

## Definition of Matrix

A matrix is a collection of numbers arranged into a fixed number of rows and columns. Usually the numbers are real numbers. In general, matrices can contain complex numbers but we won't see those here. Here is an example of a matrix with three rows and three columns:

$$\begin{matrix} & \text{col 1} & \dots & & \\ \text{row 1} & \dots & \begin{pmatrix} 1 & -2 & 3 \\ 0 & 8 & 4.6 \\ 4 & -1 & 0 \end{pmatrix} & & \end{matrix}$$

The top row is row 1. The leftmost column is column 1. This matrix is a 3x3 matrix because it has three rows and three columns. In describing matrices, the format is:

rows X columns

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Each number that makes up a matrix is called an *element* of the matrix. The elements in a matrix have specific locations.

The upper left corner of the matrix is row 1 column 1. In the above matrix the element at row 1 col 1 is the value 1. The element at row 2 column 3 is the value 4.6.

### QUESTION:

What is the value of the element at row 3 column 1?

## Matrix Dimensions(orders)

The numbers of rows and columns of a matrix are called its **dimensions**. Here is a matrix with three rows and two columns:

$$\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$$

### Types of Matrices

#### Row Matrix

A matrix having only one row is called a row-matrix.

For example: A[1 3 2 -2] is a row matrix of order 1 x 4.

#### Column Matrix

A matrix having only one column is called a column matrix.

For example:  $A = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$  and  $B = \begin{bmatrix} 3 \\ 2 \\ 5 \\ 4 \end{bmatrix}$  are column matrices of order

3 x 1 and 4 x 1 respectively.

## Square Matrix

A matrix in which the number of rows is equal to the number of columns,.

For example, the matrix  $\begin{bmatrix} 2 & 1 & -1 \\ 3 & -2 & 5 \\ 1 & 5 & -3 \end{bmatrix}$  is a square matrix of order 3.

The leading diagonal elements are 2, -2 and -3.

## Diagonal Matrix

A square matrix  $A = [a_{ij}]_{n \times n}$  is called a diagonal matrix if all the elements, except those in the leading diagonal, are zero.

i.e.,  $a_{ij} = 0$  for all  $i \neq j$

**Example:**

The matrix  $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$  is a diagonal matrix.

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## Scalar Matrix

A scalar matrix is a diagonal matrix in which all the diagonal elements are equal.

**Example:**

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1-2i & 0 & 0 \\ 0 & 1-2i & 0 \\ 0 & 0 & 1-2i \end{bmatrix}$$

The matrices are scalar matrices of order 2 and 3 respectively.

## Identity or Unit Matrix

A square matrix  $A = [a_{ij}]_{n \times n}$  is called an identity or unit matrix if

(1)  $a_{ij} = 0$  for all  $i \neq j$  and

(2)  $a_{ii} = 1$  for all  $i = j$

In other words a square matrix each of whose diagonal elements is unity and each of whose non-diagonal elements is equal to zero is called an identity or unit matrix. The identity matrix of order  $n$  is denoted by  $I_n$ .

**Example:**

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The matrices are identity matrices of order 2 and 3 respectively.

## Null Matrix or Zero Matrix

A matrix of order  $m \times n$  whose elements are all 0 is called a null matrix (or zero matrix) of order  $m \times n$ . It is usually denoted by  $O$  or more clearly  $[O]_{m,n}$ .

**Example:**

$$[0,0] \quad \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

are all zero matrices of orders 1 x 2, 2 x 1, 2 x 2 and 3 x 3 respectively.

