# Differential calculus and Trigonometry

## 1st. stage (2nd. Sec.)

## Final Revision

## Answer the following question

**1)** a) Evaluate

i) $\lim\limits_{x\to 1}\dfrac{2x^2+x-3}{x^3-1}$ ii) $\lim\limits_{x\to 4}\dfrac{\frac{1}{2}x^4-128}{x-4}$ iii) $\lim\limits_{x\to\infty}\left(\dfrac{2x^3}{2x^2+1}-x\right)$ iv) $\lim\limits_{x\to 0}\dfrac{x^2+\tan 3x^2}{3x^2-\sin 2x^2}$

**b)** ABC is an obtuse angled triangle at A in which b=5cm., $m(\angle B)=30^\circ$ and $\tan c=\dfrac{4}{3}$

Find to nearest cm. each of a,c & surface area of $\Delta$ ABC

**c)** If $y=\dfrac{\sin x}{\cos x+\sin x}$ prove that $\dfrac{dy}{dx}=\dfrac{1}{1+\sin 2x}$

## Answer only three questions of the following:-

**2)** a) Evaluate:

i) $\lim\limits_{x\to -1}\left(\dfrac{x^2}{x^2-1}-\dfrac{2x+3}{x^2-1}\right)$ ii) $\lim\limits_{x\to 0}\dfrac{(1-2x)^5-1}{5x}$ iii) $\lim\limits_{x\to\infty}\dfrac{2x^4+2x^2-1}{5-x-3x^4}$ iv) $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\cos x}{2x-\pi}$

b) Find measure of the angel which the tangent makes with the +ve. direction of X-axis to the curve $y=\sqrt{2x-1}$ at y=1 also find the equation of this tangent.

c) If $\sin^2 A=\dfrac{9}{25}$, $A\in\left]\pi,\dfrac{3\pi}{2}\right[$ , $\tan B=-\dfrac{5}{12}$, $\dfrac{\pi}{2}<B<\pi$

Without using calculator. Find the value of sin 2A, cos (A+B) and tan (A-B)

**3)** a) Evaluate

i) $\lim\limits_{x\to 0}\dfrac{2x}{\sqrt{1+x}-\sqrt{1-x}}$ ii) $\lim\limits_{x\to -\frac{1}{2}}\dfrac{32x^5+1}{64x^6-1}$ iii) $\lim\limits_{x\to\infty}\dfrac{\sqrt{4x^3+3x}}{\sqrt[4]{x^6+x^2-1}}$ iv) $\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{x\tan 2x}$

b) Find the points on the curve $y=\frac{3x^2+1}{2x-1}$ at which the tangent is // to the straight line: $4x+2y+7=0$

c) A man standing at point B observed an object C in the eastern of B and at a distance 60 metes from B after the man walked from B to A in direction 60° North of East he Found that the point C in the direction 25° South of East. Find the distance CA

**4)** a) Evaluate

i) $\lim_{h\to 0}\frac{(x+5h)^{15}-x^{15}}{3h}$ ii) $\lim_{x\to\infty}\left(\sqrt{x^2+x+1}-x\right)$ iii) $\lim_{x\to 0}\frac{x^2+\tan^2 x}{5x^2-\sin^2 2x}$ iv) $\lim_{x\to 3}\frac{\sin(x-3)}{9-x^2}$

b) If $y=\sqrt{z}$ , $z=\frac{x-2}{x+1}$. Find $\frac{dy}{dx}$ at $x=3$

c) ABCD is a parallelogram in which the length of its diagonals are 5 cm.,8 cm. and the measure of angle between them is $77°18'$ Find the length of two adjacent sides in this parallelogram.

**5)** a) Find the point on the curve $y = \tan\frac{x}{2}$ at which the tangent perpendicular to the straight line $3x+3y+7=0$. where $0 < x < 2\pi$

b) $\Delta$ ABC in which: $\sin A : \sin B : \sin C = 4 : 5 : 6$ Prove that: $m(\angle C) = 2m(\angle A)$

C) i) Prove that: $\tan^2 15° + 2\sqrt{3}\tan 15° = 1$

ii) In $\Delta$ ABC prove that $\tan\frac{A}{2}\tan\frac{B}{2}+\tan\frac{B}{2}\tan\frac{C}{2}+\tan\frac{C}{2}\tan\frac{A}{2}=1$

6 ) a)

i) Without using calculator evaluate: $2 \sin 52°30` \cos 52°30`$

ii) If $\sin A \cos A = \frac{2}{5}$. Evaluate $\sin 3A \cos A + \cos 3A \sin A$.

b)

i) If $Y = ax^3 + bx$ and the average rate of change of this function when x changes from -2 to 1 is 5 & $\frac{dy}{dx} = \frac{11}{4}$ at $x = \frac{1}{2}$. Find the values of a,b

ii ) If $Y = \frac{1}{a + bx}$ is the equation of the curve passes through the point (1,-1) and the slope of tangent to this carve at this point is equal to 2 find the value of a,b

c) i) if the perimeter of $\Delta$ ABC is 12 cm., $m(\angle A) = 47^0$ , $m(\angle B) = 53^0$
Find the length of $\overline{AB}$ to nearest cm.& Surface area of the Circumcircle of $\Delta$ ABC

ii ) from the top of a tower of height 10 meter a man observed the elevation and depression angles of the top and base of a minaret are $42^0\ 18`$ and $14^0$ respectively if the bases of each of the tower and the minaret are on the some horizontal plane. Calculate the height of minaret to the nearest meter.

