Math - 2016

Prep. (3) - Geometry

Final Revision Solutions

1 First Term

Mr. Mahmoud Esmaiel

01110882717

01006487539

Prep 3 T1 FR Geometry 2016

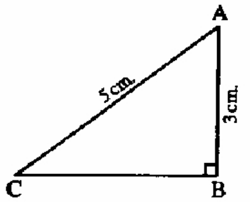
[A] Choose Problems Answers

Sn.	Answer	Sn.	Answer
1	B	22	C
2	C	23	A
3	A	24	D
4	A	25	D
5	A	26	B
6	C	27	B
7	D	28	A
8	A	29	C
9	D	30	B
10	D	31	C
11	B	32	C
12	D	33	C
13	B	34	B
14	B	35	B
15	C	36	C
16	B	37	B
17	C	38	D
18	C	39	C
19	D	40	D
20	C	41	B

21	A	42	A	
Sn.	Answer	36	Answer	
43	A	64	C	
44	B	65	B	
45	A	66	C	
46	B	67	D	
47	D	68	D	
48	C	69	C	
49	D	70	A	
50	C	71	B	
51	C	72	A	
52	A	73	A	
53	A	74	D	
54	C	75	D	
55	A	76	A	
56	C	77	A	
57	A	78	1	C
58	C	78	2	D
59	C	78	3	B
60	B	78	4	A
61	D	78	5	D
62	D	78	6	C
63	C			

[B] Essay Problems

Problem number [1]



$\because m(\angle B) = 90^\circ$

$\therefore (BC)^2 = (5)^2 - (3)^2 = 16$

$\therefore BC = 4$ cm.

① $\tan A \times \tan C = \frac{4}{3} \times \frac{3}{4} = 1$

② $\sin^2 A + \sin^2 C = \left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2 = \frac{16}{25} + \frac{9}{25} = 1$

Problem number [2]

$\cos 60^\circ \sin 30^\circ - \sin 60^\circ \cos 30^\circ$

$= \frac{1}{2} \times \frac{1}{2} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} - \frac{3}{4} = -\frac{1}{2}$

Problem number [3]

$2 \sin 45^\circ \cos 45^\circ + 4 \sin 30^\circ \cos 60^\circ$

$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + 4 \times \frac{1}{2} \times \frac{1}{2} = 2$

Problem number [4]

$\sin 45^\circ \cos 45^\circ + \sin 30^\circ \cos 60^\circ - \cos^2 30^\circ$

$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{2} - \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{2} + \frac{1}{4} - \frac{3}{4} = 0$

Problem number [5]

$\tan^2 45^\circ - 4 \cos^2 60^\circ = (1)^2 - 4 \times \left(\frac{1}{2}\right)^2$

$= 1 - 4 \times \frac{1}{4} = 0$

Problem number [6]

$$\frac{\cos^2 60^\circ + \cos^2 30^\circ + \tan^2 45^\circ}{\sin 60^\circ \tan 60^\circ - \sin 30^\circ}$$

$$= \frac{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2}{\frac{\sqrt{3}}{2} \times \sqrt{3} - \frac{1}{2}} = \frac{\frac{1}{4} + \frac{3}{4} + 1}{\frac{3}{2} - \frac{1}{2}} = 2$$

Problem number [7]

$\because \cos 60^\circ = \frac{1}{2}$ (1)

, $\cos^2 30^\circ - \sin^2 30^\circ = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4}$

$= \frac{1}{2}$ (2)

From (1) and (2) : $\therefore \cos 60^\circ = \cos^2 30^\circ - \sin^2 30^\circ$

Problem number [8]

$\because \tan^2 60^\circ - \tan^2 45^\circ = \left(\sqrt{3}\right)^2 - (1)^2$

$= 3 - 1 = 2$ (1)

, $4 \sin 30^\circ = 4 \times \frac{1}{2} = 2$ (2)

From (1) and (2) : $\therefore \tan^2 60^\circ - \tan^2 45^\circ = 4 \sin 30^\circ$

Problem number [9]

$\because 2 \sin x = \tan^2 60^\circ - 2 \tan 45^\circ$

$\therefore 2 \sin x = \left(\sqrt{3}\right)^2 - 2 \times 1$

$\therefore \sin x = \frac{3-2}{2} = \frac{1}{2}$ $\therefore x = 30^\circ$

Problem number [10]

$\because \sin^3 30^\circ = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$ (1)

, $9 \cos^3 60^\circ - \tan^2 45^\circ = 9 \times \left(\frac{1}{2}\right)^3 - (1)^2$

$= \frac{9}{8} - 1 = \frac{1}{8}$ (2)

From (1) and (2) :

$\therefore \sin^3 30^\circ = 9 \cos^3 60^\circ - \tan^2 45^\circ$

Problem number [11]

$\because \tan 60^\circ = \sqrt{3}$ (1)

, $2 \tan 30^\circ \div (1 - \tan^2 30^\circ)$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \sqrt{3} \quad (2)$$

Problem number [12]

$\because \sin 45^\circ \cos 45^\circ + \sin 30^\circ \cos 60^\circ$

$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{2} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ (1)

, $\cos^2 30^\circ = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$ (2)

From (1) and (2) :

$\therefore \sin 45^\circ \cos 45^\circ + \sin 30^\circ \cos 60^\circ = \cos^2 30^\circ$

Problem number [13]

$\because \tan 60° = \sqrt{3}$ (1)

$, \frac{2\tan 30°}{1-\tan^2 30°} = \frac{2\times\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \sqrt{3}$ (2)

From (1) and (2) : $\therefore \tan 60° = \frac{2\tan 30°}{1-\tan^2 30°}$

Problem number [14]

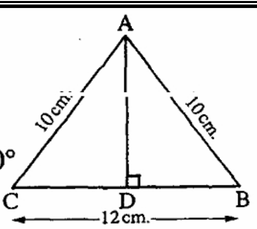
$\because 2\cos^2 30° - 1 = 2\times\left(\frac{\sqrt{3}}{2}\right)^2 - 1$

$= \frac{3}{2} - 1 = \frac{1}{2}$ (1)

$, 1 - 2\sin^2 30° = 1 - 2\times\left(\frac{1}{2}\right)^2 = 1 - \frac{1}{2} = \frac{1}{2}$ (2)

From (1) and (2) : $\therefore 2\cos^2 30° - 1 = 1 - 2\sin^2 30°$

Problem number [15]



$\because \overline{AD} \perp \overline{BC}$, AB = AC

$\therefore$ BD = CD = 6 cm.

