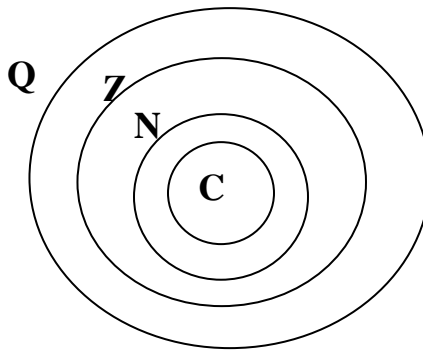


Remember the following

- 1- The counting Numbers $\mathbf{C} = \{1, 2, 3, \dots\}$.
- 2- The Natural Numbers $\mathbf{N} = \{0, 1, 2, 3, \dots\}$.
- 3- The Integers $\mathbf{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$.
- 4- The rational Numbers $\mathbf{Q} = \left\{ \frac{a}{b} : a, b \in \mathbf{Z} \text{ and } b \neq 0 \right\}$.

5- $\mathbf{C} \subset \mathbf{N} \subset \mathbf{Z} \subset \mathbf{Q}$.



- 6- Any number x has an additive inverse $(-x)$.
- 7- Any number a/b (except zero) has a multiplicative inverse $\frac{b}{a}$.
- 8- **The absolute value:** If $K \in \mathbf{Q}$, then:
 - $|K| \geq 0$.
 - If $|k| = b$, Then $k=b$ or $K = -b$.
- 9) The standard form of the rational number:
The number is written in standard form if: $a \times 10^n$ where $n \in \mathbf{Z}$, $1 \leq |a| < 10$.
- 10) **The indices (Powers):**

1- $a^m \times a^n = a^{m+n}$

For example $5^4 \times 5^3 = 5^7$.

2- $a^m \div a^n = a^{m-n}$

For example $2^5 \div 2^2 = 2^3$.

3- $(a^m)^n = a^{mn}$

For example $(3^2)^4 = 3^8$.

4- $(a b)^n = a^n b^n$ for example $(3 \times 5)^2 = 3^2 \times 5^2$.

5- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ for example $\left(\frac{2}{7}\right)^3 = \frac{2^3}{7^3}$.

6- $a^{-n} = \frac{1}{a^n}$ for example $3^{-1} = \frac{1}{3}$.

Remark:

1) $(-\text{Ve number})^{\text{even}} = +\text{ve}$.

2) $(-\text{Ve number})^{\text{odd}} = -\text{ve}$.

3) $x^0 = 1$, $x \neq 0$.

11) The square root of a rational Number

- \sqrt{k} = number its square “power 2” is k
- 25 for example has two square roots which are 5, -5

Because $(5)^2 = 25$, $(-5)^2 = 25$.

- $\sqrt{}$: This symbol means the + ve root only

Then $\sqrt{36} = |6| = 6$

• $\sqrt{49} = 7$, $-\sqrt{49} = -7$, $\pm \sqrt{49} = \pm 7$

- $\sqrt{-\text{ve}}$ is meaning less.

Exercise

1-Complete:-

- 1) The additive inverse of (-3) is.....
- 2) The additive inverse of (zero is....
- 3) The multiplicative inverse of 0.25 is
- 4) $|-4| - |4| = \dots\dots$
- 5) $\sqrt{9+16} = 3 + \dots\dots$
- 6) $(0.6)^{-2} = \dots\dots$
- 7) The standard form of 532.7 is
- 8) $(-2)^{55} \dots\dots (-2)^6$ “put $>$ or $<$ ”
- 9) The two square roots of $6\frac{1}{4}$ are,..... and their sum=.....
- 10) $(\frac{4}{3})^0 = \dots\dots$

2) Find S.S. of the following, $x \in \mathbb{Q}$:

- 1) $2x - 1 = 11.$
- 2) $11 - 5x = 6.$
- 3) $7x + 3 = 5x + 9.$
- 4) $2(x + 3) = 18.$
- 5) $4x - 11 = 0.$

(1)The cubic root of a rational Number

Remarks:-

- 1- The cubic root of a rational number is also a rational number.
 - 2- The cubic root of a + ve rational number is a + ve rational number.
 - 3- The cubic root of a –ve rational number is a –ve rational number.
-

Exercise

1) Complete the following:-

1- $\sqrt[3]{-0.027} = \dots\dots$

2- $\sqrt[3]{\frac{3}{8}} = \dots\dots$

3- If $\sqrt[3]{x} = 5$, then $x = \dots\dots\dots$

4- If $\sqrt[3]{x} = \sqrt{4}$, then $x = \dots\dots\dots$

5- $\sqrt{25} = \sqrt[3]{\dots\dots\dots}$

6- $\sqrt[3]{-1} + \sqrt{1} = \dots\dots\dots$

7- If $\sqrt[3]{X} = 2$, then $x^2 = \dots\dots\dots$

8- If $x^3 = 64$, then $\sqrt{x} = \dots\dots\dots$

2- Find S.S. of the following equations where $x \in \mathbb{Q}$:

1) $x^3 - 27 = 0$.

2) $2x^3 + 5 = 21$.

3) $(x - 7)^3 = -64$.

3- If the volume of a cube is $15\frac{5}{8}\text{cm}^3$. Find the edge length.

(2) The set of irrational numbers Q^c :

- $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{\frac{3}{7}}$,.....

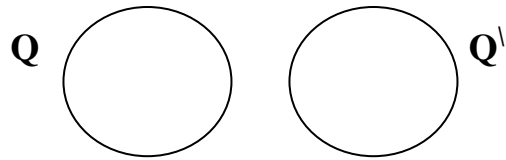
No rational numbers whose square is 2,3,5 or $\frac{3}{7}$

- $\sqrt[3]{2}$, $\sqrt[3]{3}$, $\sqrt[3]{5}$, $3\sqrt[3]{\frac{6}{11}}$,.....

No rational numbers whose cube is 2, 3, 5 or $\frac{6}{11}$

The pervious numbers \in another set called the set of irrational numbers (Q^c)

- $Q \cap Q^c = \phi$

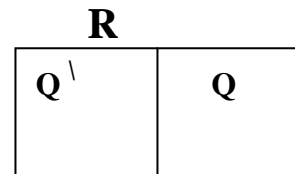


- Each irrational number lies between two rational numbers.

(3) The set of real Numbers R :

- $R = Q \cup Q^c$

- $N \subset Z \subset Q \subset R$ and $Q^c \subset R$.



(4) Ordering Numbers in R :

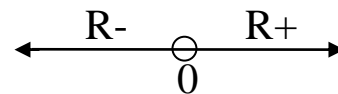
- $R_+ = \{ x : x \in R , x > 0 \}$.

- $R_- = \{ x : x \in R , x < 0 \}$.

- $R_+ \cap R_- = \phi$.

- $R = R_+ \cup \{0\} \cup R_-$.

- $R^* = R_+ \cup R_- = R - \{0\}$.



Exercise

1- Complete:

1- $Q \cap Q' = \dots\dots$

2- $Q \cup Q' = \dots\dots$

3- $R = R^+ \cup \dots\dots \cup \dots\dots$

4- $R^* = \dots\dots \cup \dots\dots = \dots\dots$

5- $R - Q = \dots\dots$

6- $R - Q' = \dots\dots$

7- IF x is an integer, $x < \sqrt{7} < x + 1$, then $x = \dots\dots\dots$

8- IF x is an integer, $x < 3\sqrt{3} < x + 1$, then $x = \dots\dots\dots$

2- Find the S.S. of the following equations:

1) $x^2 - 5 = 2$, $x \in Q$

2) $\frac{1}{2}x^3 + 3 = 7$, $x \in Q$

3) $\frac{3}{4}x^2 = 12$, $x \in R$

4) $x^2 + 25 = 0$, $x \in R$

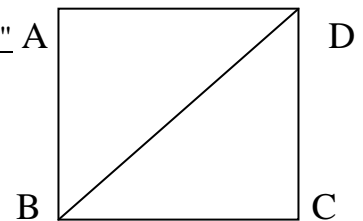
5) $1 - 0.008x^3 = -7$

3- Prove that $\sqrt{11}$ is included between 3.31, 3.32

4- In the figure:

A B C D is a square, its side length 5cm . Find the length of the diagonal \overline{BD} .

{Remember that $(BD)^2 = (BC)^2 + (CD)^2$ } "Pythagoras theorem" A



(5) The Intervals

Exercise

1) Write each of the following sets as an interval and represent it on the Number line:

1) $A = \{a: a \in \mathbb{R}, -2 \leq a \leq 7\}$

2) $B = \{b: b \in \mathbb{R}, 3 < b < 5\}$

3) $C = \{c: c \in \mathbb{R}, 0 \leq c < 2\}$

4) $D = \{d: d \in \mathbb{R}, -3 < d \leq 1\}$

5) $E = \{e: e \in \mathbb{R}, e \geq 4\}$

6) $F = \{f: f \in \mathbb{R}, f \leq 1\}$

7) $G = \{g: g \in \mathbb{R}, g < -3\}$

8) $H = \{h: h \in \mathbb{R}, h > -5\}$

9) The + ve real Numbers \mathbb{R}_+ .

10) The – ve real Numbers \mathbb{R}_- .

11) The set of non-(+ ve) real numbers.

12) The set of non - (–ve) real numbers.

13) The set of real Numbers \mathbb{R} .

2- Complete using one of the symbols \in or \notin :

1) $\sqrt{9}$ $[3, 5]$

2) -3 $]-3, 1]$

3) -3 $]-5, 1[$

4) 0 $] - 1, 4[$

5) 5 $] \sqrt{9}, \sqrt{20} [$

6) $\sqrt[3]{-27}$ $[-\sqrt{9}, \sqrt{9}]$

3) If $X = [-1, 3[$, $Y =]2, 5]$. Find using the number line:

1) $X \cap Y$

2) $X \cup Y$

3) $X - Y$

4) $Y - X$

4) If $A =]-\infty, 1]$ and $B = [-4, \infty[$. find using the number line:

1) $A \cup B$

2) $A \cap B$

3) $A - B$

4) $B - A$

5) A^c

6) B^c

5) Complete the following:

1) $[-2, 5] \cap \{-2, 5\} = \dots\dots$

2) $] -3, 4[\cap \{-3, 4\} = \dots\dots\dots$

3) $[1, 6] -]1, 6[= \dots\dots$

4) $]0, 5[\cup \{0, 5\} = \dots\dots\dots$

5) $] -2, 3] \cap \mathbb{Z}^+ = \dots\dots$

6) $[3, 7] - \{3, 7\} = \dots\dots\dots$

(6) The operations on the real Numbers

Remember that

- 1) $2x$ is the product of multiplying 2 by x .
- 2) $5a$, $7a$ are called like terms.
- 3) $5a$, $7b$ are called unlike terms.
- 4) We can add, subtract only the like terms for example:

- $3a + 4a = 7a$
 - $4a + 5a - a = 8a$
 - $-6xy - 2xy + 8xy = \text{zero}$.
 - $9a^2b + 4ab^2$ (unlike terms).
-

- 5) The multiplication of Algebraic terms.

- $3a \times 2a = 6a^2$
 - $-4x^2 \times 5y = -20x^2y$
 - $-xy \times -3xy^2 = 3x^2y^2$
-

$+ve \times +ve = +ve$
 $-ve \times -ve = +ve$
 $+ve \times -ve = -ve$
 $-ve \times +ve = -ve$

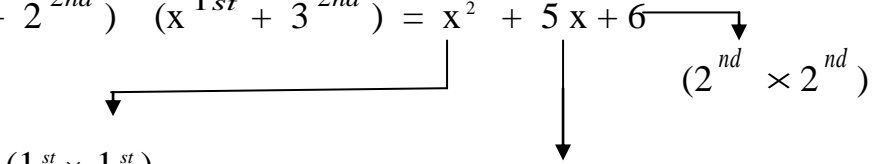
- 6) Dividing Algebraic Terms:

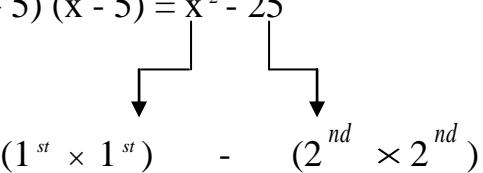
- $12a^3b^4 \div 4a^2b^2 = 3a^1b^2$.
- $(-18x^4) \div (-6x^3) = 3x$.
- $5m^3n^2 \div 5m^3n^2 = 1$.

