UNIT [1]: Real Numbers {R}

Revision:

1) **Set of Numbers:**
   a. The Set of Counting Numbers **C**= {1, 2, 3, …} = $\mathbf{N^+}$
   b. The Set of Natural Numbers **N**= {0, 1, 2, 3, …} = $\mathbf{Z^+} \cup \mathbf{\{0\}}$
   c. The Set of Integers **Z**= {…, -3, -2, -1, 0, 1, 2, 3, …}
   d. The Set of Positive Integers $\mathbf{Z^+}$= {1, 2, 3, …} = Counting No.
   e. The Set of Negative Integers $\mathbf{Z^-}$= {-1, -2, -3, …}.
   f. The Set of Rational Numbers **Q**= {a/b: a, b ∈ b≠ 0}
   g. N⊂Z⊂Q

The Standard form of a rational number is :

$a \times 10^n$ where $n \in z$, $1 \le |a| < 10$

2) **Perfect Square rational Number:** it is that positive number which can be written in the form of a square rational no. i.e. (rational no.)2 . Ex. (1, 4, 25, 9/16, 2 ¼…).

3) **Perfect Cube rational Number:** it is that number which can be written in the form of a cube rational no. i.e. (rational no.)3 . Ex. (1, 8, -27, -216, 8/125, …).

Lesson (1): the Cube root of a rational number

The cube root of the rational number a is that number whose cube is equal to a

- The cube root for the rational number a is symbolized by $\sqrt[3]{a}$
- The cube root for a positive rational number is also positive Ex: $\sqrt[3]{125} = 5$
- - The cube root for a negative rational number is also negative. Ex: $\sqrt[3]{-8} = -2$ why?
- $\sqrt[3]{zero} = zero$
- $\sqrt[3]{a^3} = a$

Remark The perfect cube rational number has one cube root which is also a rational number , why?

Lesson (2): the Set of Irrational Number Q`

The irrational number: It is that number which can not be put in the form $\frac{a}{b}$ where $a, b \in \mathbb{Z}, b \neq 0$

the following are examples to irrational numbers.

First : the square roots of the positive numbers which are not perfect squares

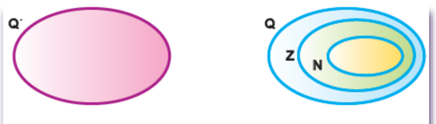
Ex : $\sqrt{2}$, $\sqrt{5}$, $-\sqrt{6}$, $\sqrt{7}$

Second: the cube roots of those numbers that are not perfect cubes

Ex : $\sqrt[3]{4}, \sqrt[3]{-2}, \sqrt[3]{11}, \ldots$

Third: the pi π (the approximation ratio)

Where it is impossible to find any exact value for any of the previous number. why?



$Q \cap Q` = \varnothing$

Lesson (3): Finding the Approximate value of an Irrational Number

Remark $\sqrt{2}$ is between $\sqrt{1}, \sqrt{4}$ i.e $1 < \sqrt{2} < 2$

i.e. $\sqrt{2} = 1 +$ a decimal fraction

Representing the irrational number on the number line.

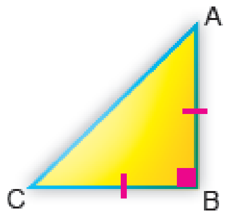
How can the point represents $\sqrt{2}$ be located on the number line?

If we draw the right triangle ABC at B which is an isosceles triangle also.

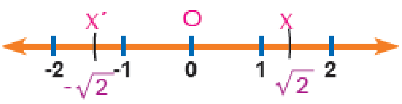
where AB = BC = one unit of length

Then $(AC)^2 = (AB)^2 + (BC)^2 = 1^2 + 1^2 = 2$

$\therefore AC = \sqrt{2}$ unit of length.



- draw the number line and place the sharp point of the compasses at point O, then adjust the compasses to a length that is equal to $\overline{AC}$ and draw an arc that intersects the number line on the right of o and at the point X, where that point represents $\sqrt{2}$
- Using the same length, we can label the point X` which represent $-\sqrt{2}$ where X` is on the left of the point o.



Solution

A $x^2 = 2$

$\therefore x = \pm\sqrt{2}$ Solution set = $\{-\sqrt{2}, \sqrt{2}\}$

B $x^3 = 5$

$\therefore x = \sqrt[3]{5}$ Solution set = $\{\sqrt[3]{5}\}$

C $\frac{4}{3}x^2 = 1$

$\therefore \frac{3}{4} \times \frac{4}{3}x^2 = \frac{3}{4} \times 1$

$x^2 = \frac{3}{4}$

$\therefore x = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{\sqrt{4}} = \pm\frac{\sqrt{3}}{2}$ Solution set $= \{-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\}$

Lesson (4): the Set of the Real number R

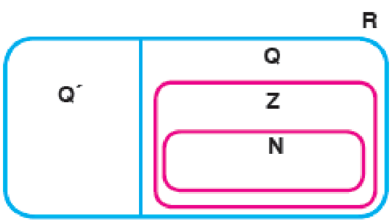
You have learned the set of rational numbers (Q), you have also found that there are other numbers that form the set of irrational number Q\ such as $\sqrt{2}$, $\sqrt[3]{2}$, π,... However, the union of these two sets forms a new set called the set of the real numbers, and it is denoted by the symbol R

