The distance between two points

The distance between two points =

$\sqrt{\text{square of the difference between x–coordinates + square of the difference between y–coordinates}}$

If $M(x_1,y_1)$ and $N(x_2,y_2)$ The distance between them =

$MN=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Examples

(1) **If a(1,2) ,b(4,6) find the distance between a and b($\overline{ab}$)**

Sol

$ab=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} =\sqrt{(1-4)^2+(2-6)^2} = \sqrt{(-3)^2+(-4)^2}$

$\underline{=5}$

---

(2) **If a(-1,2) ,b(4,6) find the length of ($\overline{ab}$)**

Sol

$ab=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} =\sqrt{(-1-4)^2+(2-6)^2} = \sqrt{(-5)^2+(-4)^2}$

$=\sqrt{41}$ length unit

---

(3) **If a(-2,2) ,b(4,-6) find the distance between a and b**

Sol

$ab=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} =\sqrt{(-2-4)^2+(2+6)^2} = \sqrt{(-6)^2+(8)^2} =\underline{10}$ length unit

(4) ***If a(-1,0) ,b(-4,6) find the length of (!ab)***

Sol

$ab=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(-1+4)^2+(0-6)^2}=\sqrt{(3)^2+(-6)^2}$

$=\sqrt{45}$ ***length unit***

---

(5) ***find the distance between (-3,4)and the origin point***

Sol

the distance $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(-3-0)^2+(4-0)^2}=$

$\sqrt{(-3)^2+(-4)^2}=5$ ***length unit***

---

(6) ***find the distance between (-5,0)and (0,12)***

Sol

the distance $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(-5-0)^2+(0-12)^2}=$

$\sqrt{(-5)^2+(-12)^2}=\sqrt{153}$ ***length unit***

---

(7) ***If a(3,1) ,b(1,2),c(5,4) prove that bc = 2ab***

Sol

$ab=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(3-1)^2+(1-2)^2}=\sqrt{(2)^2+(-1)^2}$

$=\sqrt{5}$ *length unit*

$bc=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(1-5)^2+(2-4)^2}=\sqrt{(-4)^2+(-2)^2}$

$=2\sqrt{5}$ *length unit*

Then bc = 2ab

(8) *If a(1,2) ,b(x,6) and the length of (!ab) = 5 unites*

Find the value of x

Sol

$ab=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(1\text{-}x)^2+(2\text{-}6)^2} = \sqrt{(1\text{-}x)^2+16}$

$\sqrt{(1\text{-}x)^2+16} = 5$ squaring the two sides

$(1\text{-}x)^2+16 = 25$

$1\text{-}2x + x^2 + 16 = 25$

$x^2-2x + 17\text{-}25=0$

$x^2-2x-8=0 \qquad (x-4)(x+2)=0$

$x= 4$ or $x= -2$

$(x \pm y)^2 =$

Square the first $\pm$ 2frist second

Square the second

$X^2 \pm 2xy + y^2$

---

(9) *If a(-1,2) ,b(x,6) and the length of (!ab) = $\sqrt{41}$ unites*

Find the value of x

Sol

$ab=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(\text{-}1\text{-}x)^2+(2\text{-}6)^2} = \sqrt{(\text{-}1\text{-}x)^2+(2\text{-}6)^2}$

$\sqrt{(\text{-}1\text{-}x)^2+16} = \sqrt{41}$ squaring the two sides

$(\text{-}1\text{-}x)^2+16 = 41$

$1+2x + x^2 + 16 = 41$

$x^2-2x + 17\text{-}41=0$

$x^2-2x-24=0 \qquad (x-6)(x+4)=0$

$x= 6$ or $x= -4$

$(x \pm y)^2 =$

Square the first $\pm$ 2frist second

Square the second

$X^2 \pm 2xy + y^2$

(10) ***If a(x,3) ,b(3,2),c(5,1) and bc = ab Find the value of x***

<u>Sol</u>

$ab=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(x-3)^2+(3-2)^2} = \sqrt{(x-3)^2+(1)^2}$

$bc=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(3-5)^2+(2-1)^2} = \sqrt{(-2)^2+(1)^2}$

$=\sqrt{5}$ *length unit*

Since $bc = ab$ *then* $\sqrt{(x-3)^2+(1)^2} = \sqrt{5}$ *squaring the two sides*

$(x-3)^2+(1)^2 = 5$

$x^2-6x+9+1=5$

$x^2-6x+10-5=0$

$x^2-6x+5=0 \quad (x-3)(x-2)=0$

<u>x= 3 or x= 2</u>

$(x\pm y)^2 =$

Square the first $\pm$ *2frist second*

Square the second

$X^2 \pm 2xy + y^2$

The distance between the point (3,–5)and x-axis = 5

The distance between the point (2,–3)and y-axis = 2

To prove that any three point (A,B,C)collinear(they lie on the same straight line)

The greatest distance = the sum of the two other distance

(11) ***prove that the points A(1,2),B(2,4) and C(4,8) are collinear***

<u>Sol</u>

$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(1-2)^2+(2-4)^2} = \sqrt{(-1)^2+(-2)^2} = \sqrt{5}$

$AC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(1-4)^2+(2-8)^2} = \sqrt{(-3)^2+(-6)^2}$

$=\sqrt{45} = 3\sqrt{5}$

$BC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} =\sqrt{(2-4)^2+(4-8)^2} = \sqrt{(-2)^2+(-4)^2}$

$=\sqrt{20} =2\sqrt{5}$ $AB + BC =\sqrt{5} +2\sqrt{5} =3\sqrt{5} =AC$

Then A,B and C are collinear

(12) ***prove that the points $A(4,3)$,$B(1,1)$ and $C(-5,-3)$ are collinear***

Sol

$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} =\sqrt{(4-1)^2+(3-1)^2} = \sqrt{(-3)^2+(2)^2}$

$=\sqrt{13}$

$AC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} =\sqrt{(4+5)^2+(3+3)^2} = \sqrt{(9)^2+(6)^2}$

$=\sqrt{117} =3\sqrt{13}$

$BC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} =\sqrt{(1+5)^2+(1+3)^2} = \sqrt{(6)^2+(4)^2}$

$=\sqrt{52} =2\sqrt{13}$ $AB + BC =\sqrt{13} +2\sqrt{13} =3\sqrt{13} =AC$

Then A,B and C are collinear

To prove that any three point (A,B,C)lie on the same circle whose center is M we prove $MA= MB = MC =r$(the radius of the circle)

The circumference of the circle $=2\pi r$

The area of the circle $=\pi r^2$

(13) ***prove that the points $A(-1,1)$,$B(0,4)$ and $C(3,1)$ lie on the circle whose center $M(1,2)$ and find its area where $\pi = 3.14$***

Sol

$$MA=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(1+1)^2+(2-1)^2}=\sqrt{(2)^2+(1)^2}=\sqrt{5}$$

$$MB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(1-0)^2+(2-4)^2}=\sqrt{(1)^2+(-2)^2}=\sqrt{5}$$

$$MC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(1-3)^2+(2-1)^2}=\sqrt{(-2)^2+(1)^2}=\sqrt{5}$$

$MA = MB = MC = \sqrt{5}$ then (A,B,C)lie on the same circle whose center is M

The area of the circle $=\pi r^2 = 3.14\ (\sqrt{5}\ )^2 = 15.7$ ***area unit***

---

(14) ***prove that the points A(3,-1),B(-4,6) and C(2,-2) lie on the circle whose center M(-1,2) and find its circumference where $\pi = 3.14$***

