Chapter (5)

Hydrodynamics (fluids dynamic)

Hydrodynamics: deals with fluids in motion.

Types of fluid motion

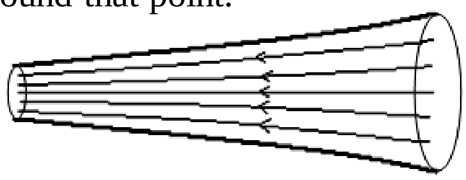
Steady flow	Turbulent flow
The flow of a liquid with small velocity such that its layers slide over each other smoothly. • The particles of the liquid follow continuous paths (streamlines). • The liquid can be visualized by a flux of streamlines representing the paths of the different particles of the liquid.	The flow of a liquid when its velocity exceeds a certain limit. • It is characterized by the existence of vortices. • Gases undergo turbulent flow as a result of diffusion from a small space to a large space or from high pressure to low pressure.

Streamlines:

The paths of the different particles of the liquid during its flow in a tube.

Density of streamlines at a point:

The number of streamlines crossing ⊥ on unit area around that point.



The properties of the streamlines:

1- Virtual lines don't intersect.

2- The tangent at any point along the streamline determines the direction of the instantaneous velocity at that point.

3- The density of streamlines expresses the velocity of the liquid flow at that point.

4- They become near at points of high velocity and keep apart at points of low velocity.

The conditions of steady flow:

1- The liquid fills the tube completely.

2- The quantity of liquid entering the tube at one end is equal to the quantity of liquid emerging from the other end in the same time (the rate of flow of the liquid is constant along its path).

3- The velocity of the liquid at each point is independent of time (doesn't change with time).

4- The flow is irrotational (no vortex motion).

- ❖ If there is no force of friction exists between the layers of the liquid, the flow is non-viscous.
- ❖ If there is force of friction, it is viscous.

The volume rate of flow: $Q_V = V_{ol} / t$

The volume of the liquid flowing through a cross section of the tube in unit time.

- Its unit is m^3/s .
- The volume rate of flow

  = cross sectional area × the distance covered by the liquid in 1 sec.

  = cross sectional area × the velocity of the liquid.

$$Q_V = A \times v$$

- The volume of the liquid flowing in time (t).

$$V_{ol} = Q_V \times t = A \times v \times t$$

The mass rate of flow: $Q_m = m / t$

The mass of the liquid flowing through a cross section of the tube in unit time.

- Its unit is kg / s .
- **The mass rate of flow**

  = the volume of the liquid flowing in unit time × liquid density.

  = the volume rate of flow × liquid density.

$$Q_m = Q_V \times \rho = A \times v \times \rho$$

- The mass of the liquid flowing in time (t).

$$m = Q_m \times t = Q_V \times \rho \times t = A \times v \times \rho \times t$$

<u>What is meant by</u>:

1- The volume rate of flow of a liquid = 2 m^3/s

It means that the volume of the liquid flowing through a cross section of the tube in unit time = $2\,m^3$.

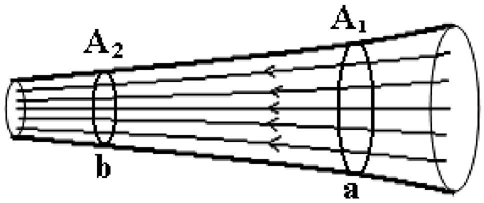
2- The mass rate of flow of a liquid = 200 kg / s

It means that the mass of the liquid flowing through a cross section of the tube in unit time = 200 kg

Continuity Equation

Choose two planes ⊥ to the streamlines at two sections (**a** & **b**).

- The cross sectional areas of the two planes are (**A_1** & **A_2**).
- The velocity of the liquid at the two planes is (**v_1** & **v_2**).
- The mass rate of flow of the liquid through **A_1**.



$$Q_m = Q_V \times \rho = A_1 \times v_1 \times \rho$$

- The mass rate of flow of the liquid through **A_2**.

$$Q_m = Q_V \times \rho = A_2 \times v_2 \times \rho$$

- As the rate of flow of the liquid along its path is constant in the steady flow.

$$A_1 \times v_1 \times \rho = A_2 \times v_2 \times \rho$$

$$\boxed{A_1 \times v_1 = A_2 \times v_2} \longrightarrow \text{Continuity equation}$$

$$\boxed{\frac{v_1}{v_2} = \frac{A_2}{A_1}}$$

<u>Continuity equation</u>:

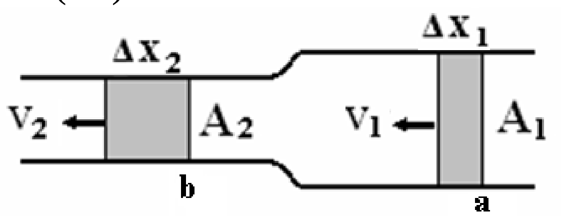
The velocity of the liquid at any point in the tube is inversely proportional to the cross section area of the tube at that point.

