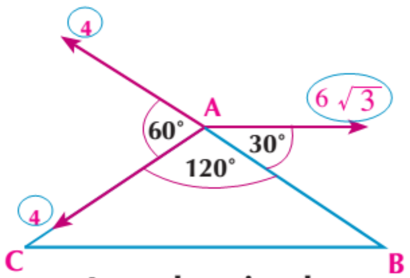
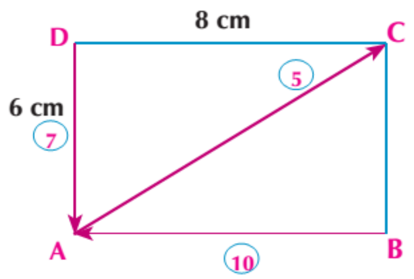


Figure (3)



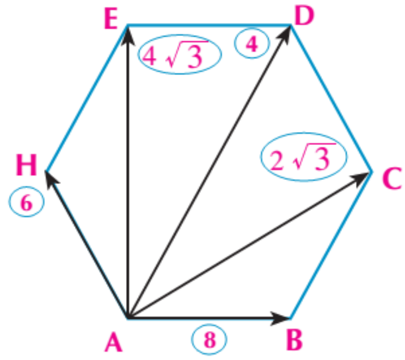
Isosceles triangle

Figure (4)



Rectangle it's dimensions are 6cm , 8cm

Figure (5)



Regular hexagon

Figure (6)

5. The forces 3 , 6 , $9\sqrt{3}$ and 12 kg.wt act on a particle and the measure of the angle between the first and the second is 60° , between the second and the third is 90° and between the third and the fourth is 150°. Find the magnitude and the direction of resultant of these forces.

6. Three forces of magnitudes 10 , 20 , 30 newton act at a particle. The first acts towards the east and the second makes an angle of measure 30° west of the north and the third makes an angle of measure 60° South of the west. Find the magnitude and the direction of resultant of these forces.

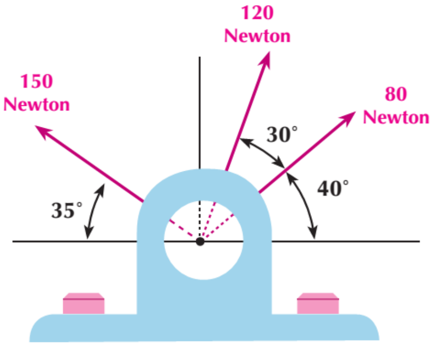
7. Four forces of magnitudes 10 , 20 , $30\sqrt{3}$ and 40 gm.wt act on a particle, the first acts in the east direction and the second acts in the direction 60° north of the east and the third acts in the direction 30° north of the west and the fourth acts in the direction making an angle of 60° South of the east. Find the magnitude and direction of resultant of these forces.

8. A B C is an equilateral triangle , M is the point of intersection of its medians. The forces of magnitudes 15 , 20 , 25 newton act on a particle in the directions of $\overrightarrow{MC}$, $\overrightarrow{MB}$, $\overrightarrow{MA}$. Find the magnitude and the direction of the resultant of these forces.

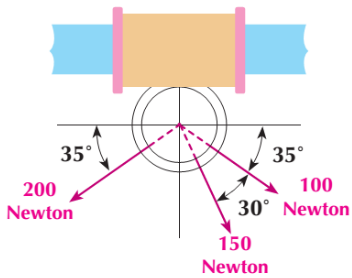
9. ABCD is a square of side length 12cm , $H \in \overline{BC}$ so BH = 5cm. Forces of magnitudes 2 , 13 , $4\sqrt{2}$ and 9 gm.wt act in directions of $\overrightarrow{AB}$, $\overrightarrow{AH}$, $\overrightarrow{CA}$, $\overrightarrow{AD}$ respectively. Find the magnitude of the resultant of these forces.

10. From the data represented in the opposite figure Find the magnitude and the direction of the resultant

a

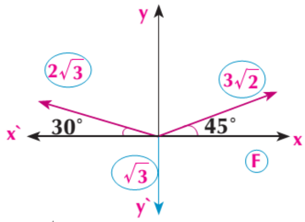


b

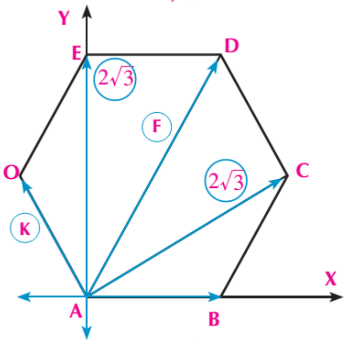


11. If $\overrightarrow{F_1} = 5\,\overrightarrow{i} + 3\,\overrightarrow{j}$, $\overrightarrow{F_2} = a\,\overrightarrow{i} + 6\,\overrightarrow{j}$ and $\overrightarrow{F_3} = 14\,\overrightarrow{i} + b\,\overrightarrow{j}$ are three coplanar forces meeting at a point and their resultant $\overrightarrow{R} = (10\sqrt{2}\ , 135°)$ Find the values of a , b

12. In the opposite figure :
If the magnitude of the resultant of the forces equals $3\sqrt{2}$ Newton, then find the value of F and the measure of the angle between the line of action of the resultant and the first force

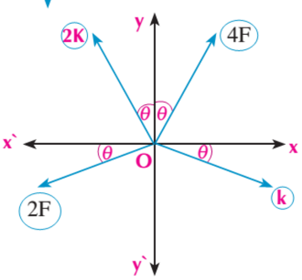


13. In the opposite figure :
If the magnitude of the resultant of the forces equals 20 Kg.wt and acts in the direction of $\overrightarrow{AD}$ Find the values of F and K.



Creative thinking:

14. **The opposite figure :** shows four coplanar forces act at the point (O) in the directions shown in the figure, where $\sin\theta = \frac{4}{5}$ and the resultant of these forces equals $8\sqrt{2}$ newton and makes an angle of measure 135° with $\overrightarrow{OX}$, then find the values of F , K.



Lesson (4)

Equilibrium of coplanar forces meeting at a point

(1) Equilibrium of a body under action of two forces

If a body is equilibrium under the action of two forces then

(i) The two forces are equal in magnitude

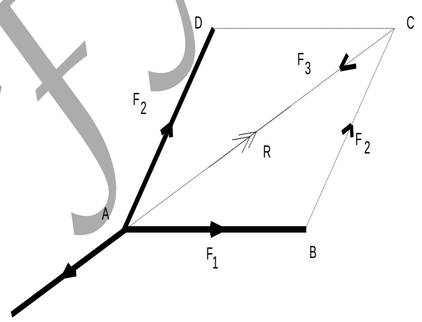
(ii) The two forces are in opposite directions

(iii) The lines of actions of the two forces on the same st. line

\- ———————— –

(1) Equilibrium of a body under action of three forces:

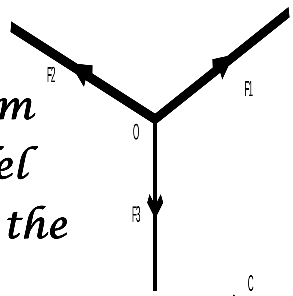
Rule(1)



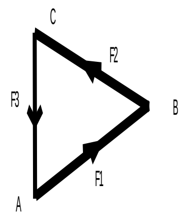
If three forces are acting at a point and can be representedby sides of a triangle taken in the same cyclic order, then theforces are in equilibrium

\- ———————— –

Rule(2) (Triangle of forces Rule)



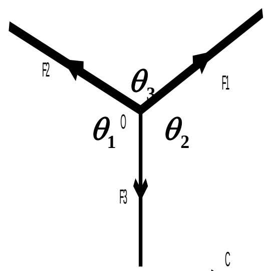
If three forces acting at a point be in equilibrium and a triangle is drawn whose sides are parallel to the lines of action of the forces and taken in the same cyclic order then the lengths of the sides of the triangle are proportional to the the magnitudes of the corresponding forces



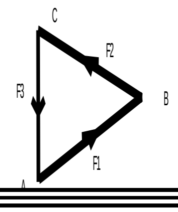
$$\boxed{\Rightarrow \frac{F_1}{AB} = \frac{F_2}{BC} = \frac{F_3}{CA}}$$

\- -——————————

Rule(3) (Lami 's Rule)



If three forces meeting at a point and acting upon aparticle are in equilibrium, then the magnitude of each force is proportional to sine of the angle between the other two forces



$$\boxed{\Rightarrow \frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}}$$

Types of questions

(1) Body and two strings
(2) Body , string and horizontal force
(3) Body string and perpendicular force
(4) Smooth inclined plane
(5) Sphere (6) Rod

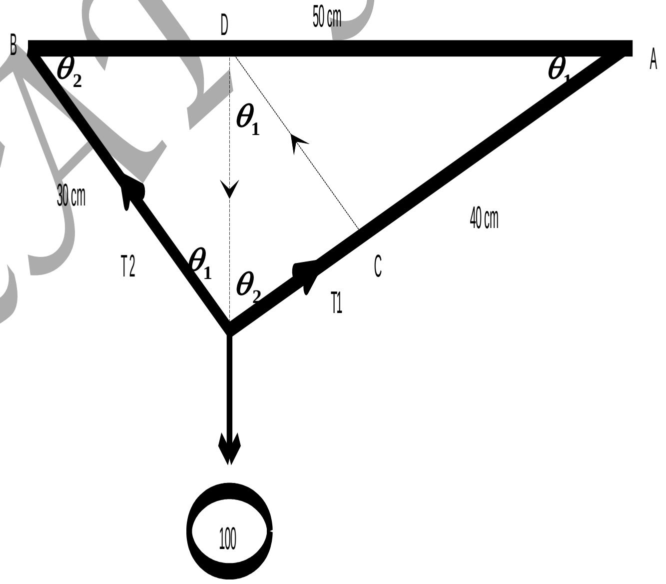
- -

Study Carefully these questions

1 A weight 100 gm.wt is suspended by two strings of lengths 30 cm., and 40 cm., from two points on the same horizontal line with 50 cm. a part. find the tension in each string.