Sol

$$MA=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(3+1)^2+(-2-1)^2}=\sqrt{(4)^2+(-3)^2}=5$$

$$MB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(-1+4)^2+(2-6)^2}=\sqrt{(3)^2+(-4)^2}=5$$

$$MC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(-1-2)^2+(2+2)^2}=\sqrt{(-3)^2+(4)^2}=5$$

$MA = MB = MC = 5$ then (A,B,C)lie on the same circle whose center is M

The circumference of the circle $=2\pi r = 2(3.14)\ (5) = 31.4$ ***length unit***

To identify the type of the triangle (ABC) we find AB , BC and AC

And if (1) AB=BC=AC (triangle ABC is equilateral

(2) AB=BC≠AC (triangle ABC is isosceles)(or any another two sides)

(3) AB ≠BC≠AC (triangle ABC is scalene)

Prove that : $\triangle ABC$ is isosceles triangle : $A(3,5)$, $B(5,1)$ $C(1,1)$

Sol

$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(3-5)^2+(5-1)^2}=\sqrt{(2)^2+(4)^2}=2\sqrt{5}$

$BC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(5-1)^2+(1-1)^2}=\sqrt{(4)^2+(0)^2}=4$

$AC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(3-1)^2+(5-1)^2}=\sqrt{(2)^2+(4)^2}=2\sqrt{5}$

Then $AB=AC\neq BC$ $\triangle ABC$ is isosceles triangle

Prove that : $\triangle ABC$ is equilateral

triangle : $A(5,0)$, $B(7,2\sqrt{3})$ $C(3,2\sqrt{3})$

Sol

$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(5-7)^2+(0-2\sqrt{3})^2}=\sqrt{(-2)^2+12}=4$

$BC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(7-3)^2+(2\sqrt{3}-2\sqrt{3})^2}=\sqrt{(4)^2+(0)^2}=4$

$AC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(5-3)^2+(0-2\sqrt{3})^2}=\sqrt{(-2)^2+12}=4$

Then $AB=BC=AC$ (triangle ABC is equilateral

Prove that : $\triangle ABC$ is isosceles triangle : $A(-2,4)$, $B(3,-1)$ $C(4,5)$

Sol

$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(-2-3)^2+(4+1)^2}=\sqrt{(-5)^2+(5)^2}=5\sqrt{2}$

$BC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(3-4)^2+(-1-5)^2}=\sqrt{(-1)^2+(-6)^2}=\sqrt{37}$

$AC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(-2-4)^2+(4-5)^2}=\sqrt{(-1)^2+(-6)^2}=\sqrt{37}$

Then $AB\neq AC=BC$ $\triangle ABC$ is isosceles triangle

To identify the type of the triangle (ABC) we find AB , BC and AC

And if (1) $(AC)^2=(BC)^2+(AB)^2$ *(triangle ABC is right angle triangle*

(2) $(AC)^2<(BC)^2+(AB)^2$ *(triangle ABC a cute angle triangle)*

(3) $(AC)^2>(BC)^2+(AB)^2$ *(triangle ABC is a obtuse angle triangle)*

Prove that :$\triangle$ *ABC is right angle triangle :A(4,5) ,B(3, 2) C(-3,4) and find its area*

Sol

$AB==\sqrt{(4-3)^2+(5-2)^2} = \sqrt{(1)^2+(3)^2} =\sqrt{10} \Rightarrow (AB)^2=10$

$BC==\sqrt{(3+3)^2+(2-4)^2} = \sqrt{(-2)^2+(6)^2} =\sqrt{40} \Rightarrow (BC)^2=40$

$AC==\sqrt{(4+3)^2+(5-4)^2} = \sqrt{(1)^2+(7)^2} =\sqrt{50} \Rightarrow (AC)^2=50$

Then $(AC)^2=(BC)^2+(AB)^2$ *(triangle ABC is a right angle triangle)*

The area of the triangle = half the base by its height

$$=\frac{1}{2}(\sqrt{40})(\sqrt{10}) =10 \text{ cm}^2$$

Prove that :$\triangle$ *ABC is a obtuse angle triangle: A(5,4) ,B(3, 2) C(1,3)* **Sol**

$AB==\sqrt{(5-3)^2+(4-2)^2} = \sqrt{(2)^2+(2)^2} =\sqrt{8} \Rightarrow (AB)^2=8$

$BC==\sqrt{(3-1)^2+(2-3)^2} = \sqrt{(2)^2+(-1)^2} =\sqrt{5} \Rightarrow (BC)^2=5$

$AC==\sqrt{(5-1)^2+(4-3)^2} = \sqrt{(4)^2+(1)^2} =\sqrt{17} \Rightarrow (AC)^2=17$

$(AC)^2>(BC)^2+(AB)^2$ (triangle ABC is a obtuse angle triangle)

Prove that : $\triangle$ ABC is a obtuse angle triangle: A(4,5) ,B(6, 2)

C(3,3) **Sol**

$AB==\sqrt{(4-6)^2+(5-2)^2} = \sqrt{(-2)^2+(3)^2} =\sqrt{13} \Rightarrow (AB)^2=13$

$BC==\sqrt{(6-3)^2+(2-3)^2} = \sqrt{(3)^2+(-1)^2} =\sqrt{10} \Rightarrow (BC)^2=10$

$AC==\sqrt{(4-3)^2+(5-3)^2} = \sqrt{(1)^2+(2)^2} =\sqrt{5} \Rightarrow (AC)^2=5$

$(AB)^2>(BC)^2+(AC)^2$ (triangle ABC is acute angle triangle)

To prove that the quadrilateral ABCD

(1) Parallelogram (AB =CD) ,,(BC=AD)

(2) Rectangle (AB =CD) ,,(BC=AD) and AC =BD)

(3) Rhombus (AB =CD=BC=AD) and(AC ≠ BD)

(4) Square (AB =CD=BC=AD) and(AC= BD)

Prove that :the points A(-2,5), B(3,3), C(-4,2), D(-9,4)

Are vertices of a Parallelogram

Sol

$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} =\sqrt{(-2-3)^2+(5-3)^2} = \sqrt{(-5)^2+(2)^2} =\sqrt{29}$

$CD=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} =\sqrt{(-4+9)^2+(2-4)^2} = \sqrt{(5)^2+(2)^2} =\sqrt{29}$

$BC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} =\sqrt{(3+4)^2+(3-2)^2} = \sqrt{(7)^2+(1)^2} =\sqrt{50}$

$AD=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} =\sqrt{(-2+9)^2+(5-4)^2} = \sqrt{(7)^2+(1)^2} =\sqrt{50}$

Then (AB =CD) ,,(BC=AD) ABCD is Parallelogram

Prove that :the points A(1,4), B(4,9), C(-1,12), D(-4,7)

Are vertices of a square and find its area

Sol

$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} =\sqrt{(1-4)^2+(4-9)^2} = \sqrt{(-5)^2+(-3)^2} =\sqrt{34}$

$CD=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} =\sqrt{(-1+4)^2+(12-7)^2} = \sqrt{(3)^2+(5)^2} =\sqrt{34}$

$BC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} =\sqrt{(4+1)^2+(9-12)^2} = \sqrt{(5)^2+(-3)^2} =\sqrt{34}$