## The Model answer

1)

a)i) $\lim\limits_{x\to 1}\dfrac{2x^2+x-3}{x^3-1}$

$f(x)=\dfrac{2x^2+x-3}{x^3-1}$ , $f(1)=\dfrac{0}{0}$ unspecified value

$\lim\limits_{x\to 1}\dfrac{2x^2+x-3}{x^3-1}=\lim\limits_{x\to 1}\dfrac{\cancel{(x-1)}(2x+3)}{\cancel{(x-1)}(x^2+x+1)}$

$=\lim\limits_{x\to 1}\dfrac{(2x+3)}{(x^2+x+1)}=\dfrac{2\times 1+3}{1^2+1+1}=\dfrac{5}{3}=1\dfrac{2}{3}$

ii)

$\lim\limits_{x\to 4}\dfrac{\frac{1}{2}x^4-128}{x-4}$

$f(x)=\dfrac{\frac{1}{2}x^4-128}{x-4}$ , $f(4)=\dfrac{0}{0}$ unspecified value

$\lim\limits_{x\to 4}\dfrac{\frac{1}{2}x^4-128}{x-4}=\lim\limits_{x\to 4}\dfrac{\frac{1}{2}\left(x^4-128\div\frac{1}{2}\right)}{x-4}$

$=\lim\limits_{x\to 4}\dfrac{\frac{1}{2}\left(x^4-128\times\frac{2}{1}\right)}{x-4}=\dfrac{1}{2}\lim\limits_{x\to 4}\dfrac{\left(x^4-256\right)}{x-4}$

$=\dfrac{1}{2}\lim\limits_{x\to 4}\dfrac{\left(x^4-(4)^4\right)}{x-4}=\dfrac{1}{2}\times 4\times(4)^3=128$

iii) $\lim\limits_{x\to\infty}\left(\dfrac{2x^3}{2x^2+1}-x\right)$

$f(x)=\left(\dfrac{2x^3}{2x^2+1}-x\right)$, $f(\infty)=\infty-\infty$ unspecified value

$\lim\limits_{x\to\infty}\left(\dfrac{2x^3}{2x^2+1}-x\right)=\lim\limits_{x\to\infty}\left(\dfrac{2x^3}{2x^2+1}-\dfrac{x(2x^2+1)}{(2x^2+1)}\right)$

$=\lim\limits_{x\to\infty}\left(\dfrac{2x^3-x(2x^2+1)}{2x^2+1}\right)=\lim\limits_{x\to\infty}\left(\dfrac{\cancel{2x^3}-\cancel{2x^3}-x}{2x^2+1}\right)$

$=\lim\limits_{x\to\infty}\left(\dfrac{-x}{2x^2+1}\right)$, Divide up & down by $x^2$

$=\lim\limits_{x\to\infty}\left(\dfrac{-\frac{x}{x^2}}{\frac{2x^2}{x^2}+\frac{1}{x^2}}\right)=\lim\limits_{x\to\infty}\left(\dfrac{-\frac{1}{x}}{2+\frac{1}{x^2}}\right)=\dfrac{0}{2}=0.$

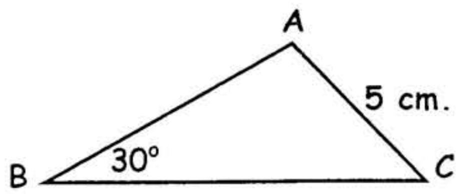
iv) $\lim\limits_{x\to 0}\dfrac{x^2+\tan 3x^2}{3x^2-\sin 2x^2}$

$f(x)=\dfrac{x^2+\tan 3x^2}{3x^2-\sin 2x^2}$ , $f(0)=\dfrac{0}{0}$ unspecified value

$\lim\limits_{x\to 0}\dfrac{x^2+\tan 3x^2}{3x^2-\sin 2x^2}$ , Divide up & down by $x^2$

$=\lim\limits_{x\to 0}\dfrac{\frac{\cancel{x^2}}{\cancel{x^2}}+\frac{\tan 3x^2}{x^2}}{\frac{3\cancel{x^2}}{\cancel{x^2}}-\frac{\sin 2x^2}{x^2}}=\lim\limits_{\substack{x\to 0\\ x^2\to 0}}\dfrac{1+\frac{\tan 3x^2}{x^2}}{3-\frac{\sin 2x^2}{x^2}}=\dfrac{1+3}{3-2}=4$

**b)**



$\because \tan C = \frac{4}{3} \Rightarrow m(\angle C) = 53°8'$

$\therefore m(\angle A) = 180° - (30° + 53°8') = 96°52'$

$\because \frac{a}{\sin 96°52'} = \frac{5}{\sin 30°} = \frac{c}{\sin 53°8'}$

$\therefore a = \frac{5 \times \sin 96°52'}{\sin 30°} \approx 10\text{cm.}$

$\therefore c = \frac{5 \times \sin 53°8'}{\sin 30°} \approx 8\text{cm.}$

$\because$ S.area of $\Delta ABC = \frac{1}{2} ac \sin B$

$\therefore$ The S.area $= \frac{1}{2} \times 10 \times 8 \times \sin 30° = 20\text{cm}^2.$

c) $\because y = \frac{\sin x}{\cos x + \sin x}$

$$\therefore \frac{dy}{dx} = \frac{\cos x(\cos x + \sin x) - \sin x(-\sin x + \cos x)}{(\cos x + \sin x)^2}$$

$$\therefore \frac{dy}{dx} = \frac{\cos^2 x + \cancel{\cos x \sin x} + \sin^2 x - \cancel{\sin x \cos x}}{\cos^2 x + \sin^2 x + 2\cos x.\sin x}$$

$$\therefore \frac{dy}{dx} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x + \sin^2 x + 2\cos x.\sin x} = \frac{1}{1 + \sin 2x}$$