## Transpose

The **transpose** of a matrix is a new matrix whose rows are the columns of the original (which makes its columns the rows of the original). Here is a matrix and its transpose:

$$\begin{pmatrix} 5 & 4 & 3 \\ 4 & 0 & 4 \\ 7 & 10 & 3 \end{pmatrix}^T = \begin{pmatrix} 5 & 4 & 7 \\ 4 & 0 & 4 \\ 3 & 4 & 3 \end{pmatrix}$$

The superscript "T" means "transpose". Another way to look at the transpose is that the element at row r column c of the original is placed at row c column r of the transpose. We will usually work with square matrices, and it is usually square matrices that will be transposed. However, non-square matrices can be transposed, as well:

$$\begin{pmatrix} 5 & 4 \\ 4 & 0 \\ 7 & 10 \\ -1 & 8 \end{pmatrix}_{4 \times 2}^T = \begin{pmatrix} 5 & 4 & 7 & -1 \\ 4 & 0 & 10 & 8 \end{pmatrix}_{2 \times 4}$$

### QUESTION:

What is the transpose of:

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}_{2 \times 3}^T = ?$$

good answer might be:

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}_{2 \times 3}^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$$

## A Rule for Transpose

If a transposed matrix is itself transposed, you get the original back:

$$\left( \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}^T \right)^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}^T = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

This illustrates the rule  $(\mathbf{A}^T)^T = \mathbf{A}$ .

### QUESTION:

What is the transpose of:

$$\begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix}^T = ?$$

A good answer might be:

$$\begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix}^T = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$$

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The transpose of a row matrix is a column matrix. And the transpose of a column matrix is a row matrix.

## Rule Summary

Here are some rules that cover what has been discussed. You should check that they seem reasonable, rather than memorize them. For each rule the matrices have the same number of rows and columns.

$$\mathbf{A} + \mathbf{0} = \mathbf{A}$$

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A} \quad \mathbf{0} + \mathbf{0} = \mathbf{0}$$

$$\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$$

$$(ab)\mathbf{A} = a(b\mathbf{A})$$

$$a(\mathbf{A} + \mathbf{B}) = a\mathbf{A} + a\mathbf{B}$$

$$a\mathbf{0} = \mathbf{0}$$

$$(-1)\mathbf{A} = -\mathbf{A}$$

$$\mathbf{A} - \mathbf{A} = \mathbf{0}$$

$$(\mathbf{A}^T)^T = \mathbf{A}$$

$$\mathbf{0}^T = \mathbf{0}$$

In the above,  $a$  and  $b$  are scalars (real numbers).  $\mathbf{A}$  and  $\mathbf{B}$  are matrices, and  $\mathbf{0}$  is the zero matrix of appropriate dimension



## Worksheet (1)

<b>1</b>	$A = \begin{pmatrix} -1 & 3 \\ 7 & -2 \end{pmatrix}$	<b>5</b>	$A = \begin{pmatrix} 3 & 7 \\ 5 & -1 \end{pmatrix}$
	<p>The matrix A of the order.....</p> <p><math>a_{12} = \dots</math> , <math>a_{11} = \dots</math> , <math>a_{22} = \dots</math></p> <p>The type of the matrix is.....</p> <p><math>A^T =</math></p>		<p>The matrix A of the order.....</p> <p><math>a_{12} = \dots</math> , <math>a_{11} = \dots</math> , <math>a_{22} = \dots</math></p> <p>The type of the matrix is.....</p> <p><math>A^T =</math></p>
<b>2</b>	$A = \begin{pmatrix} -3 & 5 & 0 \\ 1 & 4 & -2 \\ 7 & 3 & 6 \end{pmatrix}$	<b>6</b>	$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
	<p>The matrix A of the order.....</p> <p><math>a_{13} = \dots</math> , <math>a_{22} = \dots</math> , <math>a_{32} = \dots</math></p> <p>The type of the matrix is.....</p> <p><math>A^T =</math></p>		<p>The matrix A of the order.....</p> <p><math>a_{13} = \dots</math> , <math>a_{22} = \dots</math> , <math>a_{32} = \dots</math></p> <p>The type of the matrix is.....</p> <p><math>A^T =</math></p>
<b>3</b>	$A = \begin{pmatrix} -2 \\ 5 \\ 4 \end{pmatrix}$	<b>7</b>	$C = \begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix}$
	<p>The matrix A of the order.....</p> <p><math>a_{13} = \dots</math> , <math>a_{21} = \dots</math> , <math>a_{31} = \dots</math></p> <p>The type of the matrix is.....</p>		<p>The matrix C of the order.....</p> <p><math>C_{13} = \dots</math> , <math>C_{21} = \dots</math> , <math>C_{31} = \dots</math></p> <p>The type of the matrix is.....</p>
<b>4</b>	$B = (2 \quad -3 \quad 5 \quad 1)$	<b>8</b>	$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
	<p>The matrix B of the order.....</p> <p><math>B_{13} = \dots</math> , <math>b_{12} = \dots</math> , <math>b_{11} = \dots</math></p> <p>The type of the matrix is.....</p>		<p>The matrix A of the order.....</p> <p><math>a_{13} = \dots</math> , <math>a_{22} = \dots</math> , <math>a_{32} = \dots</math></p> <p>The type of the matrix is.....</p>