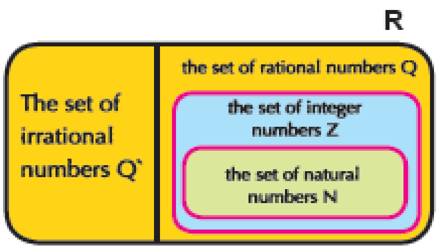
$R = Q \cup Q'$

2 Any natural, integer, rational or irrational number is a real number

3 Every real number is represented by one point on the number line.

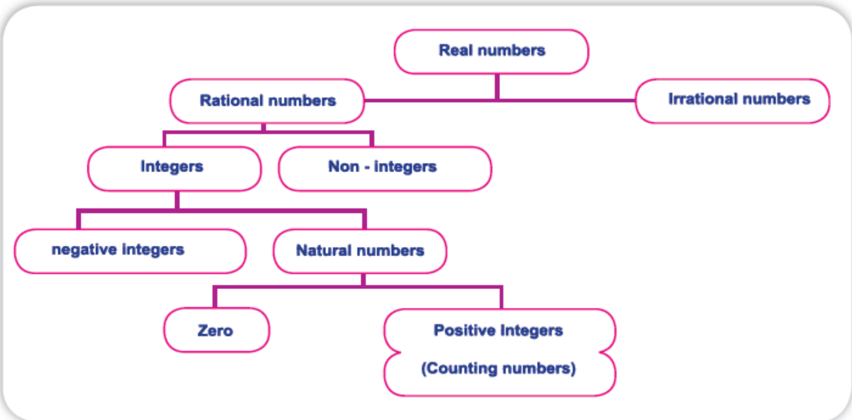
$N \subset Z \subset Q \subset R$ and so is $Q' \subset R$





Remark $\sqrt[3]{-1} = -1$ because $-1 \times -1 \times -1 = -1$

While $\sqrt{-1} \notin R$ because there is no real number If multiplyed by it self, the product is -1 .



Lesson (5): Ordering numbers at R

the set of real number is an ordered set.:

The properties of order:

1. If x , y are two real numbers represented on the number line by the two points A, B respectively, the ordering relation can be one of the following three cases:

x = y A ≡ B	y x B A	x y A B
A is congruent to B so x = y	**A follows B** so x > y	**A precedes B** so x < y

2. If x is a real number represented by the point A on the number line while O is the origin point which represents the zero, then the ordering relation can be one of the following three cases.

x = 0 O ≡ A	0 x O A	x 0 A O
A is congruent to O ∴ x = 0	**A is on the right of O** ∴ x > 0 then x is a positive real number.	**A is on the left of O** ∴ x < 0 then x is a negative real number.

Negative real number O Positive real number

The set of the positive real numbers: $R^+ = \{x : x \in R , x > 0\}$
The set of the nagative real numbers: $R^- = \{x : x \in R , x < 0\}$

$$R = R^+ \cup \{0\} \cup R^-$$

Remark : The set of non-negative real numbers = $R^+ \cup \{0\} = \{x : x \geqslant 0, x \in R\}$
The set of the non - positive real numbers = $R^- \cup \{0\} = \{x : x \leqslant 0, x \in R\}$

Example:

Arrange the following numbers ascendingly $\sqrt{27}$, - $\sqrt{45}$, $\sqrt{20}$, 6, 0, $\sqrt[3]{-1}$

Solution

$6 = \sqrt{36}$, $\sqrt[3]{-1} = -1 = -\sqrt{1}$

The ascending order is from the smallest to the greatest.

$-\sqrt{45}$, $-\sqrt{1}$, 0 , $\sqrt{20}$, $\sqrt{27}$, $\sqrt{36}$

i.e. $-\sqrt{45}$, $\sqrt[3]{-1}$, 0 , $\sqrt{20}$, $\sqrt{27}$, 6.

Lesson (6): Intervals

Interval is a subset of the set of real numbers

first: the limited intervals

If a, b ∈ R, a < b, then we can define each of:

The closed inteval [a , b]

[a, b] = { x : a ⩽ x ⩽ b, x ∈ R}

[a, b] ⊂ R in which the elements are a, b and all the real numbers between them.

When we draw that interval, we put a shaded circle at each of the two points a and b then, we shade that area between them on the number line.

The open interval]a , b[

]a, b [= { x: a < x < b, x ∈ R}

]a , b[⊂ R in which the elements are all the real numbers between the two numbers a, b

When we draw that interval, we put an unshaded circle at each of the two points which represent the two numbers a and b then, we shade that area between them on the number line.

Half openor (half closed) intervals

[a , b[	]a , b]
	
[a , b[= {x : a ⩽ x < b , x ∈ R}	]a , b] = { x: a < x ⩽ b , x ∈ R}
[a, b[⊂ R where its elements are the number a and all the numbers between a and b.	]a , b] ⊂ R where its elements are the number b and all the number between a and b .

Examples :

Represent each of the following intervals on the number line: [-1, 4] ,]-1, 4[,]-1, 4], {-1, 4}

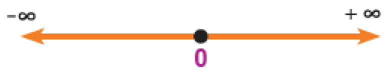
Solution

[-1, 4]	]-1, 4[	]-1, 4]	{-1, 4}
			
Closed interval	open interval	Half-open interval	Set

Second: The unlimited intervals

You know that: If the number line of real numbers is expanded on its two direction, we get more positive real numbers at the right direction and more negative real number at the left direction such all those numbers are located on that line.