2)a)i) $\lim_{x \to -1} \left( \frac{x^2}{x^2-1} - \frac{2x+3}{x^2-1} \right)$

$f(x) = \left( \frac{x^2}{x^2-1} - \frac{2x+3}{x^2-1} \right)$

$, f(-1) = \left( \frac{1}{0} - \frac{1}{0} \right) =$ unspecified value

$$\lim_{x \to -1} \left( \frac{x^2 - (2x+3)}{x^2-1} \right) = \lim_{x \to -1} \left( \frac{x^2-2x-3}{x^2-1} \right)$$

$$= \lim_{x \to -1} \frac{\cancel{(x+1)}(x-3)}{\cancel{(x+1)}(x-1)} = \lim_{x \to -1} \frac{(x-3)}{(x-1)} = \frac{-4}{-2} = 2.$$

ii) $\lim_{x \to 0} \frac{(1-2x)^5 - 1}{5x}$

$f(x) = \frac{(1-2x)^5 - 1}{5x}, f(0) = \frac{0}{0}$ unspecified value

$$\lim_{x \to 0} \frac{(1-2x)^5 - 1}{(1-2x) - 1} \times \frac{(1-2x) - 1}{5x}$$

$$\lim_{\substack{x \to 0 \\ -2x \to 0 \\ 1-2x \to 1}} \frac{(1-2x)^5 - 1^5}{(1-2x) - 1} \times \lim_{x \to 0} \frac{\cancel{1} - 2x - \cancel{1}}{5x}$$

$$\lim_{1-2x \to 1} \frac{(1-2x)^5 - 1^5}{(1-2x) - 1} \times \lim_{x \to 0} \frac{-2\cancel{x}}{5\cancel{x}} = \cancel{5} \times (1)^4 \times \frac{-2}{\cancel{5}} = -2$$

iii)

$\lim_{x \to \infty} \frac{2x^4 + 2x^2 - 1}{5 - x - 3x^4}$

$f(x) = \frac{2x^4 + 2x^2 - 1}{5 - x - 3x^4}, f(\infty) = \frac{\infty}{-\infty}$ unspecified value

Divide up and down by $x^4$

$$\lim_{x \to \infty} \frac{\frac{2x^4}{x^4} + \frac{2x^2}{x^4} - \frac{1}{x^4}}{\frac{5}{x^4} - \frac{x}{x^4} - \frac{3x^4}{x^4}} = \lim_{x \to \infty} \frac{2 + \frac{2}{x^2} - \frac{1}{x^4}}{\frac{5}{x^4} - \frac{1}{x^3} - 3}$$

$$= \frac{2 + \frac{2}{\infty} - \frac{1}{\infty}}{\frac{5}{\infty} - \frac{1}{\infty} - 3} = \frac{2+0+0}{0-0-3} = -\frac{2}{3}$$

iv) $\lim_{x \to \frac{\pi}{2}} \frac{\cos x}{2x-\pi}$

$f(x)=\frac{\cos x}{2x-\pi}$,

$f(\frac{\pi}{2})=\frac{\cos\frac{\pi}{2}}{2\left(\frac{\pi}{2}\right)-\pi}=\frac{0}{0}$ unspecified value

$$\lim_{x \to \frac{\pi}{2}} \frac{\cos x}{2x-\pi} = \lim_{\substack{x-\frac{\pi}{2}\to 0 \\ \text{both sides} \times -1 \\ \therefore \frac{\pi}{2}-x \to 0}} \frac{\sin\left(\frac{\pi}{2}-x\right)}{-2\left(\frac{\pi}{2}-x\right)}$$

$$=\frac{1}{-2}\lim_{\left(\frac{\pi}{2}-x\right)\to 0}\frac{\sin\left(\frac{\pi}{2}-x\right)}{\left(\frac{\pi}{2}-x\right)}=\frac{1}{-2}\times 1=-\frac{1}{2}$$

b) $\because y=\sqrt{2x-1} \Rightarrow y=(2x-1)^{\frac{1}{2}}$

$\frac{dy}{dx}=\frac{1}{\cancel{2}}\times(2x-1)^{-\frac{1}{2}}\times \cancel{2}=(2x-1)^{-\frac{1}{2}}=\frac{1}{(2x-1)^{\frac{1}{2}}}$

$\frac{dy}{dx}=\frac{1}{\sqrt{2x-1}}$ ....................(1)

At $y=1 \Rightarrow 1=\sqrt{2x-1}$ (By squaring both sides)

$\therefore 1=2x-1 \Rightarrow 2=2x \Rightarrow \boxed{1=x}$

$\therefore$ Slope of tangent equals $\left.\frac{dy}{dx}\right|_{\text{At } y=1, x=1}$

$\left.\frac{dy}{dx}\right|_{\text{At } x=1}=\frac{1}{\sqrt{2(1)-1}}=\frac{1}{\sqrt{1}}=1$

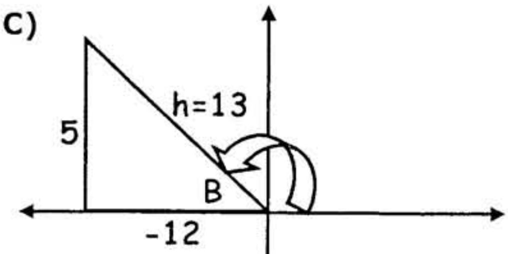
$\therefore \tan\theta = 1 \Rightarrow m(\angle\theta) = 45^\circ$

$\therefore$ The tangent makes angle of measure 45 with the +ve. direction of X-axis.