## Matrix Equality

For two matrices to be equal, they must have

1. The same dimensions.
2. Corresponding elements must be equal.

In other words, say that  $\mathbf{A}_{n \times m} = [a_{ij}]$  and that  $\mathbf{B}_{p \times q} = [b_{ij}]$ .

Then  $\mathbf{A} = \mathbf{B}$  if and only if  $n=p$ ,  $m=q$ , and  $a_{ij}=b_{ij}$  for all  $i$  and  $j$  in range.

Here are two matrices which are not equal even though they have the same elements.

$$\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2} \neq \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}_{2 \times 3}$$

### QUESTION 6:

Here is another pair of matrices.

Are these two matrices equal?

No, they have the same dimensions, but corresponding elements are not equal.

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suppose you have the following two matrices:

$$A = \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 & 0 \\ -2 & 0 & 0 \end{bmatrix}$$

These matrices cannot be the same, since they are not the same size. Even if  $A$  and  $B$  are the following two matrices:

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

- Given that the following matrices are equal, find the values of  $x$  and  $y$ .

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} x & 2 \\ 3 & y \end{bmatrix}$$

For  $A$  and  $B$  to be equal, they must have the same size and shape (which they do; they're each  $2 \times 2$  matrices)

$$x = 1, \quad y = 4$$

- Given that the following matrices are equal, find the values of  $x$ ,  $y$ , and  $z$ .

$$A = \begin{bmatrix} 4 & 0 \\ 6 & -2 \\ 3 & 1 \end{bmatrix} \quad B = \begin{bmatrix} x & 0 \\ 6 & y+4 \\ \frac{z}{3} & 1 \end{bmatrix}$$

To have  $A = B$ , I must have all entries equal. That is I must have  $a_{1,1} = b_{1,1}$ ,  $a_{1,2} = b_{1,2}$ ,  $a_{2,1} = b_{2,1}$ , and so forth. In particular, I must have:

$$\begin{aligned} 4 &= x \\ -2 &= y + 4 \\ 3 &= \frac{z}{3} \end{aligned}$$

...as you can see from the highlighted matrices:

$$\begin{bmatrix} 4 & 0 \\ 6 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} x & 0 \\ 6 & y+4 \\ \frac{z}{3} & 1 \end{bmatrix}$$

Solving these three equations, I get:

$$x = 4, y = -6, \text{ and } z = 9.$$

## Adding and Subtracting Matrices

Matrix addition is fairly simple, and is done entry-wise.

- **Add the following matrices:**

$$\begin{bmatrix} 0 & 1 & 2 \\ 9 & 8 & 7 \end{bmatrix} + \begin{bmatrix} 6 & 5 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

I need to add the pairs of entries, and then simplify for the final answer:

$$\begin{bmatrix} 0 & 1 & 2 \\ 9 & 8 & 7 \end{bmatrix} + \begin{bmatrix} 6 & 5 & 4 \\ 3 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 0+6 & 1+5 & 2+4 \\ 9+3 & 8+4 & 7+5 \end{bmatrix} = \begin{bmatrix} 6 & 6 & 6 \\ 12 & 12 & 12 \end{bmatrix}$$

So the answer is:

$$\begin{bmatrix} 6 & 6 & 6 \\ 12 & 12 & 12 \end{bmatrix}$$

Up until now, you've been able to add any two things you felt like: numbers, variables, equations, and so forth. But addition doesn't always work with matrices.

