Math - 2016

Prep. (3) - Geometry

Final Revision Rules

1

First Term

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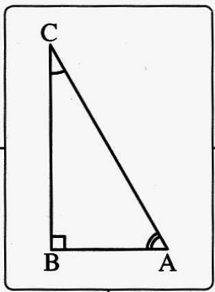
First Trigonometry

Remember — The main trigonometrical ratios of the acute angle and the important relations between them

The trigonometrical ratios of the angle A

- $\sin A = \dfrac{\text{Opposite}}{\text{Hypotenuse}} = \dfrac{BC}{AC}$
- $\cos A = \dfrac{\text{Adjacent}}{\text{Hypotenuse}} = \dfrac{AB}{AC}$
- $\tan A = \dfrac{\text{Opposite}}{\text{Adjacent}} = \dfrac{BC}{AB}$



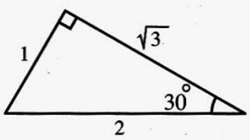
The trigonometrical ratios of the angle C

- $\sin C = \dfrac{\text{Opposite}}{\text{Hypotenuse}} = \dfrac{AB}{AC}$
- $\cos C = \dfrac{\text{Adjacent}}{\text{Hypotenuse}} = \dfrac{BC}{AC}$
- $\tan C = \dfrac{\text{Opposite}}{\text{Adjacent}} = \dfrac{AB}{BC}$

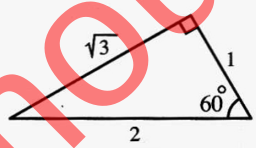
Some important relations

- $\tan A = \dfrac{\sin A}{\cos A}$
- If $m(\angle A) + m(\angle C) = 90°$, then $\sin A = \cos C$, $\cos A = \sin C$
- If $\sin A = \cos C$ or $\cos A = \sin C$, then $m(\angle A) + m(\angle C) = 90°$

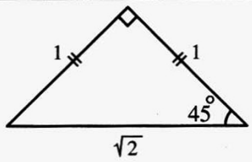
Remember — The trigonometrical ratios of some angles



- $\sin 30° = \dfrac{1}{2}$
- $\cos 30° = \dfrac{\sqrt{3}}{2}$
- $\tan 30° = \dfrac{1}{\sqrt{3}}$



- $\sin 60° = \dfrac{\sqrt{3}}{2}$
- $\cos 60° = \dfrac{1}{2}$
- $\tan 60° = \sqrt{3}$



- $\sin 45° = \dfrac{1}{\sqrt{2}}$
- $\cos 45° = \dfrac{1}{\sqrt{2}}$
- $\tan 45° = 1$

Notice that

If $\cos \theta = 0.7152$, then we use the calculator to find θ by using the keys as the following sequence from left : shift cos · 7 1 5 2 = °,,,

Then $\theta \simeq 44° \, 20' \, 25''$

Second Analytical geometry

Remember The important laws

If $A(x_1, y_1)$, $B(x_2, y_2)$

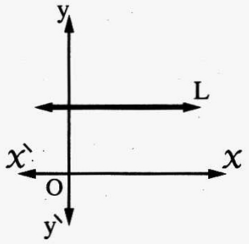
- The law of the distance between the two point A , B (the length of $\overline{AB}$) :
  $AB = \sqrt{(\text{difference between } x-\text{coordinates})^2 + (\text{difference between } y-\text{coordinates})^2}$
- The law of finding the coordinates of the midpoint of $\overline{AB}$:
  The midpoint of $\overline{AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- The law of finding the slope of the straight line $\overleftrightarrow{AB}$:
  $m = \frac{y_2 - y_1}{x_2 - x_1}$

Remember How to find the slope of the straight line

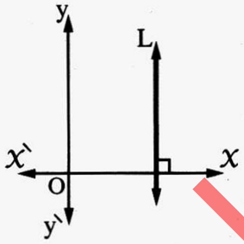
1	Given two points on the line as : $A(x_1, y_1)$, $B(x_2, y_2)$	$m = \frac{y_2 - y_1}{x_2 - x_1}$
2	Given the measure of the positive angle which the straight line makes with the positive direction of x-axis , say θ	$m = \tan\theta$
3	Given the equation of the straight line in the form : $y = bx + c$	$m = b$ where b is the coefficient of x
4	Given the equation of the straight line in the form : $ax + by + c = 0$	$m = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-a}{b}$
5	Given the slope of the parallel straight line to it , say m_1	$m = m_1$ because the two slopes are equal.
6	Given the slope of the perpendicular straight line to it , say m_2	$m = \frac{-1}{m_2}$ because : $m \times m_2 = -1$