❖ The liquid flows slowly where the cross sectional area is large and vice versa.

Explaining the continuity equation by the conservation of mass law.

Consider a small amount of liquid ($\Delta m = \rho \Delta V_{ol}$) passes by a point (**a**) with velocity (v_1) and travels a distance (Δx_1) in time (Δt).

❖ The volume that passes in time (Δt).



$\Delta V_{ol} = A_1 \Delta x_1$ where $\Delta x_1 = v_1 \Delta t$

$\therefore \boxed{\Delta V_{ol} = A_1 v_1 \Delta t} \longrightarrow (1)$

❖ Since the liquid is incompressible, the same volume must emerge from the other side of the tube at point (**b**) with velocity (v_2) and travels a distance (Δx_2) in the same time (Δt).

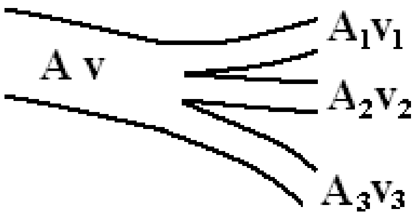
$\Delta V_{ol} = A_2 \Delta x_2$ where $\Delta x_2 = v_2 \Delta t$

$\therefore \boxed{\Delta V_{ol} = A_2 v_2 \Delta t} \longrightarrow (2)$

❖ From (1) and (2).

$$\therefore \boxed{A_1 v_1 = A_2 v_2}$$

Notes:



1- When a tube is branched into many branches:

$$Q_v = Q_{v1} + Q_{v2} + Q_{v3} + \cdots$$

$$\boxed{A\ v = A_1 v_1 + A_2 v_2 + A_3 v_3 + \cdots}$$

2- If the branches are equal: $A v = n A_1 v_1$, where (n) is the number of branches.

3- $A = \pi r^2$, where (r) is the radius of the tube.

Give reasons for:

1- **The fire hose has a narrow nozzle.**

Because the velocity of the liquid in a tube is inversely proportional to the cross sectional area, so water emerges from the nozzle with high velocity and can reach high elevations.

2- **The diameter of the water emerging from the fountain increases with height.**

Because the velocity of the liquid in a tube is inversely proportional to the cross sectional area, and the velocity of water decreases with height, so the cross sectional area increases.

3- **The velocity of blood in the blood capillaries is slower than that in the main artery.**

As $A v = n A_1 v_1$, and the cross sectional area of the main artery is smaller than the sum of the cross sectional areas of the blood capillaries ($A < n A_1$)

Then the velocity of the blood in the blood capillaries is slow to allow gas exchange with the cells and providing the cells with food.

Viscosity

Experiment

1- Hang two funnels each on a stand and put a beaker under each.

2- Pour alcohol in one funnel and a similar volume of glycerin in the other.

3- Observe the velocity of each.

Observation:

The flow of alcohol is faster than that of glycerin.

Experiment

1- Take two similar beakers, one containing water and the other containing an equal volume of honey.

2- Stir the liquids in both beakers with a glass rod.

3- Observe which of the liquids is easier to stir.

4- Remove the rod and observe which one stops faster.

Observation:

1- Water is easier to stir (water resistance to the motion of the glass rod is less than that of honey).

2- The motion of honey stops faster than that of water.

Experiment

1- Take two long similar measuring cylinders and fill them to the end, one with water and the other with glycerin.

2- Take two similar steel balls and drop one in each liquid.

3- Record the time each ball takes to hit the bottom.

Observation:

The time taken in water is less than that in glycerin (glycerin resistance to the motion of the ball through is greater).

Conclusion:

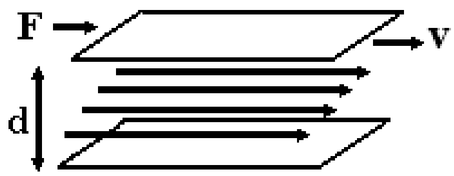
1- Some liquids such as water and alcohol offer little resistance to the motion of bodies in them; while they can flow easily (**have low viscosity**).

2- Other liquids such as honey and glycerin offer high resistance to the motion of bodies in them; while they don't flow easily (**have high viscosity**).

Viscosity:

The property which causes resistance (or friction) between the layers of the liquids hindering the easy sliding of these layers as well as the motion of bodies through.