In Δ ABD : $\because m(\angle ADB) = 90°$

$\therefore (AD)^2 = (10)^2 - (6)^2 = 64$

$\therefore$ AD = 8 cm.

(1) L.H.S = $\sin B + \cos C = \frac{8}{10} + \frac{6}{10} = 1.4$ = R.H.S

(2) L.H.S = $\sin^2 C + \cos^2 C$

$= \left(\frac{8}{10}\right)^2 + \left(\frac{6}{10}\right)^2 = \frac{64}{100} + \frac{36}{100} = 1$ = R.H.S

Problem number [16]

$\because 2\sin 30° \cos 30° = 2\times\frac{1}{2}\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$ (1)

$, \sin 60° = \frac{\sqrt{3}}{2}$ (2)

From (1) and (2) : $\therefore 2\sin 30°\cos 30° = \sin 60°$

Problem number [17]

$\because \sin\theta = \sin 45°\cos 30° + \cos 45°\sin 30°$

$\therefore \sin\theta = \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\times\frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\therefore \theta = 75°$

Problem number [18]

$\because \sin x = \tan 30° \sin 60°$

$\therefore \sin x = \frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{2} = \frac{1}{2}$ $\therefore x = 30°$

$\therefore 4\cos x \tan 2x = 4\cos 30° \tan 60°$

$= 4\times\frac{\sqrt{3}}{2}\times\sqrt{3} = 6$

Problem number [19]

$\because 2\sin\theta = \tan^2 60° - 2\tan 45°$

$\therefore 2\sin\theta = \left(\sqrt{3}\right)^2 - 2\times 1 = 1$

$\therefore \sin\theta = \frac{1}{2}$ $\therefore \theta = 30°$

Problem number [20]

$\because \sin x = 2\sin 60°\cos 30° - \tan 45°$

$\therefore \sin x = 2\times\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2} - 1 = \frac{1}{2}$ $\therefore x = 30°$

Problem number [21]

$\because x \sin 30° \cos^2 45° = \sin^2 60°$

$\therefore x \times \frac{1}{2}\times\left(\frac{1}{\sqrt{2}}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2$ $\therefore x = \frac{\frac{3}{4}}{\frac{1}{2}\times\frac{1}{2}} = 3$

Problem number [22]

$\because \sin^2 45° = \cos E \tan 30°$

$\therefore \left(\frac{1}{\sqrt{2}}\right)^2 = \cos E \times \frac{1}{\sqrt{3}}$

$\therefore \cos E = \frac{\sqrt{3}}{2}$ $\therefore E = 30°$

Problem number [23]

$\because 2\cos(x + 15°) = \sqrt{2}$ $\therefore \cos(x + 15°) = \frac{\sqrt{2}}{2}$

$\therefore x + 15° = 45°$ $\therefore x = 30°$

$\therefore \tan 2x - \sin 2x = \tan 60° - \sin 60°$

$= \sqrt{3} - \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

Problem number [24]

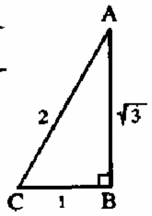
$\because \sin\theta \sin 45° \cos 45° \tan 60° = \tan^2 45° - \cos^2 60°$

$\therefore \sin\theta \times \frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\times\sqrt{3} = (1)^2 - \left(\frac{1}{2}\right)^2$

$\therefore \sin\theta = \frac{1-\frac{1}{4}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{2}$ $\therefore \theta = 60°$

Problem number [25]

$\because 2\,AB = \sqrt{3}\,AC \qquad \therefore \frac{AB}{AC} = \frac{\sqrt{3}}{2}$



let $AB = \sqrt{3}$ length unit

, $AC = 2$ length unit

$\therefore BC = 1$ length unit

$\therefore \sin C = \frac{\sqrt{3}}{2}$, $\cos C = \frac{1}{2}$, $\tan C = \sqrt{3}$

Problem number [26]

$\because m(\angle B) = 90°$

(1) $\therefore (AB)^2 = (5)^2 - (3)^2 = 16 \qquad \therefore AB = 4$ cm.

(2) $\cos A \sin C - \sin A \cos C = \frac{4}{5} \times \frac{4}{5} - \frac{3}{5} \times \frac{3}{5}$

$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

Problem number [27]

$\because m(\angle C) = 90° \qquad \therefore (AC)^2 = (10)^2 - (8)^2 = 36$

$\therefore AC = 6$ cm.

(1) $\tan B \times \tan A = \frac{6}{8} \times \frac{8}{6} = 1$

(2) $\because \cos B = \frac{8}{10}$

$\therefore m(\angle B) \simeq 36° \, 52' \, 12''$

Problem number [28]

(1) $\because \tan C = \frac{AB}{BC} \qquad \therefore \frac{3}{4} = \frac{6}{BC}$

$\therefore BC = \frac{4 \times 6}{3} = 8$ cm.

, $\because m(\angle B) = 90°$

$\therefore (AC)^2 = (8)^2 + (6)^2 = 100 \qquad \therefore AC = 10$ cm.