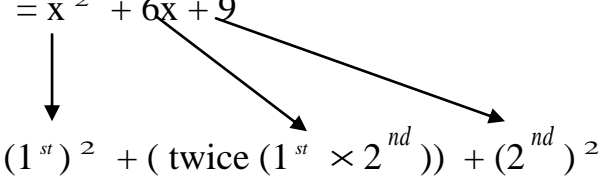
7) Multiplying of an algebraic Term \times Alg. expression.

- $3a(2a + 5) = 6a^2 + 15a$ (unlike)
 - $-5x^2y(2xy^2 - x^2y + 1) = -10x^3y^3 + 5x^4y^2 - 5x^2y.$
-

8) Multiplying of binomials “expressions consists of 2 terms”

- $(x^{1^{st}} + 2^{2^{nd}})(x^{1^{st}} + 3^{2^{nd}}) = x^2 + 5x + 6$

 $(1^{st} \times 1^{st})$ $(2^{nd} \times 2^{nd})$
 (Sum of mult. means and extremes)

- $(x + 5)(x - 5) = x^2 - 25$

 $(1^{st} \times 1^{st}) - (2^{nd} \times 2^{nd})$

- $(x + 3)^2 = x^2 + 6x + 9$

 $(1^{st})^2 + (\text{twice } (1^{st} \times 2^{nd})) + (2^{nd})^2$

- $(x - 5)^2 = x^2 - 10x + 25.$

Exercise

1) Find in the simplest form:-

1) $2\sqrt{5} + 7\sqrt{5} =$

2) $5\sqrt{3} - 7\sqrt{3} + 3\sqrt{3} =$

3) $6\sqrt{11} - 2\sqrt{2} + 4\sqrt{2} - 5\sqrt{11} + 1 =$

4) $4\sqrt{7} + \sqrt{7} - 5\sqrt{7} =$

5) $3 \times \sqrt{6} =$

6) $-4 \times \sqrt{2} =$

7) $\sqrt{3} \times \sqrt{3} =$

8) $2\sqrt{5} \times \sqrt{5} =$

9) $4\sqrt{5} \times -3\sqrt{5} =$

10) $\sqrt{7} \times \sqrt{2} =$

11) $3\sqrt{7} \times 4\sqrt{2} =$

12) $\frac{4}{5}\sqrt{5} \times \frac{1}{2}\sqrt{2}$

13) $5(\sqrt{2} + \sqrt{3})$

14) $\sqrt{3}(\sqrt{5} - \sqrt{3})$

15) $2\sqrt{6}(3\sqrt{3} + 4\sqrt{5})$

16) $3\sqrt{2}(2\sqrt{2} - 9\sqrt{3} + 11)$

17) $(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})$

18) $(5\sqrt{5} + 2\sqrt{6})(5\sqrt{5} - 2\sqrt{6})$

19) $(\sqrt{6} + 3)^2$

20) $(2\sqrt{3} - 3\sqrt{2})^2$

2) Make the denominator in each of the following an integer:-

1) $\frac{5}{\sqrt{5}}$

2) $\frac{-3}{2\sqrt{6}}$

3) $\frac{\sqrt{3} - \sqrt{2}}{\sqrt{5}}$

3) Complete the following:-

1) $\sqrt{3} + \sqrt{3} = \dots\dots\dots$ But $\sqrt{3} \times \sqrt{3} = \dots\dots\dots$

2) $(\sqrt{5})^2 = \dots\dots\dots$

3) $(3\sqrt{2})^3 = \dots\dots\dots$

4) The multiplicative inverse of the number $\frac{\sqrt{6}}{3}$ is $\frac{\sqrt{6}}{\dots\dots\dots}$

5) The additive inverse of $(5 - \sqrt{3})$ is $\dots\dots\dots$

6) $(\sqrt{3} - 2)^2 = 7 - \dots\dots\dots$

7) If $a = \sqrt{3} - 1$, $b = \sqrt{3} + 1$ then : $a + b = \dots\dots\dots$

8) The two dimensions of a rectangle are $(5 + \sqrt{2})$ cm and $(5 - \sqrt{2})$ cm , then its perimeter = $\dots\dots\dots$ cm.

(7) Operations on roots

Remember that:

$$1) \sqrt{a} \times \sqrt{b} = \sqrt{ab}.$$

$$2) \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}, \quad b \neq 0.$$

$$3) \sqrt{a^2 \pm b^2} \neq a \pm b.$$

4) The two Numbers $(\sqrt{a} + \sqrt{b})$, $(\sqrt{a} - \sqrt{b})$ are called (Conjugate numbers),

$$\text{and } (\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b.$$

$$5) \sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{ab}.$$

$$6) \frac{\sqrt[3]{a}}{\sqrt[3]{b}} = \sqrt[3]{\frac{a}{b}}.$$

$$7) \sqrt[3]{a^3 \pm b^3} \neq a \pm b.$$

$$8) \sqrt[3]{-a} = -\sqrt[3]{a}.$$

Exercises

1) Simplify to the simplest form:-

1) $\sqrt{75}$

2) $\sqrt[3]{16}$

3) $\sqrt[3]{54}$

4) $\frac{1}{5} \sqrt{1000}$

5) $4 \sqrt{\frac{5}{2}}$

6) $6 \sqrt[3]{\frac{1}{3}}$

7) $\sqrt{75} - 2\sqrt{27} + \sqrt{3}$

8) $3\sqrt{54} + \sqrt[3]{16} - \sqrt[3]{250}$

9) $2\sqrt{50} - 3\sqrt{2} - 4\sqrt{\frac{1}{8}}$

10) $\sqrt[3]{72} + \sqrt[3]{\frac{1}{3}} + \sqrt[3]{-9}$

11) $12 \sqrt{\frac{2}{3}} \times \sqrt{54}$

12) $3\sqrt{\frac{5}{4}} \div 3\sqrt{\frac{2}{5}}$

2) Write the conjugate number of each of the following numbers:

$\sqrt{6} - \sqrt{2}$, $3\sqrt{7} - 4$, $11 + 4\sqrt{5}$

$-3 + 2\sqrt{2}$, $-4\sqrt{5} - 7\sqrt{2}$.

3) Write the following such that the denominator is an integer:-

1) $\frac{\sqrt{3}}{\sqrt{2}}$

2) $3\sqrt{\frac{1}{4}}$

3) $\frac{5}{\sqrt{7}-\sqrt{2}}$

4) $\frac{4}{2-\sqrt{2}}$

5) $\frac{\sqrt{8}}{3+2\sqrt{2}}$.

4) If $a = \frac{2}{\sqrt{5}-\sqrt{3}}$, $b = \sqrt{5} - \sqrt{3}$

Find: 1) $a^2 + 2ab + b^2$.

2) $a^2 + ab + b^2$.

3) $a^2 - b^2$.

5) If $x = \frac{1}{\sqrt{7}-\sqrt{6}}$, y is the multiplicative inverse of x find the value of $(x + y)^2$.

6) If $\frac{11}{2\sqrt{5}+3} = a\sqrt{5}+b$ find a, b .

7) Complete the following:-

1) The multiplicative inverse of $\sqrt{20}$ is

2) If $x = \frac{\sqrt{10}}{\sqrt{5}}$ then $x^{-1} = \dots\dots\dots$.

3) $\sqrt{2}$, $\sqrt{8}$, $\sqrt{18}$, $\sqrt{32}$, , (In same pattern).

4) The conjugate of the fraction $\frac{1}{\sqrt{3}-\sqrt{2}}$ is

5) $(\sqrt{6}-\sqrt{5})^{11} (\sqrt{6}+\sqrt{5})^{11} = \dots\dots\dots$

6) $(\sqrt[4]{2})^3 = \dots\dots\dots$

7) $\frac{6}{\sqrt[3]{6}} \times \frac{\sqrt[3]{6}}{\sqrt[3]{6}} \times \frac{\sqrt[3]{6}}{\sqrt[3]{6}} = \sqrt[3]{\dots\dots\dots}$

8) $(\sqrt{10})^4 = \dots\dots\dots$

9) $(\sqrt{2})^4 = 2\dots\dots = \dots\dots\dots$

10) $(\sqrt[3]{4})^6 = 4 = \dots\dots\dots$

8) If $x = \sqrt{7} + \frac{1}{2}\sqrt{12}$ and $y = \frac{1}{3}\sqrt{63} - \sqrt{3}$. Prove that: $x^2 y^2 = 16$.

9) If $a = \sqrt[3]{6} + 5$, $b = \sqrt[3]{6} - 5$. Find the value of $(a - b)^3$

10) Find the result of the following: $(\sqrt[3]{5} - \sqrt[3]{4})(\sqrt[3]{25} + \sqrt[3]{20} + \sqrt[3]{16})$.

(8) Applications on the real Numbers
Exercise

1) Complete:-

- 1) The volume of a cube is 125 cm^3 , then its edge length = cm.
 - 2) T The volume of a cube is 27 cm^3 , then the area of one face = cm^2
 - 3) The volume of a cube is 8 cm^3 , then its lateral surface area = cm^2
 - 4) If the total surface area is 96 cm^2 , then its volume =..... cm^3
 - 5) The edge of a cube is $5\sqrt{2}$ cm, then its total surface area = cm^2
 - 6) The volume of a sphere whose radius $\sqrt[3]{2}$ is..... $(\pi = 3.14)$
 - 7) The volume of a cuboids with dimension $5\sqrt{2}$ cm , $2\sqrt{3}$ cm, $3\sqrt{6}$ cm is.....
 - 8) The area of a sphere whose diameter $4\sqrt[3]{3}$ is
 - 9) If the surface area of a sphere is $9\pi \text{ cm}^2$, then its diameter length =cm.
 - 10) The volume of a right circular cylinder with radius 1 cm, height = 1cm is..... .

- 2) The sum of all edges of a cube is 72cm find:
- i) Its volume.
 - ii) Its lateral surface area.
 - iii) Its total area.

3) If the radius length of a circular cylinder is $\frac{7\sqrt{2}}{2}$ cm and its height $\frac{20}{\sqrt{2}}$ cm find:

1) Its volume.

2) Its lateral surface area.

3) Its total area.

4) The circumference of the base of a right circular cylinder is 44 cm and its height = 25cm. find its volume.

5) The volume of a sphere is 4188cm^3 . Find its radius length ($\pi = 3.141$).

6) A right circular cylinder with volume $500\pi\text{cm}^3$ and its radius length = 5 cm. Find its height.

(9) Solving equations and inequalities of 1st degree in one unknown in R

Exercise

1) Find in R the S.S. of each of the following

1) $5x - 2 = 7$

2) $\sqrt{3} x + 2 = 5$

3) $x + \sqrt{6} = 1$

4) $2x - \sqrt{10} = 3\sqrt{10}$

5) $x - 1 < 2$

6) $4x + 3 \geq 3$

7) $9 - 4x \leq 13$

8) $5x - 1 > 2x + 5$

9) $-3 < 2x - 1 \leq 5$

10) $\frac{1}{4} x + 1 \leq 3$

11) $-3 < \frac{1}{2} x - 2 \leq \text{zero}$

12) $x - 1 \leq 3 - x$

13) $x - 1 \leq 3x + 1 \leq x + 5$

14) $2 + 2x \leq 3x + 3 < 5 + 2x$

15) $7 - x \leq 1 - 4x < 3 - x$

Unit (2): Statistics

(1) Collecting and organizing data

(2) Ascending and descending cumulative frequency tables

1) The following are the marks of 40 students in math exam:

10 ,21 ,32 ,42 ,52 ,33 ,40 ,46 ,27 ,36 ,25 ,30 ,41 ,12 ,35 ,50,
24 ,44 ,39 ,45 ,20 ,11 ,31 ,32 ,48 ,55 ,21 ,38 ,54 ,44 , 29 ,34,
33, 40, 58, 59, 30, 18, 28, 47.

Required:

I) Form a frequency table with sets: (10- , 20- ,...50-)

II) Form the ascending cumulative frequency table and graph it.

III) Form the descending cumulative frequency table and graph it.