$AD=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} =\sqrt{(1+4)^2+(4-7)^2} = \sqrt{(5)^2+(-3)^2} =\sqrt{34}$

The diagonals

$AC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} =\sqrt{(1+1)^2+(4-12)^2} = \sqrt{(2)^2+(-8)^2} =\sqrt{68}$

$BD=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} =\sqrt{(4+4)^2+(9-7)^2} = \sqrt{(-8)^2+(2)^2} =\sqrt{68}$

Then (AB =CD=BC=AD)and AC = BD

ABCD is square

Area of the square $=(\sqrt{34}\ )^2 = 34$

---

(15) ***Prove that :the points A(5,3), B(6,-2), C(1,-1), D(0,4)***

Are vertices of a rhombus and find its area

Sol

$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} =\sqrt{(5-6)^2+(3+2)^2} = \sqrt{(-1)^2+(5)^2} =\sqrt{26}$

$CD=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} =\sqrt{(1-0)^2+(-1-4)^2} = \sqrt{(1)^2+(-5)^2} =\sqrt{26}$

$BC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} =\sqrt{(6-1)^2+(-2+1)^2} = \sqrt{(1)^2+(5)^2} =\sqrt{26}$

$AD=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} =\sqrt{(5-0)^2+(3-4)^2} = \sqrt{(-1)^2+(5)^2} =\sqrt{26}$

The diagonals

$AC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(5-1)^2+(3+1)^2}=\sqrt{(4)^2+(4)^2}=4\sqrt{2}$

$BD=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(6-0)^2+(-2-4)^2}=\sqrt{(6)^2+(-6)^2}=6\sqrt{2}$

Then $(AB=CD=BC=AD)$ and $AC \neq BD$

$\therefore$ ABCD is rhombus

Area of the rhombus = (half the product of its two diagonal)

Area of the rhombus $=\frac{1}{2}\times 4\sqrt{2}\times 6\sqrt{2}=24$

---

(16) **Prove that :the points A(0,1), B(4,5), C(1,8), D(-3,4)**

Are vertices of a rectangle

<u>Sol</u>

$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(0-4)^2+(1-5)^2}=\sqrt{(-4)^2+(-4)^2}=4\sqrt{2}$

$CD=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(1+3)^2+(8-4)^2}=\sqrt{(4)^2+(4)^2}=4\sqrt{2}$

$BC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(4-1)^2+(5-8)^2}=\sqrt{(3)^2+(3)^2}=3\sqrt{2}$

$AD=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(0+3)^2+(1-4)^2}=\sqrt{(3)^2+(3)^2}=3\sqrt{2}$

The diagonals

$AC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(0-1)^2+(1-8)^2}=\sqrt{(-1)^2+(-7)^2}=5\sqrt{2}$

$BD=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(4+3)^2+(5-4)^2}=\sqrt{(7)^2+(1)^2}=5\sqrt{2}$

Then $(AB=CD, BC=AD)$ and $AC=BD$ ABCD is rectangle

The two co-ordinate of the mid-point line segment

If $A(x_1,y_1)$ and $B(x_2,y_2)$

The two co-ordinate of the mid-point $=(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2})$

Calculate co-ordinate of point C the mid-point of $\overline{AB}$

(1) A(2,4) ,B(6,0)

Sol

The two co-ordinate of the mid-point $=(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2})$

$$C=(\frac{2+6}{2},\frac{4+0}{2})=(4,2)$$

---

(2) A(7,-5) ,B(-3,5)

Sol

The two co-ordinate of the mid-point $=(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2})$

$$C=(\frac{7-3}{2},\frac{5-5}{2})=(\frac{4}{2},0)=(2,0)$$

---

(3) A(3,-6) ,B(-3,6)

Sol

The two co-ordinate of the mid-point $=(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2})$

$$C=(\frac{3-3}{2},\frac{-6+6}{2})=(0,0)$$

---

(4) A(7,-6) ,B(-1,0)

Sol

The two co-ordinate of the mid-point $=(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2})$

$$C=(\frac{7-1}{2},\frac{-6+0}{2})=(3,-3)$$

(5) If C(10,-4) is the mid-point of $\overline{AB}$ where A(4,-2)
(6) Find The two co-ordinate of B

Sol

The two co-ordinate of the mid-point $=(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2})$

$$(10,-4)=(\frac{4+x}{2},\frac{-2+y}{2})$$

$\frac{4+x}{2}=10$

$4+x=20$

$x=20-4=16$

$x=16$

$\frac{-2+y}{2}=-4$

$-2+y=-8$

$y=-8+2=-6$

$y=-6$

B(16,-6)

---

(7) If C(1,2) is the mid-point of $\overline{AB}$ where A(x,1) B(–3,y)Find The value of x and y

Sol

The two co-ordinate of the mid-point $=(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2})$

$$(1,2)=(\frac{x-3}{2},\frac{1+y}{2})$$

$\frac{x-3}{2}=1$

$x-3=2$

$x=2+3=5$

$x=5$

$\frac{1+y}{2}=2$

$1+y=4$

$y=4-1=-3$

$y=-3$

(8) *A(1,-6) ,B(9,2) find co-ordinate of the points which divide $\overline{AB}$ into four equal parts*

Sol

Let C is the mid-point of AB

The two co-ordinate of the mid-point $=(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2})$

$C=(\frac{1+9}{2},\frac{-6+2}{2})=(\frac{10}{2},\frac{-4}{2})=(5,-2)$

Let D is the mid-point of AC

The two co-ordinate of the mid-point $=(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2})$

$D=(\frac{1+5}{2},\frac{-6-2}{2})=(\frac{6}{2},\frac{-8}{2})=(3,-4)$

Let E is the mid-point of CB

The two co-ordinate of the mid-point $=(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2})$

$E=(\frac{5+9}{2},\frac{2-2}{2})=(\frac{14}{2},\frac{0}{2})=(7,0)$

If : $\overline{AB}$ is a diameter in the circle M then M is the mid-point of AB

(9) ***If : $\overline{AB}$ is a diameter in the circle M where A(4,-1) B(-2,7)***

Find the co-ordinate of M , then find the circumference and the area of the circle

Sol

The two co-ordinate of the mid-point AB $=(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2})$

$M=(\frac{4-2}{2},\frac{-1+7}{2})=(\frac{2}{2},\frac{6}{2})=(1,3)$

To find the circumference and the area of the circle

We find the length of the radius by the distance between two point

the radius = AM =BM =r

$r=AM=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(4-1)^2+(-1-3)^2}=\sqrt{(3)^2+(-4)^2}=5$

The circumference of the circle $=2\pi r=2(3.14)(5)=31.4$

The area of the circle $=\pi r^2=(3.14)(5)^2=78.75$

---

(10) Prove the points A(2,1),B(5,-1),C(6,5),D(3,7)are vertices of a parallelogram

Sol

$\because$ The two diagonals are $\overline{AC}$ and $\overline{BD}$

The two co-ordinate of the mid-point AC $=(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2})$

$=(\frac{2+6}{2},\frac{1+5}{2})=(\frac{8}{2},\frac{6}{2})=(4,3)$

The two co-ordinate of the mid-point BD $=(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2})$

$=(\frac{5+3}{2},\frac{-1+7}{2})=(\frac{8}{2},\frac{6}{2})=(4,3)$

$\therefore$ the mid-point AC and is the same mid-point BD $\therefore$ The two diagonals bisect each other