- The symbol (∞) is read (infinity) and it is more than any imagined real number, $\infty \notin R$
- The symbol ($-\infty$) is read (negative infinity) and it is less then any imagined real number, $-\infty \notin R$
- The two symbols ∞ , $-\infty$ can not be represented by any points on the number line and they are expansions to the number line at its two directions.



If a is a real number, then we can define the following unlimited intervals:

The interval $[a , \infty[$

$[a, \infty[= \{ x : x \geqslant a , x \in R\}$

That interval represents the number a and all the real numbers which are more than a

The interval $]-\infty , a]$

$]-\infty , a] = \{ x : x \leqslant a , x \in R\}$

That interval represents the number a and all the real number which are less than a.

the interval $]a, \infty[$

$]a,\infty[= \{x : x > a , x \in R\}$

That interval represents all the real number which are more them a

the interval $]-\infty , a[$

$]-\infty ,a[= \{ x : x < a , x \in R\}$

that interval represents all the real numbers which are less than a

Remark : The set of real numbers (R) can be represented in the form of the interval $]-\infty , \infty[$

The set of the positive real numbers $R^+ =]0, \infty[$

The set of the negative real numbers $R^- =]-\infty, 0[$

The set of non-negative real numbers = $[0, \infty[$

The set of non-positive real numbers = $]-\infty, 0]$

Operations on intervals

Since all the intervals are subsets of the set of the real number R, The operations of union, intersection, difference and complement can be applied on the intervals. The graphical representation to the intervals on the number line contributes to determine and verify the result of any operation. This can be clarified from the following examples:

Examples

1 **If X = [-2, 3] , Y = [1, 5 [, find the following using the number line:**

A X ∩ Y B X ∪ Y

Solution

A X ∩ Y = [-2, 3] ∩ [1, 5[= [1, 3]

B X ∪ Y = [-2, 3] ∪ [1, 5[= [-2, 5[

2 **If M = [2, ∞[, J =] -2, 3[, find the following using the number line:**

A M - J B M ∩ J C M ∪ J

D J ∪ {2, 3} E M` F J´

Solution

A M - J = [2 , ∞ [-] -2 , 3[= [3 , ∞[

B M ∩ J = [2 , ∞ [∩] -2 , 3 [= [2 , 3[

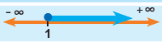
C M ∪ J = [2 , ∞ [∪] -2 , 3 [=]-2 , ∞[

D J ∪ { 2, 3} =]-2, 3 [∪ {2, 3 } =]-2 , 3]

E M` =]-∞, 2[

F J` =]-∞, -2] ∪ [3, ∞[

1 **Complete** **the following table as shown in the first example:**

Interval	Representation by using the descri•tion method	Graphical representation on the number line
[-1 , 2]	{x : -1 ⩽ x ⩽ 2, x ∈ R}	
[1 , 3[		
]-∞ , 2]		
	{x : 0 < x ⩽ 3, x ∈ R}	
	{x : x > -1, x ∈ R}	
		
		
]1, 5[		
	{x : x > 0, x ∈ R}	

Lesson (7): Operations on Real Numbers

First: The properties of adding the real numbers:

the closure property If a ∈ R, b ∈ R then (a + b) ∈ R

Par example : each of 2 + 3 , $1 + \sqrt{2}$, $-2 + \sqrt{5}$ and $2 + \sqrt[3]{3}$ are real numbers.

The commutative property If a ∈ R, b ∈ R then a + b = b + a

Par example : $2 + \sqrt{3} = \sqrt{3} + 2$, $3 - \sqrt{5} = -\sqrt{5} + 3$

The associative property If a ∈ R, b ∈ R, c ∈ R, then (a + b) + c = a + (b + c) = a + b + c

Par example : $(3 + \sqrt{2}) + 5 = 3 + (\sqrt{2} + 5)$ associative property

$= 3 + (5 + \sqrt{2})$ commutative property

$= 3 + 5 + \sqrt{2}$ associative property

$= 8 + \sqrt{2}$

Zero is the additive neutral element: If a ∈ R then a + 0 = 0 + a = a

Par example : $\sqrt{5} + 0 = 0 + \sqrt{5} = \sqrt{5}$, $-\sqrt[3]{4} + 0 = 0 + (-\sqrt[3]{4}) = -\sqrt[3]{4}$

Each real number has an additive inverse For a number a ∈ R there is (-a) ∈ R where a + (-a) = (-a) + a = zero

Par example $\sqrt{3}$ ∈ R , has additive inverse $(-\sqrt{3})$ ∈ R where

$\sqrt{3} + (-\sqrt{3}) = (-\sqrt{3}) + \sqrt{3} =$ zero.

Second: The properties of multiplying the real numbers

The closure property

If $a \in R$, $b \in R$ then $a \times b \in R$

the set of real number is closed under the operation of multiplication.

i.e the product of multiplying every two real number is a real number.