(2) *Graph the ascending and descending curves for the following frequency distribution:*

Sets	8-	12-	16-	20-	24-	28-	32-	36-	40-	Total
Frequency	4	7	12	18	20	19	11	6	3	100

(3) The Mean

Remember that:

- The mean of a set of values = $\frac{\text{The total of values}}{\text{Number of values}}$

Exercise

1) Complete:-

- 1) The mean of the values: 7 , 12 , 5 , 6 is
- 2) If the mean of the values: 10, $a + 2$, 4 is 7 , then $a =$
- 3) If the mean of the values:
 $x + 1$, $x - 2$, $x - 5$, $2x$ is 6 , then $x =$
- _____

2) *The following table shows the frequency distribution of weights of 50 children (kids) in kg:*

Sets	5-	15-	25-	35-	45-	Total
Frequency	7	10	12	13	8	50

Calculate the mean.

3) The following table shows the frequency distribution of weekly wages of workers in one factory

Sets	16-	20-	24-	28-	32-	36-	Total
Frequency	10	15	22	25	20	8	100

Find the mean of the wages.

4) The following table shows the frequency distribution of marks 30 students in algebra exam:

Sets	5-	15-	25-	35-	45-	Total
Frequency	3	4	8	x	6	30

1- Find the value of x.

2- Find the mean.

3- Find the number of students whose marks are less than 25 marks.

(4) The median

Exercise

1) Complete:-

1) The median of the values: 4 , 3 , 1, 9 , 8 is.....

2) The median of the values: 11 , 18 , 14 , 12 , 16 , 19 is.....

2) *The following table shows, the frequency distribution of 100 factories According to the number of weekly working hours:*

Sets hours	50-	60-	70-	80-	90-	100-	Total
Number of factories	5	8	12	28	33	14	100

Find using the descending cumulative frequency curve of the median number of hours of work of these factories.

3) *The following table shows, the frequency distribution of marks of 60 students in mathematics exam:*

Sets of marks	5-	10-	15-	20-	25-	30-	35-	Total
Number of students	2	5	14	20	13	5	1	60

Find the median mark.

(5) The Mode

Exercise

1) Complete:

- 1) The mode of the values: 7 , 9 , 3 , 7 is.....
 - 2) The mode of the values: 3 , 4 , 3 , 2 , 1 , 3 is.....
 - 3) If the mode of the values: 8 , 10 , $x - 1$, 7 , 9 is 10 , then $x =$
-

2) The following table shows the frequency distribution of marks of 100 pupils in an exam:

Marks sets	10-	14-	18-	22-	26-	30-	34-	Total
Number of pupils	2	10	15	40	25	6	2	100

Find the mode mark using the histogram of this distribution.

3) The following table shows the frequency distribution of ages of 45 persons:

Sets of ages in years	12-	14-	16-	18-	20-	22-	24-	Total
Number of persons	5	7	8	12	6	4	3	45

Find the mode age.

Final Revision

1) Complete:

1) If $|x| + 3 = 7$, then $x = \dots\dots\dots$ or $\dots\dots\dots$

2) $\sqrt[3]{64} = \sqrt{\dots\dots\dots}$

3) $\sqrt{36 + 64} = 6 + \dots\dots\dots$

4) If $\sqrt[3]{x} + 2 = 0$, then $x = \dots\dots\dots$

2) Choose the correct answer:

1) $\sqrt{(-5)^2} = \dots\dots\dots$

a) 5

b) -5

c) ± 5

2) The S.S. of the equation $x^2 + 9 = 0$ is $\dots\dots\dots$ ($x \in \mathbb{Q}$).

a) 3

b) -3

c) ± 3

d) ϕ

3) If $a^2 = 25$, then $a^{-1} \in \{\dots\dots, \dots\dots\}$.

a) $\{5, -5\}$

b) $\{0, 5\}$

c) $\{\frac{1}{5}, \frac{-1}{5}\}$

d) $\{-5, 0\}$

4) If the volume of a cube is $3\frac{3}{8} \text{ cm}^3$, then its edge = $\dots\dots\dots$ Cm.

a) $\frac{3}{2}$

b) $\frac{27}{8}$

c) -1.5

d) $\frac{2}{3}$

3- Find S.S. of the following equations, $x \in \mathbb{Q}$:

1) $5(x^2 - 1) = 75.$

2) $\sqrt[3]{x+2} = 3.$

4) Complete:

1) If $x \in \mathbb{Z}$, $x < \sqrt{13} < x+1$, then $x = \dots\dots\dots$

2) If $x \in \mathbb{Z}$, $x - 1 < -\sqrt[3]{6} < x$, then $x = \dots\dots\dots$

3) If $x = \sqrt{2}$, then $x^2 = \dots\dots\dots$

4) If $x - \sqrt[3]{5} = 0$, then $x^3 = \dots\dots\dots$

5) If $x \in \mathbb{Q}$, then $x^2 \in \dots\dots\dots$

6) If $a^2 \in \mathbb{Q}$, then $a \in \dots\dots\dots$ Or $\dots\dots\dots$

5) A- Prove that $\sqrt{5}$ lies between 2.2 and 2.3

B- Prove that $\sqrt[3]{12}$ lies between 2.2 and 2.3

6) Put \surd or \times :

1) If $m \in \mathbb{Q}$, then $m^2 \in \mathbb{Q}.$

2) If $m^2 \in \mathbb{Q}$, then $m \in \mathbb{Q}.$

3) If $k \geq 2$, $1 < \sqrt{k} < 2$ then $k=2$ or $3.$

4) If the area of square is 10 cm^2 , its side length is $\sqrt{10} \text{ cm}$.

5) $\sqrt[3]{3\frac{3}{8}} \in \mathbb{Q}^1$

6) If the volume of a cube is 6 cm^3 , then its edge length $\in \mathbb{Q}$.

7) Complete

1) The S.S. of the equation $x^2 + 21 = 0$ is.....

2) $\sqrt{64} _ \sqrt[3]{64} = \dots\dots\dots$

3) $[-1, 3] \cap \{-1, 3\} = \dots\dots\dots$

4) $] -2, 5[\cap \} -2, 5 \{ = \dots\dots\dots$

5) $[-3, 4] _] -3, 4[= \dots\dots\dots$

6) $\sqrt[3]{250} - \sqrt[3]{2} = \dots\dots\dots$

7) $\mathbb{Q} \cup \mathbb{Q}^1 = \dots\dots\dots$

8) $\mathbb{Q} \cap \mathbb{Q}^1 = \dots\dots\dots$

9) $\mathbb{R} = \mathbb{R}^+ \cup \dots\dots \cup$

10) $\mathbb{R} _ \mathbb{Q} = \dots\dots\dots$

11) $\mathbb{R} - \mathbb{Q}^1 = \dots\dots\dots$

12) $\sqrt{6\frac{1}{4}} = \dots\dots\dots$

13) The conjugate of $\frac{1}{\sqrt{5}-2}$ is

14) $\sqrt{2} + \sqrt{2} = \dots\dots\dots$

15) The mulct inverse of $\sqrt{6}$ is

16) $\sqrt{8} - \sqrt{2} = \dots\dots\dots$

17) $x < \sqrt{5} < x + 1$, then $x = \dots\dots\dots$

18) $-\sqrt{25} = \sqrt[3]{b}$, then $b = \dots\dots\dots$

19) $(\sqrt{3} + \sqrt{2})^2 = \dots\dots\dots + 2\sqrt{6}$

20) The additive inverse of $\frac{-1}{\sqrt{2}}$ is

8) If $x = \sqrt{3} + \sqrt{2}$, $y = \frac{1}{\sqrt{3} + \sqrt{2}}$.Find $x^2 + 2xy + y^2$.

9) If $x = \sqrt{5} + \sqrt{3}$, $y = \frac{2}{\sqrt{5} + \sqrt{3}}$. Find $(x + y)^2$.

10) If $x = \frac{1}{\sqrt{5}-2}$, $y = \frac{20}{\sqrt{5}}$. Find $x^2 - y$.

11) Simplify: $\sqrt{48} - 2\sqrt{27} + 6\sqrt{\frac{1}{3}}$

12) Simplify: $\sqrt[3]{16} - \sqrt[3]{54} - \sqrt[3]{128}$

13) Simplify: $\sqrt[3]{108} - 2\sqrt[3]{\frac{1}{2}} - |3\sqrt{-4}|$

14) If $A =] -3 , 1]$, $B =] -1 , 5 [$,use the number line to find:-

1) $A \cap B$

2) $A \cup B$

3) $A - B$

4) $B - A$

15) Find S.S. of the following, $x \in \mathbf{R}$:

1) $2x^2 - 1 = 17$

4) $3x < 2x - 1$

2) $(x - 2)^2 = 49$

5) $7x - 3 \geq 11$

3) $(x + 1)^3 = -8$

6) $3 \leq 3 - 5x \leq 13$

16) If $x = \frac{1}{\sqrt{5-2}}$, $xy = 1$. Prove that: $(x + y)^2 = 20$.

17) Complete:-

1) The mean of 3, 5 is.....

2) If the mean of 11, x, 14 is 12 then $x = \dots\dots\dots$

3) If the mean of $x - 2$, $2x - 1$, $x + 2$, $x + 3$ is 13, then $x = \dots\dots\dots$

4) The median of 11, 14, 13, 10, 8 is.....

5) The median of 4, 7, 3, 2, 9, 6

6) The mode is.....

7) The mode of 14, 18, 17, 17, 14, 16, 14 is

8) If the mode of 2a, 3, 6, 5, 6 is 6 then $a = \dots\dots\dots$

18) The following table shows the frequency distribution of weekly wages in a factory:

Sets	50-	60-	70-	80	90-	Total
Freq	5	15	25	45	10	100

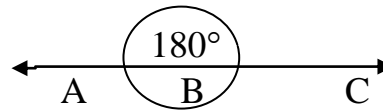
Find: 1) The mean.

2) The median.

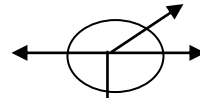
3) The mode.

Remember that

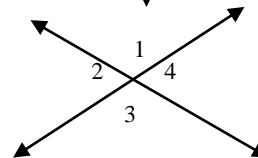
- 1) The measure of the Straight angle = 180° .
 $m(\angle ABC) = 180^\circ$.



- 2) The sum of accumulative angles at a point = 360° .



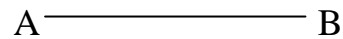
- 3) Each two vertically opposite angles are equal in measure $m(\angle 1) = m(\angle 3)$,
 $m(\angle 2) = m(\angle 4)$.



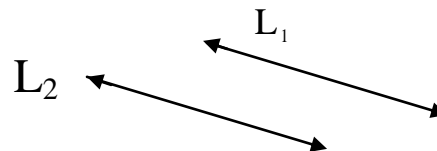
- 4) Complementary angles : Their sum = 90° .

- 5) Supplementary angles: Their sum = 180° .

- 6) The length of \overline{AB} is AB.

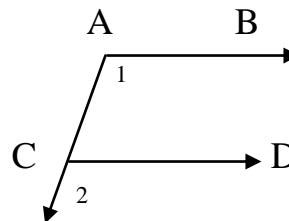


- 7) $L_1 \parallel L_2$ if $L_1 \cap L_2 = \phi$.



- 8) If $\overrightarrow{AB} \parallel \overrightarrow{CD}$

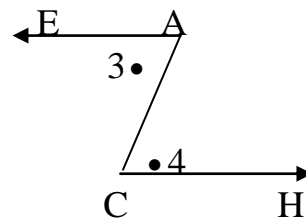
$$\therefore m(\angle 1) = m(\angle 2).$$



“Each two corresponding angles are equal in measure”.

2) If $\overrightarrow{AE} \parallel \overrightarrow{CH}$

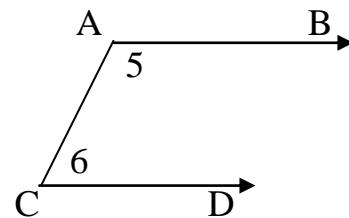
$$\therefore m(\angle 3) = m(\angle 4).$$



“Each two alternate angles are equal in measure”.

3) If $\overrightarrow{AB} \parallel \overrightarrow{CD}$

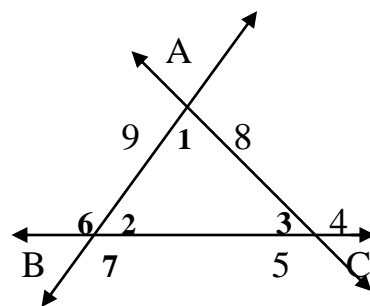
$$\therefore m(\angle 5) + m(\angle 6) = 180^\circ.$$



**“Each two interior angles on the same
Side of the transversal are supplementary”.**

The Triangle

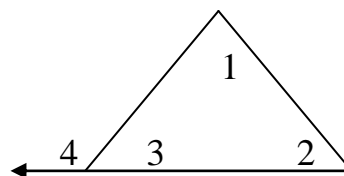
- Any triangle contains 3 interior angles and 6 exterior angles.