$\therefore$ ABCD is parallelogram

(11) If ABCD is parallelogram where A(3,-1),B(-5,2),C(-2,4)

Find The two co-ordinate of D

Sol

Let $D(x,y)$

$\because$ $ABCD$ is parallelogram $\therefore$ The two diagonals bisect each other

$\therefore$ the mid-point AC is the same mid-point BD

The two co-ordinate of the mid-point AC $=(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2})$

$=(\frac{3-2}{2},\frac{-1+4}{2})=(\frac{1}{2},\frac{3}{2})$

The two co-ordinate of the mid-point BD $=(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2})$

$=(\frac{-5+x}{2},\frac{2+y}{2})$

$\therefore$ the mid-point AC is the same mid-point BD

$$(\frac{1}{2},\frac{3}{2})=(\frac{-5+x}{2},\frac{2+y}{2})$$

$\frac{1}{2}=\frac{-5+x}{2}$	$\frac{3}{2}=\frac{2+y}{2}$
$2=2(-5+x)$	$6=2(2+y)$
$-5+x=1$	$3=2+y$
$x=1+5=6$	$y=3-2=1$
$x=6$	$y=1$

The two co-ordinate of $D(6,1)$

(12) If ABCD is rhombus where A(3,2),B(4,-3),C(-1,-2) D(-2,3) find

(1) The two co-ordinate of the point of intersection The two diagonals

(2) the area of rhombus

Sol

$\because$ *ABCD is rhombus*

$\therefore$ *The two diagonals bisect each other*

$\therefore$ *the mid-point AC is the same mid-point BD*

The two co-ordinate of the mid-point $AC = (\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$

$= (\frac{3-1}{2}, \frac{2-2}{2}) = (1,0)$

To find the area of rhombus the length of The two diagonals by the distance between two points

$Ac = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(3+1)^2+(2+2)^2} = \sqrt{(4)^2+(4)^2} = 4\sqrt{2}$

$BD = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(4+2)^2+(-3-3)^2} = \sqrt{(6)^2+(-6)^2} = 6\sqrt{2}$

the area of rhombus = half the product of its two diagonal s

the area of rhombus $= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} = 24$

---

(13) Prove the points A(6,0),B(2,-4),C(-4,2) are vertices of a right angle triangle at B then find The two co-ordinate of D that make the figure ABCD is rectangle

Sol

$AB==\sqrt{(6-2)^2+(0+4)^2} = \sqrt{(4)^2+(4)^2} = \sqrt{32} \Rightarrow (AB)^2=32$

$BC==\sqrt{(2+4)^2+(-4-2)^2} = \sqrt{(-6)^2+(6)^2} = \sqrt{72} \Rightarrow (BC)^2=72$

$AC==\sqrt{(6+4)^2+(2-0)^2} = \sqrt{(10)^2+(2)^2} = \sqrt{104} \Rightarrow (AC)^2=104$

Then $(AC)^2=(BC)^2+(AB)^2$ (triangle ABC is a right angle triangle)

Let $D(x,y)$

$\because$ $ABCD$ is rectangle $\therefore$ The two diagonals bisect each other

$\therefore$ the mid-point AC is the same mid-point BD

The two co-ordinate of the mid-point AC $=(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2})$

$=(\frac{6-4}{2},\frac{0+2}{2})=(1,1)$

The two co-ordinate of the mid-point BD $=(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2})$

$=(\frac{2+x}{2},\frac{-4+y}{2})$

$\therefore$ the mid-point AC is the same mid-point BD

$$(1,1)=(\frac{2+x}{2},\frac{-4+y}{2})$$

$\frac{2+x}{2}=1$	$\frac{-4+y}{2}=1$
$2+x=2$	$-4+y=2$
$x=2-2=0$	$y=2+4$
$x=0$	$y=6$

The two co-ordinate of D $=(0,6)$

Prove the points A(-3,0),B(3,4),C(1,-6) are vertices of Isosceles triangle then find The length of the line segment drown from A to BC

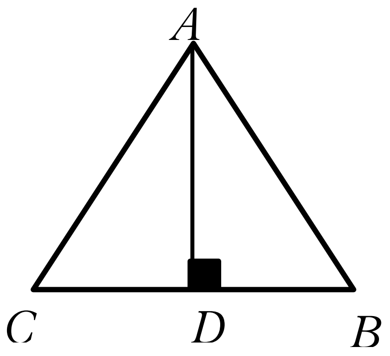
<u>***Sol***</u>

$AB=\sqrt{(-3-3)^2+(0-4)^2}=\sqrt{(-6)^2+(-4)^2}=\sqrt{52}$

$BC=\sqrt{(3-1)^2+(4+6)^2}=\sqrt{(2)^2+(10)^2}=\sqrt{104}$

$AC=\sqrt{(-3-1)^2+(0+6)^2}=\sqrt{(-4)^2+(6)^2}=\sqrt{52}$

$\because AB = AC \quad \therefore \Delta\ ABC$ Isosceles



$\because \quad \Delta ABC$ Isosceles and $\overline{AD} \perp \overline{BC}$

$\therefore D$ the mid-point CB

The two co-ordinate of the mid-point CB $=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$=\left(\frac{3+1}{2},\frac{4-6}{2}\right)=(2,-1)$

$AD=\sqrt{(-3-2)^2+(0+1)^2}=\sqrt{(5)^2+(1)^2}=\sqrt{26}$

The slope of straight line

The slope of straight line passing through two points

$(x_1,y_1)\ (x_2,y_2) = \dfrac{y_2 - y_1}{x_2 - x_1}$

Find The slope of straight line passing through (3,1) (4,2)

sol

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{4-3} = \frac{1}{1} = 1$$

Find The slope of straight line passing through (2,–1) (2,3)

sol

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3+1}{2-2} = \frac{4}{0} = \text{undefined}$$

Find The slope of straight line passing through (4,0) (2,2)

sol

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-0}{2-4} = \frac{2}{-2} = -1$$

Find The slope of straight line passing through $(-2,\sqrt{3})\ (1,4\sqrt{3})$

sol

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4\sqrt{3}-\sqrt{3}}{1+2} = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

Find The slope of straight line passing through (–1,3) (2,3)

sol

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-3}{2+1} = \frac{0}{1} = 0$$

Find The slope of straight line passing through (-3,4) and the origin point

sol

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-0}{0+3} = \frac{4}{3}$$

if The slope of straight line passing through (–1,2) ,(3,a) = 2 find the value of a

<u>*sol*</u>

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{a-2}{3+1}=\frac{a-2}{4} \qquad \text{and } m = 2$$

$$\frac{a-2}{4} = 2 \quad \text{then } a-2 = 8 \qquad \text{then } a = 10$$

The slope of a straight line = ***tan E*** *(where E is the positive angle That the straight line makes with the positive direction of the x axis)*

(1) Find the slope of the straight line which makes an angle of a measure 56° 12` 48`` in the positive direction to the x-axes.