Solution $\because (50)^2=(30)^2+(40)^2$

$\therefore$ AOC is right angled triangle at O



$\therefore m(\angle\theta_1)+m(\angle\theta_2)=90^\circ \rightarrow (1)$

$In\Delta DOC$:

$m(\angle A)+m(\angle\theta_2)=90^\circ \rightarrow (2)$

$from (1)\,\&\,(2) \Rightarrow m(\angle\theta_1)+m(\angle A)$

$Similar \Rightarrow m(\angle\theta_2)+m(\angle B)$

By using Lami's Rule :

$$\frac{F_1}{\sin\theta_1}=\frac{F_2}{\sin\theta_2}=\frac{F_3}{\sin\theta_3}$$

$$\frac{T_1}{\sin(180-\theta_1)}=\frac{T_2}{\sin(180-\theta_2)}\frac{100}{\sin 90^\circ}$$

$$\boxed{\begin{array}{l} \because \sin(180-\theta)=\sin\theta \\ from\Delta AOB \Rightarrow \sin\theta_1=\frac{opp}{hyp}=\frac{30}{50} \\ \qquad\qquad \Rightarrow \sin\theta_2=\frac{opp}{hyp}=\frac{40}{50} \end{array}}$$

$$T_1=100\sin\theta_1=100\times\frac{30}{50}=60\ gm.wt$$

$$T_2=100\sin\theta_2=100\times\frac{40}{50}=80\ gm.wt$$

- -

<u>Another solution</u>: by using the triangle of forces :

$\because$ ODC is the triangle of forces

$$\therefore \frac{F_1}{OC}=\frac{F_2}{DC}=\frac{F_3}{OD} \quad \Rightarrow \quad \frac{T_1}{OC}=\frac{T_2}{DC}=\frac{100}{OD}$$

$\therefore T_1 = 100 \times \frac{OC}{OD}$ *But* $\frac{OC}{OD} = \sin\theta_1 = \frac{opp}{hyp} = \frac{30}{50}$

$\therefore T_1 = 100 \times \frac{30}{50} = 60\ gm.wt$

$\therefore T_2 = 100 \times \frac{CD}{OD}$ *But* $\frac{CD}{OD} = \sin\theta_2 = \frac{opp}{hyp} = \frac{40}{50}$

$\therefore T_1 = 100 \times \frac{40}{50} = 80\ gm.wt$

- -

$\boxed{2}$ *If the three forces* $\vec{F}_1 = 2\hat{i} + 5\hat{j}$, $\vec{F}_2 = \hat{i} - 8\hat{j}$, $\vec{F}_3 = m\hat{i} + n\hat{j}$, *meeting at a point and in equilibrium, find the value of "m" and "n".*

Solution $\because$ *The system is in equilibrium*

$\therefore \vec{R} = 0 \Rightarrow \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$

$(2+1+m)\hat{i} + (5-8+n)\hat{j} = 0 \quad \Rightarrow (m+3)\hat{i} + (n-3)\hat{j} = 0\hat{i} + 0\hat{j}$

$\therefore m+3 = 0 \quad \Rightarrow \boxed{m=-3}$ *and* $\therefore n-3 = 0 \quad \Rightarrow \boxed{n=3}$

- -

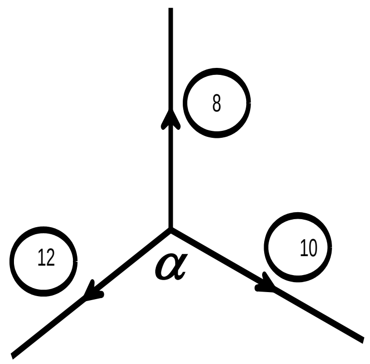
$\boxed{3}$ *Three coplanar forces 8, 10, and 12 newtons act at a partical, if the forces are in equilibrium, find the measure of the angle between the last two forces.*

Solution

$\because$ *the required is the angle between the two forces 10 and 12 , then we need to find the resultant of the two forces 10 and 12*

$\because$ *The forces are in equilibrium*

Then this resultant = 8



$R^2 = F_1^{\,2} + F_2^{\,2} + 2F_1F_2\cos\alpha$

$(8)^2 = (10)^2 + (12)^2 + 2 \times 10 \times 12 \times \cos\alpha$

$240\cos\alpha = -180 \Rightarrow \therefore \cos\alpha = \frac{-3}{4}$

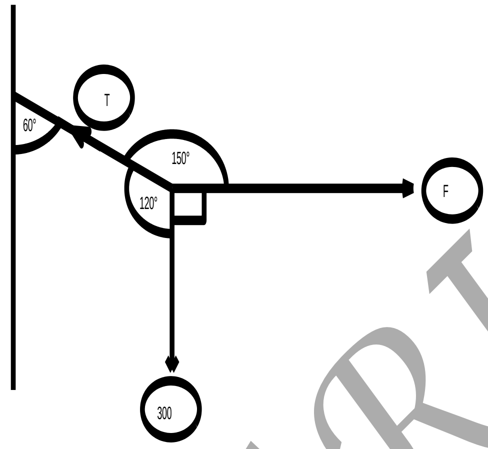
$\Rightarrow \therefore m(\angle\alpha) = 138^\circ 35^\circ$

- -

$\boxed{4}$ *A body of weight 300 gm.wt is tide to an end of a string and the other end of the string is fixed at a vertical wall. If the body is pulled by a horizontal force until the string makes an angle of measure* 60° *with the wall in the state of equilibrium, Find the magnitude of the force and the tension in the string.*

Solution

By using Lami's rule :



$$\frac{F}{\sin 120^\circ} = \frac{T}{\sin 90^\circ} = \frac{300}{\sin 150^\circ}$$

$$\therefore T = \frac{300 \times \sin 90^\circ}{\sin 150^\circ} = 600 \ gm.wt$$

$$\therefore F = \frac{300 \times \sin 120^\circ}{\sin 150^\circ} = 300\sqrt{3} \ gm.wt$$

- -

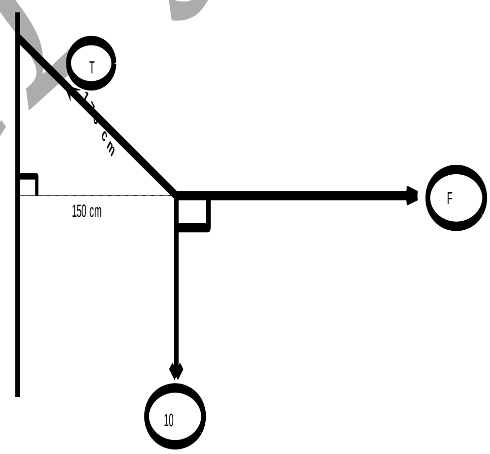
5 *A body of weight 10 kg.wt is tide to an end of a string of length 170 cm., the other end of the string is fixed at a vertical wall, the body is pulled way of the wall by a horizontal force until it becomes 150 cm. apart from the wall in the state of equilibrium, Find the magnitude of* F *and* T

Solution *In ΔOAB is rigth at A*

$$(OA)^2 = (OB)^2 - (AB)^2 = (170)^2 - (150)^2$$

$$\therefore OA = 80 \ cm.$$

ΔOAB is the triangle of forces



$$\therefore \frac{10}{80} = \frac{T}{170} = \frac{F}{150}$$

$$\therefore T = \frac{10 \times 170}{80} = \frac{85}{4}$$

$$\therefore F = \frac{10 \times 150}{80} = \frac{75}{4}$$

- -

6 *A light string of length 24 cm, its end* A *is fixed to a fixed point. a body of weight 100 gm.wt is hung at the other end* B. *Find the magnitude of the force required to keep the weight at a distance 12 cm from the horizontal straight line through* A *in the following cases :*

first : If the effective force is horizontal

second : If the direction of the force is perpendicular to $\overline{AB}$.

find the tension in the string in each case

Solution

first : the effective force is horizontal

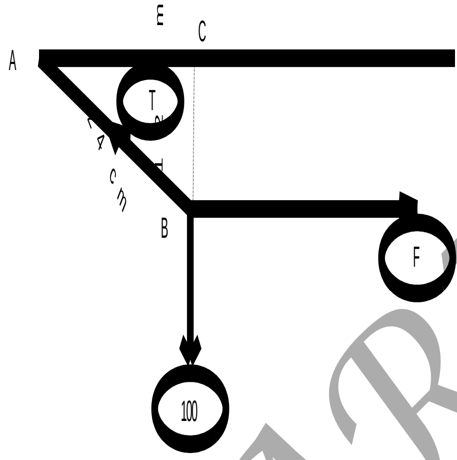
$$(AC)^2 = (24)^2 - (12)^2 \Rightarrow \therefore AC = 12\sqrt{3} \ cm$$

By using Lami's rule :

$$\frac{F}{\sin(180^\circ-\theta)}=\frac{T}{\sin 90^\circ}=\frac{100}{\sin(90^\circ+\theta)}$$

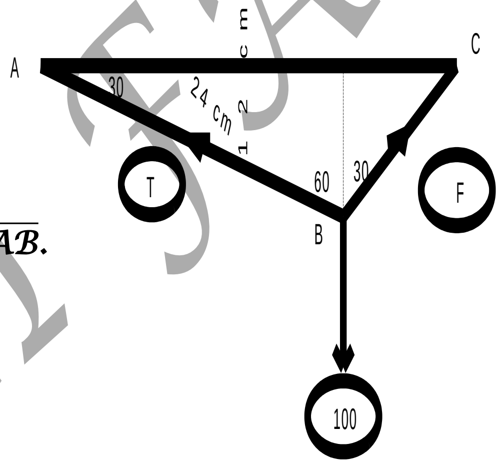
$$\frac{F}{\sin\theta}=\frac{T}{1}=\frac{100}{\cos\theta}$$

$$\therefore\Rightarrow \boxed{\begin{array}{l}\because \sin(180^\circ-\theta)=\sin\theta\\ \because \sin(90^\circ+\theta)=\cos\theta\\ from\Delta AOB\Rightarrow \sin\theta=\dfrac{opp}{hyp}=\dfrac{12\sqrt{3}}{24}\\ \qquad\qquad\Rightarrow \cos\theta=\dfrac{adj}{hyp}=\dfrac{12}{24}\end{array}}$$



$$\therefore T=\frac{100}{\cos\theta}=\frac{100}{\frac{12}{24}}=200\ gm.wt$$

$$\therefore F=\frac{100\times\sin\theta}{\cos\theta}=100\sqrt{3}\ gm.wt$$



second : If the direction of the force is $\perp\ \overline{AB}$.

By using Lami's rule :

$$\frac{F}{\sin 120^\circ}=\frac{T}{\sin 150^\circ}=\frac{100}{\sin 90^\circ}$$

$$\therefore T=100\times\sin 150^\circ=50\ gm.wt$$

$$\therefore F=100\times\sin 120^\circ=50\sqrt{3}\ gm.wt$$

- -

[7] A body of mass 6 kg.wt is plased on smoth inclined plane 30° with the horizontal, is supported by force, Find this force and the reaction of the plane in each of the following cases.

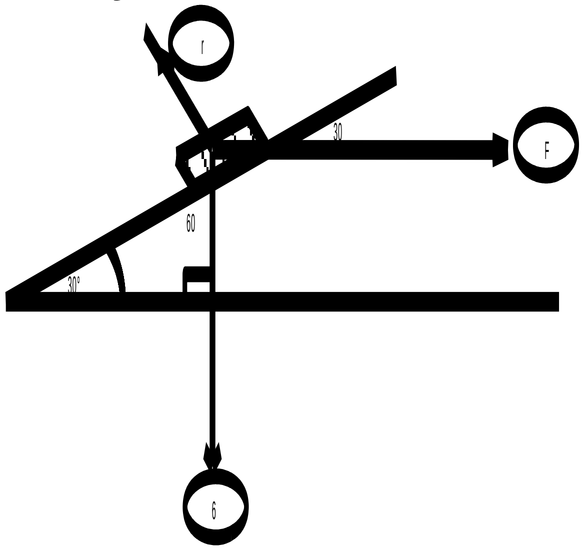
first: the force is horizontal

second: the force is inclined by 30° to the plane

Solution

first: the force is horizontal

By using Lami's rule :