Par example : $5 \times \sqrt{2} = 5\sqrt{2} \in R$, $\sqrt{3} \times \sqrt{3} = 3 \in R$

$-2 \times \sqrt[3]{5} = -2\sqrt[3]{5} \in R$, $\frac{2}{3} \times \pi = \frac{2}{3}\pi \in R$

$2\sqrt{3} \times \sqrt{3} = 6 \in R$, $2\sqrt{3} \times 5 = 10\sqrt{3} \in R$

Commutative property

Si $a \in R$ et $b \in R$, alors $a \ast b = b \ast a$

Par example : $\sqrt{2} \times 3 = 3 \times \sqrt{2} = 3\sqrt{2}$

The associative property

Si $a \in R$, $b \in R$ et $c \in R$, alors

$(a \ast b) \ast c = a \ast (b \ast c) = a \ast b \ast c$

Par example : $\sqrt{2} \times (5 \times \sqrt{2}) = (\sqrt{2} \times 5) \times \sqrt{2} = (5 \times \sqrt{2}) \times \sqrt{2}$

$= 5 \times \sqrt{2} \times \sqrt{2} = 5 \times 2 = 10$

One is the multiplicative neutral

Si $a \in R$, alors $a \ast 1 = 1 \ast a = a$

Par example : $2\sqrt{5} \times 1 = 1 \times 2\sqrt{5} = 2\sqrt{5}$

Every real number $\neq 0$ has a multiplicative inverse

Si $a \neq 0$

il existe un nombre réel $\frac{1}{a}$

où $a \ast \frac{1}{a} = \frac{1}{a} \ast a = 1$ (l'élément neutre pour la multiplication)

Par example: the multiplicative inverse for $\frac{\sqrt{3}}{2}$ is $\frac{2}{\sqrt{3}}$

where $\frac{\sqrt{3}}{2} \times \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = 1$

Remark : $\frac{a}{b} = a \times \frac{1}{b}$, $b \neq 0$

i.e. $\frac{a}{b} = a \times$ the multiplicative inverse of b.

Distribution of multiplication on addition

For any three real numbers a, b, c.

$a \times (b + c) = (a \times b) + (a \times c) = a\,b + a\,c$

$(a + b) \times c = (a \times c) + (b \times c) = a\,c + b\,c$

2 **Give an estimation to the result of $(3 + \sqrt{5}) \times (1 + \sqrt{8})$, then check your answer using the calculator.**

Solution

First: The estimate of $\sqrt{5}$ is 2 $\therefore (3 + \sqrt{5})$ the estimate of $3 + 2 = 5$

the estimate of $\sqrt{8}$ is 3 $\therefore (1 + \sqrt{8})$ the estimate of $1 + 3 = 4$

$\therefore (3 + \sqrt{5})(1 + \sqrt{8})$ the estimate of $5 \times 4 = 20$

Second: when we use the calculator to find $(3 + \sqrt{5}) \times (1 + \sqrt{8})$

We find that the result is 20.0459

Therefore, the estimate is reasonable.

Lesson (8): Operations on the square roots

If a , b are two non-negative real numbers, then

First: $\boxed{\sqrt{a} \times \sqrt{b} = \sqrt{ab}}$

For example : $\sqrt{2} \times \sqrt{3} = \sqrt{2 \times 3} = \sqrt{6}$

Second: $\boxed{\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \text{ where } b \neq 0}$

For example : $\sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{\sqrt{9}} = \frac{1}{3}\sqrt{5}$

Third: $\boxed{\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{b} \quad b \neq 0}$

For example : $\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{\frac{18}{2}} = \sqrt{9} = 3$

The two conjugate numbers

If a and b are two positive rational numbers.

Then each of the two number $\boxed{(\sqrt{a} + \sqrt{b}), (\sqrt{a} - \sqrt{b})}$ is a conjugate to the other one.

then, their sum is $= \boxed{2\sqrt{a}}$ twice the first term

and their product is $= (\sqrt{a} + \sqrt{b}) , (\sqrt{a} - \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = \boxed{a - b}$

= The square of the first term - The square of the second term

The product of two conjugates is always a rational number

If we have a real number whose denominator is written in the form $(\sqrt{a} \pm \sqrt{b})$, we should put it in the simplest form by multiplying both the numerator and denominator by the conjugate of the denominator.

2 Given $x = \frac{4}{\sqrt{7} - \sqrt{3}}$, $y = \sqrt{7} - \sqrt{3}$,

prove that **x and y are conjugates, then find the values of:**

$x^2 - 2xy + y^2$, $(x - y)^2$. What do you observe?

Solution

$x = \frac{4}{\sqrt{7} - \sqrt{3}} \times \frac{\sqrt{7} + \sqrt{3}}{\sqrt{7} + \sqrt{3}} = \frac{4(\sqrt{7} + \sqrt{3})}{7 - 3} = \sqrt{7} + \sqrt{3}$

$y = \sqrt{7} - \sqrt{3}$ $\therefore$ x , y (two conjugate numbers)

$x^2 - 2xy + y^2 = (\sqrt{7} + \sqrt{3})^2 - 2(\sqrt{7} + \sqrt{3})(\sqrt{7} - \sqrt{3}) + (\sqrt{7} - \sqrt{3})^2$

$= (7 + 2\sqrt{21} + 3) - 2(7 - 3) + (7 - 2\sqrt{21} + 3)$

$= 10 + 2\sqrt{21} - 8 + 10 - 2\sqrt{21}$

$= 12$

$(x - y)^2 = [(\sqrt{7} + \sqrt{3}) - (\sqrt{7} - \sqrt{3})]^2$

$= [\sqrt{7} + \sqrt{3} - \sqrt{7} + \sqrt{3}]^2 = (2\sqrt{3})^2$

$= 4 \times 3 = 12$

Remark : $x^2 - 2xy + y^2 = (x - y)^2$

Lesson (9): Operations on the Cube roots

For any two real numbers a, b:

1. $$\boxed{\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{a \times b}}$$

For example :
$$\sqrt[3]{5} \times \sqrt[3]{2} = \sqrt[3]{5 \times 2} = \sqrt[3]{10}$$
$$\sqrt[3]{3} \times \sqrt[3]{-4} = \sqrt[3]{3 \times -4} = \sqrt[3]{-12}$$

2. $$\boxed{\sqrt[3]{a \times b} = \sqrt[3]{a} \times \sqrt[3]{b}}$$

For example :
$$\sqrt[3]{40} = \sqrt[3]{8 \times 5} = \sqrt[3]{8} \times \sqrt[3]{5} = 2\sqrt[3]{5}$$
$$\sqrt[3]{-128} = \sqrt[3]{-64 \times 2} = \sqrt[3]{-64} \times \sqrt[3]{2} = -4\sqrt[3]{2}$$

3. $$\boxed{\frac{\sqrt[3]{a}}{\sqrt[3]{b}} = \sqrt[3]{\frac{a}{b}} \quad \text{where } b \neq o,\ a,\ b \in R}$$

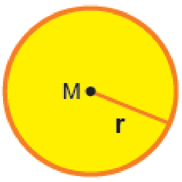
For example :
$$\frac{\sqrt[3]{12}}{\sqrt[3]{3}} = \sqrt[3]{\frac{12}{3}} = \sqrt[3]{4}$$

4. $$\boxed{\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}} \quad \text{where } b \neq o,\ a,\ b \in R}$$

For example :
$$\sqrt[3]{\frac{3}{2}} = \frac{\sqrt[3]{3}}{\sqrt[3]{2}}$$

Lesson (10): Applications on the Real Numbers

The circle:



Circumference of a circle = 2 π r length unit

area of a circle = π r^2 square unit

where r is the length of the radius in a circle,
π is the (approximate ratio).

Examples

1. **Find** the circumference of acircle whose area is 38.5 cm^2 $(\pi = \frac{22}{7})$

Solution

The area of the circle = πr^2

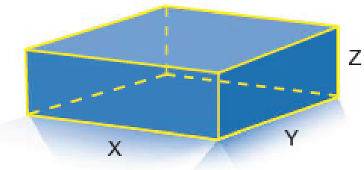
$$38.5 = \frac{22}{7} r^2 \qquad \therefore r^2 = \frac{38.5 \times 7}{22} = \frac{49}{4}$$

$$\therefore r = \sqrt{\frac{49}{4}} = \frac{7}{2} = 3.5 \text{ cm}$$

The cuboid

It is a body whose six faces are of a rectangular shape such that every two opposite faces are congruent.:

If the lengths of its edges were x, y, z, then:



The lateral area = the perimeter of the base × the height

The lateral area = 2 (x + y) × z square unit

The lateral area = the lateral area + 2 × the area of the base

The total area = 2 (xy + yz + Xz) square unit

The volume of the cuboid = the area of the base × the height

The volume of the cuboid = x × y × z cubic unit

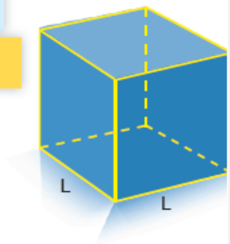
A special case : the cube

It is a cuboid whose edges are equal in length. If the length of one edge = L length unit, then:

The area of each face = L^2 square unit

The lateral area of each face = $4L^2$ square unit

The total area = $6L^2$ square unit, the volume of the cube = L^3 cubic unit



Examples

Find the total area of a cube whose volume is 125 cm^3

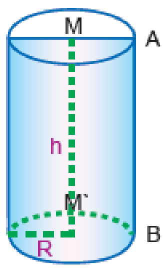
Solution

The volume of the cube = L^3 $\therefore 125 = L^3$ $\therefore L = \sqrt[3]{125} = 5$cm

The total area = $6L^2 = 6 \times (5)^2 = 150$ cm^2

The right circular cylinder :

It is a body that has two parallel congruent bases each is a circular shaped surface, while its lateral surface is a curved surface called cylindrical surface.

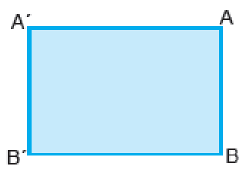


- If M, M` are the bases of the cylinder, then M M` is the height of cylinder.

Let's think If A ∈ the circle M, B ∈ the circle M`,

$\overline{AB} \parallel \overline{MM`}$

- Then, if we cut the lateral cylindrical surface at AB and we stretch That surface, we get the surface of the rectangle A B B` A`



Then, AB = height of cylinder, A A` = the perimeter of the base of the cylinder.

The area of the rectangle A B B` A` = the lateral area of the cylinder

The lateral area of the cylinder = the perimeter of the base × height = 2π r h (square unit)

the total area of the cylinder = area of lateral surface + sum of the areas of the two bases

$= 2\pi r h + 2\pi r^2$ (square unit)

the volume of the cylinder = base area × height = $\pi r^2 h$ (cubic unit)

The sphere:

It is a body of curved surface in which the points have the same distance (r) from a constant point inside it (the center of the sphere)..