$\because$ Equation of tangent $\frac{y-y_1}{x-x_1}=\frac{dy}{dx}$

$\therefore \frac{y-1}{x-1}=1 \Rightarrow y-1=x-1 \Rightarrow \boxed{y=x}$

C)

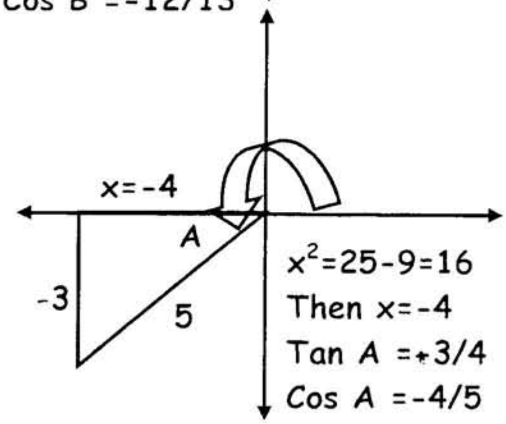



$h^2=25+144=169$

Then h=13

Sin B =5/13

Cos B =-12/13




$x^2=25-9=16$

Then x=-4

Tan A =+3/4

Cos A =-4/5

$\because \sin^2 A = \frac{9}{25} \Rightarrow \sin A = \pm\frac{3}{5}$

$\because \angle A \in \left]\pi, \frac{3\pi}{2}\right[ \Rightarrow \boxed{\sin A = -\frac{3}{5}}$

$\because \sin 2A = 2\sin A\cos A$

$\therefore \sin 2A = 2\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right)=\frac{24}{25}$

$\because \cos(A+B) = \cos A\cos B - \sin A\sin B$

$\therefore \cos(A+B) = \left(-\frac{4}{5}\right)\left(-\frac{12}{13}\right)-\left(-\frac{3}{5}\right)\left(\frac{5}{13}\right)$

$=\left(\frac{48}{65}\right)+\left(\frac{15}{65}\right)=\frac{63}{65}$

$\tan(A-B)=\frac{\tan A - \tan B}{1+\tan A\tan B}=\frac{\frac{3}{4}-\left(-\frac{5}{12}\right)}{1+\left(\frac{3}{4}\times-\frac{5}{12}\right)}$

$=\frac{\frac{3}{4}+\frac{5}{12}}{1+\left(-\frac{15}{48}\right)}=\frac{\left(\frac{3}{4}+\frac{5}{12}\right)\times 48}{\left(1-\frac{15}{48}\right)\times 48}$

$=\frac{36+20}{48-15}=\frac{56}{33}=1\frac{23}{33}$

3) a) i) $\lim\limits_{x\to 0}\dfrac{2x}{\sqrt{1+x}-\sqrt{1-x}}$

$f(x)=\dfrac{2x}{\sqrt{1+x}-\sqrt{1-x}}$, $f(0)=\dfrac{0}{0}$ unspecified value

$$\lim_{x\to 0}\frac{2x}{\sqrt{1+x}-\sqrt{1-x}}\times\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$$

$$\lim_{x\to 0}\frac{2x\left(\sqrt{1+x}+\sqrt{1-x}\right)}{(1+x)-(1-x)}=\lim_{x\to 0}\frac{2x\left(\sqrt{1+x}+\sqrt{1-x}\right)}{\cancel{1}+x-\cancel{1}+x}$$

$$=\lim_{x\to 0}\frac{2\cancel{x}\left(\sqrt{1+x}+\sqrt{1-x}\right)}{2\cancel{x}}$$

$$=\lim_{x\to 0}\frac{\cancel{2}\left(\sqrt{1+x}+\sqrt{1-x}\right)}{\cancel{2}}=\left(\sqrt{1}+\sqrt{1}\right)=2$$

ii) $\lim\limits_{x\to -\frac{1}{2}}\dfrac{32x^5+1}{64x^6-1}$

$f(x)=\dfrac{32x^5+1}{64x^6-1}$, $f\left(-\dfrac{1}{2}\right)=\dfrac{0}{0}$ unspecified value

$$\lim_{x\to -\frac{1}{2}}\frac{32x^5+1}{64x^6-1}=\lim_{x\to -\frac{1}{2}}\frac{(2x)^5+1}{(2x)^6-1}$$

$$=\lim_{\substack{x\to -\frac{1}{2}\\ 2x\to -1}}\frac{(2x)^5-(-1)^5}{(2x)^6-(-1)^6}=\frac{5}{6}\times(-1)^{5-6}$$

$$=\frac{5}{6}\times(-1)^{-1}=\frac{5}{6}\times\frac{1}{-1}=-\frac{5}{6}$$

iii) $\lim\limits_{x\to\infty}\dfrac{\sqrt{4x^3+3x}}{\sqrt[4]{x^6+x^2-1}}$

$f(x)=\dfrac{\sqrt{4x^3+3x}}{\sqrt[4]{x^6+x^2-1}}$, $f(\infty)=\dfrac{\infty}{\infty}$ unspecified value