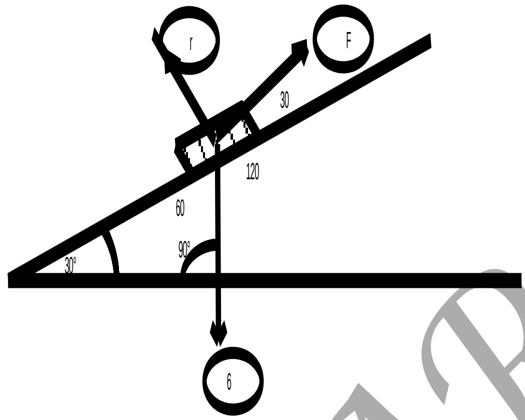
$$\frac{F}{\sin 150^\circ}=\frac{r}{\sin 90^\circ}=\frac{6}{\sin 120^\circ}$$

$$\therefore F=\frac{6\sin 150^\circ}{\sin 120^\circ}=2\sqrt{3}\ kg.wt$$

$$\therefore r=\frac{6\sin 90^\circ}{\sin 120^\circ}=4\sqrt{3}\ kg.wt$$

<u>second:</u> the force is inclined by 30° to the plane

By using Lami's rule :



$$\frac{F}{\sin 150^\circ} = \frac{r}{\sin 150^\circ} = \frac{6}{\sin 60^\circ}$$

$$\therefore F = \frac{6\sin 150^\circ}{\sin 60^\circ} = 2\sqrt{3}\ kg.wt$$

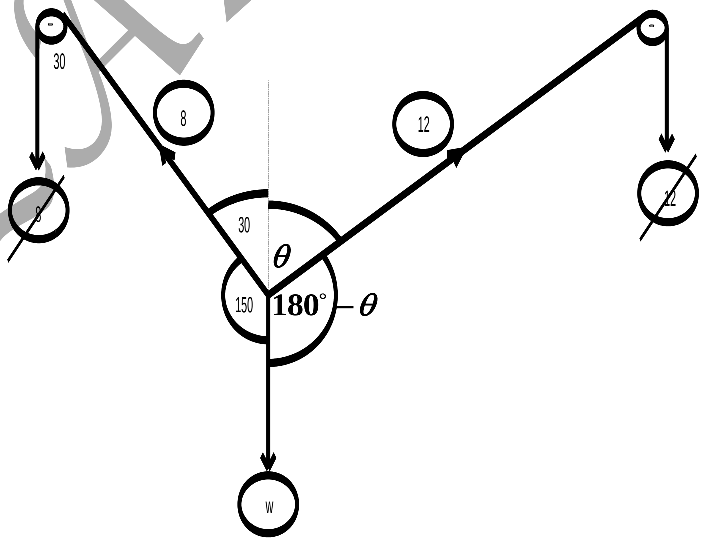
$$\therefore r = \frac{6\sin 150^\circ}{\sin 60^\circ} = 2\sqrt{3}\ kg.wt$$

- ... - -

$\boxed{8}$ *A body of weight "w" newtons is suspended by two stringth, the first inclines by angle of measure θ to the vertical passing over a smooth pulley and carries at its other end a body of weight 12 newtons, the other string form with the vertical 30°, passing over a smooth pulley and carries at its other end a body of weight 8 newtons. Find θ and F*

Solution

By using Lami's rule :



$$\frac{w}{\sin(30^\circ + \theta)} = \frac{12}{\sin 150^\circ} = \frac{8}{\sin(180^\circ - \theta)}$$

$$\therefore \sin\theta = \frac{8\sin 150^\circ}{12} = \frac{1}{3}$$

$$\therefore m(\angle\theta) = 19^\circ\ 28^\backslash$$

$$\therefore \frac{w}{\sin(30^\circ + 19^\circ\ 28^\backslash)} = \frac{12}{\sin 150^\circ}$$

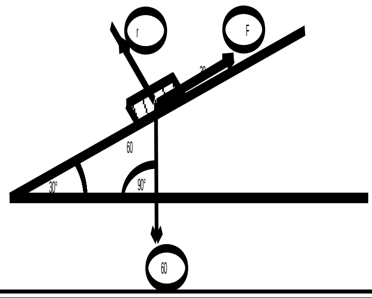
$$w = \frac{12\sin 49^\circ\ 28^\backslash}{\sin 150^\circ} \simeq 18\ newtons$$

- ... - -

$\boxed{9}$ *A body of weight 60 newtons is placed on a smooth inclined plane to the horizontal by 30. it is pulled up the plane by a stringwhich coincides the line of the greatest slope. Find the tension in the string and the reaction of the plane on the body*

Solution

By using Lami's rule :



$$\frac{T}{\sin 150^\circ} = \frac{r}{\sin 120^\circ} = \frac{60}{\sin 90^\circ}$$

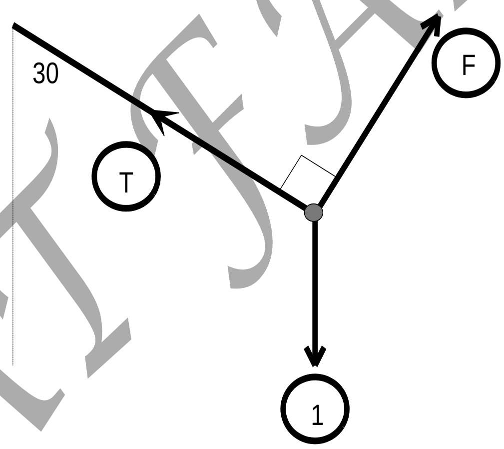
$\therefore T = \dfrac{60\sin 150^\circ}{\sin 90^\circ} = 30$ *newtons*

$\therefore r = \dfrac{60\sin 120^\circ}{\sin 90^\circ} = 30\sqrt{3}$ *newtons*

$\boxed{10}$ *The ball of a pandulum of weight "1" newton is plased until the string made 30 with the vertical under the action of a force perpendicular to the string. Find the magnitude of the force and the tension in the string*

$\boxed{\textit{Solution}}$

By using Lami's rule :



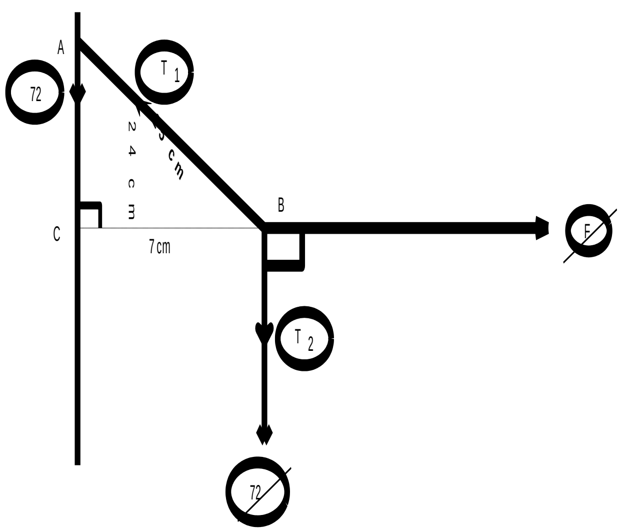
$$\frac{F}{\sin 150^\circ} = \frac{T}{\sin 120^\circ} = \frac{1}{\sin 90^\circ}$$

$\therefore F = \dfrac{1\sin 150^\circ}{\sin 90^\circ} = \dfrac{1}{2}$ *newtons*

$\therefore T = \dfrac{1\times\sin 120^\circ}{\sin 90^\circ} = \dfrac{\sqrt{3}}{2}$ *newtons*

$\boxed{11}$ *A body of 72 gm.wt, is suspended at one end of a string, its other end of the string is fixed at a point "A" on a vertical wall. another string is attached to the first one at a point "B" 25 cm. a part of "A" and pulled horizontally until the point B becomes 7 cm. a part the wall. Find the tension in the horizontal string and in each part of the second string*

$\boxed{\textit{Solution}}$ *In* Δ *ABC*



$(AC)^2 = (25)^2 - (7)^2 = 576$

$\therefore AC = 24cm$

ΔCAB *is the triangle of forces*

$$\frac{F}{7} = \frac{T}{25} = \frac{72}{24}$$

$\therefore T = \dfrac{72\times 25}{24} = 75$ *gm.wt*

2^{nd} *part* $= 72$ *gm.wt*

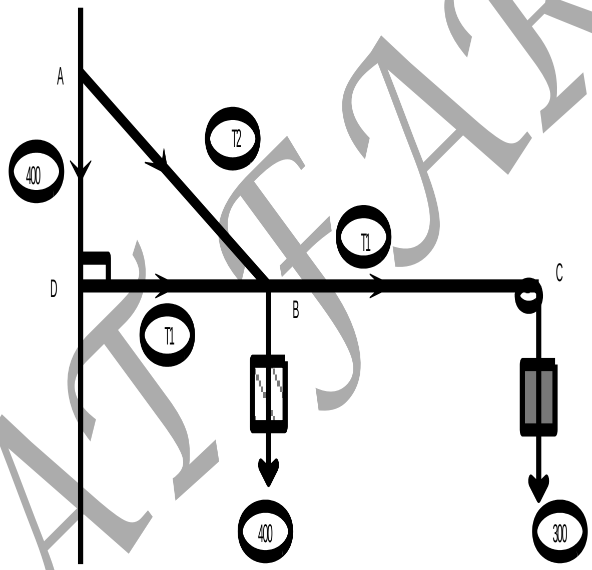
$\therefore F = \dfrac{72\times 7}{24} = 21$ *gm.wt*

12 *A body weight 400 gm.wt, is suspended by a string at a point A, from a point B on the string, another string is attached and pulled horizontally by second string BC passing over a smooth fixed pulley and carries at its other end a body 300 gm.wt, find the inclination of $\overline{AB}$ with vertical and the tension in each of the two strings AB , BC*

Solution

By using triangle of forces :

ΔADB is the triangle of forces



$$\frac{T_1}{BD} = \frac{T_2}{AB} = \frac{W}{AD}$$

$$\frac{300}{BD} = \frac{T_2}{AB} = \frac{400}{AD} \Rightarrow \therefore \frac{300}{BD} = \frac{400}{AD}$$

$$\therefore \frac{300}{400} = \frac{BD}{AD} = \frac{3}{4}$$

$$\therefore \tan A = \frac{3}{4} \Rightarrow \therefore m(\angle A) = 36^\circ 52'$$

$\therefore$ *angle of inclination* $= 36^\circ 52'$

By using Lami's rule

$$\therefore \frac{300}{\sin 36^\circ 52'} = \frac{T_2}{\sin 90^\circ}$$

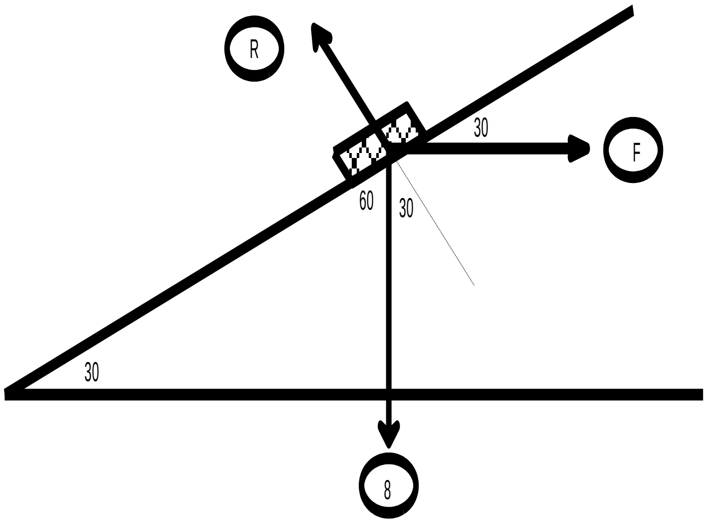
$$\therefore T_2 = \frac{300 \times \sin 90^\circ}{\sin 36^\circ 52'} = 500 \ gm.wt$$

---

13 *A body weight 8 km.wt, is placed on a smooth plane inclined to the horizontal at an angle of measure 30° , find the magnitude of the horizontal force that act on the body to keep it in equilibrium, and also find the reaction of the plane.*

Solution

By using Lami's rule :



$$\therefore \frac{F}{\sin 150^\circ} = \frac{R}{\sin 90^\circ} = \frac{8}{\sin 120^\circ}$$