Solution

(1) ∵ m = tan E ∴ m = Tan 56° 12` 48`` = 1.494534405

Start → tan 56 °'" 12 °'" 48 °'" =

---

(1) Find The slope of the straight line which make an angle of measure (0) with the positive direction of the x axis

<u>*sol*</u>

∵ *The slope of a straight line = tan E , E = 0*

∴ *The slope of a straight line = tan 0 = 0*

∴ *The slope of a straight line = 0*

(2) ***Find The slope of the straight line which make an angle of measure (30) with the positive direction of the x axis***

<u>sol</u>

$\because$ *The slope of a straight line* $= \tan E$, $E = 30$

$\therefore$ *The slope of a straight line* $= \tan 30 = \frac{\sqrt{3}}{3}$

$\therefore$ *The slope of a straight line* $= \frac{\sqrt{3}}{3}$

---

(3) ***Find The slope of the straight line which make an angle of measure (45) with the positive direction of the x axis***

<u>sol</u>

$\because$ *The slope of a straight line* $= \tan E$, $E = 45$

$\therefore$ *The slope of a straight line* $= \tan 45 = 1$

$\therefore$ *The slope of a straight line* $= 1$

---

(4) ***Find The slope of the straight line which make an angle of measure (60) with the positive direction of the x axis***

<u>sol</u>

$\because$ *The slope of a straight line* $= \tan E$, $E = 60$

$\therefore$ *The slope of a straight line* $= \tan 60 = \sqrt{3}$

$\therefore$ *The slope of a straight line* $= \sqrt{3}$

(5) ***Find The slope of the straight line which make an angle of measure (90) with the positive direction of the x axis***

<u>sol</u>

$\because$ *The slope of a straight line* $= \tan E$, $E = 90$

$\therefore$ *The slope of a straight line* $= \tan 90 =$ *unknown*

$\therefore$ *The slope of a straight line* $=$ *unknown*

---

(6) ***Find The slope of the straight line which make an angle of measure (135) with the positive direction of the x axis***

<u>sol</u>

$\because$ *The slope of a straight line* $= \tan E$, $E = 135$

$\therefore$ *The slope of a straight line* $= \tan 135 = -1$

$\therefore$ *The slope of a straight line* $= -1$

---

2 Find the measure of the positive angle that the straight line makes to the x - axis if m =1.4865 (where m is the slope) .

Solution

2 $\because m = \tan E$ $\therefore \tan E = 1.4865$ $\therefore m(\angle E) = 56^\circ\, 4'\, 13''$

Start → shift tan 1,4865 = °'''

To prove that three points (a ,b, c) are lying on the same straight line : we prove The slope of the straight line (ab) = The slope of the straight line (bc)

(1) prove that the points A(1,2),B(2,4) and C(4,8) are collinear

sol

$\because M(ab)=\frac{y_2-y_1}{x_2-x_1}=\frac{4-2}{2-1}=2 \qquad \therefore M(ab)=2$

$\because M(bc)=\frac{y_2-y_1}{x_2-x_1}=\frac{8-4}{4-2}=2 \qquad \therefore M(bc)=2$

$\because M(ab)=M(bc)$

$\therefore$ (a ,b, c) are lying on the same straight line

---

(2) If the point A(4,1),B(–2,7) and C(3,y) are lying on the same straight line find the value of y

sol

$\because M(AB)=\frac{y_2-y_1}{x_2-x_1}=\frac{7-1}{-2-4}=-1 \qquad \therefore M(AB)=-1$

$\because M(BC)=\frac{y_2-y_1}{x_2-x_1}=\frac{y-7}{3+2}$

$\because$ (A,B, C) are lying on the same straight line

$\therefore M(AB)=M(BC)$

$\therefore \frac{y-7}{3+2}=-1 \qquad \therefore Y-7=-5 \qquad \therefore Y=2$

If The slope of straight line passing through (1,–2) (5,y) =3 find the value of y

<u>***sol***</u>

$\because m=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{y+2}{5-1}$ $\quad\because$ and the slope $=3$

$\therefore \dfrac{y+2}{5-1}=3 \qquad \therefore y+2=12 \qquad \therefore y=10$

If the point A(1,7),B(–1,5) and C(4,2), prove that : $C\notin\overleftrightarrow{AB}$

<u>***sol***</u>

$\because M(AB)=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{5-7}{-1-1}=2 \qquad \therefore \quad M(AB)=2$

$\because M(BC)=\dfrac{y_2-y_1}{x_2-x_1}=\dfrac{2-5}{4+1}=\dfrac{-3}{5} \qquad \therefore \quad M(AB)=\dfrac{-3}{5}$

$\because M(AB)\neq M(BC)$

$\therefore \quad C\notin\overleftrightarrow{AB}$

The relation between the two slopes of the two parallel straight lines

If $L_1 // L_2$ *then* $m_1 = m_2$

(if two straight lines are parallel then their slopes are equal)

If $m_1 = m_2$ *then* $L_1 // L_2$

(if two straight lines have equal slopes then the two straight lines are parallel

Prove that ***the straight line passing through (4,2) (5,6)parallel the straight line passing through (0,5) (–1,1)***

sol

$\because m_1 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{6-2}{5-4} = 4$

$\because m_2 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-5}{-1-0} = 4$

$\because m_1 = m_2 \quad \therefore L_1 // L_2$

Prove that ***the straight line passing through (2,–1) (6,3)parallel the straight line which make an angle of measure (45) with the positive direction of the x axis***

sol

$\because m_1 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3+1}{6-2} = 1$

$\because m_2 = \tan 45 = 1$

$\because m_1 = m_2 \quad \therefore L_1 // L_2$

The slope of the straight line parallel the x–axis =0

The slope of the straight line parallel the y–axis is unknown $=\frac{1}{0}$

(1) ***If the slope of the straight line passing through (a,7) (3,5)parallel the y–axis then find the value of a***

sol

$\because m_1=\frac{y_2-y_1}{x_2-x_1}=\frac{5-7}{3-a}$

$\because$ *The slope of the straight line parallel the y–axis is unknown* $=\frac{1}{0}$

$\therefore \frac{5-7}{3-a}=\frac{1}{0} \quad \therefore 3-a=0 \quad \therefore a=3$

---

(1) ***If the slope of the straight line passing through (4,2) (–5,b)parallel the x–axis then find the value of b***

sol

$\because m_1=\frac{y_2-y_1}{x_2-x_1}=\frac{b-2}{-5-4}$

$\because$ *The slope of the straight line parallel the x–axis =0*

$\therefore \frac{b-2}{-5-4}=0 \quad \therefore b-2=0 \quad \therefore b=2$

---

the point A(1,5),B(x–1,3), C(4,7), D(2,1) four points satisfy $\overleftrightarrow{AD}//\overleftrightarrow{BC}$ ***find the value of x***

sol

$\because m_1 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-5}{2-1} = -4$

$\because m_2 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{7-3}{5-x}$

$\overleftrightarrow{AD} // \overleftrightarrow{BC} \quad \therefore\ m_1 = m_2$

$\dfrac{7-3}{5-x} = -4 \quad \therefore \dfrac{4}{5-x} = -4 \quad \therefore 5-x = -1 \quad \therefore x = 6$

If : $L_1 \perp L_2 \quad \therefore m_1 \times m_2 = -1$

If the two straight line are perpendicular then The product of the slopes of the perpendicular straight line $= -1$

If $m_1 \times m_2 = -1 \quad \therefore L_1 \perp L_2$

If The product of the slopes of the perpendicular straight line $= -1$ then the two straight line are perpendicular

***Prove that** the straight line passing through (–3,4) (–3,–2) perpendicular the straight line passing through (1,2) (–3,2)*

sol

$\because m_1 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2-4}{-3+3} = \dfrac{-6}{0} = \dfrac{1}{0}$