Divide up and down by $\sqrt[4]{x^6}=x^{\frac{6}{4}}=x^{\frac{3}{2}}=\sqrt{x^3}$

$$\lim_{x\to\infty}\frac{\dfrac{\sqrt{4x^3+3x}}{\sqrt{x^3}}}{\dfrac{\sqrt[4]{x^6+x^2-1}}{\sqrt[4]{x^6}}}=\lim_{x\to\infty}\frac{\sqrt{\dfrac{4x^3+3x}{x^3}}}{\sqrt[4]{\dfrac{x^6+x^2-1}{x^6}}}$$

$$=\lim_{x\to\infty}\frac{\sqrt{4+\dfrac{3}{x^2}}}{\sqrt[4]{1+\dfrac{1}{x^4}-\dfrac{1}{x^6}}}=\frac{\sqrt{4+\dfrac{3}{\infty}}}{\sqrt[4]{1+\dfrac{1}{\infty}-\dfrac{1}{\infty}}}$$

$$=\frac{\sqrt{4+0}}{\sqrt[4]{1+0-0}}=\frac{\sqrt{4}}{\sqrt[4]{1}}=\frac{2}{1}=2$$

iv) $\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{x\tan 2x}$

$f(x)=\dfrac{1-\cos 2x}{x\tan 2x}$, $f(0)=\dfrac{0}{0}$ unspecified value

$$\lim_{x\to 0}\frac{1-\cos 2x}{x\tan 2x}=\lim_{x\to 0}\frac{1-\left(1-2\sin^2 x\right)}{x\tan 2x}$$

$$=\lim_{x\to 0}\frac{\cancel{1}-\cancel{1}+2\sin^2 x}{x\tan 2x}=\lim_{x\to 0}\frac{2\sin^2 x}{x\tan 2x}$$

Divide up and down by $x^2$

$$=\lim_{x\to 0}\frac{\dfrac{2\sin^2 x}{x^2}}{\dfrac{\cancel{x}\tan 2x}{x^{\cancel{2}}}}=\lim_{x\to 0}\frac{2\left(\dfrac{\sin x}{x}\right)^2}{\dfrac{\tan 2x}{x}}=\frac{2}{2}=1$$

b) $\because y=\dfrac{3x^2+1}{2x-1} \Rightarrow \dfrac{dy}{dx}=\dfrac{6x(2x-1)-2(3x^2+1)}{(2x-1)^2}$

$\dfrac{dy}{dx}=\dfrac{12x^2-6x-6x^2-2}{(2x-1)^2}=\dfrac{6x^2-6x-2}{(2x-1)^2}$

$\because$ the tangent is // to the line: $4x+2y+7=0$

$\therefore \dfrac{dy}{dx}$ = Slope of the line $4x+2y+7=0$

$\because$ Slope of $4x+2y+7=0$ equals $\dfrac{-4}{2}=-2$

$-2=\dfrac{6x^2-6x-2}{(2x-1)^2} \Rightarrow -2(2x-1)^2=6x^2-6x-2$

$-2(4x^2-4x+1)=6x^2-6x-2$

$-8x^2+8x-2=6x^2-6x-2 \Rightarrow 14x^2-14x=0$

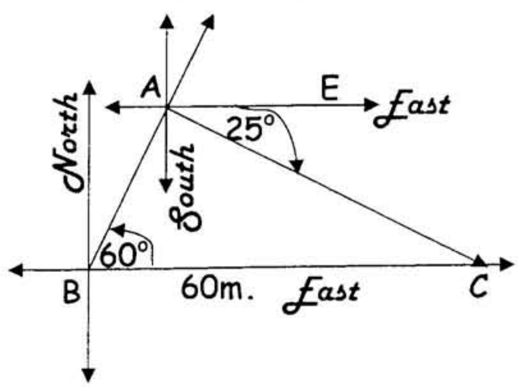
$14x(x-1)=0$

Either $x=0 \Rightarrow y=\dfrac{3(0)^2+1}{2(0)-1}=-1 \Rightarrow (0,-1)$

Or $x-1=0 \Rightarrow x=1 \Rightarrow y=\dfrac{3(1)^2+1}{2(1)-1}=4 \Rightarrow (1,4)$

$\therefore$ The points are $(0,-1)$, $(1,4)$

c)




$\because m(\angle ACB)=m(\angle CAE)=25^\circ$ Alternate.

In $\triangle ACB$

$\because m(\angle BAC)=180^\circ-(60^\circ+25^\circ)=95^\circ$

$\because \dfrac{BC}{\sin(\angle BAC)}=\dfrac{AC}{\sin(\angle ABC)}$

$\therefore \dfrac{60}{\sin(95^\circ)}=\dfrac{AC}{\sin(60^\circ)}$

$\therefore AC=\dfrac{60\sin(60^\circ)}{\sin(95^\circ)}\approx 52$m.

4)a)i) $\lim\limits_{h\to o}\dfrac{(x+5h)^{15}-x^{15}}{3h}$

$f(h)=\dfrac{(x+5h)^{15}-x^{15}}{3h}$, $f(0)=\dfrac{0}{0}$ unspecified value

$\lim\limits_{h\to o}\dfrac{(x+5h)^{15}-x^{15}}{5h}\times\lim\limits_{h\to o}\dfrac{5h}{3h}$

$\lim\limits_{(x+h)\to x}\dfrac{(x+5h)^{15}-x^{15}}{(x+5h)-x}\times\lim\limits_{h\to o}\dfrac{5\cancel{h}}{3\cancel{h}}$

$=15x^{14}\times\dfrac{5}{3}=\overset{5}{\cancel{15}}x^{14}\times\dfrac{5}{\cancel{3}}=25x^{14}$

ii) $\lim\limits_{x\to\infty}\left(\sqrt{x^2+x+1}-x\right)$

$f(x)=\left(\sqrt{x^2+x+1}-x\right)$

$f(\infty)=\infty-\infty$ unspecified value

$\lim\limits_{x\to\infty}\left(\sqrt{x^2+x+1}-x\right)\times\dfrac{\left(\sqrt{x^2+x+1}+x\right)}{\left(\sqrt{x^2+x+1}+x\right)}$