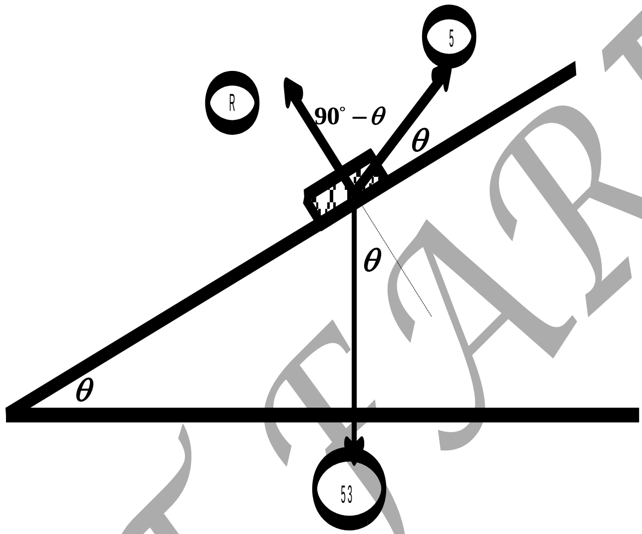
$$\therefore F = \therefore \frac{8 \times \sin 150^\circ}{\sin 120^\circ} = \frac{8\sqrt{3}}{3} \ kg.wt$$

$$\therefore R = \therefore \frac{8 \times \sin 90^\circ}{\sin 120^\circ} = \frac{16\sqrt{3}}{3} \ kg.wt$$

12 *A body weight $5\sqrt{3}$ gm.wt, is placed on a smooth plane inclined to the horizontal by an angle of measure θ if the body is kept in equilibrium by $5\sqrt{3}$ gm.wt, a force inclined to the line of greatest of slope of the plane at an angle of measure θ upeards, find θ and the reaction of the plane.*

Solution

By using Lami's rule



$$\therefore \frac{W}{\sin(90^\circ-\theta)}=\frac{F}{\sin(180^\circ-\theta)}=\frac{R}{\sin(90^\circ+2\theta)}$$

$$\therefore \frac{5\sqrt{3}}{\cos\theta}=\frac{5}{\sin\theta}=\frac{R}{\cos 2\theta}$$

$$\therefore \frac{\sin\theta}{\cos\theta}=\frac{\cancel{5}}{\cancel{5}\sqrt{3}}=\frac{1}{\sqrt{3}}$$

$$\therefore \tan\theta=\frac{1}{\sqrt{3}}\ \therefore\Rightarrow m(\angle\theta)=30^\circ$$

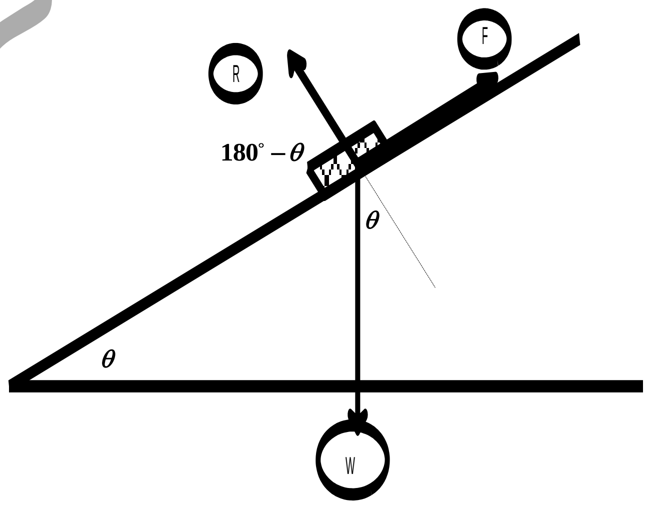
$$\therefore \frac{5}{\sin 30^\circ}=\frac{R}{\cos 60^\circ}\Rightarrow R=5\ kg.wt$$

\- -

13 *A body is in equilibrium on a smooth inclined plane under the action of a force acting up the plane, its magnitude equals falf the weight of the body find angle of inclination of the plane to the horizontal and the reaction of the plane.*

Solution

By using Lami's rule :



$$\therefore \frac{F}{\sin(180^\circ-\theta)}=\frac{W}{\sin 90^\circ}=\frac{R}{\sin(90^\circ+\theta)}$$

$$\therefore \frac{\frac{1}{2}W}{\sin\theta}=\frac{W}{1}=\frac{R}{\cos\theta}$$

$$\therefore \sin\theta=\frac{\frac{1}{2}\cancel{W}}{\cancel{W}}=\frac{1}{2}\ kg.wt$$

$$\therefore R=W\cos 30=\frac{\sqrt{3}}{2}W$$

Study Carefully these questions

1 *From a point on the surface of a homogenous sphere, a light string is tied whose other end is attached to a point in a vertical smooth wall. if the sphere is stable on the wall, find the tension in the string and the reaction of the wall if the weight of the sphere is W newtons and acts at its center and the string inclined to the vertical by 30°*

Solution *In this problem we find that the sphere is in equilibrium state under the action of three forces :*

(1) The weight (W) act at M

(2) The reaction of the wall (R)

$\because$ the wall is smooth, then the reaction is perpendicular to the plane , therefore it passes through the center of the sphere M

(3) The tension in the string (T)

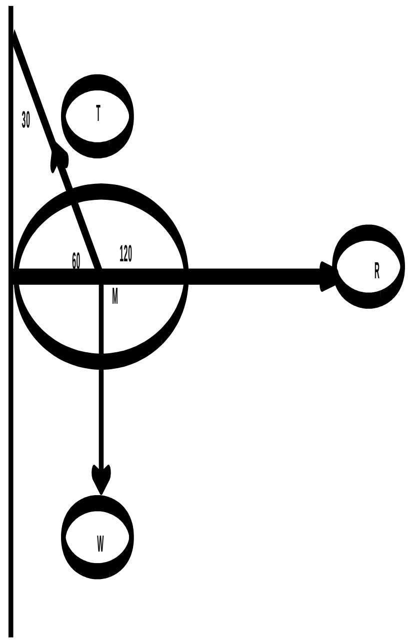
Since the line of the action of the weight and the reaction of the wall passes through the center M, then the line of action of the third force (the tension) must be pass through the point M

$\therefore$ *By using Lami's Rule :*

$$\frac{W}{\sin 120^\circ} = \frac{T}{\sin 90^\circ} = \frac{R}{\sin 150^\circ}$$

$$T = \frac{W \times \sin 90^\circ}{\sin 120^\circ} = \frac{2\sqrt{3}}{3} W \quad newtons$$

$$R = \frac{W \times \sin 150^\circ}{\sin 120^\circ} = \frac{\sqrt{3}}{3} W \quad newtons$$



Another solution : By analizing

By taking two perp. axis passing through the center M one of them is horizontal and the other is vertical

$$\therefore X = R - T\cos 60^\circ = 0$$

$$\therefore R - T\cos 60^\circ = 0 \Rightarrow \boxed{\therefore R = \frac{1}{2} T} \longrightarrow (1)$$

$$\therefore Y = T\sin 60^\circ - W = 0$$

$$\therefore W = T\sin 60^\circ \Rightarrow \boxed{\therefore T = \frac{2\sqrt{3}}{3} W} \longrightarrow (2)$$

From (1) & (2)

$$T = \frac{2\sqrt{3}}{3} W \quad newtons \quad , \quad R = \frac{\sqrt{3}}{3} W \quad newtons$$

2 *A smooth sphere of weight 15 newtons is on a smooth vertical wall and suspended by a light string from a point on its surface the other end of the string is attached to a point on the wall above the point of contact between the wall and the sphere, if the length of the string equals the radius of the sphere, find the pressure on the wall and the tension in the string*

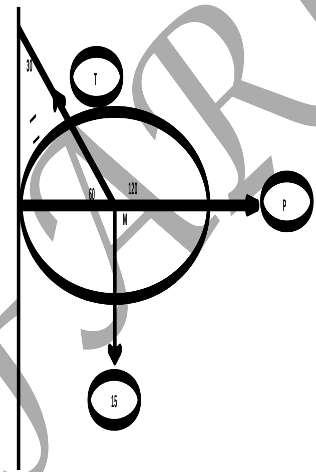
Solution

$\therefore$ *By using Lami's Rule* :

$$\frac{15}{\sin 120^\circ}=\frac{P}{\sin 150^\circ}=\frac{T}{\sin 90^\circ}$$

$$P=\frac{15\times\sin 150^\circ}{\sin 120^\circ}=5\sqrt{3}\ \text{newtons}$$

$$T=\frac{15\times\sin 90^\circ}{\sin 120^\circ}=10\sqrt{3}\ \text{newtons}$$



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3 *A smooth iron sphere of weight 30 newtons rests between a vertical smooth wall and an inclined smooth plane inclined by 60 with the horizontal, find the pressure on each of the wall and the plane*

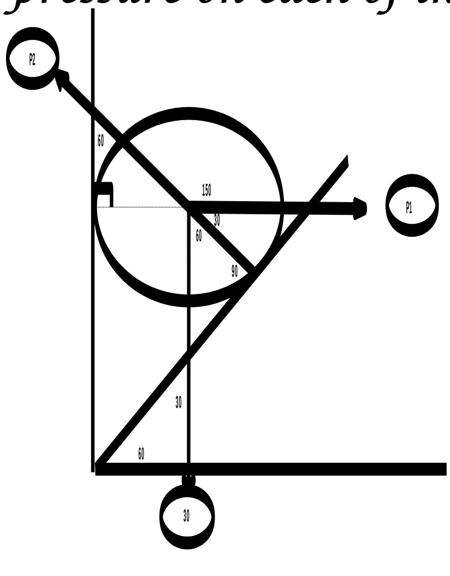
Solution

$\therefore$ *By using Lami's Rule* :

$$\frac{P_1}{\sin 120^\circ}=\frac{P_2}{\sin 90^\circ}=\frac{30}{\sin 150^\circ}$$

$$P_1=\frac{30\times\sin 120^\circ}{\sin 150^\circ}=30\sqrt{3}\ \text{newtons}$$

$$P_1=\frac{30\times\sin 90^\circ}{\sin 150^\circ}=60\ \text{newtons}$$



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4 *A sphere rests between two parallel rods lie in the same horizontal plane, the distance between them equals the radius of the sphere find the pressure on each rod given that the weight of the sphere is 10 newtons.*

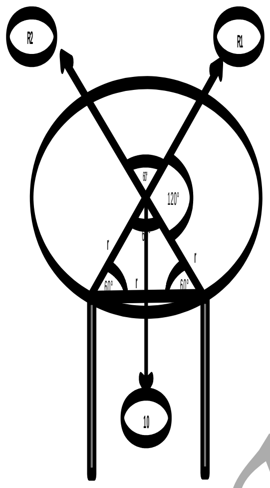
Solution

$\therefore$ *By using Lami's Rule* :

$$\frac{R_1}{\sin 150^\circ} = \frac{R_2}{\sin 50^\circ} = \frac{10}{\sin 60^\circ}$$

$$R_1 = R_2 = \frac{10 \times \sin 150^\circ}{\sin 60^\circ}$$

$$= \frac{10\sqrt{3}}{3} \ \ newtons$$



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5 *A homogenous sphere of radius 30 cm., and of weight 200 kg.wt rests on a smooth vertical wall and suspended from a point on its surface by a string of length 20 cm., and the other end of the string is fixed at a point on the wall vertically above the point of contact of the sphere with the wall. Find the tension in the string and the reaction of the wall on the sphere.*

Solution

$$AB = \sqrt{(50)^2 - (30)^2}$$

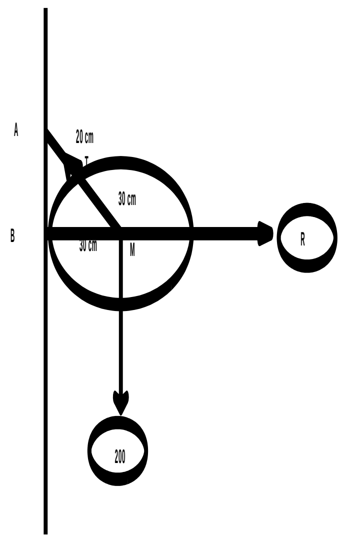
$$\therefore AB = 40 \ cm$$

ΔABM *is the triangle of forces*

$$\frac{T}{50} = \frac{200}{40} = \frac{R}{30}$$

$$\therefore R = \frac{200 \times 30}{40} = 150 \ gm.wt$$

$$\therefore T = \frac{200 \times 50}{40} = 250 \ gm.wt$$



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6 *A uniform rod of 1 metre long and 30 newtons weight is suspended from both its ends by two strings their other ends are fixed at one point in the ceiling. if the two strings are perp. one of them is 60 cm., find the tension in each string when it is suspended freely and in equilibrium.*