(2) $\sin A + \cos A = \frac{8}{10} + \frac{6}{10} = \frac{14}{10} = 1.4$

Problem number [29]

(1) In ΔABC : $\because m(\angle B) = 90°$

, $\because m(\angle C) = 2\,m(\angle A)$

$\therefore m(\angle A) + 2\,m(\angle A) = 90°$

$\therefore 3\,m(\angle A) = 90° \qquad \therefore m(\angle A) = 30°$

$\therefore m(\angle C) = 60°$

(2) $\sin A + \cos C = \sin 30° + \cos 60° = \frac{1}{2} + \frac{1}{2} = 1$

Problem number [30]

In ΔABC :

$\because m(\angle B) = 90°$ (properties of rectangle)

$\therefore (BC)^2 = (25)^2 - (15)^2 = 400 \qquad \therefore BC = 20$ cm.

(1) $\because \sin(\angle ACB) = \frac{15}{25} = \frac{3}{5}$

$\therefore m(\angle ACB) \simeq 36° \, 52' \, 12''$

(2) The area of the rectangle $ABCD = 15 \times 20$

$= 300\ cm^2$.

Problem number [31]

In ΔXYZ :

$\because m(\angle Y) = 90°$ (properties of rectangle)

$\therefore \sin 43° = \frac{XZ}{XZ} = \frac{XY}{10}$

$\therefore XY = 10 \sin 43° \simeq 6.8$ cm.

, $\cos 43° = \frac{XZ}{XZ} = \frac{YZ}{10}$

$\therefore YZ = 10 \cos 43° \simeq 7.3$ cm.

$\therefore$ The perimeter of $\Delta XYZ = 10 + 6.8 + 7.3$

$= 24.1$ cm.

Problem number [32]

In ΔABC : $\because m(\angle B) = 90°$ (properties of rectangle)

$\therefore (BC)^2 = (5)^2 - (3)^2 = 16 \qquad \therefore BC = 4$ cm.

(1) The area of the rectangle $ABCD = 4 \times 3 = 12\ cm^2$.

(2) $\because \sin(\angle ACB) = \frac{AB}{AC} = \frac{3}{5}$

$\therefore m(\angle ACB) \simeq 36° \, 52' \, 12''$

Problem number [33]

In ΔABC :

$\because AB = AC$, $\overline{AD} \perp \overline{BC}$

$\therefore D$ is the midpoint of $\overline{BC} \qquad \therefore BD = CD = 6$ cm.

In ΔADC : $\because m(\angle ADC) = 90°$

$\therefore AD = \sqrt{(10)^2 - (6)^2} = 8$ cm.

(1) $\sin^2 C + \cos^2 C = \left(\frac{8}{10}\right)^2 + \left(\frac{6}{10}\right)^2$

$= \frac{64}{100} + \frac{36}{100} = 1$

(2) $\sin B + \cos C = \frac{8}{10} + \frac{6}{10} = \frac{14}{10} > 1$

Problem number [34]

$MN = \sqrt{(0-7)^2 + (4+3)^2}$

$= \sqrt{49 + 49} = \sqrt{98} = 7\sqrt{2}$ length unit

Problem number [35]

$\because AB = \sqrt{(-4-3)^2 + (1-2)^2}$

$= \sqrt{49 + 1} = \sqrt{50} = 5\sqrt{2}$ length unit

, $BC = \sqrt{(2+4)^2 + (-1-1)^2}$

$= \sqrt{36 + 4} = \sqrt{40} = 2\sqrt{10}$ length unit

, $AC = \sqrt{(2-3)^2 + (-1-2)^2}$

$= \sqrt{1 + 9} = \sqrt{10}$ length unit

, $\because (AB)^2 = (BC)^2 + (AC)^2$

$\therefore \Delta ABC$ is a right-angled triangle at C

, its area $= \frac{1}{2} \times 2\sqrt{10} \times \sqrt{10} = 10$ square unit

Problem number [36]

$\sin 30° = \frac{AD}{12}$

$AD = 12 \times \sin 30° = 6$ cm.

The area of $(\Delta ABC) = \frac{1}{2} \times AD \times BC$

The area of $(\Delta ABC) = \frac{1}{2} \times 6 \times 16 = 48$ cm^2.

Problem number [37]

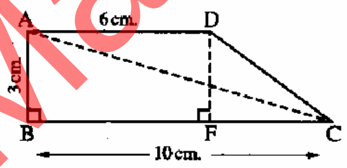
Draw $\overline{DF} \perp \overline{BC}$

$\therefore \overline{AD} \,//\, \overline{BC}$, $\overline{AB} \perp \overline{BC}$, $\overline{DF} \perp \overline{BC}$

$\therefore$ ABFD is a rectangle $\therefore$ BF = AD = 6 cm.

$\therefore$ FC = 4 cm. ,

DF = AB = 3 cm.



From Δ DFC which is right-angled at F :

$(DC)^2 = 3^2 + 4^2 = 25$ $\therefore$ DC = 5 cm.

$\therefore \cos(\angle DCB) - \tan(\angle ACB) = \frac{4}{5} - \frac{3}{10} = \frac{1}{2}$

Problem number [38]

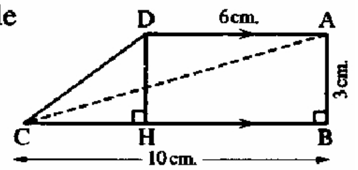
$\because \overline{AD} \,//\, \overline{BH}$, $\overline{AB} \perp \overline{BH}$, $\overline{DH} \perp \overline{BH}$

$\therefore$ ABHD is a rectangle

$\therefore$ BH = AD = 6 cm.

, CH = 10 – 6 = 4 cm.

, DH = AB = 3 cm.



In Δ DHC : $\because m(\angle CHD) = 90°$

$\therefore (CD)^2 = (4)^2 + (3)^2 = 25$ $\therefore$ CD = 5 cm.