- The sum of measures of the interior angles of a triangle = -----°

- The measure of the exterior angle of a Δ equals the sum of measures of two interior Angles not the adjacent one.

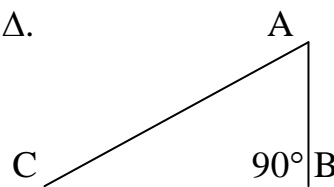
$$\therefore m(\angle 4) = m(\angle 1) + m(\angle 2).$$



- If the measure of an angle in a Δ equals the sum of Measures of other two angles, then :the Δ is a right angled Δ .

$$\therefore \text{ If } m(\angle B) = m(\angle A) + m(\angle C)$$

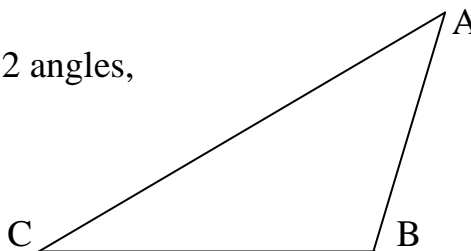
$$\therefore m(B) = 90^\circ.$$

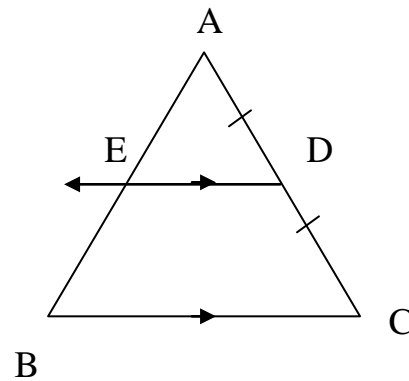
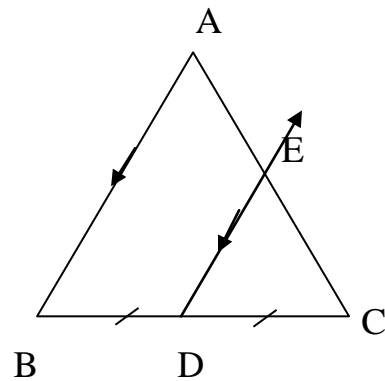


- If the measure of an angle in a Δ is greater than the sum of measures of the other 2 angles, then the triangle is obtuse.

$$\therefore \text{ If } m(\angle B) > m(\angle A) + m(\angle C).$$

$$\therefore m(\angle B) > 90^\circ \Rightarrow \Delta \text{ is obtuse at B.}$$





- The ray drawn from the mid point of a side of a triangle parallel to another side bisects the third side.

If D is the mid – point of a side \overrightarrow{BC} , $DE \parallel$ to another side then :
E is the midpoint of the 3rd side.

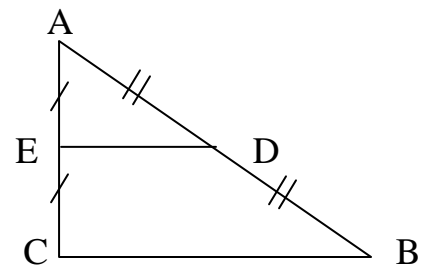
- The line segment joining the mid points of two sides of a triangle parallel to the 3rd side and equal half its length.

If D , E are the mid – points of \overrightarrow{AB} , \overrightarrow{AC}

in ΔABC , then:

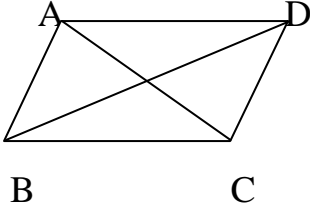
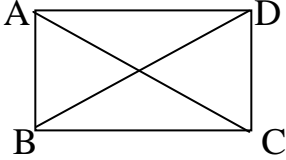
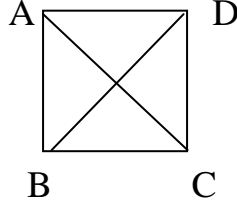
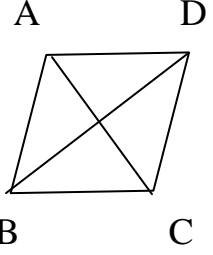
(i) $\overline{DE} \parallel \overline{BC}$.

(ii) $DE = \frac{1}{2} BC$.



1) The parallelogram

Remember that

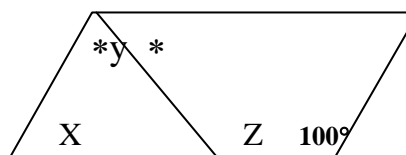
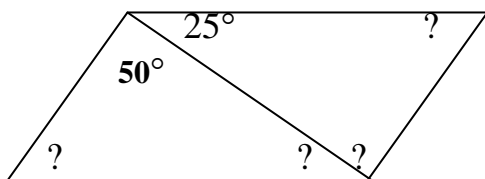
	Parallelogram	Rectangle	Square	Rhombus
	 <ul style="list-style-type: none"> • It is a quadrilateral in which each two opposite sides are //. 	 <ul style="list-style-type: none"> • It is a // gram in which one of its angle is right. 	 <ul style="list-style-type: none"> • It is a rectangle in which two adjacent sides are equal in length. 	 <ul style="list-style-type: none"> • It is a // gram in which 2 adjacent sides are equal in length.
Sides	Each two opposite sides are equal in length.	Each two opposite sides are equal in length.	All the sides are equal in length.	All the sides are equal in length.
Angles	<ul style="list-style-type: none"> • Each two opp. Angles are equal in measure. • Each two consecutive angles are supplementary. 	All the angles are equal in measure each = 90° .	All the angles are equal in measure each = 90° .	Each two opposite Angles are equal in measure.
Diagonals	<ul style="list-style-type: none"> • Bisect each other • Not equal in length. 	<ul style="list-style-type: none"> • Bisect each other • Equal in length. 	<ul style="list-style-type: none"> • Bisect each other • Equal in length • Perpendicular • Bisect vertex angle. 	<ul style="list-style-type: none"> • Bisect each other • Not equal • Perpendicular • Bisect vertex angle.

Exercise

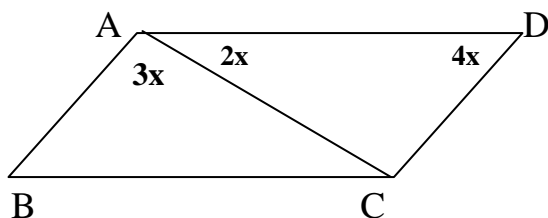
1) Complete the following :

- 1) In the parallelogram, each two opposite sides are
- 2) In the parallelogram, each tow opposite angles are.....
- 3) In the parallelogram, each two consecutive angles are.....
- 4) In the parallelogram, the diagonals

2) Find the unknown angles:



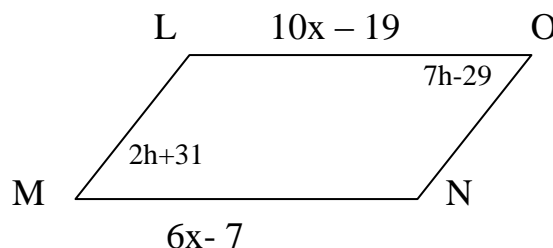
3)



- Find $m(\angle B)$, $m(\angle ACD)$

4) Find the length of \overline{MN} ,

$m(\angle O)$ and $m(\angle L)$.



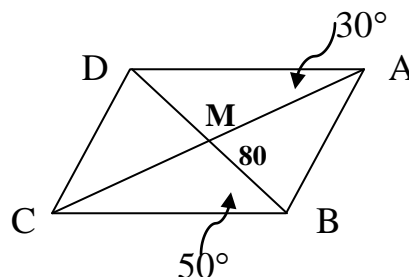
2) Complete the following:-

- 1) The parallelogram whose diagonals are perpendicular is called.....
 - 2) The parallelogram whose diagonals are equal in length is called.....
 - 3) The parallelogram whose diagonals are equal in length and perpendicular is called.....
 - 4) The parallelogram whose all sides are equal in length is called.....
 - 5) The rhombus in which one angle is right is.....
 - 6) The rhombus whose diagonals are equal in length is called.....
 - 7) The rectangle whose diagonals are perpendicular is called.....
 - 8) The rectangle in which two adjacent sides are equal in length is called.....
 - 9) If ABCD is a square, then $m(\angle CBA) = \dots\dots\dots^\circ$, $m(\angle CAB) = \dots\dots\dots$
 - 10) If ABCD is a rectangle $AC = 5\text{cm}$. then $BD = \dots\dots\dots$
 - 11) The perimeter of the square =,
 - The perimeter of the rectangle =,
 - The perimeter of the rhombus =
-

3) In the opposite figure:-

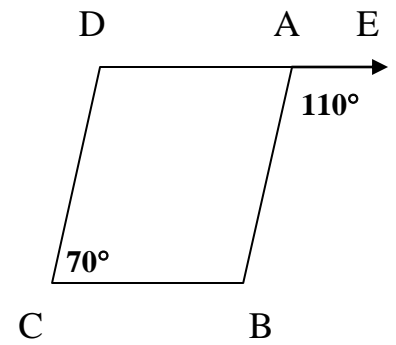
ABCD is a quadrilateral; its diagonals intersect at M,
 $\overline{AB} \parallel \overline{DC}$, $m(\angle AMB) = 80^\circ$ and $m(\angle MBC) = 50^\circ$
 $m(\angle MAD) = 30^\circ$.

Prove that: ABCD is a parallelogram.



4) In the opposite figure:

ABCD is a quadrilateral in which $\overline{AD} \parallel \overline{BC}$, $E \in \overrightarrow{DA}$,
 $m(\angle BAE) = 110^\circ$ and $m(\angle DCB) = 70^\circ$.
Prove that: The quadrilateral ABCD is a parallelogram.



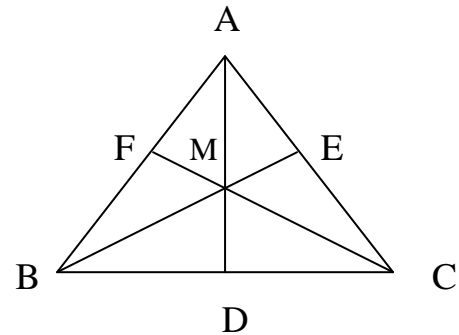
2) Medians Triangle

Remember that:-

1) **Definition:** The median of a triangle is a line Segment drawn from any vertex of the triangle to the mid – point of the opposite side.

2) Any Δ contains 3 medians.

3) The medians of the triangle intersect at one point (M).



4) The point of intersection of the medians in the triangle (M) divides each of them by the ratio 1 : 2 from the base (or 2 : 1 from the vertex).

$$\text{i) } \therefore MD : MA = 1 : 2 \Rightarrow MD = \frac{1}{2} MA \Rightarrow MD = \frac{1}{3} AD$$

$$\text{and : } MA = \frac{2}{3} AD, \quad MA = 2MD.$$

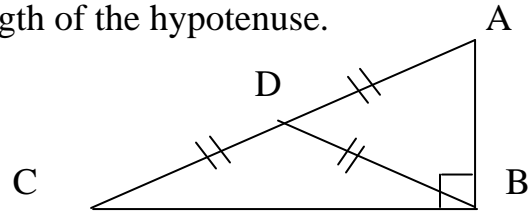
$$\text{ii) } ME : MB = 1 : 2 \Rightarrow ME = \frac{1}{2} MB \Rightarrow ME = \frac{1}{3} BE$$

$$, \quad MB = \frac{2}{3} BE, \quad MB = 2 ME$$

$$\text{iii) } MF : MC = 1 : 2 \Rightarrow MF = \frac{1}{2} MC \Rightarrow MF = \frac{1}{3} CF$$

$$, \quad MC = \frac{2}{3} CF, \quad MC = 2 MF$$

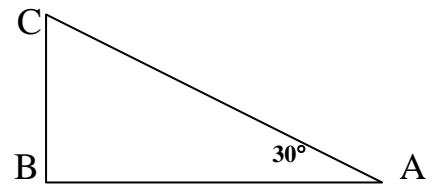
- 5) In the right angled Δ , the length of the median from the right angle equals $\frac{1}{2}$ the length of the hypotenuse.