Explaining the viscosity:



- Imagine layers of a liquid trapped two parallel plates, one is stationary and the other is moving with velocity (**v**).
- The liquid layer next to the stationary plate is stationary, while the layer next to the moving plate is moving at (v).
- The layers in between move at velocities varying from (**0**) to (**v**), then.

  a) The friction force between the stationary plate and the liquid layer in contact with it due to the adhesive force between the molecules of the solid surface and the contacting liquid molecules lead to zero velocity of this layer. Similarly, the upper layer moves at the same velocity of the moving plate.

  b) The friction (shear) force between each liquid layer and the adjacent one resists the sliding of the liquid layers with respect to each other and produces a relative change in velocity between any two adjacent layers.

❖ **<u>The velocity gradient</u>:** the ratio between the velocity of a liquid layer and its distant from the stationary layer. (velocity gradient $= \frac{v}{d}$).

❖ This type of flow is called non-turbulent viscous laminar flow (viscous steady flow), since no vortices are formed.

Coefficient of viscosity

❖ For the moving plate to keep its constant velocity, a force (**F**) must exist

❖ This force depends on:

1- The velocity of the moving plate: directly proportional ($F \propto v$).

2- The area of the moving plate: directly proportional ($F \propto A$).

3- The distance between the moving plate and stationary plate:

inversely proportional ($F \propto \frac{1}{d}$).

$$\therefore \quad F \propto \frac{A\,v}{d} \quad \Rightarrow \quad \boxed{F = \eta_{vs} \frac{A\,v}{d}}$$

Where η_{vs} is the coefficient of viscosity.

$$\boxed{\eta_{vs} = \frac{F\,d}{A\,v} = \frac{F}{A\,v/d}}$$

<u>Coefficient of viscosity</u>:

The tangential force acting on unit area, to produce a unit velocity difference between two layers, separated by unit distance apart.

❖ **The measuring units of the coefficient of viscosity:**

1- $\frac{N.m}{m^2.m/s} = N.s/m^2$

2- $\frac{kg.m/s^2.m}{m^2.m/s} = kg.m^{-1}.s^{-1}$

3- $N.s/m^2 = \frac{N}{m^2}s = pascal.s$

Applications of viscosity

1) Lubrication:

The metallic parts of machines are lubricated to;

a. Reduce the heat generated by friction.

b. Protect the machine parts from corrosion (wearing away).

❖ **Lubrication is carried out using highly viscous liquids.**
Because they have high adhesive forces, so they remain in contact with the moving parts of machines and reduce friction.

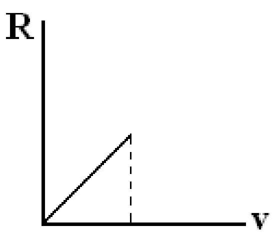
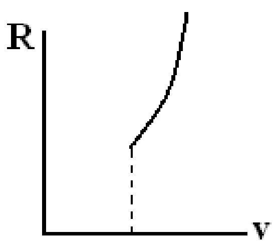
❖ **Water is not used in lubrication.**
Because water has low viscosity, so it will seep away from the moving parts of machines due to its low adhesive forces.

❖ **In summer, lubrication is carried out using very high viscous liquids.**
Because the viscosity decreases with rising temperature.

2) Economizing fuel consumption in a car:

❖ Air has resistance on the bodies move in it.

❖ When a car moves, the work done by the car motor which is supplied from burnt fuel acts against air resistance and friction with the road.

At low and medium velocities (below a certain limit)	At high velocities (above a certain limit)
Air resistance is directly proportional to the velocity of the car ($R \propto v$). 	Air resistance is directly proportional to the velocity square ($R \propto v^2$). The work needed to overcome air resistance increases, so the fuel consumption increases. 

- **It is advisable not to exceed the car velocity above a certain limit.**

  To save fuel consumption, because in low and medium velocities, air resistance is directly proportional to the velocity of the car ($R \propto v$).

  While in high velocities, air resistance is directly proportional to the velocity square ($R \propto v^2$), hence the work needed to overcome air resistance increases, so the fuel consumption increases.

3) Blood precipitation (sedimentation) rate test:

Measuring the final velocity of falling of the red blood cells in the plasma.

- When a ball undergoes s free fall in a liquid, it is under the effect of 3 forces:
  1. The weight of the ball.
  2. The buoyancy of the liquid on the ball.
  3. The friction force between the ball and the liquid due to viscosity.
- The final velocity of the red blood cells in the plasma is proportional to the radius square ($v \propto r^2$).
- The doctor may decide if the size of the red blood cells is normal or not by measuring the precipitation rate, so he can diagnose some diseases:
  1. In **rheumatic fever** and **gout**, the red blood cells adhere together, their volume and their radius increase, hence the precipitation rate increases.
  2. In **anemia**, the red blood cells break up, their volume and their radius decrease, hence the precipitation rate decreases.
- The normal precipitation rate = 15 mm after 1 hour and 30 mm after 2 hours.

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