Solution

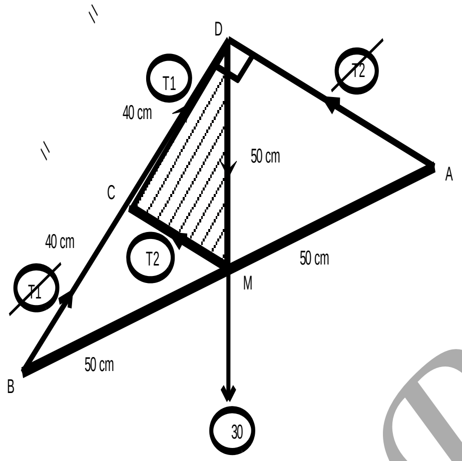
Δ *OMC is the triangle of forces*

$$\frac{30}{OM}=\frac{T_1}{OC}=\frac{T_2}{MC}$$

$$\frac{30}{50}=\frac{T_1}{40}=\frac{T_2}{30}$$

$$\therefore T_1=\frac{40\times 30}{50}=24 \text{ newtons}$$

$$\therefore T_2=\frac{30\times 30}{50}=18 \text{ newtons}$$



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7 *A uniform rod of 130 cm long and 26 newtons weight is suspended from both its ends by two strings their other ends are fixed at one point in the ceiling. if the lenfth of one of them 50 cm and the length of the other one is 120 cm., what is the position in which the rod is in equilibrium and what is the tension in each of the two strings.*

Solution

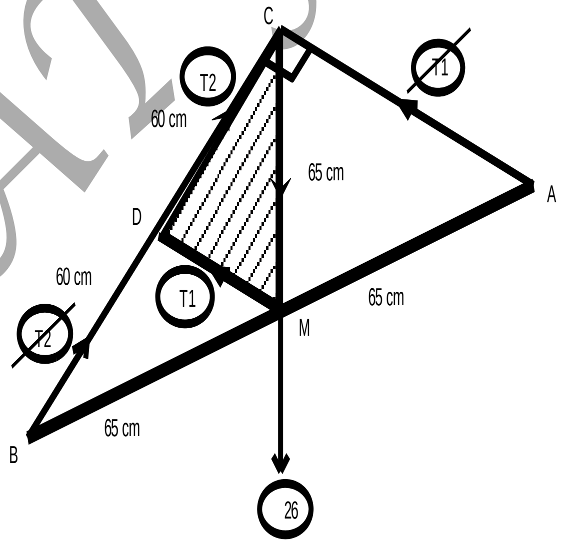
Δ *DMC is the triangle of forces*

$$\frac{26}{CM}=\frac{T_1}{DM}=\frac{T_2}{DC}$$

$$\frac{26}{65}=\frac{T_1}{25}=\frac{T_2}{60}$$

$$\therefore T_1=\frac{25\times 26}{65}=10 \text{ newtons}$$

$$\therefore T_2=\frac{60\times 26}{65}=24 \text{ newtons}$$



and the position in which the rod is equilibrium when the weight equals the tension in the two strings

---

8 *AB is a uniform rod of length 120 cm., and weight 300 gm.wt its end A is attached to a hing fixed at a vertical wall, the other end B is attached to a string $\overline{BC}$ of length 150 cm., this string is fixed at a point C on the wall and lies vertically above A, the rod is in equilibrium when it is in horizontal position, find the tension in the string and the reaction of the hing at A.*

Solution

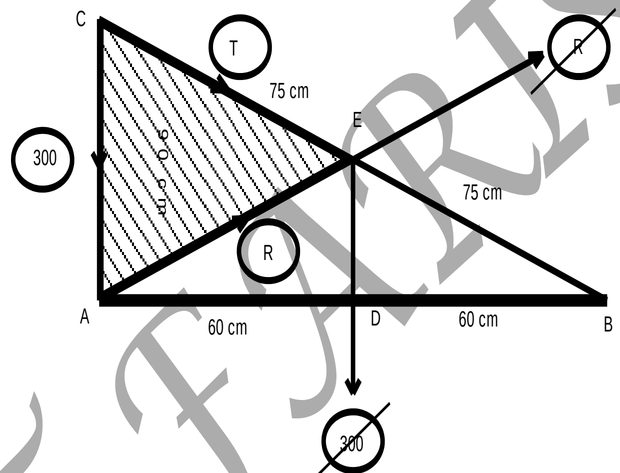
Δ *ABC is right at A* $\Rightarrow \therefore AC = \sqrt{(150)^2 - (120)^2} = 90cm.$

$\because$ *is in equilibrium when it is in horizontal position*

$\therefore$ *D is midpoint of* $\overline{AB}$, *E is midpoint of* $\overline{BC}$

$\because \overline{AE}$ *is a median drawn from the vertix of right angle*

$\therefore AE = \frac{1}{2} BC = 75\ cm.$



Δ *AEC is the triangle of forces*

$$\frac{300}{AC} = \frac{R}{AE} = \frac{T}{CE}$$

$$\frac{300}{90} = \frac{R}{75} = \frac{T}{75}$$

$$\therefore T = R = \frac{75 \times 300}{90} = 250\ gm.wt$$

---

9 *AB is a uniform rod of length 120 cm., and weight 18 km.wt it rotates about a hing at its end A and its end B is pulled by a light string whose other ends is tied to a point D above A, the rod in equilibrium when it inclined to the horizontal at 30°, if the reaction of the hing is horizontal. Find the tansion of the string and the reaction of the rod in the wall*

Solution

$\because \overrightarrow{BF} // \overrightarrow{AD} \quad \therefore m(\angle ABF) = m(\angle DAB) = 30^\circ \quad (alt.)$

In ΔAOE, $\overline{OE}$ is opposite 30°

$\therefore OE = \frac{1}{2} AE = 30\ cm$

InΔABD $\because O, E$ are midpoints of $\overline{BD}$, $\overline{BA}$

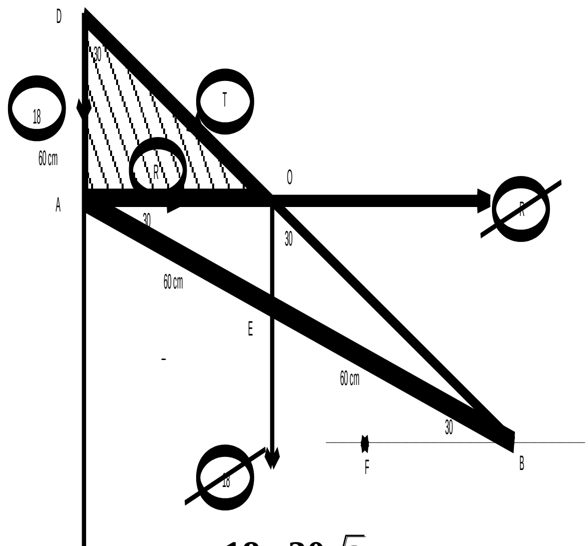
$\therefore OE = \frac{1}{2} AD \quad \therefore \boxed{AD = 60\ cm} \rightarrow (1)$

In ΔAOE, $AO = \sqrt{(60)^2 - (30)^2} = 30\sqrt{3}$

$\therefore \boxed{AO = 30\sqrt{3}} \rightarrow (2)$

In ΔAOD, $DO = \sqrt{(60)^2 + (30\sqrt{3})^2} = 30\sqrt{7}$

$\therefore \boxed{DO = 30\sqrt{7}} \rightarrow (3)$



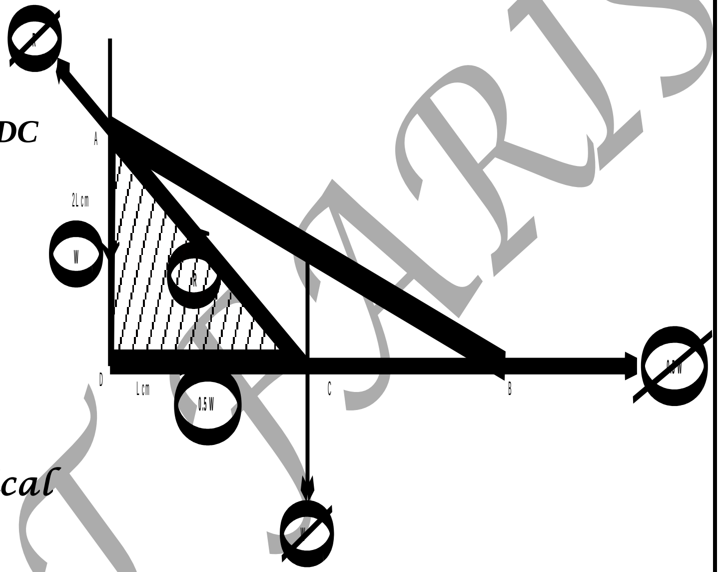
Δ *AOD is the triangle of forces*

$$\frac{18}{60} = \frac{R}{30\sqrt{3}} = \frac{T}{30\sqrt{7}} \Rightarrow \therefore T = \frac{18 \times 30\sqrt{7}}{60} = 9\sqrt{7}\ km.wt, \text{ and } R = \frac{18 \times 30\sqrt{3}}{60} = 9\sqrt{3}\ km.wt$$

$\boxed{10}$ *A uniform rod which is movable around one of its ends is pulled a side by horizontal force acting on the other end and equals half the weight of the rod, find the angle of inclination of the rod to the vertical when it is equilibrium and also find the reaction at the end.*

$\boxed{\textbf{Solution}}$

Δ *ADC is the triangle of forces*



$$\frac{R}{AC}=\frac{W}{AD}=\frac{\frac{1}{2}W}{DC} \quad \therefore\Rightarrow \frac{\cancel{W}}{AD}=\frac{\frac{1}{2}\cancel{W}}{DC} \therefore\Rightarrow AD=2DC$$

Let $DC = L$ cm., $AD=2L$ cm.,

$AC=\sqrt{4L^2+L^2} \therefore\Rightarrow AC=\sqrt{5}L$

and $CB=L$ cm.,

$$\therefore \frac{R}{\sqrt{5}L}=\frac{W}{2L}=\frac{\frac{1}{2}W}{L}$$

$\therefore$ *the rod inclined by* 45° *with the vertical*

$$R=\frac{W\times\sqrt{5}L}{2L}=\frac{\sqrt{5}}{2}W \text{ newtons}$$

$\boxed{11}$ *AB is a uniform rod of weight 50N its end A is fixed at hinge in a vertical wall the rode is in equilibrium when a horizontal force acts at its end B such the rod inclined to the vertical at 60 , find the magnitude of this force and the reaction of the hinge in state of equilibrium.*