- 6) In the right angled Δ , the length of the side opposite the angle of measure 30°

equals $\frac{1}{2}$ the length of the hypotenuse.

$$\therefore BC = \frac{1}{2} AC.$$



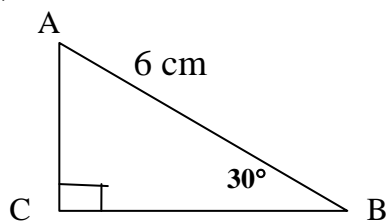
Exercise

1) Complete the following:

- 1- In $\triangle ABC$, if D is the midpoint of \overline{BC} , then \overline{AD} is called.....
- 2- The medians of the triangle intersect at.....
- 3- The point of concurrence of the medians of the triangle divides each median in the ratio..... From its base.
- 4- The point of concurrence of the medians of the triangle divides each median in the ratio From the vertex.
- 5- The number of medians in the right - angled triangle is.....
- 6- The length of the median from the vertex of the right angle in the right - angled triangle =
- 7- The length of the side opposite the angle whose measure is 30° in the right - angled triangle =
- 8- The length of the hypotenuse in thirty and sixty triangle equals the length of the side opposite the angle whose measure is 30° .
- 9- The line segment drawn between the two midpoints of two sides in a triangle is and its length equals.....

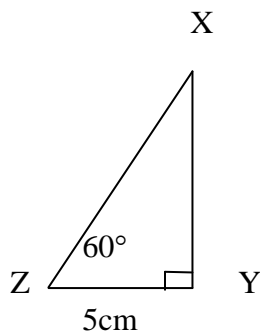
2) Using data given for each of the following figures , find the required below each figure:

1)



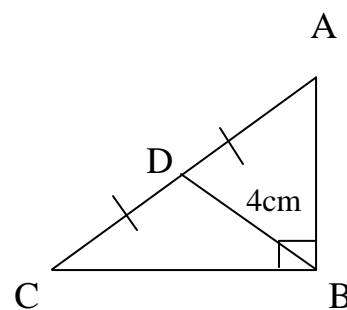
AC = cm.

2)



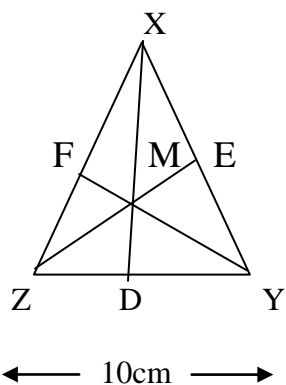
XZ =cm

3)



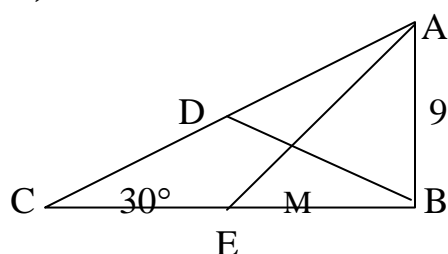
AC =cm

4)



E, D, F are the mid-points of \overline{XY} , \overline{YZ} , \overline{ZX} . If MD = 4cm. then:
XM =cm and
YD =cm

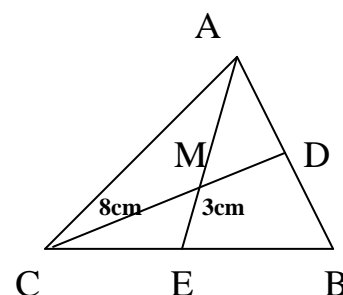
5)



If \overline{D} , \overline{E} are the mid-points of \overline{AC} , \overline{BC} . Then :

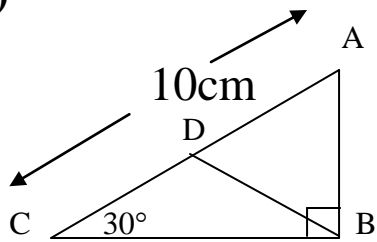
AC =cm.
BD =cm.
MD =cm.

6)



If \overline{D} , \overline{E} are the mid-points of \overline{AB} , \overline{BC} . Then:
MA = cm,
MD =cm,
ME =AE and
MC =CD.

7)



If D is the mid-point of \overline{AC} , then :

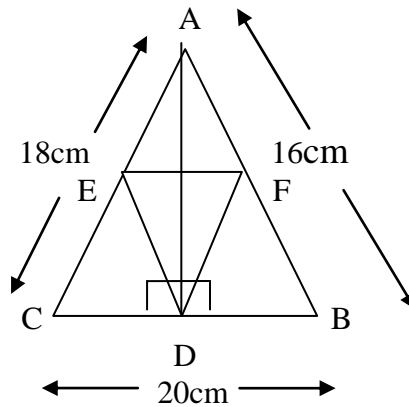
$BD = \dots\dots\dots\text{cm}$,

$AB = \dots\dots\dots\text{cm}$ and the perimeter of $\triangle ABD$

$\dots\dots\dots\text{cm}$,

$BC = \dots\dots\dots\text{cm}$

8)



If F , D , E are the mid-points of AB , BC , CA. Then:

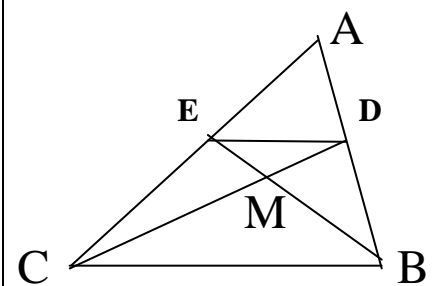
$DF = \dots\dots\dots\text{cm}$,

$DE = \dots\dots\dots\text{cm}$,

$FE = \dots\dots\dots\text{cm}$.

and the perimeter of $\triangle DEF = \dots\dots\dots\text{cm}$.

9)



If D , E are the mid-points of AB , AC , If $BC = 12\text{ cm}$., $BE = 9\text{ cm}$.

And $MC = 8\text{ cm}$.,

Then:

$DE = \dots\dots\dots\text{cm}$,

$ME = \dots\dots\dots\text{cm}$ and

$MD = \dots\dots\dots\text{cm}$.

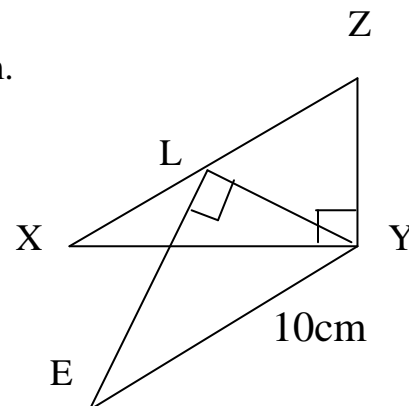
3) In the opposite figure:-

$m(\angle YLE) = 90^\circ$, $m(\angle E) = 30^\circ$, $YE = 10\text{ cm}$.

$m(\angle XYZ) = 90^\circ$ and

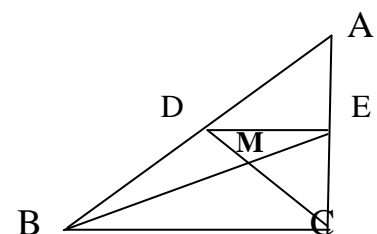
L is the midpoint of \overline{XZ}

Find by proof the length of : XZ .



4) In the opposite figure:

If D is the midpoint of \overline{AB} , E is the midpoint of \overline{AC} and $\overline{BE} \cap \overline{DC} = \{M\}$ If $DE = 4\text{cm}$, $DM = 3\text{ cm}$. and $BE = 6\text{ cm}$, find the perimeter of $\triangle BMC$



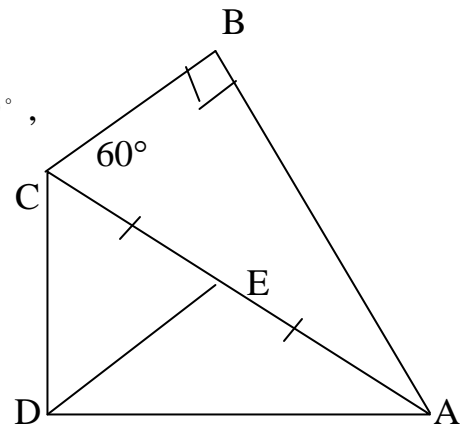
5) In the opposite figure:

ABC is a right – angled triangle at B, $m(\angle ACB) = 60^\circ$,

E is the midpoint of AC and

$DE = BC$

Prove that: $m(\angle ADC) = 90^\circ$.

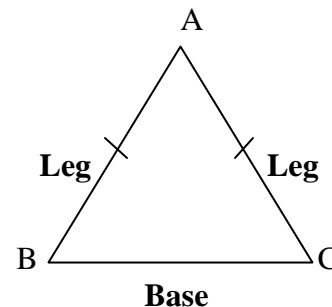


3) The isosceles Triangle

Remember that:-

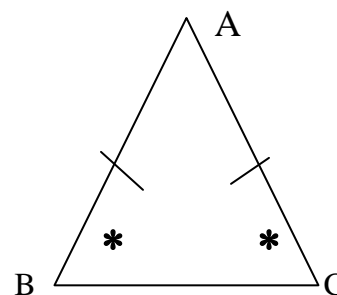
Definition: The isosceles triangle:

It is a triangle in which 2 sides equal in length.



Theorem:

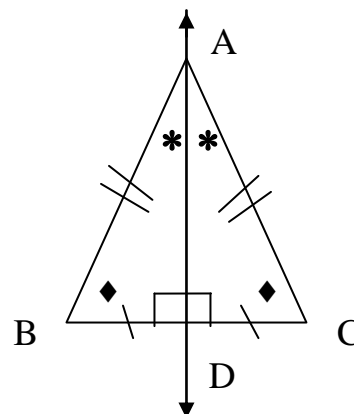
- The base angles of the isosceles Δ are equal in measure.
- If $AB = AC \Leftrightarrow m(\angle B) = m(\angle C)$.



Results: -

- The st. line passing through the vertex of an Isosceles Δ , perpendicular on the base:

- Bisect the base.
- Bisect the vertex angle.
- Axis of symmetry.

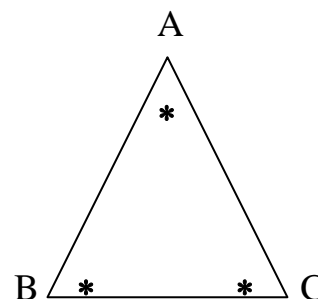


- The bisector of the vertex angle of an isosceles Δ :
 - Bisect the base
 - Perpendicular on the base.

- In the equilateral Δ : All the angles

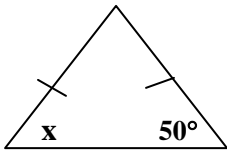
Are equal in measure, each one = 60°

- Remark: - If one of the angles in isosceles $\Delta = 60^\circ$
Then the Δ will be equilateral.

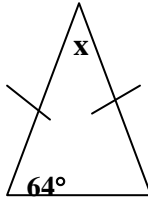


Exercise

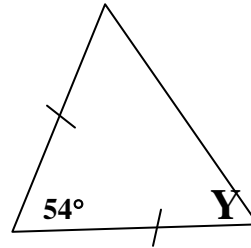
1) In each of the following find the value of the symbol used for the measure of the angle:



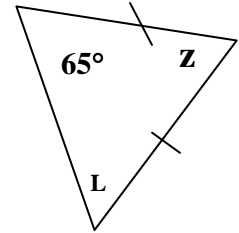
X =



X =



y =



L =, Z =

2) In the opposite figure:

$AB = AC = CD$ and $m(\angle ACD) = 130^\circ$

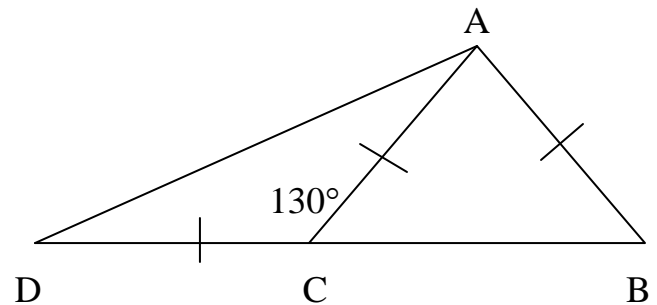
Complete the following:

1) $m(\angle ACD) = \dots\dots^\circ$.

2) $m(\angle ACB) = \dots\dots^\circ$.