If the sphere is cut by a plane passing by its center, then the resulted section is a circle whose center is the center of a sphere where its radius is the radius of a sphere (r).

Volume of the sphere = $\frac{4}{3}\pi r^3$ **cubic units.**

area of the sphere = $4\pi r^2$ **square units.**

Lesson (11): Solving Equations & Inequalities of first Degree in one Variable in R

First: Solving Equations of First degree in one variable in R

We know that: The equation $3X - 2 = 4$ is called an equation of first degree where the exponent of the (unknown) variable X is 1. To solve that equation in R

$3x - 2 = 4$ **By adding 2 to the sides of the equation**

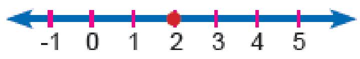
$3x = 6$ **(we can multiply by the multiplicative inverse of the coefficient of X)**

$\frac{1}{3} \times 3x = \frac{1}{3} \times 6$

$\therefore x = 2$

i.e the solution set { 2 }

This solution can be graphed on the number line as shown in the figure opposite .



Examples

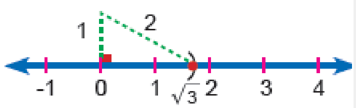
1 **Find the solution set of the equation** $\sqrt{3}x - 1 = 2$**, in R, then graph the solution on the number line.**

Solution

$\sqrt{3}x - 1 = 2$ $\qquad \therefore \sqrt{3}x = 3$

$\therefore x = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ $\qquad \therefore x = \sqrt{3} \in \mathbb{R}$

The solution set is $\{\sqrt{3}\}$



This solution can be graphed on the number line as shown in the figure opposite .

Second: Solving Inequalities of the First degree in one Variable in R, graphing the solution on the number line:

The following properties are used to solve the inequality in R. The solution set is written in the form of an interval

If A , B، C were real number where A< B, then:

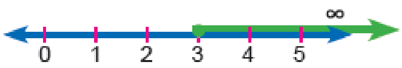
1	$A + C < B + C$.	addition property.
2	If $C > 0$ then $A \times C < B \times C$.	property of multiplication by a positive real number
3	If $C < 0$ then $A \times C > B \times C$.	property of multiplication by negative real number.

Examples

1 **Find** the solution set for the inequality $2x - 1 \geq 5$ in R and represent the solution set graphically.

Solution

By adding 1 to the sides of the inequality it becomes $2x \geq 6$
by multiplying the side of the in equality by $(\frac{1}{2} > 0)$ $x \geq 3$
$\therefore$ The solution set in R is $[3 , \infty[$
and it is graphed by green color ray on the number line.

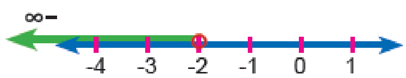


2 **Find** the solution set for the inequality $5 - 3x > 11$, in R, then represent the solution graphically.

Solution

By adding (-5) to the sides of the inequality then $-3x > 6$
by multiplying the sides of the inequality by $(-\frac{1}{3} < 0)$ we get :
$\therefore x < -2$

i.e., the solution set in R is est $]-\infty , -2[$

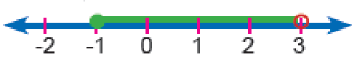


and it is represented by the green color ray on the number line.

3 **Find** the solution set for the in equality $-3 \leqslant 2x - 1 < 5$ in R and represent the solution graphically.

Solution

by adding (1) to the sides of the inequality $-3 + 1 \leqslant 2x - 1 + 1 < 5 + 1$
Namely, $-2 \leqslant 2x < 6$, and by multiplying the sides of the inequality by $(\frac{1}{2} > 0)$ $-1 \leqslant x < 3$
$\therefore$ the solution set in R is $[-1, 3[$ and it is graphed on the number line by the green color.



in example 3 What is the solution set for the inequality in N?
What is the solution set for the inequality in Z?

UNIT [2]: Relation Between Two Variables

Lesson (1): Linear Relation of two variables

Think and Discuss

A person has some bills of LE 50 and LE 20. He bought an electrical apparatus for LE 390.

Think: **How many bills of each type does he give to the seller?**

Suppose : x represents the number of fifties bills, then the value of what he has of these bills is L.E 50x, y represents the number of Twenties bills, then the value of what he has of these bills is L.E 20y.

Required is to know: x and y that verify the equation:

$$50x + 20y = 390$$

This relation represents a linear equation in two variables. Dividing both sides over 10 produces the following equivalent equation:

$$5x + 2y = 39$$

$$\therefore y = \frac{39 - 5x}{2}$$

Note that : x and y are natural numbers. Therefore, x should be an odd number.

The following table can be created to know the different possibilities of giving bills to the seller: a bill of L.E50 and 17 bills of L.E 20, or 3 bills of L.E 50 and 12 bills of L.E 20, or 5 bills of L.E 50 and 7 bills of L.E 20, or 7 bills of 50 and 2 bills of L.E 20.

x	y	(x , y)
1	17	(1 , 17)
3	12	(3 , 12)
5	7	(5 , 7)
7	2	(7 , 2)
9	negative	refused

2. The perimeter of an isosceles triangle is 19cm. What are the different possible lengths of its sides? Side length $\in Z_+$

Remember : The sum of the lengths of any two sides of a triangle is greater than the length of the third side .