$\therefore \cos(\angle DCB) - \tan(\angle ACB) = \frac{4}{5} - \frac{3}{10} = \frac{1}{2}$

Problem number [39]

$\because AB = \sqrt{(-1-1)^2 + (-2-4)^2}$

$= \sqrt{4 + 36} = \sqrt{40} = 2\sqrt{10}$ length unit

, $BC = \sqrt{(2+1)^2 + (-3+2)^2}$

$= \sqrt{9 + 1} = \sqrt{10}$ length unit

, $AC = \sqrt{(2-1)^2 + (-3-4)^2}$

$= \sqrt{1 + 49} = \sqrt{50} = 5\sqrt{2}$ length unit

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$\therefore \Delta ABC$ is a right-angled triangle at B

, its area $= \frac{1}{2} \times 2\sqrt{10} \times \sqrt{10} = 10$ square units.

Problem number [40]

$\because AB = \sqrt{(-4-1)^2 + (2+2)^2}$

$= \sqrt{25 + 16} = \sqrt{41}$ length unit

, $BC = \sqrt{(1+4)^2 + (6-2)^2}$

$= \sqrt{25 + 16} = \sqrt{41}$ length unit

, $AC = \sqrt{(1-1)^2 + (6+2)^2} = \sqrt{64} = 8$ length unit

$\therefore$ AB = BC $\therefore \Delta ABC$ is an isosceles triangle.

Problem number [41]

$\because AB = \sqrt{(1+2)^2 + (-1-3)^2}$

$= \sqrt{9 + 16} = \sqrt{25} = 5$ length unit

, $BC = \sqrt{(1-1)^2 + (7+1)^2} = \sqrt{64} = 8$ length unit

, $AC = \sqrt{(1+2)^2 + (7-3)^2}$

$= \sqrt{9 + 16} = \sqrt{25} = 5$ length unit

$\therefore$ AB = AC

$\therefore \Delta ABC$ is an isosceles triangle

, the perimeter = 5 + 8 + 5 = 18 length unit

Problem number [42]

$\because AB = \sqrt{(3+2)^2 + (-1-4)^2}$

$= \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}$ length unit

, $BC = \sqrt{(4-3)^2 + (5+1)^2}$

$= \sqrt{1 + 36} = \sqrt{37}$ length unit

, $AC = \sqrt{(4 + 2)^2 + (5 - 4)^2}$

$= \sqrt{36 + 1} = \sqrt{37}$ length unit

$\therefore BC = AC$ $\therefore \Delta ABC$ is an isosceles triangle

Problem number [43]

$\because MA = \sqrt{(-1 - 3)^2 + (2 + 1)^2}$

$= \sqrt{16 + 9} = \sqrt{25} = 5$ length unit

, $MB = \sqrt{(-1 + 4)^2 + (2 - 6)^2}$

$= \sqrt{9 + 16} = \sqrt{25} = 5$ length unit

, $MC = \sqrt{(-1 - 2)^2 + (2 + 2)^2}$

$= \sqrt{9 + 16} = \sqrt{25} = 5$ length unit

$\therefore MA = MB = MC$

$\therefore$ A , B and C lie on the circle M which its radius length is 5 length units

$\therefore$ The circumference of the circle

$= 2\pi r = 2 \times 3.14 \times 5 = 31.4$ length unit

Problem number [44]

$\because \sqrt{(2a - a)^2 + (-5 - 7)^2} = 13$

$\therefore \sqrt{a^2 + 144} = 13$ "squaring both sides"

$\therefore a^2 + 144 = 169$ $\therefore a^2 = 169 - 144$

$\therefore a^2 = 25$ $\therefore a = \pm\sqrt{25} = \pm 5$

Problem number [45]

$\because \sqrt{(x - 6)^2 + (5 - 1)^2} = 2\sqrt{5}$ "squaring the two sides"

$\therefore (x - 6)^2 + (4)^2 = 20$

$\therefore x^2 - 12x + 36 + 16 - 20 = 0$

$\therefore x^2 - 12x + 32 = 0$ $\therefore (x - 4)(x - 8) = 0$

$\therefore x = 4$ or $x = 8$

Problem number [46]

$\because \sqrt{(x + 2)^2 + (7 - 3)^2} = 5$ "squaring the two sides"

$\therefore (x + 2)^2 + (4)^2 = 25$

$\therefore x^2 + 4x + 4 + 16 - 25 = 0$

$\therefore x^2 + 4x - 5 = 0$ $\therefore (x + 5)(x - 1) = 0$

$\therefore x = -5$ or $x = 1$

Problem number [47]

$BC = \sqrt{(5 - 3)^2 + (1 - 2)^2} = \sqrt{4 + 1} = \sqrt{5}$ length unit

$\therefore AB = \sqrt{5}$ length unit

$\therefore \sqrt{(x - 3)^2 + (3 - 2)^2} = \sqrt{5}$ "squaring the two sides"

$\therefore (x - 3)^2 + (1)^2 = 5$ $\therefore x^2 - 6x + 9 + 1 - 5 = 0$

$\therefore x^2 - 6x + 5 = 0$

$\therefore (x - 5)(x - 1) = 0$ $\therefore x = 5$ or $x = 1$

Problem number [48]

The coordinates of $C = \left(\frac{3 - 5}{2}, \frac{-7 - 3}{2}\right) = (-1, -5)$

Problem number [49]

(1) $AB = \sqrt{(5 - 2)^2 + (3 + 1)^2} = \sqrt{9 + 16} = \sqrt{25}$

$= 5$ length unit

(2) $c = \left(\frac{2 + 5}{2}, \frac{-1 + 3}{2}\right) = \left(3\frac{1}{2}, 1\right)$

Problem number [50]

$\because$ C is the midpoint of $\overline{AB}$

$\therefore (-3, k) = \left(\frac{h + 9}{2}, \frac{-6 - 11}{2}\right)$

$\therefore k = \frac{-6 - 11}{2} = -8\frac{1}{2}$, $\frac{h + 9}{2} = -3$

$\therefore h + 9 = -6$ $\therefore h = -15$

Problem number [51]