Important remarks on the slope of the straight line

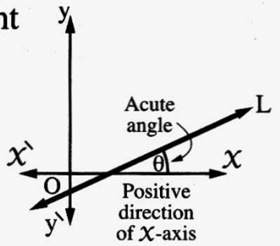
- The slope of x-axis = 0
- The slope of the straight line parallel to x-axis equals 0



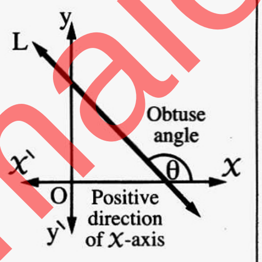
- The slope of y-axis is undefined.
- The slope of the straight line parallel to y-axis is undefined.



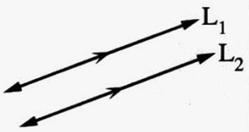
- The slope of the straight line which makes an acue angle with the positive direction of x-axis is positive.



- The slope of the straight line which makes an obtuse angle with the positive direction of x-axis is negative.

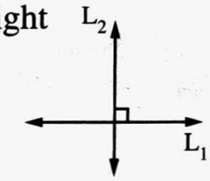


- The two parallel straight lines their slopes are equal.



i.e. If $L_1 // L_2$, then $m_1 = m_2$

- The two perpendicular straight lines the product of their slopes equals – 1

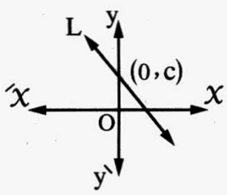


i.e. If $L_1 \perp L_2$, then $m_1 \times m_2 = -1$

Remember The equation of the straight line

- The equation of the straight line whose slope = m and cuts y-axis at the point (0 , c) is :

$y = m x + c$



For example :

- The equation of the straight line whose Slope is – 2 and cuts from the positive part of y-axis 7 units is : $y = -2x + 7$
- To find the equation of the striaght line whose slope is 3 and passes through the point (1 , – 2) :

$\because$ The slope = 3 $\therefore$ The equation of the straight line is : $y = 3x + c$

, then substitute by the point (1 , – 2) to find the value of c as the following :

$-2 = 3 \times 1 + c$, then : $c = -5$

$\therefore$ The equation of the straight line is : $y = 3x - 5$

Important remarks on the equation of the straight line

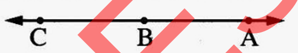
1. The equation of the straight line which passes through the origin point O (0 , 0) is : $y = m\,x$ where m is the slope.
2. The equation of x-axis is : $y = 0$ and the equation of y-axis is : $x = 0$
3. The equation of the straight line parallel to x-axis and cuts y-axis at the point (0 , c) is : $y = c$
4. The equation of the straight line parallel to y-axis and cuts x-axis at the point (a , 0) is : $x = a$

Remember

Some rules and remarks which help you to solve the exercises

1 To prove that the points A , B and C are collinear

We will prove that :



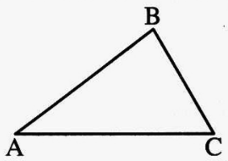
• The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$ **or** • AB + BC = AC (where AC is the greatest length)

2 To prove that the points A , B and C are vertices of a triangle

We prove that :

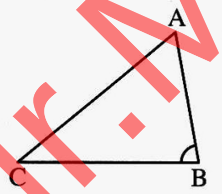
• The slope of $\overleftrightarrow{AB} \neq$ the slope of $\overleftrightarrow{BC}$

or • AB + BC > AC (where AC is the greatest length)



3 To determine the type of the triangle ABC according to its angle measures

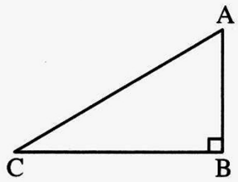
We compare between : $(AC)^2$, $(AB)^2 + (BC)^2$ where $\overline{AC}$ is the longest side , if :



$(AC)^2 < (AB)^2 + (BC)^2$

, then :

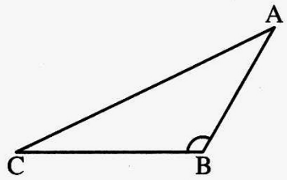
Δ ABC is acute-angled.