$\boxed{\textbf{Solution}}$

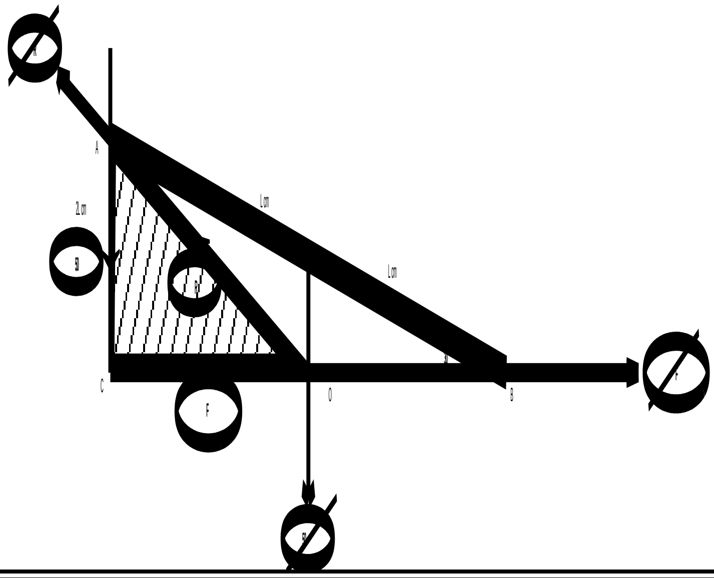
Let the length of rod $AB = 2L$

$\because m(\angle B)=30 \therefore\Rightarrow BC = L\sqrt{3}$

$\therefore BO = OC = \frac{1}{2}\sqrt{3}$

In $\Delta AOC \Rightarrow AO=\sqrt{(L)^2+(\frac{\sqrt{3}}{2}L)^2}=\frac{\sqrt{7}}{2}L$

Δ *ACO is the triangle of forces*



$$\frac{50}{AC}=\frac{F}{CO}=\frac{R}{AO}$$

$$\therefore\Rightarrow \frac{50}{L}=\frac{F}{\frac{\sqrt{3}}{2}L}=\frac{R}{\frac{\sqrt{7}}{2}L}$$

$$R=\frac{50\times\sqrt{7}}{2}=25\sqrt{7} \text{ newtons}$$

$$F=\frac{50\times\sqrt{3}}{2}=25\sqrt{3} \text{ newtons}$$

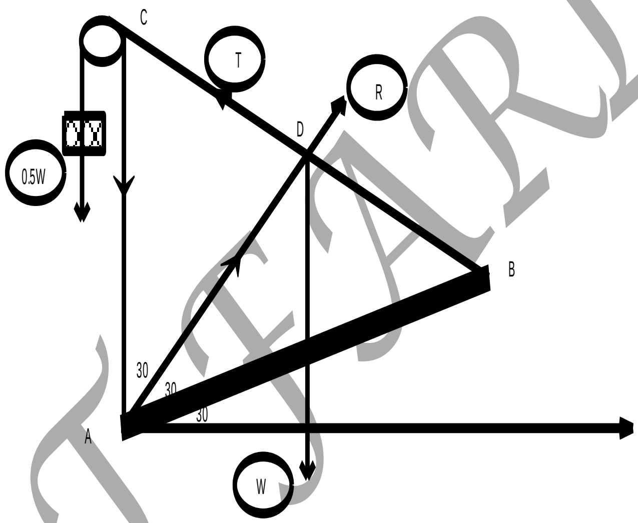
$\boxed{12}$ *AB is a uniform rod hinged at A, the other end B is tied to a string passes over a smooth pulley C exactly above A and attached to a weight equals half the weight of the rod. Find the angle of inclination of the rod to the horizontal in state of equilibrium given that AC = AB*

Solution $T = \frac{1}{2}W \quad , AB = AC \quad ,$

D is midpoint of $\overline{BC}$

$\therefore \overline{AD} \perp \overline{BC}$

Δ *ADC is the triangle of forces*



$$\frac{R}{AD} = \frac{W}{AC} = \frac{\frac{1}{2}W}{DC}$$

$$\therefore \Rightarrow \frac{\cancel{W}}{AC} = \frac{\frac{1}{2}\cancel{W}}{DC}$$

$$\therefore \Rightarrow AC = 2DC$$

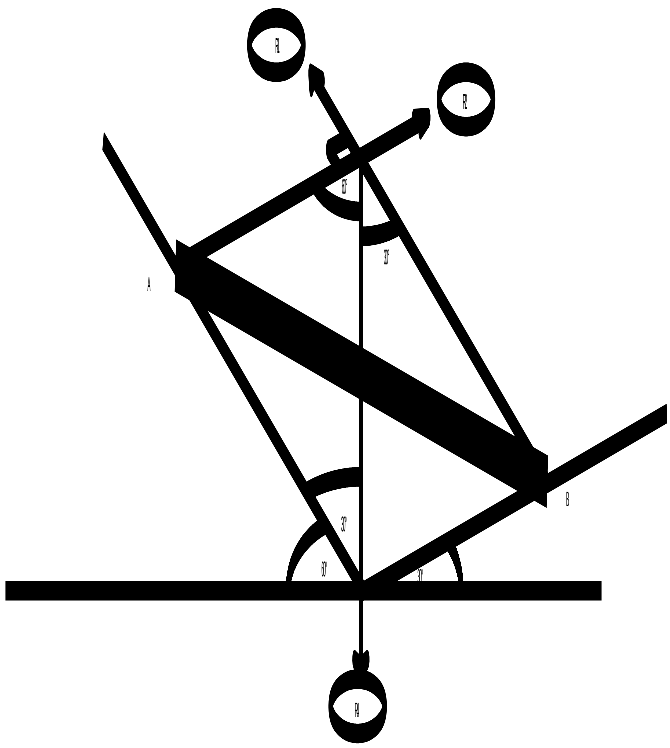
$$\because DC = \frac{1}{2}AC \Rightarrow \therefore m(CAD) = 30^\circ$$

$$\therefore \text{ the angle of inclination } = 30^o$$

$\boxed{13}$ *A uniform rod of weight 4 newtons is places on two smooth inclined planes. they inclined to the horizontal at 30°, 60°, find the magnitude of the pressure on each plane and the angle of inclination of the rod to the horizontal in state of equilibrium.*

Solution

$\because$ *Smooth inclined plane its reaction is always perpendicular*



$$\text{In } \Delta AOC \Rightarrow AO = \sqrt{(L)^2 + (\frac{\sqrt{3}}{2}L)^2} = \frac{\sqrt{7}}{2}L$$

Δ ACO is the triangle of forces

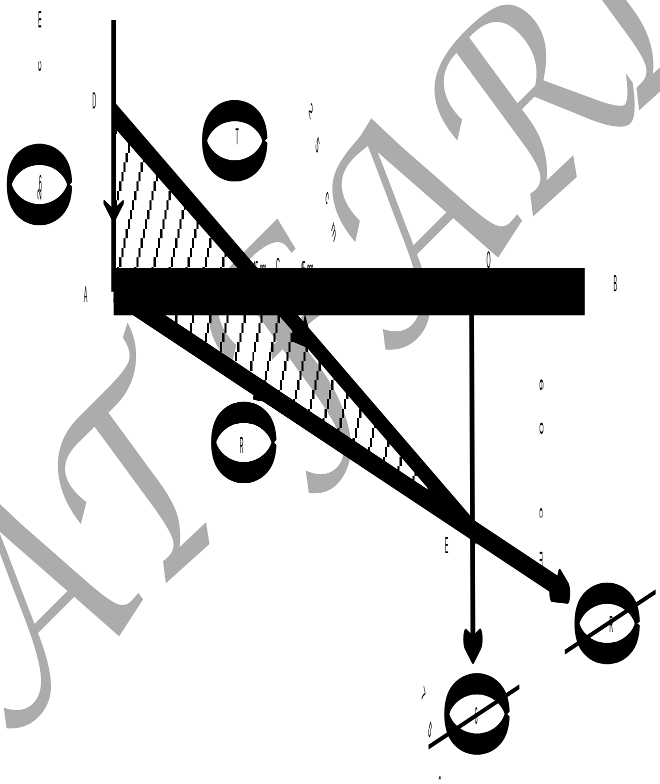
$$\frac{R_1}{\sin 120^\circ} = \frac{R_2}{\sin 150^\circ} = \frac{4}{\sin 90^\circ}$$

$$\therefore \Rightarrow R_1 = \frac{4 \times \sin 120^\circ}{\sin 90^\circ} = 2\sqrt{3} \text{ newtons}$$

$$\therefore \Rightarrow R_2 = \frac{4 \times \sin 150^\circ}{\sin 90^\circ} = 2 \text{ newtons}$$

14 *AB is a uniform rod of length 129 cm., and weight 6 kg.wt its end A is attached to a hinge fixed to a vertical wall, the rod is kept in equilibrium horizontally by means of a light string, one of its ends is tied to a point C, on the rod, where AC= 15 cm., and the other end is fixed at point D on the wall lying above A, and at distance 20 cm., from it. Find the magnitude of the tension in the string, and the reaction of the hinge.*

Solution



$$DC = \sqrt{(20)^2 + (15)^2} = 25 \ cm.$$

$$\because \Delta DAC \sim \Delta EOC \Rightarrow \therefore \frac{15}{45} = \frac{20}{OE}$$

$$\therefore OE = \frac{20 \times 45}{15} = 60 \ cm.$$

$$AE = \sqrt{(60)^2 + (60)^2} = 60\sqrt{2} \ cm.$$

$$CE = \sqrt{(45)^2 + (60)^2} = 75 \ cm.$$

Δ ADE is the triangle of forces

$$\frac{R}{AE} = \frac{W}{AD} = \frac{T}{DE}$$

$$\therefore \Rightarrow \frac{R}{60\sqrt{2}} = \frac{6}{20} = \frac{T}{100}$$

$$\therefore \Rightarrow R = \frac{6 \times 60\sqrt{2}}{20} = 18\sqrt{2} \ kg.wt$$

$$\therefore \Rightarrow T = \frac{6 \times 100}{20} = 30 \ kg.wt$$

---

15 *AB is a uniform rod of length 120 cm., and weight 5 kg.wt its end A is attached to a hinge, fixed to a vertical wall, the rod is kept in equilibrium in horizontal position by means of a light string, one of its ends is tied to a point C, on the rod, at a distance 80 cm., from A and the other end is fixed at D on the wall exactly above A and at distance $80\sqrt{3}$ cm. from it Find the magnitude of the tension in the string, and the reaction of the hinge.*

Solution

$$DC = \sqrt{(8\sqrt{3})^2 + (80)^2} = 160 \ cm.$$

$$\because \Delta CHO \sim \Delta CDA \Rightarrow \therefore \frac{20}{80} = \frac{HO}{80\sqrt{3}} = \frac{CH}{160}$$

$$\therefore HO = \frac{20 \times 80\sqrt{3}}{80} = 20\sqrt{3} \ cm.$$

$$\therefore HC = \frac{20 \times 160}{80} = 40 \ cm.$$

$\therefore DH = 160 - 40 = 120 \ cm.$

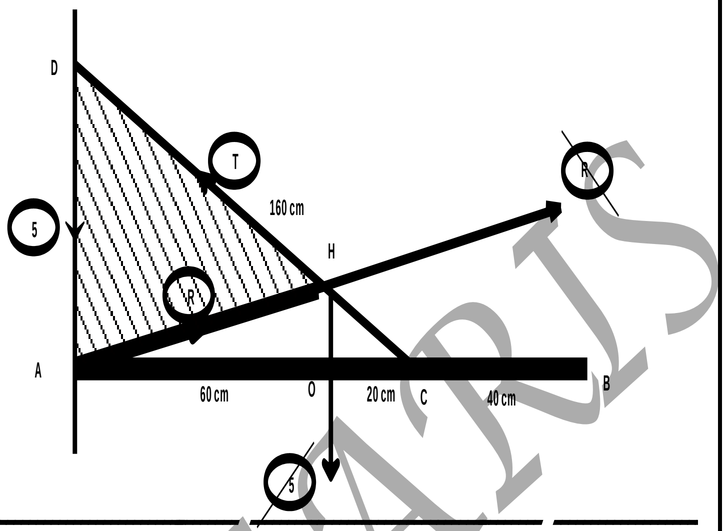
$HA = \sqrt{(60)^2 + (20\sqrt{3})^2} = 40\sqrt{3} \ cm.$