$\because$ C is the midpoint of $\overline{AB}$

$\therefore (4, 6) = \left(\frac{x + 6}{2}, \frac{3 + y}{2}\right)$

$\therefore \frac{x + 6}{2} = 4$ $\therefore x + 6 = 8$ $\therefore x = 2$

, $\frac{3 + y}{2} = 6$ $\therefore 3 + y = 12$ $\therefore y = 9$

Problem number [52]

$\because \overline{AB}$ is a diameter in the circle M

$\therefore$ M is the midpoint of $\overline{AB}$

Let $A(x, y)$ $\therefore (5, 7) = \left(\frac{x + 8}{2}, \frac{y + 11}{2}\right)$

$\therefore \frac{x + 8}{2} = 5$ $\therefore x + 8 = 10$ $\therefore x = 2$

, $\frac{y + 11}{2} = 7$ $\therefore y + 11 = 14$

$\therefore y = 3$ $\therefore A(2, 3)$

Problem number [53]

$\because \overline{AD}$ is a median in ΔABC

$\therefore$ D is the midpoint of $\overline{BC}$

$\therefore D = \left(\frac{3-3}{2}, \frac{2+6}{2}\right) = (0, 4)$

, $\because$ M is the midpoint of $\overline{AD}$

$\therefore M = \left(\frac{0+0}{2}, \frac{8+4}{2}\right) = (0, 6)$

Problem number [54]

$\because AB = \sqrt{(3+3)^2 + (4-0)^2} = \sqrt{36+16} = \sqrt{52}$

$= 2\sqrt{13}$ length unit

$, BC = \sqrt{(1-3)^2 + (-6-4)^2} = \sqrt{4+100} = \sqrt{104}$

$= 2\sqrt{26}$ length unit

$, AC = \sqrt{(1+3)^2 + (-6-0)^2}$

$= \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$ length unit

$\therefore AB = AC$ $\quad \therefore \Delta ABC$ is an isosceles triangle.

Let $\overline{AD} \perp \overline{BC}$

$, \because AB = AC$ $\quad \therefore$ D is the midpoint of $\overline{BC}$

$\therefore D = \left(\frac{3+1}{2}, \frac{4-6}{2}\right) = (2, -1)$

$\therefore AD = \sqrt{(2+3)^2 + (-1-0)^2}$

$= \sqrt{25+1} = \sqrt{26}$ length unit

Problem number [55]

$\because$ In the parallelogram the two diagonals bisect each other.

$\therefore$ Let M be the point of intersection of the two diagonals.

$\therefore$ The coordinates of $M = \left(\frac{3+0}{2}, \frac{2-3}{2}\right)$

$= \left(1\frac{1}{2}, -\frac{1}{2}\right)$

Let $D(x, y)$

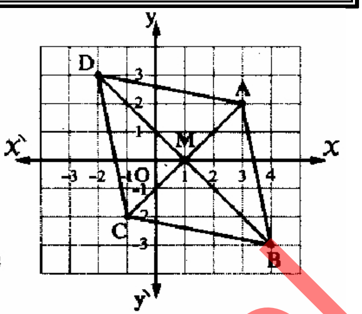
$\therefore \left(1\frac{1}{2}, -\frac{1}{2}\right) = \left(\frac{4+x}{2}, \frac{-5+y}{2}\right)$ $\quad \therefore \frac{4+x}{2} = 1\frac{1}{2}$

$\therefore 4 + x = 3$ $\quad \therefore x = -1$

$, \frac{-5+y}{2} = -\frac{1}{2}$ $\quad \therefore -5 + y = -1$ $\quad \therefore y = 4$

$\therefore D(-1, 4)$

Problem number [56]



$\because$ The two diagonals of the rhombus bisect each other

(1) Let M be the point of intersection of the two diagonals

$\therefore$ the coordinates of $M = \left(\frac{3-1}{2}, \frac{2-2}{2}\right) = (1, 0)$

(2) $\because AC = \sqrt{(-1-3)^2 + (-2-2)^2}$

$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$ length unit

$, BD = \sqrt{(-2-4)^2 + (3+3)^2}$

$= \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$ length unit

The area of the rhombus ABCD

$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} = 24$ square unit

(3) $\because$ The two diagonals of the rhombus are perpendicular

$\therefore$ In ΔAMB which is right at M

$\tan(\angle ABM) = \frac{AM}{BM} = \frac{2\sqrt{2}}{3\sqrt{2}} = \frac{2}{3}$

$\therefore m(\angle ABM) \simeq 33° \, 41' \, 24''$

$\because$ The diagonals of the rhombus bisect its angles.

$\therefore m(\angle ABC) = 2\, m(\angle ABM) = 2 \times 33° \, 41' \, 24''$

$= 67° \, 22' \, 48''$

Problem number [57]

$\because m_1 = \frac{3\sqrt{3} - 2\sqrt{3}}{5-4} = \sqrt{3}$ $, m_2 = \tan 60° = \sqrt{3}$

$\therefore m_1 = m_2$

$\therefore$ The two straight lines are parallel.

Problem number [58]

$\because m_1 = \frac{6-5}{2-3} = -1$ $, m_2 = \tan 45° = 1$

$\therefore m_1 \times m_2 = -1 \times 1 = -1$

$\therefore$ The two straight lines are perpendicular.

Problem number [59]

$\because m_1 = \frac{-5-2}{4+3} = -1$, $m_2 = \tan 45° = 1$

$\therefore m_1 \times m_2 = -1 \times 1 = -1$

$\therefore$ The two straight lines are perpendicular.