$\because m_2 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-2}{-3-1} = \dfrac{0}{1}$

$\because\ m_1 \times m_2 = -1 \quad \therefore\ L_1 \perp L_2$

__Prove that__ the straight line passing through $(4,3\sqrt{3}\,)\ (5,2\sqrt{3}\,)$

Perpendicular the straight line which make an angle of measure (30) with the positive direction of the x axis

__sol__

$m_1=\frac{y_2-y_1}{x_2-x_1}=\frac{2\sqrt{3}-3\sqrt{3}}{5-4}=\frac{-\sqrt{3}}{1}=-\sqrt{3}$

$m_2=\tan E \quad \text{and } E=30$

$m_2=\tan 30 \ =\frac{1}{\sqrt{3}}$

$\because\ m_1 \times m_2=-1 \quad \therefore\ L_1 \perp L_2$

If L_1 straight line passing through (3,1) (2,x)and L_2 straight line which make an angle of measure (45) with the positive direction of the x axis find the value of x if (L_1 , L_2)

(1) Parallel (2) Perpendicular

__sol__

$\because m_1=\frac{y_2-y_1}{x_2-x_1}=\frac{x-1}{2-3}=\frac{x-1}{-1}$

$\because m_2=\tan E=\tan 45=1$

$\because\ L_1 // L_2 \quad \therefore\ m_1=m_2 \quad \therefore\ \frac{x-1}{-1}=1 \quad \therefore\ x-1=-1 \quad \therefore\ x=0$

$\because\ L_1 \perp L_2 \quad \therefore\ m_1 \times m_2=-1 \quad \therefore\ \frac{x-1}{-1} \times 1=-1 \quad \therefore\ x-1=1$

$X=2$

Find the measure of the positive angle which the straight line passing through (4,3) (2,-5)

<u>sol</u>

$\because m=\frac{y_2-y_1}{x_2-x_1}=\frac{-5-3}{2-4}=4$

$\because E=$ shift tan $4=$

If: the points A(3,-1), B(x,3), C(5,3) Are vertices of a right angle triangle at A find the value of x

<u>sol</u>

$\because$ $\triangle ABC$ is a right angle triangle at A $\therefore$ $AC \perp AB$

$\therefore$ $m_1 \times m_2=-1$

$\because m_1=\frac{y_2-y_1}{x_2-x_1}=\frac{3+1}{5-3}=2$

$\because m_2=\frac{y_2-y_1}{x_2-x_1}=\frac{3+1}{x-3}$

$\because m_1 \times m_2=-1$

$\therefore 2 \times \frac{3+1}{x-3}=-1$ $\therefore$ $x-3=-8$ $\therefore$ $x=-5$

To prove that ABCD is parallelogram we prove that each two opposite sides are parallel (m(AB)= m(CD) and m(BC)=m(AD))

Prove that :the points A(-2,5), B(3,3), C(-4,2), D(-9,4)

Are vertices of a Parallelogram

Sol

$\because\ m(AB) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-5}{3+2} = \frac{-2}{5}$

$\because\ m(CD) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{-9+4} = \frac{-2}{5}$

$\therefore\ m(AB) = m(CD)$ ………………..(1)

$\because\ m(BC) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-3}{-4-3} = \frac{1}{7}$

$\because\ m(AD) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-5}{-9+2} = \frac{1}{7}$

$\therefore\ m(BC) = m(AD)$ ………………..(2)

From (1) and (2) ABCD is parallelogram

Two prove that ABCD is rectangle we prove ABCD is parallelogram and $m(AB) \times m(BC) = -1$ *and* $m(AD) \times m(DC) = -1$

Prove that :the points A(-1,3), B(5,1), C(6,4), D(0,6)

Are vertices of a rectangle

Sol

$\because\ m(AB) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-3}{5+1} = \frac{-1}{3}$

$\because\ m(CD) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-4}{0-6} = \frac{-1}{3}$

$\therefore\ m(AB) = m(CD)$ ....(1)

$\because\ m(BC) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-1}{6-5} = 3$

$\because\ m(AD) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-3}{0+1} = 3$

$\therefore\ m(BC) = m(AD)$ ....(2)

From (1) and (2) ABCD is parallelogram

$\because m(AB) \times m(BC) = 3 \times \frac{-1}{3} = -1 \quad \therefore\ m(AB) \times m(BC) = -1$

$\because m(AD) \times m(DC) = 3 \times \frac{-1}{3} = -1 \quad \therefore\ m(AD) \times m(DC) = -1$

$\therefore$ ABCD is rectangle

Two prove that ABCD is trapezium we prove there is two opposite sides are parallel and the other two opposite sides not parallel

Prove that : the points A(-1,0), B(7,4), C(5,8), D(1,6) Are vertices of a trapezium

<u>Sol</u>

$\because\ m(AB) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-0}{7+1} = \frac{1}{2}$

$\because\ m(CD) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-8}{1-5} = \frac{1}{2} \quad \because\ m(AB) = m(CD) \quad \therefore\ AB // CD$

$\because\ m(BC) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-4}{5-7} = -2$

$\because\ m(AD) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-0}{1+1} = 3 \quad \because\ m(AB) \neq m(CD) \quad \therefore\ AB \not\parallel CD$

From (1) and (2) ABCD is a trapezium

Two prove that ABCD is square we prove ABCD is rectangle and its diagonals are perpendicular

Prove that :the points A(-1,–1), B(2,3), C(6,0), D(3,-4) Are vertices of a square

Sol

$\because m(AB) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3+1}{2+1} = \frac{4}{3}$

$\because m(CD) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-0}{3-6} = \frac{4}{3}$

$\because m(BC) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-3}{6-2} = \frac{-3}{4}$

$\because m(AD) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4+1}{3+1} = \frac{-3}{4}$

$\because m(AB) = m(CD) \quad \therefore \quad AB // CD$

$\because m(BC) = m(AD) \quad \therefore \quad BC // AD$

$\therefore$ *ABCD is parallelogram*

$\because m(AB) \times m(BC) = \frac{4}{3} \times \frac{-3}{4} = -1 \quad \therefore m(AB) \times m(BC) = -1$

$\because m(AD) \times m(DC) = \frac{4}{3} \times \frac{-3}{4} = -1 \quad \therefore m(AD) \times m(DC) = -1$

$\therefore$ *ABCD is rectangle……..(1)*

Its *two diagonal*

$\because m(AC) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0+1}{6+1} = \frac{1}{7}$

$\because m(BD) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-3}{3-2} = -7$

$\because m(AC) \times m(BD) = \frac{1}{7} \times -7 = -1 \quad \therefore m(AD) \times m(DC) = -1 \ldots (2)$

ABCD is a square

The equation of the straight line given its slope and its y-intercept

Remember

The slope of straight line passing through two points

$(x_1, y_1)\ (x_2, y_2) = \frac{y_2 - y_1}{x_2 - x_1}$

The slope of a straight line = **tan E**

If the equation of the straight line in the form : $y = mx + c$

(1) The slope of the straight line = m

(2) The length of the intercepted part of y-axis = $|c|$

(3) the straight line passes through (0,c)

Find the slope and the intercepted of y-axis for each the following straight line