- **Perform the indicated operation, or explain why it is not possible.**

$$\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Since matrices are added entry-wise, I have to add the 1 and the 4, the 2 and 5, the 0 and the 7, and the 3 and the 8. But what do I add to the 6 and to the 9? There are no corresponding entries in the first matrix that can be added to these entries in the second matrix. So the answer is:

**I can't add these matrices, because they're not the same size.**

This is always the case: To be able to add two matrices, they must be of the same size. If they are not the same size (if they do not have the same "dimensions"), then the addition is "not defined" (doesn't make mathematical sense).

Subtraction works entry-wise, too.

- Given the following matrices, find  $A - B$  and  $A - C$ , or explain why you can not.

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 3 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 0 & -4 & 3 \\ 9 & -4 & -3 \end{bmatrix} \quad C = \begin{bmatrix} 0 & -4 \\ 9 & -4 \end{bmatrix}$$

$A$  and  $B$  are the same size, each being  $2 \times 3$  matrices, so I can subtract, working entry-wise:

$$\begin{aligned} A - B &= \begin{bmatrix} -1 & 2 & 0 \\ 0 & 3 & 6 \end{bmatrix} - \begin{bmatrix} 0 & -4 & 3 \\ 9 & -4 & -3 \end{bmatrix} \\ &= \begin{bmatrix} -1-0 & 2-(-4) & 0-3 \\ 0-9 & 3-(-4) & 6-(-3) \end{bmatrix} \\ &= \begin{bmatrix} -1 & 2+4 & -3 \\ -9 & 3+4 & 6+3 \end{bmatrix} = \begin{bmatrix} -1 & 6 & -3 \\ -9 & 7 & 9 \end{bmatrix} \end{aligned}$$

However,  $A$  and  $C$  are not the same size, since  $A$  is  $2 \times 3$  and  $C$  is  $2 \times 2$ . So this subtraction is not defined.

$$A - B = \begin{bmatrix} -1 & 6 & -3 \\ -9 & 7 & 9 \end{bmatrix}$$

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$A - C$  is not defined, because  $A$  and  $C$  are not the same size.

Matrix addition and subtraction, where defined (that is, where the matrices are the same size so addition and subtraction make sense), can be turned into homework problems.

- Find the values of  $x$  and  $y$  given the following equation:

$$\begin{bmatrix} -3 & x \\ 2y & 0 \end{bmatrix} + \begin{bmatrix} 4 & 6 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 7 \\ -5 & 1 \end{bmatrix}$$

First, I'll simplify the left-hand side a bit by adding entry-wise:

$$\begin{aligned} \begin{bmatrix} -3 & x \\ 2y & 0 \end{bmatrix} + \begin{bmatrix} 4 & 6 \\ -3 & 1 \end{bmatrix} &= \begin{bmatrix} -3+4 & x+6 \\ 2y-3 & 0+1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & x+6 \\ 2y-3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 7 \\ -5 & 1 \end{bmatrix} \end{aligned}$$

Since matrix equality works entry-wise, I can compare the entries to create simple equations that I can solve. In this case, the 1,2-entries tell me that  $x + 6 = 7$ , and the 2,1-entries tell me that  $2y - 3 = -5$ . Solving, I get:

$$\begin{aligned} x + 6 &= 7 \\ x &= 1 \end{aligned}$$

$$\begin{aligned} 2y - 3 &= -5 \\ 2y &= -2 \\ y &= -1 \end{aligned}$$



The first rule says that matrix addition is *commutative*. This is because ordinary addition is being done on the corresponding elements of the two matrices, and ordinary (real) addition is commutative:

$$\begin{pmatrix} 1 & 3 & 5 \\ 7 & 9 & 11 \\ 13 & 15 & 17 \end{pmatrix} + \begin{pmatrix} 0 & 2 & 4 \\ 6 & 8 & 10 \\ 12 & 14 & 16 \end{pmatrix} = \begin{pmatrix} 1+0 & 3+2 & 5+4 \\ 7+6 & 9+8 & 11+10 \\ 13+12 & 15+14 & 17+16 \end{pmatrix}$$

$$= \begin{pmatrix} 0+1 & 2+3 & 4+5 \\ 6+7 & 8+9 & 10+11 \\ 12+13 & 14+15 & 16+17 \end{pmatrix} = \begin{pmatrix} 0 & 2 & 4 \\ 6 & 8 & 10 \\ 12 & 14 & 16 \end{pmatrix} + \begin{pmatrix} 1 & 3 & 5 \\ 7 & 9 & 11 \\ 13 & 15 & 17 \end{pmatrix}$$

## Practice with Matrix Addition

Here is another matrix addition problem. Mentally form the sum (or use a scrap of paper):

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} + \begin{pmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{pmatrix} = ?$$

Hint: this problem is not as tedious as it might at first seem.

### QUESTION 10:

What is the sum?

Each element of the 3x3 result is 10.

## Scalar and Matrix Multiplication (page 1 of 3)

There are two types of multiplication for matrices: scalar multiplication and matrix multiplication. Scalar multiplication is easy. You just take a regular number (called a "scalar") and multiply it on every entry in the matrix.

- For the following matrix  $A$ , find  $2A$  and  $-1A$ .

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

To do the first scalar multiplication to find  $2A$ , I just multiply a 2 on every entry in the matrix:

$$2A = 2 \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 \cdot 1 & 2 \cdot 2 \\ 2 \cdot 3 & 2 \cdot 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$$

The other scalar multiplication, to find  $-1A$ , works the same way:

$$-1A = -1 \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -1 \cdot 1 & -1 \cdot 2 \\ -1 \cdot 3 & -1 \cdot 4 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -3 & -4 \end{bmatrix}$$

So the final answer is:

$$2A = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$$

$$-1A = \begin{bmatrix} -1 & -2 \\ -3 & -4 \end{bmatrix}$$

## Matrix Multiplication / The Identity Matrix (page 3 of 3)

Here are a couple more examples of matrix multiplication:

- Find  $CD$  and  $DC$ , if they exist, given that  $C$  and  $D$  are the following matrices:

$$C = \begin{bmatrix} 2 & -1 \\ 0 & 3 \\ 1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 1 & 4 & -1 \\ -2 & 0 & 0 & 2 \end{bmatrix}$$

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$C$  is a  $3 \times 2$  matrix and  $D$  is a  $2 \times 4$  matrix, so first I'll look at the dimension product for  $CD$ :

product is defined  
(3×2)(2×4)  
product will be 3×4

So the product  $CD$  is defined (that is, I can do the multiplication); also, I can tell that I'm going to get a  $3 \times 4$  matrix for my answer. Here's the multiplication:

$$\begin{aligned} CD &= \begin{bmatrix} 2 & -1 \\ 0 & 3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 4 & -1 \\ -2 & 0 & 0 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 2 \cdot 0 + (-1)(-2) & 2 \cdot 1 + (-1) \cdot 0 & 2 \cdot 4 + (-1) \cdot 0 & 2 \cdot (-1) + (-1) \cdot 2 \\ 0 \cdot 0 + 3 \cdot (-2) & 0 \cdot 1 + 3 \cdot 0 & 0 \cdot 4 + 3 \cdot 0 & 0 \cdot (-1) + 3 \cdot 2 \\ 1 \cdot 0 + 0 \cdot (-2) & 1 \cdot 1 + 0 \cdot 0 & 1 \cdot 4 + 0 \cdot 0 & 1 \cdot (-1) + 0 \cdot 2 \end{bmatrix} \\ &= \begin{bmatrix} 0+2 & 2+0 & 8+0 & -2-2 \\ 0-6 & 0+0 & 0+0 & 0+6 \\ 0+0 & 1+0 & 4+0 & -1+0 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 8 & -4 \\ -6 & 0 & 0 & 6 \\ 0 & 1 & 4 & -1 \end{bmatrix} \end{aligned}$$