Problem number [60]

$\because AB = \sqrt{(1-5)^2 + (-3-1)^2}$

$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$ length unit

, $BC = \sqrt{(-5-1)^2 + (3+3)^2}$

$= \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$ length unit

, $CD = \sqrt{(-1+5)^2 + (7-3)^2}$

$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$ length unit

, $AD = \sqrt{(-1-5)^2 + (7-1)^2} = \sqrt{36+36}$

$= \sqrt{72} = 6\sqrt{2}$ length unit

$\therefore AB = CD$, $AD = BC$

$\therefore ABCD$ is a parallelogram

, $\because AC = \sqrt{(-5-5)^2 + (3-1)^2}$

$= \sqrt{100+4} = \sqrt{104} = 2\sqrt{26}$ length unit

, $BD = \sqrt{(-1-1)^2 + (7+3)^2}$

$= \sqrt{4+100} = \sqrt{104} = 2\sqrt{26}$ length unit

$\therefore AC = BD$ $\quad\quad$ $\therefore ABCD$ is a rectangle

Problem number [61]

$\because \overleftrightarrow{AB}$ // the x-axis $\quad$ $\therefore$ The slope of $\overleftrightarrow{AB} = 0$

$\therefore \frac{y+4}{-2-5} = 0$ $\quad$ $\therefore y+4 = 0$ $\quad$ $\therefore y = -4$

Problem number [62]

$\because m_1 = \frac{3-k}{1-0} = 3-k$, $m_2 = \frac{5-3}{2-1} = 2$

$\because m_1 = m_2$ $\quad$ $\therefore 3-k = 2$ $\quad$ $\therefore k = 1$

Problem number [63]

$\because L_1 // L_2$ $\quad$ $\because m_1 = m_2$

$\therefore \tan 45° = \frac{-a}{-2}$ $\quad$ $\therefore 1 = \frac{a}{2}$ $\quad$ $\therefore a = 2$

Problem number [64]

$m_1 = \frac{k-1}{2+3} = \frac{k-1}{5}$, $m_2 = \tan 45° = 1$

, $\because L_1 \perp L_2$ $\quad$ $\therefore m_1 \times m_2 = -1$

$\therefore \frac{k-1}{5} \times 1 = -1$ $\quad$ $\therefore k-1 = -5$ $\quad$ $\therefore k = -4$

Problem number [65]

$y = \frac{1}{2}x + 2$

Problem number [66]

$\because$ The slope $= \frac{1}{2}$

$\therefore$ The equation of the straight line is : $y = \frac{1}{2}x + c$

, $\because (4 , 7)$ satisfies the equation

$\therefore 7 = \frac{1}{2} \times 4 + c$ $\quad$ $\therefore c = 5$

$\therefore$ The equation of the straight line is : $y = \frac{1}{2}x + 5$

Problem number [67]

$\because$ The slope $= \frac{3}{4}$

$\therefore$ The equation of the straight line is : $y = \frac{3}{4}x + c$

$\because (3 , -5)$ satisfies the equation

$\therefore -5 = \frac{3}{4} \times 3 + c$ $\quad$ $\therefore c = -7\frac{1}{4}$

$\therefore$ The equation of the straight line is : $y = \frac{3}{4}x - 7\frac{1}{4}$

Problem number [68]

$\because$ The slope of the straight line $= \frac{-1+3}{5-2} = \frac{2}{3}$

$\therefore$ The equation of the straight line is : $y = \frac{2}{3}x + c$

, $\because (2 , -3)$ satisfies the equation

$\therefore -3 = \frac{2}{3} \times 2 + c$ $\quad$ $\therefore c = -4\frac{1}{3}$

$\therefore$ The equation of the straight line is :

$y = \frac{2}{3}x - 4\frac{1}{3}$

Problem number [69]

$\because$ The slope of $\overleftrightarrow{AB} = \frac{5-3}{3-1} = 1$

$\therefore$ The slope of the axis of symmetry of $\overleftrightarrow{AB} = -1$

$\therefore$ The equation of the axis of symmetry of $\overleftrightarrow{AB}$ is :

$y = -x + c$

, $\because$ The midpoint of $\overline{AB} = \left(\frac{1+3}{2} , \frac{3+5}{2}\right)$

$= (2 , 4)$

$\therefore (2 , 4)$ satisfies the equation : $y = -x + C$

$\therefore 4 = -2 + c$ $\quad$ $\therefore c = 6$

$\therefore$ The equation of the axis of symmetry of $\overleftrightarrow{AB}$ is :

$y = -x + 6$

Problem number [70]

$\because$ The slope of the straight line $= \frac{6-2}{-1-1} = -2$

$\therefore$ The equation of the straight line is : $y = -2x + c$

, $\because$ The straight line passes through the point $(1 , 2)$

$\therefore 2 = -2 \times 1 + c$ $\qquad \therefore c = 4$

$\therefore$ The equation of the straight line is :

$y = -2x + 4$

Problem number [71]

$\because$ The slope of the straight line $= \frac{2-3}{-3-2} = \frac{1}{5}$

$\therefore$ The equation of the straight line is :

$y = \frac{1}{5}x + c$

, $\because (2 , 3)$ satisfies the equation

$\therefore 3 = \frac{1}{5} \times 2 + c$ $\qquad \therefore c = 2\frac{3}{5}$

$\therefore$ The equation of the straight line is :

$y = \frac{1}{5}x + 2\frac{3}{5}$

Problem number [72]

$\because$ The slope of $\overleftrightarrow{AB} = \frac{-2-4}{-1-1} = 3$

$\therefore$ The slope of $\overleftrightarrow{BC} = -\frac{1}{3}$

$\therefore$ The equation of $\overleftrightarrow{BC}$ is : $y = -\frac{1}{3}x + c$

, $\because B(-1 , -2)$ satisfies the equation of $\overleftrightarrow{BC}$

$\therefore -2 = -\frac{1}{3} \times -1 + c$ $\qquad \therefore c = -2\frac{1}{3}$