(1) $y = 5x - 3$

sol

we should write any equation at the form $y = mx + c$

$\because y = 5x - 3$

$\therefore$ The slope of the straight line = m = 5

$\therefore$ The length of the intercepted part of y-axis = c = 3

(2) $2y=4-x$

sol

we should write any equation at the form $y = mx + c$

$\because$ $2y=4-x$ $\div 2$ $\therefore$ $y = 2 - \frac{1}{2}x$

$\therefore$ The slope of the straight line $= m = -\frac{1}{2}$

$\therefore$ The length of the intercepted part of y-axis $= c = 2$

---

(3) $2x-3y-6=0$

sol

we should write any equation at the form $y = mx + c$

$2x-3y-6=0$ $\therefore$ $-3y = 6-2x$ $\div -3$

$\therefore$ $y = -2 + \frac{2}{3}x$

$\therefore$ The slope of the straight line $= m = \frac{2}{3}$

$\therefore$ The length of the intercepted part of y-axis $= c = 2$

---

(4) $\frac{x}{2}+\frac{y}{3}=1$

sol

we should write any equation at the form $y = mx + c$

$\therefore \frac{x}{2}+\frac{y}{3}=1$ $\therefore$ $\frac{y}{3} = 1-\frac{x}{2}$ x 3

$\therefore$ $y = 3-\frac{3}{2}x$

$\therefore$ The slope of the straight line $= m = -\frac{3}{2}$

$\therefore$ The length of the intercepted part of y-axis $= c = 3$

If the equation of the straight line in the form : ax +by +c=0

1. *The slope of the straight line* $=\dfrac{-\text{coefficient of } x}{\text{coefficient of } y} = \dfrac{-a}{b}$
2. *The length of the intercepted part of y-axis* $=\dfrac{-\text{absolute tearm}}{\text{coefficient of } y} = \left|-\dfrac{c}{b}\right|$
3. *the straight line passes through* $(0, -\dfrac{c}{b})$

Find the slope and the intercepted of y-axis for each the following straight line

(1) $y+x-1=0$

sol

$\therefore$ *The slope of the straight line* $=\dfrac{-\text{coefficient of } x}{\text{coefficient of } y} = \dfrac{-a}{b} = -1$

$\therefore$ *The length of the intercepted part of y-axis* $=\dfrac{-\text{absolute tearm}}{\text{coefficient of } y} = \left|-\dfrac{c}{b}\right| = 1$

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(2) $2x+5y-15=0$

sol

$\therefore$ *The slope of the straight line* $=\dfrac{-\text{coefficient of } x}{\text{coefficient of } y} = \dfrac{-a}{b} = \dfrac{-2}{5}$

$\therefore$ *The length of the intercepted part of y-axis* $=\dfrac{-\text{absolute tearm}}{\text{coefficient of } y} = \left|\dfrac{15}{5}\right| = 3$

Prove that the two straight line :
$2x - 3y + 5 = 0$,, $4x - 6y + 1 = 0$ are parallel

<u>Sol</u>

$\because\ m_1 = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-a}{b} = \frac{-2}{-3} = \frac{2}{3}$

$\because\ m_2 = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-a}{b} = \frac{-4}{-6} = \frac{2}{3}$

$\because\ m_1 = m_2 \quad \therefore$ the two straight line are parallel

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Prove that the two straight line :
$6x - 3y + 5 = 0$,, $y = 2x + 7$ are parallel

<u>Sol</u>

$\because\ m_1 = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-a}{b} = \frac{-6}{-3} = 2$

$\because\ m_2 = 2 \qquad (y = mx + c)$

$\because\ m_1 = m_2 \quad \therefore$ the two straight line are parallel

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<u>Prove that</u> the straight line passing through (2,5) (-3,1)parallel

the straight line which its equation : $4x - 5y + 7 = 0$

<u>Sol</u>

$\because\ m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-5}{-3-2} = \frac{4}{5}$

$\because\ m_2 = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-a}{b} = \frac{-4}{-5} = \frac{4}{5}$

$\because\ m_1 = m_2 \quad \therefore$ the two straight line are parallel

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Prove that ***the straight line passing through (2,3) (-2,1)perpendicular to the straight line which its equation : 2x +y +8 =0***

Sol

$\because\ m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-3}{-2-2} = \frac{1}{2}$

$\because m_2 = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-a}{b} = -2$

$\because m_1 \times m_2 = \frac{1}{2} \times -2 = -1$

$\therefore$ *the two straight line are perpendicular*

If the straight line passing through (0,1) (3,3)parallel the straight line which its equation : ax –6y +5 =0

Find the value of a

Sol

$\because\ m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{3-0} = \frac{2}{3}$

$\because m_2 = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-a}{b} = \frac{-a}{-6} = \frac{a}{6}$

$\because$ *the two straight line are parallel* $\therefore m_1 = m_2$

$\therefore \frac{2}{3} = \frac{a}{6} \quad \therefore 3a = 12 \quad \div 3$

$\therefore a = 4$

If the straight line passing through (-1,3) (1,y)parallel the straight line which its equation : $6x - 4y + 1 = 0$

Find the value of a

Sol

$\because m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y-3}{1+1} = \frac{y-3}{2}$

$\because m_2 = \frac{\text{-coefficient of } x}{\text{coefficient of } y} = \frac{-a}{b} = \frac{3}{2}$

$\because$ the two straight line are parallel $\therefore m_1 = m_2$

$\therefore \frac{3}{2} = \frac{y-3}{2} \quad \therefore 2y - 6 = 6 \quad \therefore 2y = 12 \qquad \div 2$

$Y = 6$

If the straight line $ax - 4y + 1 = 0$ parallel the straight line : $5x - 2y + 3 = 0$ Find the value of a

Sol

$\because m_1 = \frac{\text{-coefficient of } x}{\text{coefficient of } y} = \frac{-a}{b} = \frac{a}{4}$

$\because m_2 = \frac{\text{-coefficient of } x}{\text{coefficient of } y} = \frac{-a}{b} = \frac{5}{2}$

$\because$ the two straight line are parallel $\therefore m_1 = m_2$

$\therefore \frac{a}{4} = \frac{5}{2} \quad \therefore a = 10$

If the straight line passing through (0,1) (3,3) perpendicular to the straight line : $ax - 6y + 5 = 0$ Find the value of a

Sol

$\because m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{3-0} = \frac{2}{3}$

$\because m_2 = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-a}{b} = \frac{-a}{-6} = \frac{a}{6}$

$\because$ the two straight line are perpendicular $\therefore m_1 \times m_2 = -1$

$\therefore \quad \frac{2}{3} \times \frac{a}{6} = 2a = 18$

$\therefore \quad a = 9$

finding the equation of the straight line given its slope and the length of intercepted part of y –axis

the straight line whose slope = m and cuts y-axis at the point (0,c) its equation is in the form : $y = mx + c$

Find the equation of the straight line whose slope = 2 and cuts y-axis at the point (0,3)

sol

$\because m = 2$ and $c = 3$

$\therefore \quad y = mx + c \qquad \therefore \quad y = 2x + 3$

Find the equation of the straight line whose slope = 2 and cuts y-axis at the point (0,3)

sol

$\because\ m = 2$ and $c = 3$

$\therefore\ y = mx + c \qquad \therefore\ y = 2x + 3$

Find the equation of the straight line whose slope = 3 and intercepts from the positive part of y-axis 3 unit

sol

$\because\ m = 3$ and $c = 3$

$\therefore\ y = mx + c \qquad \therefore\ y = 3x + 3$

Find the equation of the straight line whose slope = 2 and intercepts from the negative part of y-axis 3 unit

sol

$\because\ m = 2$ and $c = -3$

$\therefore\ y = mx + c \qquad \therefore\ y = 2x - 3$

Remark

(1) the equation of the straight line passing through the origin point is $y = mx$