The Relation of two variables

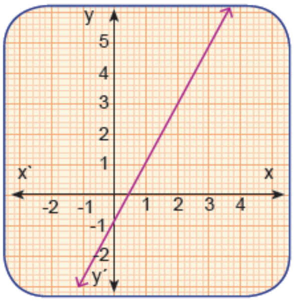
a x + by = c where a ≠ 0 , b ≠ 0 is called a linear relation of two variable **x** and **y** and can be described by a set of ordered pairs (**x**, **y**) verifying this relation.

Example:

Refer to the relation **2x – y = 1**

If x = 1, ∴ y = 1	∴ (1 , 1)	satisfies the relation
If x = 0, ∴ y = -1	∴ (0 , -1)	satisfies the relation
If x = 3, ∴ y = 5	∴ (3 , 5)	satisfies the relation
If x = -1, ∴ y = -3	∴ (-1 , -3)	satisfies the relation

Thus, there are an infinite number of ordered pairs satisfying the relation.



Note that:

- The linear relation 2x –y = 1, can be represented graphically by using any of the ordered pairs obtained before.
- Each point ∈ the straight line (in red) is represented by an ordered pair whose elements satisfy the linear relation 2x – y = 1.

Graphing the Relation of two Variables

The relation **ax + by = c** where a and b or both are not equal zero. is called a linear relation of two variables x and y and can be represented graphically by a straight line.

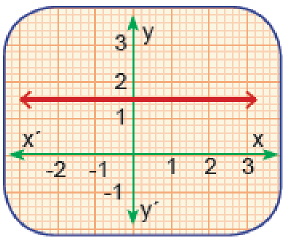
for **a = 0**

The relation is represented by a straight line parallel to x-axis.

Example : 2 y = 3

i.e. : $y = \frac{3}{2}$

is represented by the red line which passes through the point $(0. \frac{3}{2})$ and is parallel to x-axis.



Special case:

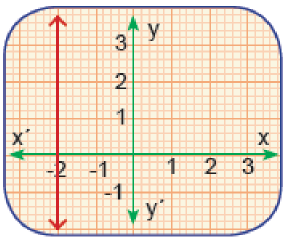
the relation y = 0 represents the x-axis

for **b = 0**

The relation is represented by a straight line parallel to y-axis.

Example : x = -2

is represented by the red line which passes through the point (-2, 0) and is parallel to y-axis.



Special case:

the relation x = 0 represents the y-axis.

Example :

Graph the relation: $x + 2y = 3$

Solution

Choose some ordered pairs that satisfy the relation:

Example : For $y = 2$ $\therefore x = -1$ $(-1, 2)$ **satisfies the relation**

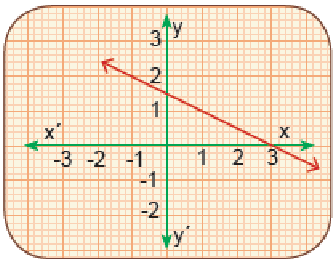
$y = 0$ $\therefore x = 3$ $(3, 0)$ **satisfies the relation**

$y = -1$ $\therefore x = 5$ $(5, -1)$ **satisfies the relation**

and so on ....

The following table lists these data:

x	-1	3	5	0
y	2	0	-1	$\frac{3}{2}$



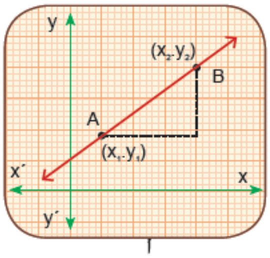
The red line represents this relation.

Discuss with your teacher:

Lesson (2): the Slope of a line & real-life Applications

Think and Discuss

When observing the motion of a point on a straight line from the location A (x_1, y_1) to the location B (x_2, y_2), where $x_2 > x_1$ and A, B $\in$ line, then:

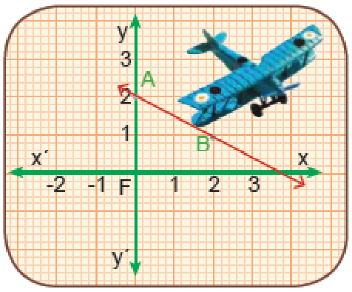


1. the change in x-coordinate $= x_2 - x_1$, and is called the horizontal change.
2. the change in y-coordinate $= y_2 - y_1$ is called the vertical change and may be positive, negative or zero.

The slope of a line = $\frac{\text{change in y-coordinate}}{\text{change in x-coordinate}} = \frac{\text{vertical change}}{\text{horizontal change}}$

$$S = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{where } x_2 \neq x_1$$

Example (2) :



If: A (0, 2), B (2, 1);

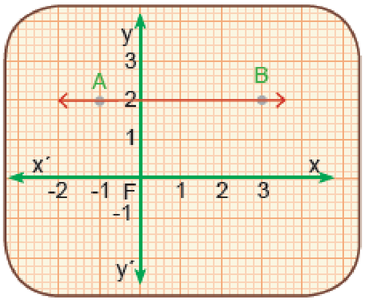
then: the slope of $\overleftrightarrow{AB} = \frac{1-2}{2-0} = -\frac{1}{2}$

Not that :

The point A moves on the line downwards to the point B

2. $y_2 < y_1$
3. The slope of the line is negative.