$\therefore$ The equation of $\overleftrightarrow{BC}$ is : $y = \frac{-1}{3}x - 2\frac{1}{3}$

Problem number [73]

$\because$ The slope of the given straight line $= \frac{-1}{2}$

$\therefore$ The slope of the required straight line $= -\frac{1}{2}$

$\therefore$ The equation of the required straight line is :

$y = -\frac{1}{2}x + c$

$\because$ The straight line passes through the point :

$(3 , -5)$

$\therefore -5 = \frac{-1}{2} \times 3 + c$ $\qquad \therefore c = -3\frac{1}{2}$

$\therefore$ The equation of the required straight line is :

$y = -\frac{1}{2}x - 3\frac{1}{2}$

Problem number [74]

$\because$ The slope of the given straight line $= \frac{-2}{-1} = 2$

$\therefore$ The slope of the required straight line $= 2$

$\therefore$ The equation of the required straight line is :

$y = 2x + c$

$\because (2 , 3)$ satisfies the equation

$\therefore 3 = 2 \times 2 + c$ $\qquad \therefore c = -1$

$\therefore$ The equation of the required straight line is :

$y = 2x - 1$

Problem number [75]

$\because$ The slope of the given straight line $= \frac{-1}{2}$

$\therefore$ The slope of the required straight line $= 2$

$\therefore$ The equation of the required straight line is :

$y = 2x + c$

$\because (3 , -5)$ satisfies the equation

$\therefore -5 = 2 \times 3 + c$ $\qquad \therefore c = -11$

$\therefore$ The equation of the required straight line is :

$y = 2x - 11$

Problem number [76]

$\because$ The slope of the given straight line $= \frac{-5}{-2} = \frac{5}{2}$

$\therefore$ The slope of the required straight line $= \frac{-2}{5}$

$\therefore$ The equation of the required straight line is :

$y = \frac{-2}{5}x + c$

, $\because (3 , 4)$ satisfies the equation

$\therefore 4 = -\frac{2}{5} \times 3 + c$ $\qquad \therefore c = 5\frac{1}{5}$

$\therefore$ The equation of the required straight line is :

$y = \frac{-2}{5}x + 5\frac{1}{5}$

Problem number [77]

$\therefore$ The slope of the required straight line $= 2$

$\therefore$ The equation of the required straight line is :

$y = 2x + c$

, $\because (1 , 5)$ satisfies the equation

$\therefore 5 = 2 \times 1 + c$ $\qquad \therefore c = 3$

$\therefore$ The equation of the required straight line is :

$y = 2x + 3$

Problem number [78]

$\because$ The slope of the given straight line $= \frac{-4+3}{5-2}$

$= -\frac{1}{3}$

$\therefore$ The slope of the required straight line $= 3$

$\therefore$ The equation of the required straight line is :

$y = 3x + c$

, $\because$ (1 , 2) satisfies the equation

$\therefore 2 = 3 \times 1 + c \qquad \therefore c = -1$

$\therefore$ The equation of the required straight line is :

$y = 3x - 1$

Problem number [79]

$\because$ The midpoint of $\overline{AB} = \left(\frac{1+3}{2}, \frac{-2-4}{2}\right) = (2, -3)$

$\therefore$ The slope of the straight line $= \frac{6+3}{1-2} = -9$

$\therefore$ The equation of the straight line is : $y = -9x + c$

, $\because$ (1 , 6) satisfies the equation

$\therefore 6 = -9 \times 1 + c \qquad \therefore c = 15$

$\therefore$ The equation of the straight line is :

$y = -9x + 15$

Problem number [80]

(1) $y = \frac{1}{2}x + 2$

(1) Put $y = 0$ $\qquad \therefore 0 = \frac{1}{2}x + 2$

$\therefore \frac{1}{2}x = -2 \qquad \therefore x = -4$

$\therefore$ The intersection point with the x-axis is $(-4, 0)$

Problem number [81]

$\because$ The slope of $\overleftrightarrow{AC} = \frac{6-4}{-1-5} = -\frac{2}{6} = -\frac{1}{3}$

, $\because$ The two diagonals of the square are perpendicular.

$\therefore$ The slope of $\overleftrightarrow{BD} = 3$

$\therefore$ The equation of $\overleftrightarrow{BD}$ is : $y = 3x + c$

$\because$ The coordinates of the midpoint of $\overline{AC}$

$= \left(\frac{5-1}{2}, \frac{6+4}{2}\right) = (2, 5)$

$\therefore$ (2 , 5) satisfies the equation of $\overleftrightarrow{BD}$

$\therefore 5 = 2 \times 3 + c \qquad \therefore c = -1$

$\therefore$ The equation of $\overleftrightarrow{BD}$ is : $y = 3x - 1$

Problem number [82]

(1) $\because$ $\overline{AB}$ is a diameter of the circle

, M is the midpoint of $\overline{AB}$ let A (x, y)

$\therefore (5, 7) = \left(\frac{x+8}{2}, \frac{y+11}{2}\right) \qquad \therefore \frac{x+8}{2} = 5$

$\therefore x + 8 = 10 \qquad \therefore x = 2 \qquad , \frac{y+11}{2} = 7$

$\therefore y + 11 = 14 \qquad \therefore y = 3 \qquad \therefore A(2, 3)$

(2) $\because$ The slope of $\overleftrightarrow{AB} = \frac{11-3}{8-2} = \frac{4}{3}$

$\therefore$ The slope of the required straight line $= \frac{-3}{4}$

$\therefore$ The equation of the required straight line is :

$y = \frac{-3}{4}x + c$

$\because$ B (8 , 11) satisfies the equation

$\therefore 11 = \frac{-3}{4} \times 8 + c \qquad \therefore c = 17$

$\therefore$ The equation of the required straight line is :

$y = \frac{-3}{4}x + 17$

Problem number [83]

$\because$ In the parallelogram the two diagonals bisect each other.