3) $m(\angle BAC) = \dots\dots^\circ$.

4) $m(\angle BAD) = \dots\dots^\circ$.



3) Complete the following:

1) The two base angles in the isosceles triangle are

2) The measure of each angle in the equilateral triangle =

3) In $\triangle DEF$, if $DE = DF$, then $m(\angle E) = m(\angle \dots\dots)$.

4) In the isosceles triangle, if the measure of one of the two base angles is 65° then the measure of its vertex angle =

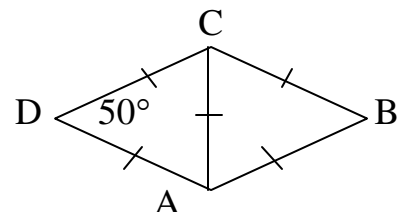
5) In the isosceles triangle, if the measure of the vertex angle = 40° , then the measure of one of the two base angles equals..... $^\circ$.

6) In $\triangle ABC$, if $AB = AC$ and $m(\angle A) = 80^\circ$, then $m(\angle B) = m(\angle \dots\dots) = \dots\dots^\circ$.

7) In the opposite figure:

If $AB = BC = AC$, $m(\angle D) = 50^\circ$ and $AD = DC$

, then $m(\angle B) = \dots\dots^\circ$, $(\angle BAD) = \dots\dots$.



4) Find the value of the symbol used in each figure:

<p>1)</p> <p>X = Y =</p>	<p>2)</p> <p>Y =°, L =° Z =°</p>	<p>3)</p> <p>X =° Y =°</p>	<p>4)</p> <p>X =° Y =°</p>
--	--	--	--

<p>5)</p> <p>X =° Y =°</p>	<p>6)</p> <p>X =° Y =°</p>	<p>7)</p> <p>m (∠ EAF) =°</p>
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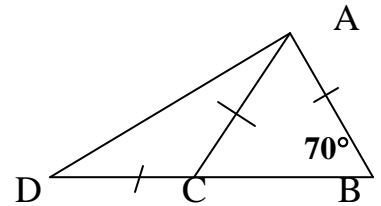
5) In each of the following figures, write the equal sides in length:

--	--	--

6) In the opposite figure:

$$AB = AC = CD \text{ and } m(\angle B) = 70^\circ.$$

Find by proof: $m(\angle BAD)$.

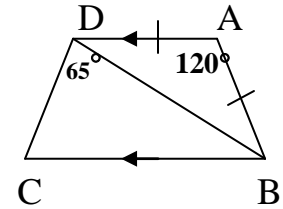


7) In the opposite figure:

$$AB = AD, \overline{AD} \parallel \overline{BC},$$

$$m(\angle BAD) = 120^\circ \text{ and } m(\angle BDC) = 65^\circ$$

Find: 1) $m(\angle ADB)$ 2) $m(\angle C)$.

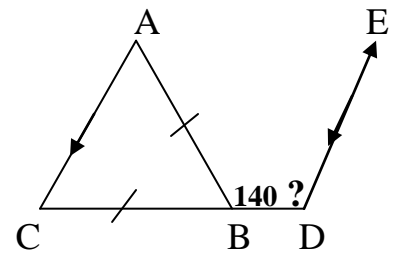


8) In the opposite figure:

$$AB = BC, m(\angle ABD) = 140^\circ$$

$$\text{And } \overline{AC} \parallel \overline{DE}$$

Find: $m(\angle EDC)$.

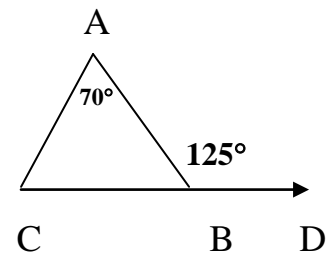


9) In the opposite figure:

$$D \in \overrightarrow{CB}, m(\angle ABD) = 125^\circ$$

$$\text{And } m(\angle A) = 70^\circ$$

Prove that : $\triangle ABC$ is an isosceles triangle .



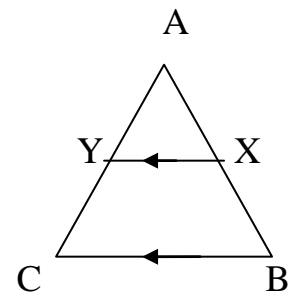
10) In the opposite figure:

$$ABC \text{ is a triangle in which } AB = AC, X \in \overline{AB}$$

$$Y \in \overline{AC} \text{ and } \overline{XY} \parallel \overline{BC}$$

Prove that : 1) $\triangle AXY$ is an isosceles triangle.

2) $XB = YC$.



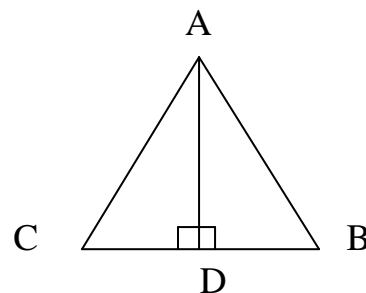
11) Complete the following:

- 1) The straight line drawn from the vertex of the isosceles triangle perpendicular to the base is called.....
- 2) The number of axes of symmetry in the equilateral triangle =.....
- 3) The number of axes of symmetry in the isosceles triangle =.....
- 4) The number of axes of symmetry in the scalene isosceles =.....
- 5) The median of the isosceles drawn from the vertex.....
- 6) The bisector of the vertex angle of the isosceles triangle =.....
- 7) The straight line drawn from the vertex of the isosceles triangle perpendicular to its base.....
- 8) The axis of the line segment is.....
- 9) Any point belonging to the axis of a line segment is..... from its two terminals
- 10) If C belongs to the axis of symmetry of \overline{AB} , then =

12) In the opposite figure:

If $AB = AC$, $AD \perp BC$, $BC = 4\text{cm}$ and
 $m(\angle DAC) = 35^\circ$, complete the following :

- 1) $m(\angle BAD) = \dots\dots\dots$
- 2) $m(\angle BAC) = \dots\dots\dots$
- 3) $m(\angle B) = \dots\dots\dots$
- 4) $BD = \dots\dots\dots\text{cm}$
- 5) The axis of symmetry of $\triangle ABC$ is.....



(4) Geometric constructions

Exercise

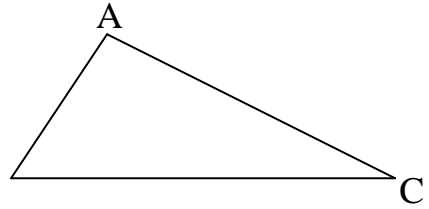
- 1) Draw \overline{AB} of length 7cm using the geometric instruments; draw the axis of symmetry of AB.
 - 2) Draw the equilateral $\triangle ABC$ with side length 6cm, then draw $\overline{BD} \perp \overline{AC}$ and find $M(\angle ABD)$. (Don't remove the arcs).
 - 3) Draw $\angle ABC$ where $m(\angle ABC) = 90^\circ$, then divide it to 4 equal angles. (Don't remove the arcs).
 - 4) Draw $\triangle ABC$ which is right- angled at B such that $\overline{BC} = 4\text{cm}$, $\overline{AB} = 3\text{cm}$, Then bisect \overline{AC} at D, and then find the length of \overline{BD} . (Don't remove the arcs).
-

Unit (4): Inequality

Remember that

- 1) In a triangle, if two sides have unequal lengths, the longer is opposite to the angle of the greater measure.

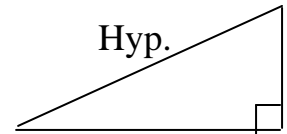
• In $\triangle ABC$, If $AC > AB$, then: $m(\angle B) > m(\angle C)$. B



- 2) In a triangle, if two angles are unequal in measure, then the greater angle in measure is opposite to a side greater in length than that opposite to the other angle.

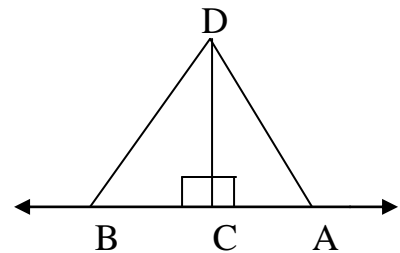
• In $\triangle ABC$, If $m(\angle B) > m(\angle C)$, then $AC > AB$.

- 3) In the right-angled triangle, the hypotenuse is the longest side.
-



- 4) The length of the perpendicular line segment drawn from a point outside a straight line to this line is shorter than any line segment drawn from this point to the given straight line.

• $DC < DA$, $DC < DB$.



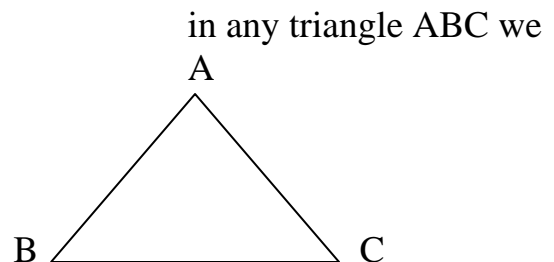
- 5) **Triangle inequality:** In any triangle, the sum of the length of any two sides is greater than the length of the third side.

•
get:

$$AB + BC > AC.$$

$$BC + AC > AB.$$

$$AC + AB > CB.$$



Exercise

- 1) Arrange the measures of the angles of ΔABC if: $AB = 11\text{cm}$, $BC = 15\text{cm}$, $AC = 10$ in descending order.
 - 2) Arrange the lengths of the sides of the ΔXYZ if: $m(\angle X) = 40^\circ$, $m(\angle Y) = 110^\circ$ and $m(\angle Z) = 30^\circ$ in ascending order.
-

3) Complete the following:

- 1) If the lengths of two sides of a triangle are unequal, then greater in length is opposite to.....
- 2) The perpendicular bisector of a line segment is called.....of it.
- 3) If in ΔABC : $AB = 4\text{cm}$. $BC = 5\text{cm}$. and $AC = 6\text{cm}$. then
 $m(\angle \dots) > m(\angle \dots) > m(\angle \dots)$.
- 4) In the isosceles triangle, if the measure of one of the two base angles 50° , then the measure of the vertex angle =.....
- 5) If two angles in a triangle are unequal in measure, then the greater angle in measure is opposite to..... and if the two lengths of two sides in a triangle are unequal then the greater side in length is opposite to the angle which is.....
- 6) The smallest angle of a triangle (in measure) is opposite to.....
- 7) The longest side in the right-angled triangle is.....
- 8) The shortest distance between a given point and a given straight line is.....
- 9) ABC is a triangle in which $m(\angle C) = 110^\circ$, then its longest side is.....
- 10) If in ΔABC , $m(\angle A) = 50^\circ$, $m(\angle B) = 30^\circ$, then the shortest side in the triangle is.....
- 11) If in ΔABC , $m(\angle A) = m(\angle B) + m(\angle C)$, then the longest side in the triangle is.....

4) Choose the correct answer from those ones:

- 1) If \overline{AD} is a median of $\triangle ABC$, M is the point of intersection of its medians and $AM = 12\text{cm}$. then $AD = \dots\dots\dots$
 - (a) 8cm.
 - (b) 4cm.
 - (c) 18cm.
 - (d) 9cm.
- 2) If in $\triangle ABC$: $AB > BC$, then: $m(\angle A) \dots\dots\dots m(\angle C)$
 - (a) $>$
 - (b) $<$
 - (c) $=$
 - (d) \equiv
- 3) If the length of any side in a triangle $= \frac{1}{3}$ the perimeter of this triangle, then the number of axes of symmetry of this triangle $= \dots\dots\dots$
 - (a) 1
 - (b) 2
 - (c) 3
 - (d) Zero
- 4) ABCD is a parallelogram in which $m(\angle A) + m(\angle C) = 140^\circ$, then $m(\angle B) = \dots\dots\dots$
 - (a) 70
 - (b) 40
 - (c) 110
 - (d) 220
- 5) If in $\triangle ABC$: $AB = 6\text{cm}$ and $AC = 7\text{cm}$., then $BC \in \dots\dots\dots$
 - (a) $] 6, 13]$
 - (b) $[6, 7]$
 - (c) $] 1, 13 [$
 - (d) $[1, 7 [$
- 6) An isosceles triangle in which the measure of the vertex angle $= 100^\circ$, then the measure of the base angle $= \dots\dots\dots$
 - (a) 80°
 - (b) 40°
 - (c) 50°
 - (d) 100°
- 7) If the length of the median drawn from a vertex of the triangle equals half the length of the opposite side to this vertex, then the angle at this vertex is $\dots\dots\dots$
 - (a) acute
 - (b) obtuse
 - (c) right
 - (d) reflex
- 8) The numbers which can be the lengths of the sides of a triangle are $\dots\dots\dots$
 - (a) 7, 7, 14
 - (b) 3, 4, 9
 - (c) 4, 5, 12
 - (d) 5, 5, 5
- 9) The sum of lengths of any two sides in a triangle $\dots\dots\dots$ the length of third side.
 - (a) less than
 - (b) greater than
 - (c) equal
 - (d) half

10) The lengths of any side in a triangle..... the sum of lengths of the two other sides.