(2) the equation of x- axis is $y = 0$

(3) the equation of y- axis is $x = 0$

Find the equation of the straight line passing through the origin point and makes an angle of measure (135) with the positive direction of x –axis

sol

$\because\ m = \tan E$ and $E = 135 \qquad \therefore\ m = \tan 135 = -1$

the equation of the straight line passing through the origin point is $y = mx$

$\therefore\ y = -x$

Find the equation of the straight line passing through (0,5)
And its slope = 2

sol

$\because\ m = 2 \quad and\ y = mx + c$

$\therefore\ y = 2x + c$ and passing through $(0,5)$

$\therefore\ 5 = 2(0) + c \qquad \therefore\ c = 5 \qquad \therefore\ y = 2x + 5$

Find the equation of the straight line passing through (3,-5)
And parallel the straight line $x + 2y - 7 = 0$

sol

$\because$ they are parallel straight lines then there slopes are equal

$$m = \frac{-coefficient\ of\ x}{coefficient\ of\ y} = \frac{-a}{b} = \frac{-1}{2}$$

$and\ y = mx + c$

$\therefore\ y = \frac{-1}{2} x + c$ and passing through $(3,-5)$

$\therefore\ -5 = \frac{-1}{2}(3) + c \qquad \therefore\ c = \frac{7}{2}$

$\therefore\ y = \frac{-1}{2} x + \frac{7}{2}$

Find the equation of the straight line passing through (3,2) And parallel the straight line passing through the two points (5,6) ,(-1,2)

sol

$\because$ they are parallel straight lines then there slopes are equal

$$\because\ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-6}{-1-5} = \frac{2}{3}$$

$$\text{and } y = mx + c$$

$$\therefore\ y = \frac{2}{3}x + c \text{ and passing through}(3,2)$$

$$\therefore\ 2 = \frac{2}{3}(3) + c \qquad \therefore\ c = 0$$

$$\therefore\ y = \frac{2}{3}x$$

Find the equation of the straight line passing through (1,2) And perpendicular to the straight line passing through the two points (2,-3) ,(5,-4)

sol

$\because$ they are perpendicular straight lines

$$\therefore\ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4+3}{5-2} = \frac{-1}{3} \text{ the perpendicular slope } = 3$$

$$\text{and } y = mx + c$$

$$\therefore\ y = 3x + c \text{ and passing through}(1,2)$$

$$\therefore\ 2 = 3(1) + c \qquad \therefore\ c = -1$$

$$\therefore\ y = 3x - 1$$

Find the equation of the straight line passing through the two points (2,-1) ,(1,1)

sol

$\because m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1+1}{1-2} = -2$ and $y = mx + c$

$\therefore y = -2x + c$ and any point satisfy the equation

$\therefore 1 = -2(1) + c \qquad \therefore c = 3$

$\therefore y = -2x + 3$

Find the equation of the straight line the perpendicular to $\overline{AB}$ From its midpoint :A(-3,6),B(2,1)

<u>sol</u>

$\because m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-6}{2+3} = -1$ the perpendicular slope $= 1$

$Y = mx + c \qquad y = x + c$

the straight line the perpendicular to $\overline{AB}$ From its midpoint

The two co-ordinate of the mid-point $= \left(\frac{-3+2}{2}, \frac{6+1}{2}\right) = \left(\frac{-1}{2}, \frac{7}{2}\right)$

$\because y = x + c \quad \therefore \frac{7}{2} = \frac{-1}{2} + c \quad \therefore c = \frac{9}{2} \therefore y = x + \frac{9}{2}$

Find the equation of the axis of symmetry of $\overline{XY}$

: X=(3,-2) and Y = (-5,6)

<u>sol</u>

$\because m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6+2}{-5-3} = -1$ the perpendicular slope $= 1$

$Y = mx + c \qquad y = x + c$

the straight line the perpendicular to $\overline{XY}$ From its midpoint

The two co-ordinate of the mid-point $= \left(\frac{3-5}{2}, \frac{-2+6}{2}\right) = (-1,2)$

$\because y = x + c \quad \therefore 2 = -1 + c \quad \therefore c = 3 \therefore y = x + 3$

If A (- 3, 4), B (5, - 1), C (3, 5) find the equation of the straight line passing through the vertex A and bisecting $\overline{BC}$.

Solution

The midpoint of $\overline{BC} = (\frac{3+5}{2}, \frac{5-1}{2}) = (\frac{8}{2}, \frac{4}{2}) = (4, 2)$

$\therefore$ The slope of the required straight line $= \frac{2-4}{4+3} = \frac{-2}{7}$

$\because y = mx + c$ $\quad\quad \therefore y = \frac{-2}{7}x + c$

$\because$ The point of A (-3, 4) passes through the straight line, so it satisfies the equation.

$\therefore 4 = \frac{-2}{7} \times -3 + c$ $\quad\quad \therefore 4 = \frac{6}{7} + c$ $\quad\quad \therefore c = \frac{22}{7}$

$\therefore$ The equation of the straight line is written as in the formula: $y = \frac{-2}{7}x + \frac{22}{7}$ and by the multyipling two sides in 7

$\therefore 7y = -2x + 22$ $\quad\quad$ i.e the equation is : $2x + 7y - 22 = 0$

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8 When its slope is equal to slope of the straight line $\frac{y-1}{x} = \frac{1}{3}$ and intersects a part from the negative direction 3

We should write the equation at the form $ax + by + c = 0$

$\because \frac{y-1}{x} = \frac{1}{3}$ $\quad\quad \therefore 3y - 3 = x$ $\quad\quad \therefore x - 3y + 3 = 0$

$\because m = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-a}{b} = \frac{1}{3}$

And there slopes are equal

$Y = \frac{1}{3}x - 3$

Find the equation of the straight line which intersects from the x - y axes two positive parts both lengths are 4 and 9 respectively.

$Let\ \frac{x}{a} + \frac{y}{b} = 1$ and $a=4$ and $y=9$

$\frac{x}{4} + \frac{y}{9} = 1$ x 36 $9x + 4y - 36 = 0$

A B C is a triangle where A (1, 2), B (5, - 2), C (3, 4), D is the midpoint of $\overline{AB}$, drawn $\overrightarrow{DE}$ // $\overleftrightarrow{BC}$ and intersects $\overline{AC}$ in E, find the equation of the straight line $\overleftrightarrow{DE}$.

5 The following table represents linear relation:

A) Find the equation of the straight line.

B) Find the length of the intersected part from the y - axis.

C) Find the value of A.

x	1	2	3
y= f (x)	1	3	A

Sol

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-3}{1-2} = 2$ and $y = mx + c$

$\because y = 2x + c$ and the straight line passes through $(1,1)$

$\because 1 = 2 + c \quad c = 1-2 = -1$

$\therefore y = 2x - 1$ ……..(1)

$\therefore$ the length of the intercepted part from y –axis $= 1$ …..(2)

$\because (3, A)$ belong to $y = 2x - 1$

$\therefore A = 6 - 1 = 5 \quad \therefore A = 5$