Δ ADH is the triangle of forces

$\frac{5}{80\sqrt{3}} = \frac{R}{40\sqrt{3}} = \frac{T}{120}$

$\therefore \Rightarrow R = \frac{5 \times 40\sqrt{3}}{80\sqrt{3}} = \frac{5}{2}$ *kg.wt*

$\therefore \Rightarrow T = \frac{5 \times 120}{80\sqrt{3}} = \frac{5\sqrt{3}}{2}$ *kg.wt*



$\boxed{16}$ *A uniform ladder XY of weight 60 kg.wt, and length 4 meters, rests wit its end X on vertical smooth wall, and with its end Y on a rough horizontal ground. the ladder is in equilibrium when it inclined at angle og measure 60° to the ground, Find magnitude of the wall reaction, and magnitude and direction of the ground reaction*

Solution

$\because$ *O and D are midpoints of $\overline{XY}$ and $\overline{ZY}$*

$\therefore XO = YO = 2 \ cm.$

In Δ XYZ, m(∠Y)=60° and m(∠X)=30°

$\therefore ZD = DY = 1 \ cm.$

$ZY = 2 \ cm \qquad \Rightarrow \therefore XZ = 2\sqrt{3} \ cm.$

$CD = 2\sqrt{3}$ and $DY = 1 \ cm.$

$CY = \sqrt{(2\sqrt{3})^2 + (1)^2} = \sqrt{13} \ cm.$

Δ YCD is the triangle of forces

$\frac{R}{AE} = \frac{W}{AD} = \frac{T}{DE}$

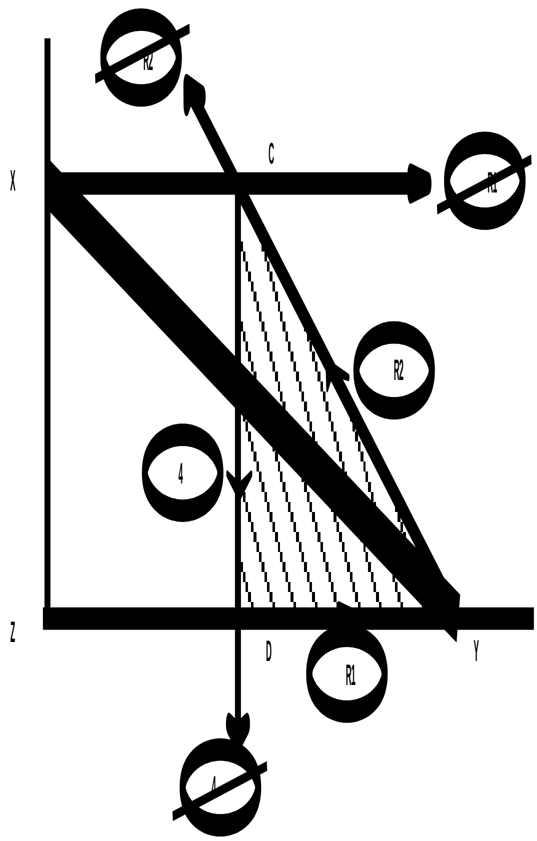
$\therefore \Rightarrow \frac{R_1}{1} = \frac{R_2}{\sqrt{13}} = \frac{4}{2\sqrt{3}}$

$\therefore \Rightarrow R_1 = \frac{1 \times 4}{2\sqrt{3}} = \frac{2\sqrt{3}}{3}$ *kg.wt*

$\therefore \Rightarrow R_2 = \frac{\sqrt{13} \times 4}{2\sqrt{3}} = \frac{2\sqrt{39}}{3}$ *kg.wt*

$\tan(\angle CYD) = \frac{DC}{DY} = \frac{2\sqrt{3}}{1}$

$\therefore m(\angle CYD) = 73^\circ \ 54'$



Homework

Complete the following:

1. The necessary and sufficient condition for equilibrium of a set of coplanar forces meeting at a point is to be represented geometrically by ....

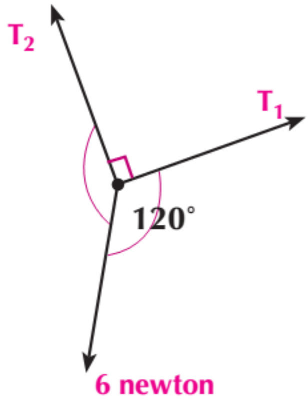
2. The condition for equilibrium of a set of coplanar forces, meeting at a point is to be .... , ...

3. If $\overrightarrow{F_1} = 4\overrightarrow{i} + b\overrightarrow{j}$, $\overrightarrow{F_2} = -7\overrightarrow{i} - 2\overrightarrow{j}$, $\overrightarrow{F_3} = a\overrightarrow{i} - 3\overrightarrow{j}$ are in equilibrium, so: a = ... , b = ...

4. If the force of magnitude F is in equilibrium with two perpendicular forces of magnitude 3, 4 newton so, the magnitude of F = ...

5. If three coplanar and equilibrium forces are completely represented by the sides of triangle taken in one cyclic order, then the lengths of the sides of the triangle are proportional with ..

6. If the body is in equilibrium under the effect of three coplanar forces and they met at a point then the magnitude of each force is proportional with ....



7. If the body is in equilibrium under the effect of three forces non-parallel and coplanar then, the lines of action of these forces ....

8. Three forces equal in magnitude and meeting at a point are in equilibrium then, the measure of the angle between any two forces equals ....

9. **In the opposite figure:** A set of forces are in equilibrium and meeting at a point. T_1 = ... newton , T_2 = ...newton.

10. Each figure from the following figures represents a set of coplanar equilibrium force meeting at a point. Find the value of the unknown either it is a force or a measure of angle .

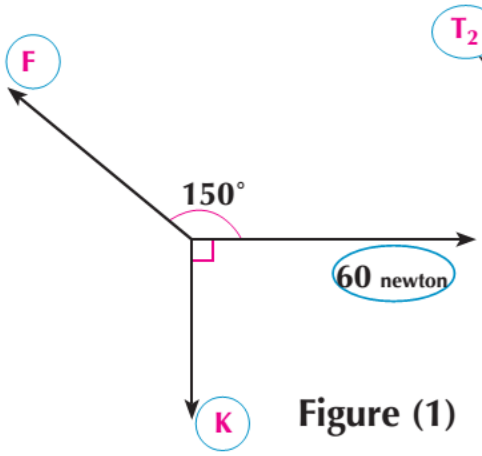


Figure (1)

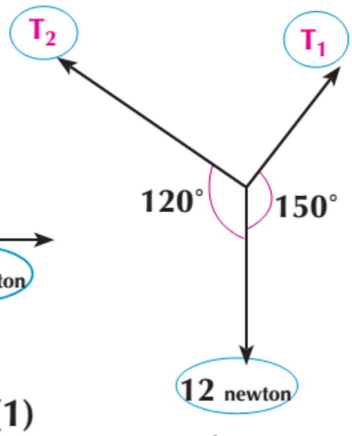


Figure (2)

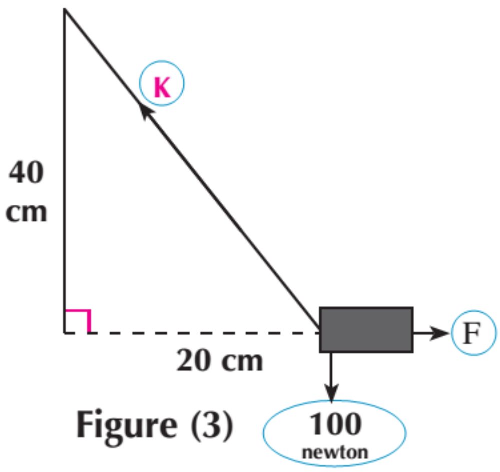


Figure (3)

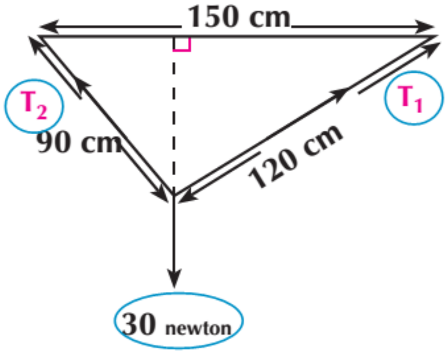


Figure (4)

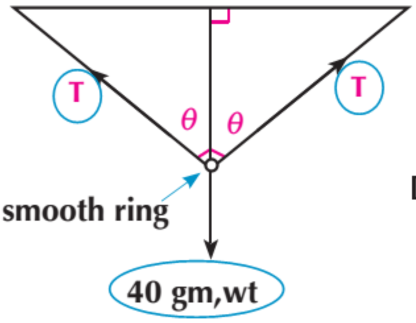


Figure (5)

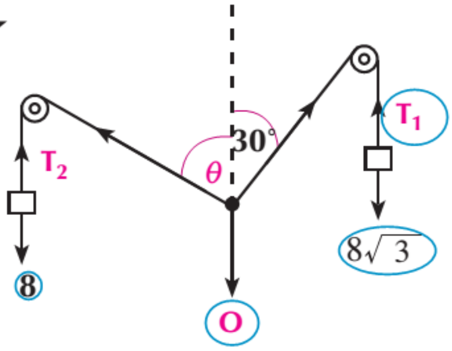
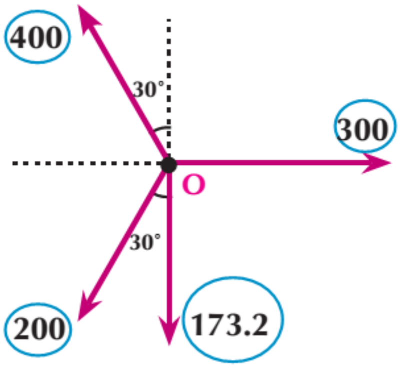


Figure (6)

11 The opposite figure represents four coplanar forces meeting at the point O

a Prove analytical that the forces are equilibrium.

b **Activity:** use (Geo-Gebra) program to represent the forces by a closed polygon of forces in a suitable drawing scale



12 A weight of magnitude 60 gm.wt is suspended from one end of a string of length 28 cm. The other end is fixed at a point in the ceiling of the room, A force acts on the body so that the body became in equilibrium when it is about 14 cm vertically down the ceiling, If the force is in equilibrium position when it is normal to the string . Find the magnitude of each of the force and the tension in the string.

13 A weight of magnitude 200gm.wt is hanged (suspended) by two strings of lengths 60 cm, 80cm from two points on one horizontal line. The distance between them is 100 cm. Find the magnitude of the tension in each of the two strings.

14 A particle of weight 200 gm.wt is hanged (suspended) by two light strings. One of the them inclines to the vertical with an angle of measure θ and the other string inclines to the vertical with an angle of measure 30°. If the magnitude of the tension in the first string equals 100 gm.wt, find θ and magnitude of the tension of the second string.

15 A body of weight 800 gm.wt is placed on a smooth plane inclined to the horizontal by angle of measure θ so that $\sin\theta = 0.6$. The body is kept in equilibrium by a horizontal force. Find the magnitude of this force and the reaction of the plane on the body.

16 A body of weight (W) newton is placed on a smooth plane inclined to the horizontal by an angle of measure 30° and the body is kept in equilibrium by the effect of force of magnitude 36 newton acts in direction of the line of the greatest slope upwards. Find the magnitude of the weight of the body and the magnitude of the reaction of the plane.

17 A smooth metal sphere of weight 3 Newton at rest (stable) between a smooth vertical wall and a smooth plane inclined to the vertical wall with angle of measure 30°. Find the pressure on each of the vertical wall and the inclined plane.