$=\lim\limits_{x\to\infty}\dfrac{\left(\cancel{x^2}+x+1-\cancel{x^2}\right)}{\left(\sqrt{x^2+x+1}+x\right)}=\lim\limits_{x\to\infty}\dfrac{x+1}{\sqrt{x^2+x+1}+x}$

Divide up and down by $x$, $\boxed{x=\sqrt{x^2}}$

$=\lim\limits_{x\to\infty}\dfrac{\frac{x}{x}+\frac{1}{x}}{\frac{\sqrt{x^2+x+1}+x}{x}}=\lim\limits_{x\to\infty}\dfrac{\frac{x}{x}+\frac{1}{x}}{\frac{\sqrt{x^2+x+1}}{\sqrt{x^2}}+\frac{x}{x}}$

$=\lim\limits_{x\to\infty}\dfrac{1+\frac{1}{x}}{\sqrt{\frac{x^2+x+1}{x^2}}+1}=\lim\limits_{x\to\infty}\dfrac{1+\frac{1}{x}}{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+1}=\dfrac{1}{1+1}=\dfrac{1}{2}$

iii) $\lim\limits_{x\to o}\dfrac{x^2+\tan^2x}{5x^2-\sin^2 2x}$

$f(x)=\dfrac{x^2+\tan^2x}{5x^2-\sin^2 2x}$, $f(0)=\dfrac{0}{0}$ unspecified value

Divide up and down by $x^2$

$\lim\limits_{x\to o}\dfrac{\frac{\cancel{x^2}}{\cancel{x^2}}+\frac{\tan^2x}{x^2}}{\frac{5\cancel{x^2}}{\cancel{x^2}}-\frac{\sin^2 2x}{x^2}}=\lim\limits_{x\to o}\dfrac{1+\left(\frac{\tan x}{x}\right)^2}{5-\left(\frac{\sin 2x}{x}\right)^2}=\dfrac{1+1}{5-4}=2$

iv) $\lim\limits_{x\to 3} \dfrac{\sin(x-3)}{9-x^2}$

$f(x)=\dfrac{\sin(x-3)}{9-x^2}$ , $f(3)=\dfrac{0}{0}$ unspecified value

$$\therefore \lim_{\substack{x\to 3\\ x-3\to 3-3\\ x-3\to 0}} \frac{\sin(x-3)}{-\left(x^2-9\right)} = \lim_{\substack{x\to 3\\ x-3\to 0}} \frac{\sin(x-3)}{-(x+3)(x-3)}$$

$$= \lim_{x\to 3} \frac{1}{-(x+3)} \times \lim_{(x-3)\to 0} \frac{\sin(x-3)}{(x-3)} = \frac{1}{-6} \times 1 = -\frac{1}{6}$$

b) $\because\ y=\sqrt{z}$ , $z=\dfrac{x-2}{x+1}$

$$\therefore y=\sqrt{\frac{x-2}{x+1}}=\left(\frac{x-2}{x+1}\right)^{\frac{1}{2}}$$

$$\therefore \frac{dy}{dx}=\frac{1}{2}\left(\frac{x-2}{x+1}\right)^{-\frac{1}{2}} \times \left(\frac{1(x+1)-1(x-2)}{(x+1)^2}\right)$$

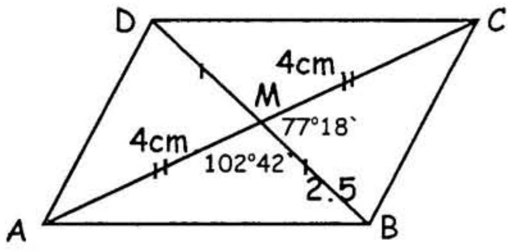
$$\therefore \frac{dy}{dx}=\frac{1}{2}\left(\frac{x-2}{x+1}\right)^{-\frac{1}{2}} \times \left(\frac{x+1-x+2}{(x+1)^2}\right)$$

$$\therefore \frac{dy}{dx}=\frac{1}{2}\left(\frac{x+1}{x-2}\right)^{\frac{1}{2}} \times \frac{3}{(x+1)^2}$$

$$\therefore \left.\frac{dy}{dx}\right|_{x=3}=\frac{1}{2}\left(\frac{3+1}{3-2}\right)^{\frac{1}{2}} \times \frac{3}{(3+1)^2}=\frac{3}{16}$$



C)




Let $\overline{AC} \cap \overline{BD} = \{M\}$

From properties of $\square$

$\therefore AM=CM=\dfrac{8}{2}=4$cm., $BM=DM=\dfrac{5}{2}=2.5$cm.

$\therefore m(\angle AMB)=180^\circ - 77^\circ 18' = 102^\circ 42'$

In $\Delta AMB$

$\because (AB)^2=(MA)^2+(MB)^2-2\,MA\,MB\cos\widehat{M}$

$\therefore (AB)^2=(4)^2+(2.5)^2-2\times 4\times 2.5\cos 102^\circ 42'$

$\therefore (AB)^2 \approx 26.65 \Rightarrow \boxed{AB \approx 5.2\text{cm.}}$

$\because (BC)^2=(MB)^2+(MC)^2-2\,MB\,MC\cos\widehat{M}$

$\therefore (BC)^2=(2.5)^2+(4)^2-2\times 2.5\times 4\cos 77^\circ 18'$

$\therefore (BC)^2 \approx 17.85 \Rightarrow \boxed{BC \approx 4.2\text{cm.}}$

5) a)

$\because$ The tangent is $\perp$ to the line $3x+3y+7=0$

Slope of the line $3x+3y+7=0$ equals $\frac{-3}{3}=-1$

$\therefore$ Slope of tangent $\frac{dy}{dx}=1$

$\because y=\tan\frac{x}{2} \Rightarrow \frac{dy}{dx}=\frac{1}{2}\sec^2\frac{x}{2} \Rightarrow \frac{1}{2}\sec^2\frac{x}{2}=1$