- (a) $>$ (b) $<$ (c) $=$ (d) twice

11) Which of the following numbers cannot be side lengths of triangle.....

- (a) 7, 7, 5 (b) 9, 9, 9 (c) 3, 6, 12 (d) 3, 4, 5

12) If the lengths of two sides in a triangle 7cm. and 4 cm., then the length of the third side can be.....

- (a) 1cm (b) 2cm (c) 3cm (d) 4cm

13) If the lengths of two sides of an isosceles triangle 3cm.and 7 cm, then the length of the third side =.....

- (a) 7cm (b) 3cm (c) 4cm (d) 10cm

14) A triangle has one axis of symmetry; the lengths of two sides in it are 4 cm. and 8 cm, then its perimeter =.....

- (a) 16cm (b) 20cm (c) 24cm (d) 30 cm

15) In ΔABC : if $AB = 3\text{cm.}$, $BC = 5\text{cm.}$ and $AC = x\text{ cm.}$, then $x \in$

- (a) $] 3,5 [$ (b) $] 2,5 [$ (c) $] 5,8 [$ (d) $] 2,8 [$

16) If the lengths of two sides of a triangle are 5 cm. and 10 cm. then the length of the third side belongs to.....

- (a) $[10,15 [$ (b) $] 5,15 [$ (c) $] 5,10]$ (d) $[10,15]$

17) In ΔABC : $AB + BC - AC$

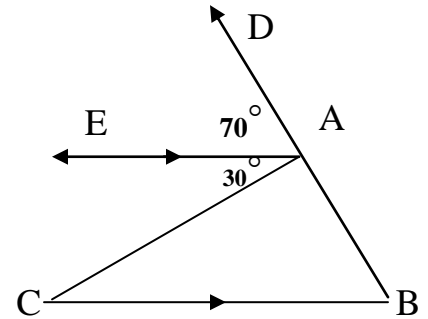
- (a) $>$ Zero (b) $<$ Zero
(c) $=$ Zero (d) $=$ the perimeter of the triangle ABC.

5) In the opposite figure:

$\overline{AE} \parallel \overline{BC}$, $m(\angle DAE) = 70^\circ$

And $m(\angle EAC) = 30^\circ$

Prove that: $AC > AB$



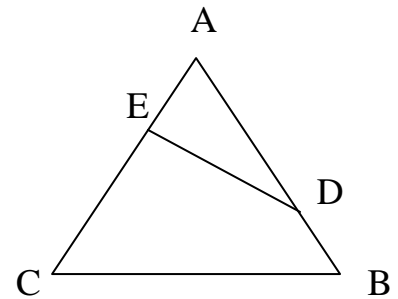
6) In the opposite figure:

ABC is an equilateral triangle

whose side length = 7cm., $D \in AB$ such that

$AD = 5\text{cm.}$ and $E \in AC$ such that $CE = 4\text{cm.}$

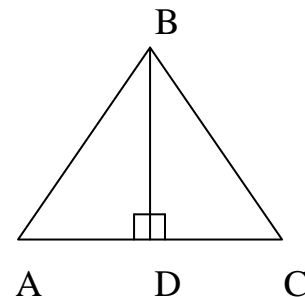
Prove that: $m(\angle AED) > 60^\circ$.



7) In the opposite figure:

$\overline{BD} \perp \overline{AC}$, proves that:

$AB + BC > 2BD$

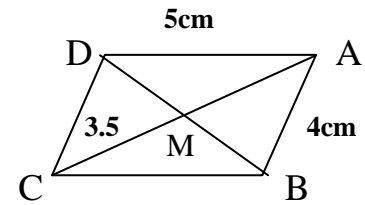


8) Complete the following:-

- 1) If the measures of two angles of a triangle are unequal, then the greater in measure is opposite to.....
- 2) In $\triangle ABC$: If $AB = AC$ and $m(\angle A) = 2m(\angle B)$, then $m(\angle C) = \dots\dots\dots$
- 3) ABC is a triangle in which $m(\angle A) = 50^\circ$, $m(\angle B) = 60^\circ$,
then the length of < the length of

4) In the opposite figure:

If ABCD is a parallelogram
 , then the perimeter of $\Delta ABC = \dots\dots\dots$



5) In the right-angled triangle, the longest side is the.....

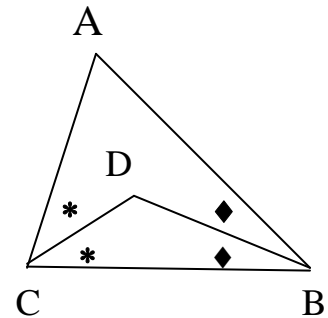
6) If in ΔABC : $m(\angle A) = 60^\circ$ and $m(\angle B) = 70^\circ$, then the shortest side is.....

7) The bisector of the vertex angle of the isosceles triangle is.....

9) In the opposite figure:

$\triangle ABC$ is a triangle in which $\overline{AB} > \overline{AC}$
 ,BD bisects $\angle ABC$ and CD bisects $\angle ACB$

Prove that: $BD > DC$.



10) Find the interval to which the length of the third side of each of the following triangles belongs if the two lengths of the other two sides are:

(1) 6cm, 5cm

(2) 7.5cm, 7.5cm

11) ABCD is a quadrilateral. Its diagonals intersect at M.

Prove that: $AC + BD > BC + AD$.

Revision

1) Choose the correct answer from the given ones:

1) If the lengths of two sides of an isosceles triangle 11 cm. and 5 cm. then the length of the third side is.....

- (a) 11 cm (b) 5cm (c) 4cm (d) 6cm

2) The number of axes of symmetry of the triangle in which the measure of two angles are 60° and 70° equals.....

- (a) 1 (b) 2 (c) Zero (d) 3

3) The length of the hypotenuse of the right-angled triangle =..... The length of the median drawn from the vertex of the right angle.

- (a) half (b) twice (c) third (d) quarter

4) ABC is a triangle in which $m(\angle A) = 54^\circ$, $m(\angle B) = 72^\circ$, then.....

- (a) $AB < BC$ (b) $\underline{AB} = \underline{BC}$
 (c) $AB > BC$ (d) $AB \perp BC$

5) The point of intersection of the medians of the triangle bisects each of them with ratio..... from the vertex.

- (a) 1 : 3 (b) 3 : 1 (c) 1 : 2 (d) 2 : 1

6) In ΔABC : if $m(\angle B) = 90^\circ$ and $m(\angle A) = 60^\circ$, then.....

- (a) $BC = \frac{1}{2} AC$ (b) $AC = \frac{1}{2} AB$ (c) $AC = 2 AB$ (d) $AC = 2 BC$

7) ABC is a triangle in which $(\angle A) = 30^\circ$, $m(\angle B) = 90^\circ$ then $AC =$

- (a) 2BC (b) $\frac{1}{2} BC$ (c) 2AB (d) $\frac{1}{2} AB$

8) If ΔABC has one axis of symmetry and $m(\angle B) = 120^\circ$, then $(\angle A) = \dots\dots\dots$

- (a) 60° (b) 120° (c) 30° (d) 40°

9) If in ΔABC : $m(\angle A) = 50^\circ$ and $m(\angle B) = 60^\circ$, then $AB \dots\dots\dots AC$

- (a) $>$ (b) $<$ (c) \leq (d) $=$

10) The point of intersection of the medians of the triangle divides each of them with ratio..... from the vertex.

- (a) $1 : 3$ (b) $3 : 1$ (c) $1 : 2$ (d) $2 : 1$

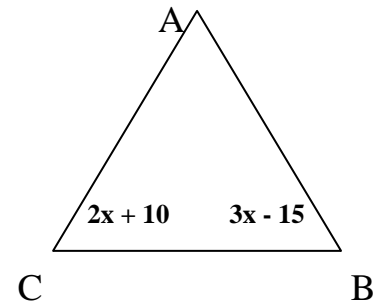
11) If the lengths of two sides of a triangle are 3 cm. and 7cm. then the smallest integer which represents the length of the third side =.....cm.

- (a) 3 (b) 4 (c) 5 (d) 6

12) In the opposite figure:

ABC is a triangle in which $AB = AC$, then $m(\angle A) = \dots\dots\dots$

- (a) 40° (b) 50° (c) 60° (d) 70°



13) An isosceles triangle, one of its base angles has measure 70° , then the measure of the vertex angle =.....

- (a) 70° (b) 110° (c) 20° (d) 40°

14) If in ΔXYZ : $XY > XZ$ then $m(\angle Y) \dots\dots\dots m(\angle Z)$

- (a) $>$ (b) $<$ (c) \geq (d) $=$

15) ΔABC is right-angled at B, then $AB \dots\dots\dots AC$

- (a) \equiv (b) \perp (c) $<$ (d) $>$

16) If A lies on the axis of symmetry of \overline{XY} , then $AX \dots\dots AY$.

- (a) \perp (b) \equiv (c) $//$ (d) $=$

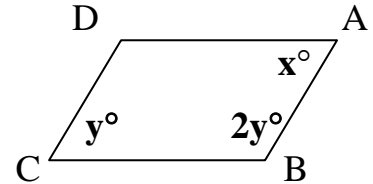
17) An isosceles triangle, the lengths of two sides of it are 4 cm. and 9 cm, then the length of the third side =cm.

- (a) 4 (b) 5 (c) 9 (d) 13

18) In the opposite figure:

ABCD is a parallelogram, then $X = \dots\dots$

- (a) 70° (b) 120° (c) 60° (d) 50°



19) If A \in the axis of symmetry of \overline{BC} , then.....

- (a) $AB > BC$ (b) $AB = BC$ (c) $AB = AC$ (d) $\overline{AB} \perp \overline{BC}$

20) XYZ is a triangle in which $m(\angle Y) > m(\angle Z)$ then $XZ \dots\dots XY$

- (a) $>$ (b) $=$ (c) $<$ (d) \leq

21) If \overline{AD} is a median of $\triangle ABC$, then.....

- (a) $AB = BC$ (b) $BD = DC$ (c) $\overline{AD} \perp \overline{BC}$ (d) $AB = AC$

22) In $\triangle ABC$: $AB = 2\text{cm}$., $BC = 7\text{cm}$., then AC may equal.....

- (a) 2cm (b) 5cm (c) 9cm (d) 8cm

23) The number of axes of symmetry of the scalene triangle =.....

- (a) 1 (b) 2 (c) zero (d) 3

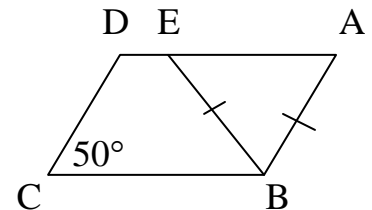
24) In the opposite figure:

ABCD is a parallelogram, $BA = BE$

$m(\angle C) = 50^\circ$,

Then $m(\angle ABE) = \dots\dots$

- (a) 50° (b) 60° (c) 70° (d) 80°



25) If the measure of one angle of the two base angles of the isosceles triangle = 75° then the measure of the vertex angle =

- (a) 50° (b) 75° (c) 30° (d) 105°

26) \overline{AD} is a median of $\triangle ABC$ where M is the point of intersection of its medians, then $AM = \dots\dots\dots AD$.

- (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) 2

27) The number of axes of symmetry of the equilateral triangle =

- (a) 1 (b) 2 (c) 3 (d) 4

28) The numbers 6, 3, can be lengths of sides of an isosceles triangle

- (a) 3 (b) 6 (c) 9 (d) 11

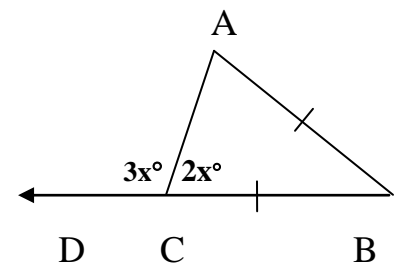
29) In the triangle, the sum of lengths of any two sides the length of the third side.