$(AC)^2 = (AB)^2 + (BC)^2$

, then :

Δ ABC is right-angled at B



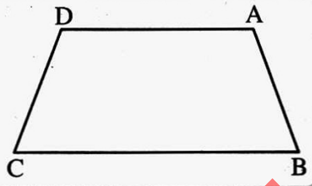
$(AC)^2 > (AB)^2 + (BC)^2$

, then :

Δ ABC is obtuse-angled at B

4 To prove that : the quadrilateral ABCD is a trapezium

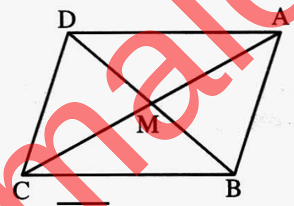
We prove that :



The slope of $\overleftrightarrow{AD}$ = the slope of $\overleftrightarrow{BC}$, then $\overline{AD} // \overline{BC}$

, the slope of $\overleftrightarrow{AB} \neq$ the slope of $\overleftrightarrow{DC}$, then $\overline{AB}$ is not parallel to $\overline{DC}$

5 To prove that : the quadrilateral ABCD is a parallelogram



- **By using the slope , we prove that :**

  The slope of $\overleftrightarrow{AD}$ = the slope of $\overleftrightarrow{BC}$, then $\overline{AD} // \overline{BC}$

  , the slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{DC}$, then $\overline{AB} // \overline{DC}$

- **By using the distance between two points , we prove that :**

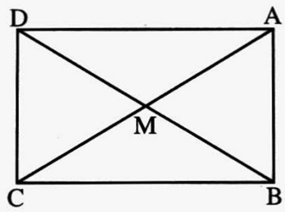
  The length of $\overline{AD}$ = the length of $\overline{BC}$, the length of $\overline{AB}$ = the length of $\overline{DC}$

- **By using the coordinates of the midpoint of a line segment , we prove that :**

  The coordinates of the midpoint of $\overline{AC}$ is the coordinates of the midpoint of $\overline{BD}$,

  then : $\overline{AC}$, $\overline{BD}$ bisect each other.

6 To prove that : the quadrilateral ABCD is a rectangle



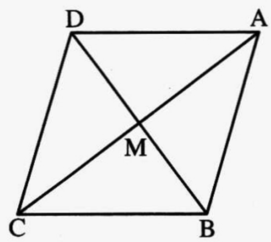
First we prove that : the quadrilateral ABCD is a parallogram by one of the previous methods , then

prove that :

- AC = BD (By using the distance between two points)

or • The slope of $\overleftrightarrow{AB}$ × the slope of $\overleftrightarrow{BC} = -1$, then : $\overline{AB} \perp \overline{BC}$

7 To prove that : the quadrilateral ABCD is a rhombus



* First we prove that : the quadrilateral ABCD is a parallelogram , then

Prove that :

- AB = BC (By using the distance between two points)

or • The slope of $\overleftrightarrow{AC}$ × the slope of $\overleftrightarrow{BD} = -1$, then $\overline{AC} \perp \overline{BD}$

* We can prove that the quadrialateral ABCD is a rhombus directly by using the distance between two points

  we prove that :

  AB = BC = CD = DA

8 To prove that : the quadrilateral ABCD is a square

* First we prove that : the quadrilateral ABCD is a parallelogram , then

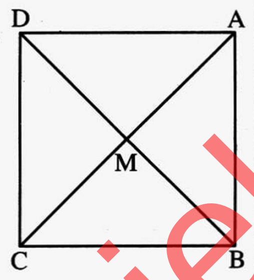
prove that :

• AB = BC (By using the distance between two points)

and the slope of $\overleftrightarrow{AB}$ × the slope of $\overleftrightarrow{BC} = -1$, then $\overline{AB} \perp \overline{BC}$

or • AC = BD (By using the distance between two points)

and the slope of $\overleftrightarrow{AC}$ × the slope of $\overleftrightarrow{BD} = -1$ then : $\overline{AC} \perp \overline{BD}$



* We can prove that the quadrilateral ABCD is a square by using the distance between two points

we prove that :