$\therefore \frac{1}{2\cos^2\frac{x}{2}}=1 \Rightarrow 2\cos^2\frac{x}{2}=1 \Rightarrow 2\cos^2\frac{x}{2}-1=0$

Remember that:

$$2\cos^2\frac{x}{2}-1=\cos\cancel{2}\left(\frac{x}{\cancel{2}}\right)=\cos x$$

$\therefore \cos\cancel{2}\left(\frac{x}{\cancel{2}}\right)=0 \Rightarrow \cos x=0$

| Either | OR |
|---|---|
| $x=\frac{\pi}{2}$ | $x=\frac{3\pi}{2}$ |
| $\therefore y=\tan\frac{\frac{\pi}{2}}{2}$ | $\therefore y=\tan\frac{\frac{3\pi}{2}}{2}$ |
| $=\tan\frac{\pi}{4}=1$ | $=\tan\frac{3\pi}{4}=-1$ |
| The 1st point$(\frac{\pi}{2},1)$ | The point 2nd$(\frac{3\pi}{2},-1)$ |

b) $\Delta$ ABC in which:

$\because \sin A : \sin B : \sin C = 4 : 5 : 6$

$\& \because \sin A : \sin B : \sin C = a : b : c$

$\therefore a : b : c = 4 : 5 : 6$

Let $\boxed{a=4x} \Rightarrow \boxed{b=5x}$ & $\boxed{c=6x}$

$\because \cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{(4x)^2+(5x)^2-(6x)^2}{2\times 4x\times 5x}$

$\therefore \cos C = \frac{16x^2+25x^2-36x^2}{40x^2} = \frac{5\cancel{x^2}}{40\cancel{x^2}} = \boxed{\frac{1}{8}}$

$\therefore \boxed{\cos C = \frac{1}{8}}$..................(1)

$\because \cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{(5x)^2+(6x)^2-(4x)^2}{2\times 5x\times 6x}$

$\therefore \cos A = \frac{25x^2+36x^2-16x^2}{60x^2} = \frac{45\cancel{x^2}}{60\cancel{x^2}} = \frac{3}{4}$

$\therefore \cos 2A = 2\cos^2 A-1 = 2\left(\frac{3}{4}\right)^2-1 = 2\times\frac{9}{16}-1$

$\therefore \boxed{\cos 2A = \frac{9}{8}-1=\frac{1}{8}}$..........(2)

From (1) & (2) we get

$\cos 2A = \cos C \Rightarrow \boxed{2m(\angle A) = m(\angle C)}$

C) i) $\because 30^\circ = 2\times 15^\circ$

$\because \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$\therefore \tan 30^\circ = \frac{2\tan 15^\circ}{1-\tan^2 15^\circ}$

$\therefore \frac{1}{\sqrt{3}} = \frac{2\tan 15^\circ}{1-\tan^2 15^\circ}$

$\therefore 1\left(1-\tan^2 15^\circ\right) = \sqrt{3}\left(2\tan 15^\circ\right)$

$\therefore 1-\tan^2 15^\circ = 2\sqrt{3}\tan 15^\circ$

$\therefore 1 = \tan^2 15^\circ + 2\sqrt{3}\tan 15^\circ$

ii) $\because m(\angle A) + m(\angle B) + m(\angle C) = 180^\circ$

$\therefore \frac{A}{2} + \frac{B}{2} + \frac{C}{2} = 90^\circ \Rightarrow \frac{A}{2} + \frac{B}{2} = 90^\circ - \frac{C}{2}$

$\therefore \text{Tan}\left(\frac{A}{2} + \frac{B}{2}\right) = \text{Tan}\left(90^\circ - \frac{C}{2}\right)$

$\therefore \dfrac{\text{Tan}\frac{A}{2} + \text{Tan}\frac{B}{2}}{1 - \text{Tan}\frac{A}{2}\,\text{Tan}\frac{B}{2}} = \cot\frac{C}{2} = \dfrac{1}{\tan\frac{C}{2}}$

$\therefore \tan\frac{C}{2}\left(\text{Tan}\frac{A}{2} + \text{Tan}\frac{B}{2}\right) = 1 - \text{Tan}\frac{A}{2}\,\text{Tan}\frac{B}{2}$

$\therefore \tan\frac{C}{2}\,\text{Tan}\frac{A}{2} + \tan\frac{C}{2}\,\text{Tan}\frac{B}{2} + \text{Tan}\frac{A}{2}\,\text{Tan}\frac{B}{2} = 1$

6 ) a) i)

$\because 2\sin 52^\circ 30` \cos 52^\circ 30` = \sin 2(52^\circ 30`)$

$= \sin 105^\circ$

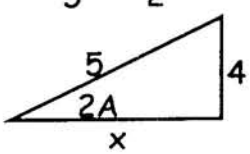
$= \sin(45^\circ + 60^\circ) = \sin 45^\circ \cos 60^\circ + \cos 45^\circ \sin 60^\circ$

$= \frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} = \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$

$= \frac{1 + \sqrt{3}}{2\sqrt{2}} = \frac{\left(1 + \sqrt{3}\right)}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2} + \sqrt{6}}{4}$

ii)

$\because \sin A \cos A = \frac{2}{5} \Rightarrow \frac{1}{2}\sin 2A = \frac{2}{5} \Rightarrow \sin 2A = \frac{4}{5}$




$\because x^2 = 25 - 16 = 9 \Rightarrow x = \pm 3 \Rightarrow \cos 2A = \pm\frac{3}{5}$