- (a) $<$ (b) $>$ (c) $=$ (d) \leq

30) In the opposite figure:

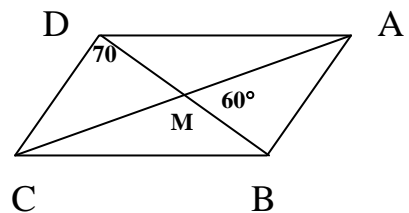
$D \in \overrightarrow{BC}$ and $BA = BC$,
then $m(\angle B) = \dots\dots\dots$

- (a) 36° (b) 30° (c) 108° (d) 72°



(2) Complete the following:

- 1) In $\triangle ABC$ if $m(\angle B) - m(\angle A) > m(\angle C)$, then $AC \dots\dots\dots AB$.
- 2) If the lengths of two sides of a triangle are not equal, then the greater in lengths opposite.....
- 3) The bisector of the vertex angle of the isosceles triangle.....
- 4) If the length of any side of a triangle $= \frac{1}{3}$ the perimeter of the triangle, then the number of axes of symmetry of the triangle is.....
- 5) If the lengths of two sides of a triangle are 5 cm. and 7 cm then length of the third side $\in] \dots\dots\dots, \dots\dots\dots[$.
- 6) In the parallelogram, the two diagonals are
- 7) In $\triangle ABC$, $AB + BC - AC > \dots\dots\dots$
- 8) The bisector of the vertex angle of the isosceles triangle.....
- 9) If in $\triangle ABC$: $AB = AC$ and $m(\angle A) = 2m(\angle B)$, then $m(\angle C) = \dots\dots\dots$
- 10) The isosceles triangle in which the measure of one of its angles is 60° , has.....axes of symmetry.
- 11) If the lengths of two sides of a triangle are not equal, then the longer side is opposite an angle..... than the measure of the angle opposite the other.
- 12) In the opposite figure:
 $ABCD$ is a parallelogram,
 Than $m(\angle BAC) = \dots\dots\dots$



13) The longest side in the right-angled triangle is.....

14) In ΔXYZ : $m(\angle Y) = 110^\circ$, then the longest is.....

15) The point of intersection of the medians of the triangle divides each of them with the ratio 1 : 2 from.....

16) In ΔABC : $m(\angle A) = 55^\circ$, $m(\angle B) = 70^\circ$ then the number of axes of symmetry of the triangle is.....

17) The median which is drawn from the vertex of an isosceles triangle bisects.....and it is..... to the base.

18) ΔABC is right-angled at B, $AB = 3$ cm. $BC = 4$ cm. If BD is a median of ΔABC , then $BD = \dots\dots\dots$ cm.

19) The length of the side opposite the angle whose measure 30° in the right-angled triangle equals.....

20) ΔABC is right-angled at B then $AC \dots\dots BC$.

21) If $\Delta ABC \cong \Delta XYZ$, then $\overline{AC} \equiv \dots\dots\dots$

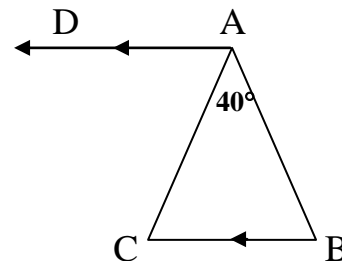
22) If the measure of one of the angles of an isosceles triangle is 60° , then the triangle is

23) The bisector of the vertex angle of the isosceles triangle bisects the base and it is.....

24) In the opposite figure:

$AB = AC$, $\overrightarrow{AD} \parallel \overrightarrow{BC}$

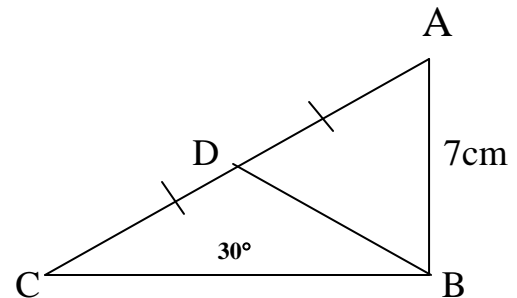
, $m(\angle BAC) = 40^\circ$ then $m(\angle BAD) = \dots\dots\dots$



- 25) The measure of the exterior angle of the equilateral triangle = °
- 26) If the measures of two angles of a triangle are not equal, then the greater in measure is opposite.....
- 27) The length of the side opposite the angle of measure 30° in the right-angled triangle equals.....
- 28) If $\triangle ABC$ is an obtuse-angled triangle at, then AB BC
- 29) The perpendicular bisector of a line segment is called.....
- 30) In the parallelogram, each two opposite sides are.....

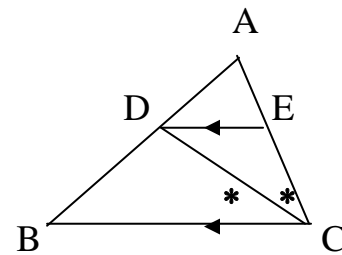
3) In the opposite figure:

$\triangle ABC$ is right-angled at B,
D bisects AC, $m(\angle C) = 30^\circ$ and
 $AB = 7\text{cm}$. Find the length
Of each of AC, BD and BC.



4) In the opposite figure:

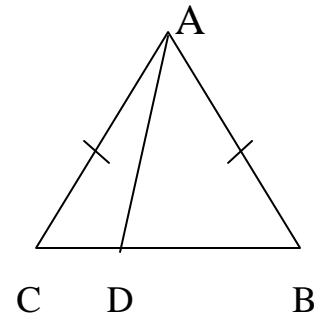
\overrightarrow{CD} bisects $\angle ACB$, $\overline{DE} \parallel \overline{CB}$
Prove that: $\triangle ECD$ is an isosceles triangle.



5) In the opposite figure

ABC is a triangle in which $AB = AC$

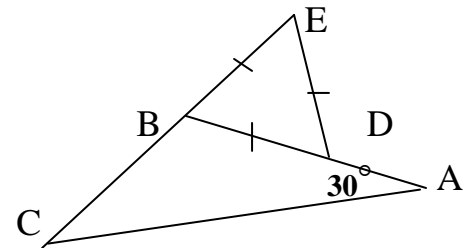
$D \in \overline{BC}$ Prove that: $AB > AD$

**6) In the opposite figure**

$E \in \overrightarrow{CB}$, $D \in \overline{AB}$

$ED = DB = EB$ and $m(\angle A) = 30^\circ$

Prove that: ABC is an isosceles triangle.

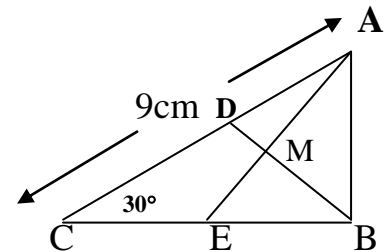
**7) In the opposite figure:**

ΔABC is right-angled at B

$m(\angle C) = 30^\circ$, D bisects \overline{AC}

, E bisects \overline{BC} and $AC = 9\text{cm}$

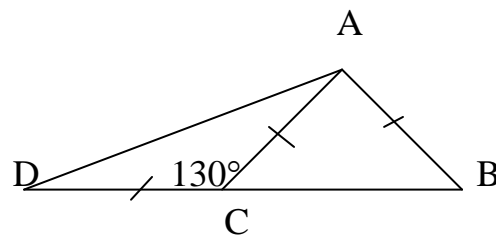
Find by proof of the length of each of BD, BM and AB.

**8) In the opposite figure:**

$CD = CA = AB$, $C \in \overline{BD}$

And $m(\angle ACD) = 130^\circ$

Find by proof $m(\angle BAD)$.

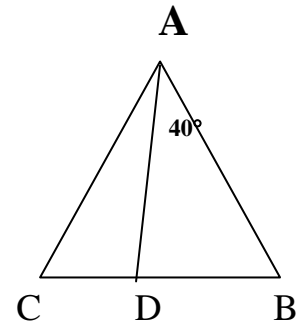


9) Using the ruler and the compasses, draw $\triangle ABC$ which is equilateral and side length = 6cm. , then draw $\overrightarrow{BD} \perp \overline{BC}$ and find $m(\angle DBA)$. (Don't remove the arcs)

10) In the opposite figure:

$\triangle ABC$ is an equilateral triangle

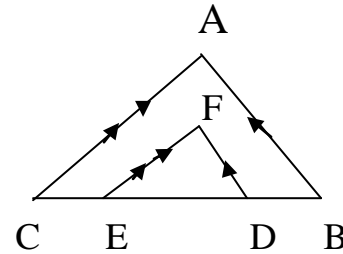
$D \in \overline{BC}$, $m(\angle DAB) = 40^\circ$, prove that $AB > AD$.



11) In the opposite figure:

$\overline{AB} \parallel \overline{DF}$, $\overline{AC} \parallel \overline{EF}$,

$AC > AB$, prove that: $FE > DF$.



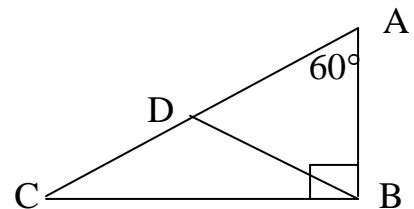
12) In the opposite figure:

$\triangle ABC$ is right-angled at B

$m(\angle A) = 60^\circ$, $AB = 5$ cm. and

D bisects \overline{AC} . Find by proof

The length of each of \overline{AC} and \overline{BD} .



13) Using the ruler and the compasses, only draw $\triangle ABC$ which is right-angled at B
 $AB = 3$ cm. $BC = 4$ cm. bisect \overline{AC} at D Then find the length of \overline{BD} .

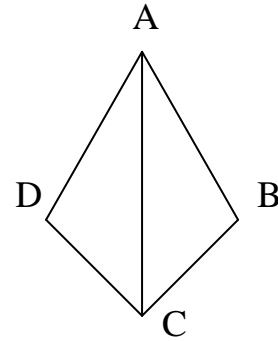
(Don't remove the arcs)

14) In the opposite figure:

$AB > BC$ and

$AD > DC$. Prove that:

$$m(\angle BCD) > m(\angle BAD).$$

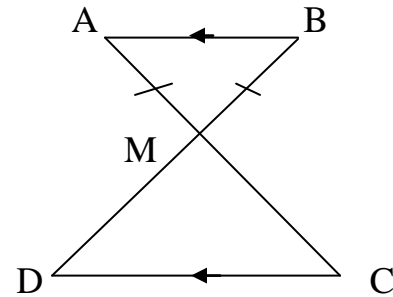
**15) In the opposite figure:**

$\overline{AB} \parallel \overline{CD}$,

$\overline{AD} \cap \overline{BC} = \{M\}$

and $MA = MB$.

Prove that: $\triangle MCD$ is an isosceles triangle.

**16) In the opposite figure:**

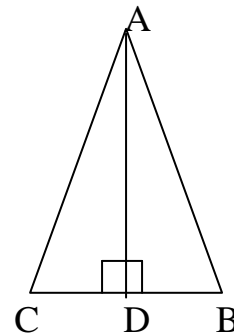
In $\triangle ABC$: $m(\angle B) = m(\angle C)$

$\overline{AD} \perp \overline{BC}$,

$AB = 13$ cm .and $BD = 5$ cm.

Find: 1- The length of \overline{BC}

2- The area of $\triangle ABC$

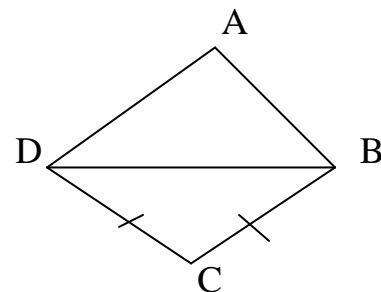
**17) In the opposite figure:**

ABCD is a quadrilateral

In which $AD > AB$ and

$BC = CD$

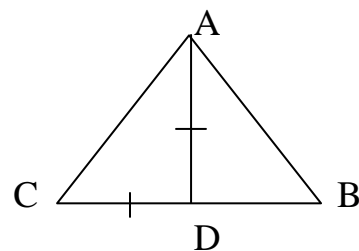
Prove that: $m(\angle ABC) > m(\angle ADC)$.



18) In the opposite figure:

$AD = DC$

Prove that: $BC > AB$



Good luck