$\therefore$ The coordinates of M $= \left(\frac{3+0}{2}, \frac{2-3}{2}\right)$

$= \left(1\frac{1}{2}, -\frac{1}{2}\right)$

Let D (x, y)

$\therefore \left(1\frac{1}{2}, -\frac{1}{2}\right) = \left(\frac{4+x}{2}, \frac{-5+y}{2}\right)$

$\therefore \frac{4+x}{2} = 1\frac{1}{2} \qquad \therefore 4 + x = 3 \qquad \therefore x = -1$

$, \frac{-5+y}{2} = -\frac{1}{2} \qquad \therefore -5 + y = -1 \qquad \therefore y = 4$

$\therefore$ D $(-1, 4)$

Problem number [84]

$\because$ The slope of $\overleftrightarrow{BC} = \frac{4+2}{3-5} = -3$

$\therefore$ The slope of $\overleftrightarrow{DE} = -3$

$\therefore$ The equation of $\overleftrightarrow{DE}$ is : $y = -3x + c$

$\because$ D is the midpoint of $\overline{AB} = \left(\frac{1+5}{2}, \frac{2-2}{2}\right) = (3, 0)$

$\therefore$ (3 , 0) satisfies the equation of $\overleftrightarrow{DE}$

$\therefore 0 = -3 \times 3 + c \qquad \therefore c = 9$

$\therefore$ The equation of $\overleftrightarrow{DE}$ is : $y = -3x + 9$

Problem number [85]

$\because m_1 = \frac{-1}{1} = -1$, $m_2 = \frac{-k}{3}$,

$\because$ The two straight lines are parallel

$\therefore m_1 = m_2$ $\quad \therefore -1 = -\frac{k}{3}$ $\quad \therefore k = 3$

Problem number [86]

The slope $= \frac{-3}{4}$

, the length of the intercepted part of y-axis

$= \left|\frac{-5}{4}\right| = \frac{5}{4}$ length unit

Problem number [87]

$\because$ Let the measure of the two angles be : $3x$, $5x$

$\therefore 3x + 5x = 180^\circ$ $\quad \therefore 8x = 180^\circ$ $\quad \therefore x = 22^\circ\, 30'$

$\therefore$ The measure of the two angles are :

$67^\circ\, 30'$, $112^\circ\, 30'$

Problem number [88]

$\because \frac{x}{2} + 3y = 6$ $\quad \therefore 3y = -\frac{x}{2} + 6$

$\therefore y = -\frac{x}{6} + 2$ $\quad \therefore$ The slope $= -\frac{1}{6}$

and the intercepted part is 2 units from the positive part of y-axis.

Problem number [89]

$\because \frac{x}{3} + \frac{y}{2} = 1$ "multiplying by 2"

$\therefore \frac{2x}{3} + y = 2$ $\quad \therefore y = -\frac{2x}{3} + 2$

$\therefore$ The slope $= \frac{-2}{3}$

, the intercepted part = 2 units from the positive part of y-axis

Problem number [90]

(1) $\because$ The slope of the straight line $= \frac{3-1}{2-1} = 2$

$\therefore$ The equation of the straight line is : $y = 2x + c$

$\because$ The point $(1 , 1) \in$ the straight line

$\therefore 1 = 2 \times 1 + c$ $\quad \therefore c = -1$

$\therefore$ The equation of the straight line is : $y = 2x - 1$

(2) One unit of the negative part of y-axis

(3) $\because$ The point $(3 , a)$ satisfies the equation

$\therefore a = 2 \times 3 - 1 = 5$

Problem number [91]

(1) $\because$ The slope of $L_1 = \tan 45^\circ = 1$

, $\because L_1$ passes through the origin point :

$\therefore$ The equation of L_1 is : $y = x$

(2) $\because L_1 \parallel L_2$ $\quad \therefore$ The slope of $L_2 = 1$

$\therefore$ The equation of L_2 is : $y = x + c$

, $\because (1 , 5)$ satisfies the equation of L_2 :

$\therefore 5 = 1 + c$ $\quad \therefore c = 4$

$\therefore$ The equation of L_2 is : $y = x + 4$

(3) Let $B (x , y)$

$\because$ B satisfies the equation of L_1 : $\therefore x = y$

, $\because \overline{AB} \perp L_1$ $\quad \therefore$ The slope of $\overleftrightarrow{AB} = -1$

$\therefore \frac{y-5}{x-1} = -1$ $\quad \therefore y - 5 = 1 - x$

, $\because x = y$ $\quad \therefore x - 5 = 1 - x$

$\therefore 2x = 6$ $\quad \therefore x = 3$

$\therefore y = 3$ $\quad \therefore B (3 , 3)$

$\therefore AB = \sqrt{(3-1)^2 + (3-5)^2}$

$= \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$ length unit

Problem number [92]

(1) Let $A (x , 0)$, $B (0 , y)$

$\because$ C is the midpoint of $\overline{AB}$

$\therefore (4 , 3) = \left(\frac{x+0}{2} , \frac{0+y}{2}\right)$

$\therefore \frac{x}{2} = 4$ $\quad \therefore x = 8$ $\quad \therefore A (8 , 0)$

$\therefore \frac{y}{2} = 3$ $\quad \therefore y = 6$ $\quad \therefore B (0 , 6)$

(2) The slope of $\overleftrightarrow{AB} = \frac{0-6}{8-0} = -\frac{3}{4}$

$\therefore$ The equation of $\overleftrightarrow{AB}$ is : $y = -\frac{3}{4}x + c$

$\because (0 , 6)$ statisfies the equation of $\overleftrightarrow{AB}$

$\therefore 6 = -\frac{3}{4} \times 0 + c$ $\quad \therefore c = 6$

$\therefore$ The equation of $\overleftrightarrow{AB}$ is : $y = -\frac{3}{4}x + 6$