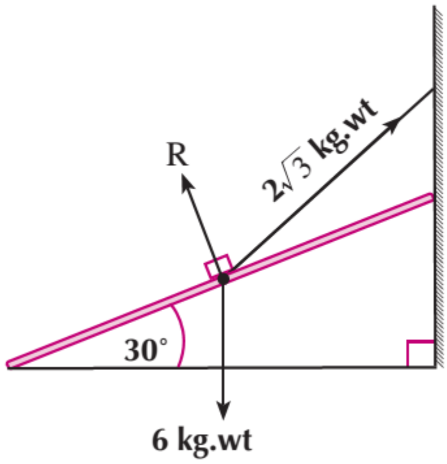
(18) A rod of length 50 cm and weight 20 newton was hanged (suspended) from its terminals with two strings such that the two ends are fixed in one point. If the length of the two strings are 30 cm , 40 cm respectively. Find the tension in each of the two strings.

(19) Five forces of magnitudes F , 6 , $4\sqrt{2}$, $5\sqrt{2}$, K kg.wt are in equilibrium and act on a particle in the directions of the east, the north, the western north, the western south and the south respectively. Find the magnitudes of F , K.

(20) Coplanar forces of magnitudes 5 , 4 , F , 3 , K , 7 kg.wt act on a particle and the angle between every two consecutive forces from them is 60°. Find the magnitude of each of F , K that make the set in a state of equilibrium.

Creative thinking:

(21) **In the opposite figure** A body of weight 6 kg.wt is placed on a smooth plane inclined to the horizontal by an angle of measure 30° and kept in equilibrium by a tension force (T) of magnitude $2\sqrt{3}$ kg.wt the tension force acts in a string one of its ends fixed to the body and the other in a vertical wall. Find the measure of the angle between the string and the plane and the magnitude of the reaction of the plane on the body.



General Exercises on unit one

Complete the following:

1. Two forces of magnitude 4 , F dyne and the measure of the angle between them $\alpha \in]0, \pi[$, their resultant bisects the angle between them So, F = ... dyne.

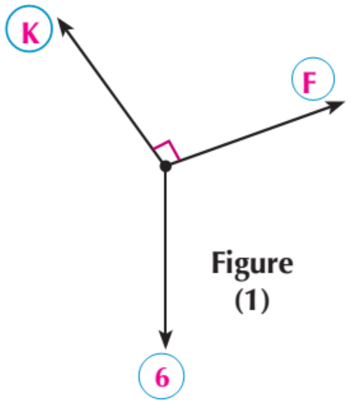
2. Two forces of magnitude 5 , 8 newton act at a particle so the maximum value for the resultant = ... newton , the minimum value for the resultant = ... newton.

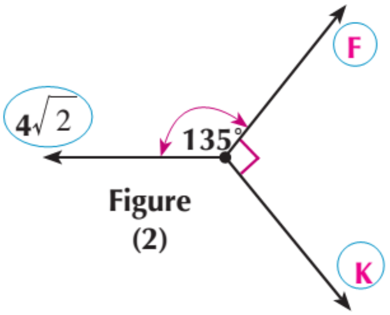
3. If a body of weight (W) placed on a smooth plane inclined to the horizontal by angle θ, so the component of its weight in direction of the plane equals ....

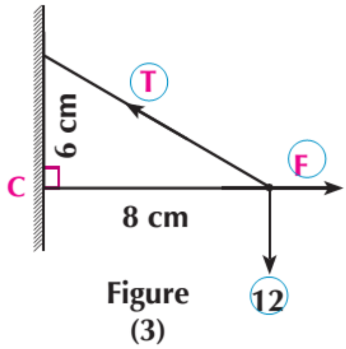
4. If the force $\overrightarrow{F}$ is in equilibrium with two perpendicular forces of magnitude 6 , 8 kg.wt so, the magnitude of the force F equals .... kg.wt.

5. If the forces $\overrightarrow{F_1} = a\,\overrightarrow{i} - 6\,\overrightarrow{j}$, $\overrightarrow{F_2} = -3\,\overrightarrow{i} + 4\,\overrightarrow{j}$, $\overrightarrow{F_3} = 9\,\overrightarrow{i} + b\,\overrightarrow{j}$ are in equilibrium, so a = ..., b = ...

6. Each figure from the following consists of three forces in equilibrium and meeting in a point. Complete the following:



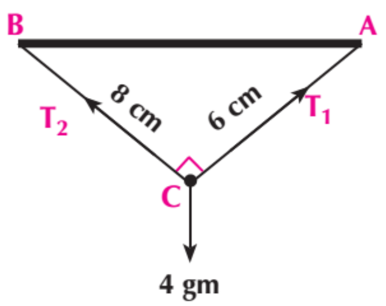


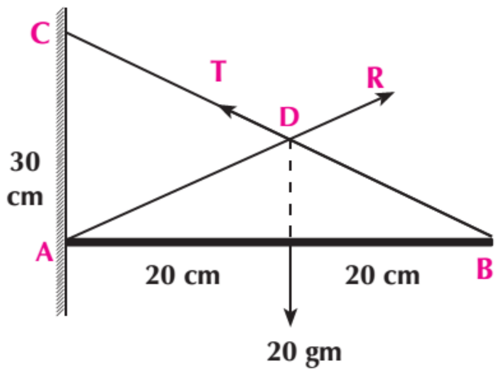


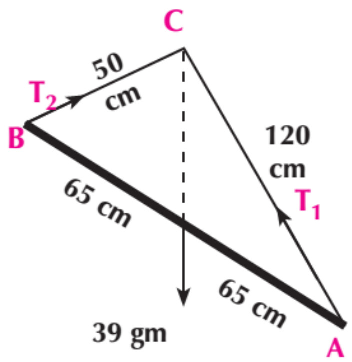
F = ... F = ... F = ...

K = ... K = ... K = ...

7. $\overline{AB}$ is a uniform rod under effect of three coplanar forces as it is shown in each figure. **Complete:**







A T_1 = ... gm.wt **B** T = ... gm.wt **C** T_1 = ... gm.wt

D T_2 = ... gm.wt **E** R= .... gm.wt **F** T_2 = ... gm .wt

8 If $\overrightarrow{R}$ is the resultant of the two forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ which enclose between them an angle of measure α and the measure of the angle of inclination of the resultant to F_1 equals θ find:

A The magnitude of $\overrightarrow{R}$, when $F_1 = 8$ newton , $F_2 = 15$ newton $\alpha = 90°$.

B The magnitude of $\overrightarrow{R}$ and the measure of angle θ when $F_1 = F_2 = 60$ dyne , $\alpha = 60°$.

C The magnitude of $\overrightarrow{R}$ and the measure of angle θ when $F_1 = 6$ newton , $F_2 = 3$ newton and the resultant is perpendicular to F_2.

D The magnitude of $\overrightarrow{R}$ and measure of angle α If $F_1 = 3\sqrt{3}$ newton , $F_2 = 6$ newton and the resultant is perpendicular to F_1

E Value of F_1 when $\overrightarrow{R} = 12$ newton , $F_2 = 12\sqrt{3}$ newton , $\alpha = 150°$

9 Find magnitude of the resultant and its inclination angle with x-axis is each figure from the following figures

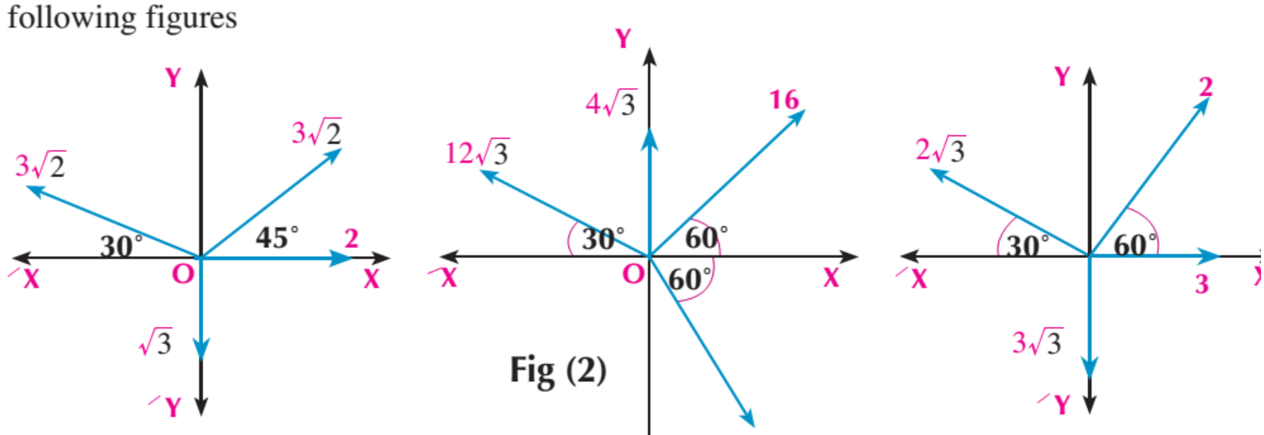


Fig (1) Fig (2) Fig (3)

10 Find the magnitude of each F , K so that, each set of forces from the following became in equilibrium..

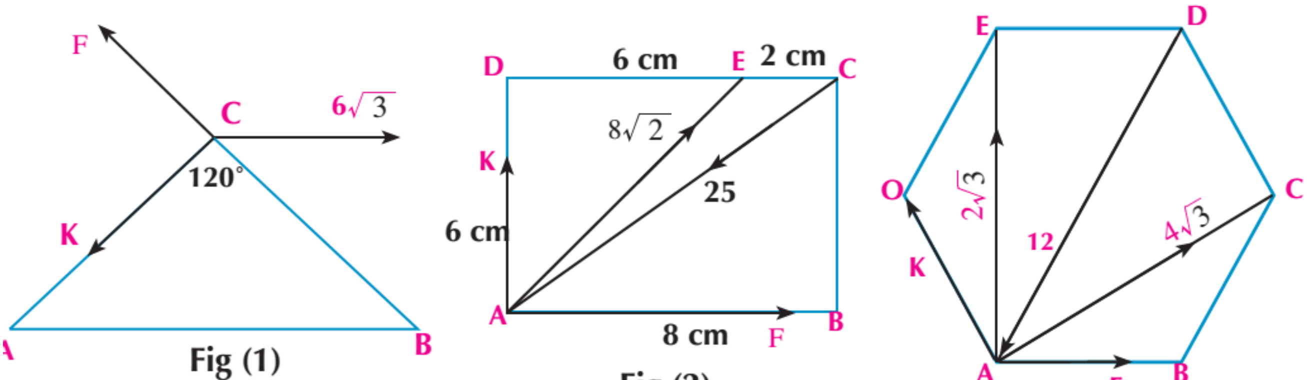
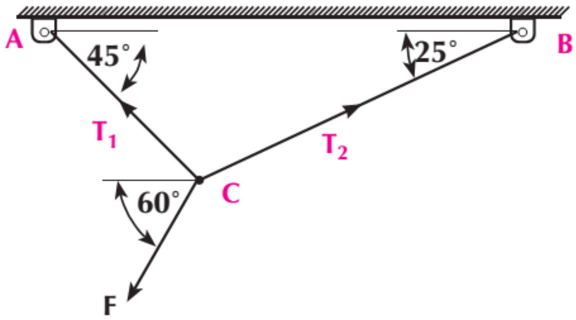


Fig (1) Fig (2) Fig (3)

11 In the opposite figure: the force $\overrightarrow{F}$ of magnitude 500 Newton acts on a particle at the point C and makes an angle of measure 60° with the horizontal . If the particle is connected by two strings at C and the other two ends of the strings are connected to two points A and B on the same horizontal line. The angle between the strings and the horizontal lines are 45° and 25° respectively.



Find in the state of equilibrium the tension in the two strings to the nearest newton