However, look at the dimension product for  $DC$ :

product is not defined  
(2×4)(3×2)

Since the inner dimensions don't match, I can't do the multiplication. (The columns of  $C$  aren't the same length as the rows of  $D$ ; the columns of  $C$  are too short, or, if you prefer, the rows of  $D$  are too long.) Then the answer is:

$$CD = \begin{bmatrix} 2 & 2 & 8 & -4 \\ -6 & 0 & 0 & 6 \\ 0 & 1 & 4 & -1 \end{bmatrix}$$

**$DC$  is not defined.**

## Multiplying by the identity

Multiplying by the identity matrix  $I$  doesn't change anything, just like multiplying a number by 1 doesn't change anything. This property is why  $I$  and 1 are each called the "multiplicative identity". But while there is only one "multiplicative identity" for regular numbers (namely the number 1), there are lots of different identity matrices. Why? Because the identity matrix you need will depend upon the size of the matrix that it is being multiplied on! For instance, suppose you have the following matrix  $A$ :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

To multiply  $A$  on the right by the identity (that is, to do  $AI$ ), you have to use the  $3 \times 3$  identity,  $I_3$ , as the right number of rows for the multiplication to work:

$$\begin{aligned} AI_3 &= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 \cdot 1 + 2 \cdot 0 + 3 \cdot 0 & 1 \cdot 0 + 2 \cdot 1 + 3 \cdot 0 & 1 \cdot 0 + 2 \cdot 0 + 3 \cdot 1 \\ 4 \cdot 1 + 5 \cdot 0 + 6 \cdot 0 & 4 \cdot 0 + 5 \cdot 1 + 6 \cdot 0 & 4 \cdot 0 + 5 \cdot 0 + 6 \cdot 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = A \end{aligned}$$

On the other hand, to multiply  $A$  on the left by the identity, you have to use  $I_2$ , the  $2 \times 2$  identity, in order to have the right number of columns:

$$\begin{aligned} I_2 A &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \\ &= \begin{bmatrix} 1 \cdot 1 + 0 \cdot 4 & 1 \cdot 2 + 0 \cdot 5 & 1 \cdot 3 + 0 \cdot 6 \\ 0 \cdot 1 + 1 \cdot 4 & 0 \cdot 2 + 1 \cdot 5 & 0 \cdot 3 + 1 \cdot 6 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = A \end{aligned}$$

That is, if you are dealing with a non-square matrix (such as  $A$  in the above example), the identity matrix you use will depend upon the side that you're multiplying on. This is just another example of matrix weirdness. Don't let it scare you. Matrices aren't bad; they're just different... really, really different.



quiz. write true or false

1. A  $2 \times 3$  matrix has three columns and two rows.
2. The transpose of a  $5 \times 6$  matrix has five columns and six rows.
3. If A is a  $2 \times 3$  matrix and B is a  $3 \times 2$  matrix, then  $A+B$  is defined.
4. If A is a  $2 \times 3$  matrix and B is a  $3 \times 2$  matrix, then  $A-B$  is defined.
5. If A is a  $2 \times 3$  matrix and B is a  $3 \times 2$  matrix, then  $AB$  is defined.
6. If A is a  $3 \times 4$  matrix and B is a  $3 \times 4$  matrix, then  $A+B$  is defined.
7. If A is a  $3 \times 4$  matrix and B is a  $3 \times 4$  matrix, then  $A-B$  is defined.
8. If A is an invertible  $3 \times 3$  matrix and B is a  $3 \times 4$  matrix, then  $A^{-1}B$  is defined.
9. It is never true that  $A+B$ ,  $A-B$ , and  $AB$  are all defined.
10. If  $AB$  is defined, then  $BA$  must also be defined.
11. If  $AB$  and  $BA$  are both defined, they may have different dimensions.
12. If  $AB$  and  $BA$  are both defined and have the same dimensions, then  $A=B$ .

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