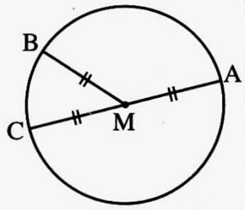
AB = BC = CD = DA , then the quadrilateral is a rhombus , then

prove that : AC = BD

9 To prove that : the points A , B , C lie on one circle of centre M

By using the distance between two points

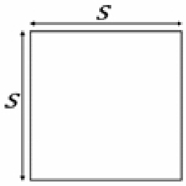
we prove that : MA = MB = MC



GEOMETRY SHAPES AND SOLIDS

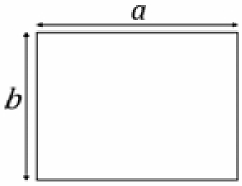
SQUARE

$P = 4s$
$A = s^2$



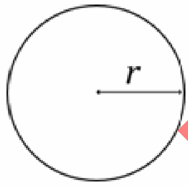
RECTANGLE

$P = 2a + 2b$
$A = ab$



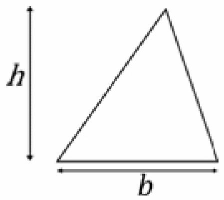
CIRCLE

$P = 2\pi r$
$A = \pi r^2$



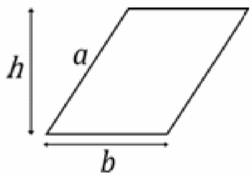
TRIANGLE

$P = a + b + c$
$A = \frac{1}{2}bh$



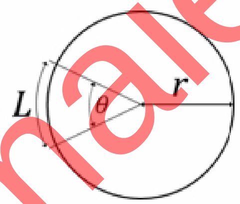
PARALLELOGRAM

$P = 2a + 2b$
$A = bh$



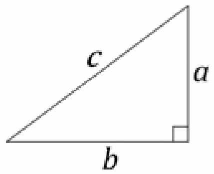
CIRCULAR SECTOR

$L = \pi r \frac{\theta}{180°}$
$A = \pi r^2 \frac{\theta}{360°}$



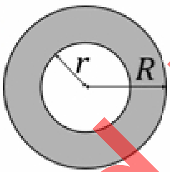
PYTHAGOREAN THEOREM

$a^2 + b^2 = c^2$
$c = \sqrt{a^2 + b^2}$



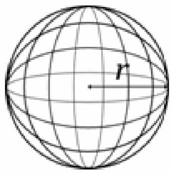
CIRCULAR RING

$A = \pi(R^2 - r^2)$



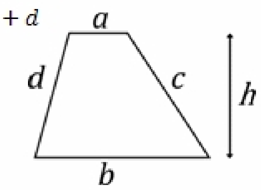
SPHERE

$S = 4\pi r^2$
$V = \frac{4\pi r^3}{3}$



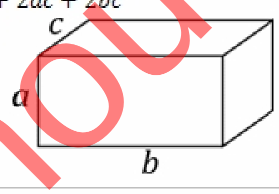
TRAPEZOID

$P = a + b + c + d$
$A = h\frac{a+b}{2}$



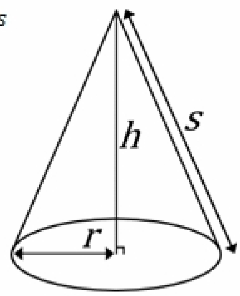
RECTANGULAR BOX

$A = 2ab + 2ac + 2bc$
$V = abc$



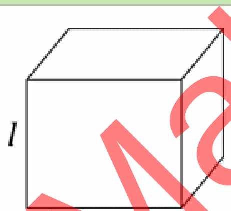
RIGHT CIRCULAR CONE

$A = \pi r^2 + \pi rs$
$s = \sqrt{r^2 + h^2}$
$V = \frac{1}{3}\pi r^2 h$



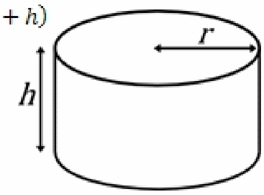
CUBE

$A = 6l^2$
$V = l^3$



CYLINDER

$A = 2\pi r(r + h)$
$V = \pi r^2 h$



FRUSTUM OF A CONE

$V = \frac{1}{3}\pi h(r^2 + rR + R^2)$