$\because \sin 3A \cos A + \cos 3A \sin A = \sin(3A + A) = \sin 4A$

$\because \sin 4A = 2 \sin 2A \cos 2A$

$\therefore \sin 4A = 2\left(\frac{4}{5}\right) \times \left(\pm\frac{3}{5}\right) = \pm\frac{24}{25}$

b) i) $\because Y = ax^3 + bx \Rightarrow \frac{dy}{dx} = 3ax^2 + b$

$\because \frac{dy}{dx} = \frac{11}{4}$ At $x = \frac{1}{2} \Rightarrow \frac{11}{4} = 3a\left(\frac{1}{2}\right)^2 + b$

$\therefore \frac{11}{4} = \frac{3}{4}a + b$ (both sides $\times$ 4)

$\therefore \boxed{11 = 3a + 4b}$...........(1)

$\because A(h) = 5$ where x changs from -2 to 1

$\therefore A(h) = \frac{f(1) - f(-2)}{1 - (-2)} = 5$

$\therefore \frac{\left(a(1)^3 + b(1)\right) - \left(a(-2)^3 + b(-2)\right)}{3} = 5$

$\therefore \frac{(a + b) - (-8a - 2b)}{3} = 5$

$\therefore a + b + 8a + 2b = 15$

$\therefore 9a + 3b = 15 \Rightarrow \boxed{3a + b = 5}$....(2)

(1)-(2) $\Rightarrow 3b = 6 \Rightarrow \boxed{b = 2} \Rightarrow \boxed{a = 1}$

ii) $\because Y = \dfrac{1}{a+bx}$

$\therefore \dfrac{dy}{dx} = \dfrac{0\times(a+bx) - b\times 1}{(a+bx)^2} = \dfrac{-b}{(a+bx)^2}$

$\because$ The curve passes through the point $(1,-1)$

$\therefore y=-1$ at $x=1 \Rightarrow -1 = \dfrac{1}{a+b(1)}$

$\therefore -1 = \dfrac{1}{a+b} \Rightarrow -1(a+b) = 1 \Rightarrow \boxed{a+b=-1}$

$\because$ Slope of tangent $=2$ at the point $(1,-1)$

$\therefore \dfrac{dy}{dx} = 2$ at $x=1$ & $\because \dfrac{dy}{dx} = \dfrac{-b}{(a+bx)^2}$

$\therefore 2 = \dfrac{-b}{(a+b(1))^2} \Rightarrow 2 = \dfrac{-b}{(a+b)^2} \Rightarrow 2 = \dfrac{-b}{(-1)^2}$

$\therefore \boxed{b=-2} \Rightarrow a-2=-1 \Rightarrow \boxed{a=1}$

c) i) In $\triangle ABC$

$\because m(\angle C) = 180^\circ - (47^\circ + 53^\circ) = 80^\circ$

$\because \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2r$

$\therefore \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = \dfrac{a+b+c}{\sin A+\sin B+\sin C}$

$\therefore \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = \dfrac{\text{perimeter of } ABC}{\sin A+\sin B+\sin C}$

$\therefore \dfrac{c}{\sin 80^\circ} = \dfrac{12}{\sin 47^\circ + \sin 53^\circ + \sin 80^\circ} = 2r$

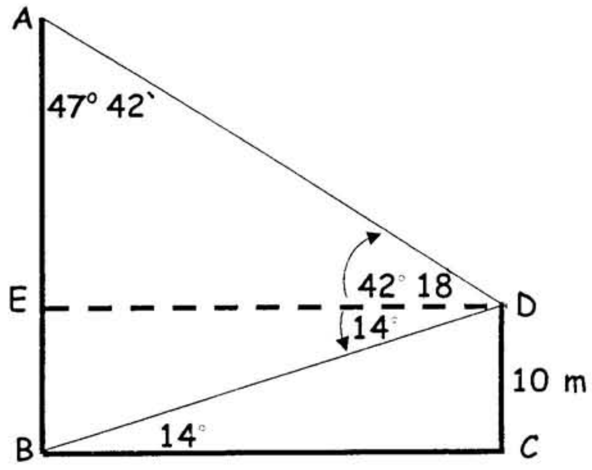
$\therefore c = \dfrac{12\times\sin 80^\circ}{\sin 47^\circ + \sin 53^\circ + \sin 80^\circ} \approx 5\text{cm}.$

$\therefore 2r = 4.77176 \Rightarrow r \approx 2\text{cm}.$

$\because$ S. area of a Circle $=\pi r^2$

$\therefore$ S. area of the circumCircle $=\pi(2)^2 = 4\pi\ \text{cm}^2$.

ii)




in $\Delta$ BCD

$\dfrac{BD}{\sin(\angle BCD)} = \dfrac{CD}{\sin(\angle CBD)}$

$\dfrac{BD}{\sin 90^\circ} = \dfrac{10}{\sin 14^\circ}$

$BD = \dfrac{10\sin 90^\circ}{\sin 14^\circ} \approx 41\text{m}.$

in $\Delta$ ADE

$m(\angle EAD) = 90^\circ - 42^\circ 18' = 47^\circ 42'$

in $\Delta$ ADB

$m(\angle BDA) = 14^\circ + 42^\circ 18' = 56^\circ 18'$

$\dfrac{AB}{\sin(\angle ADB)} = \dfrac{BD}{\sin(\angle BAD)}$

$\dfrac{AB}{\sin 56^\circ 18'} = \dfrac{41}{\sin 47^\circ 42'}$

$\therefore AB = \dfrac{41\times\sin 56^\circ 18'}{\sin 47^\circ 42'} \approx 46\text{m}.$