Example (3) :



If: A (-1, 2) and B (3, 2),

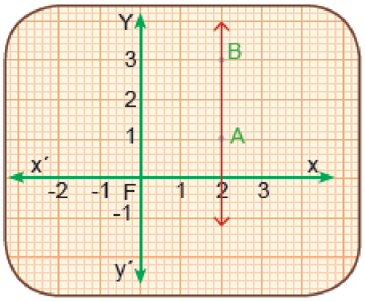
then: the slope of the line

$\overleftrightarrow{AB} = \frac{2-2}{3-(-1)} = \frac{0}{4} = 0$

Not that :

1. The point A moves horizontally to point B.
2. $y_2 = y_1$
3. The slope of the line = zero

Example (4) :



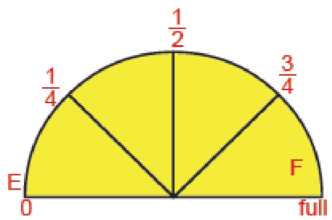
If: A = (2, 1) and B(2, 3) then: we can not calculate the slope. Because the definition of the slope is conditioned to have a change in the x-coordinate i.e. $x_2 - x_1 \neq 0$

Not that :

1. The point A moves vertically to point B.
2. $x_2 = x_1$
3. The slope of the line is an underfined number.

Application (2) :

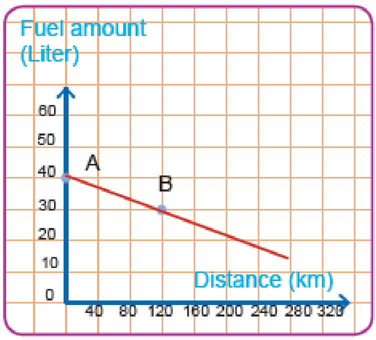
Hazem filled up the 40 Litres tank of his car. As covering a distance of 120 km, the fuel gage shows the rest of fuel is $\frac{3}{4}$ of the tank. Draw a diagram to show the relation between the amount of fuel in the tank and coverd distance (This relation is linear).Calculate the coverd distance as the tank is totally getting empty.



Solution

On the starting point: A (0, 40)

traveled distance — the amount of used fuel



After covering 120 km B = (120, 30)

The slope of $\overleftrightarrow{AB} = \frac{30-40}{120-0} = \frac{-1}{12}$

This slope means the fuel amount decreases with a rate of 1L per 12 km ,which means 1L is enough to cover a distance of 12 km.

The coverd distance that make the tank empty = $\frac{\text{Fuel Amount}}{\text{Decreasing Rate}} = \frac{40}{\frac{1}{12}}$

$= 40 \times \frac{12}{1} = 480\text{km}.$

Note that : $\overrightarrow{AB}$ intersects the distance-axis in the point (480, 0) **which gives the required distance.**

UNIT [3]: Statistiques

Lesson (1): Collecting & Organizing Data

Organizing data and representing them in frequency tables

Example

Below are the scores of 30 students in an examination

7	10	7	4	5	8	6	7	13	12
2	9	11	12	11	9	15	12	13	9
5	14	19	3	9	14	3	13	8	17

Required: forming a frequency table with sets that represents that data .

Solution

To form a frequency table with sets, follow the following steps:

First: find the highest and the lowest values of the collected data?

let the previous collected data be X

then: X = {x : 2G x G 19}

i.e: X values begins with 2 and ends in 19

i.e: the range = the highest value - the lowest value = 19 – 2 = 17

Second: divide set X into a number of separate subsets each of them is equal in range.

let them be 6 sets. ⊠ The range of the set = $\frac{17}{6}$ i.e approximated to 3

Third: the subsets are as follow.

The first set	2 –	**the third set**	8 –
The second set	5 –	**The Fourth set**	11 –

and so on

Remark : 2- means the set of data greater than or equal to 2 and less than 5 and so on.

Fourth: Record the data in the following table:

Set	tally	frequency
2 –	////	4
5 –	~~////~~ /	6
8 –	~~////~~ //	7
11 –	~~////~~ ///	8
14 –	///	3
17 –	//	2
Total		30

Fifth: Delete the tally column from the table to get the frequency table with sets. It can be written either verticaly or horizontaly. The following is the horizontal form of the table:

Sets	2 –	5 –	8 –	11 –	14 –	17 –	total
Frequency	4	6	7	8	3	2	30

Lesson (2): the Ascending & Descending Cumulative Frequency Table & their Graphical Representation

Examples

The following table shows the frequency distribution for the heights of 100 students in a school in centimeters.

Tall (sets) in c.m	115–	120–	125–	130–	135–	140–	145–	Total
Number of students (frequency)	8	12	19	23	18	13	7	100

1. How many students are with height less than 115cm?
2. How many students are with height less than 135cm?
3. How many students are with height less than 145cm?

Form the ascending cumulative frequency table for these data and represent them graphically.

Solution

- Are there students with height less than 115c.m? **No**
- Are there students with height less than 135c.m? How many? **yes, 62 student.**
- How can you calculate the number of students with height less than 145 cm? **Add the number of students in the sets of height less than the set 145.**

Now, to answer the previous questions in an easier way, form an ascending cumulative frequency table as follows:

Upper boundaries of sets	Ascending cumulative frequency
Less than 115	0
Less than 120	0 + 8 = 8
Less than 125	8 + 12 = 20
Less than 130	20 + 19 = 39
Less than 135	39 + 23 = 62
Less than 140	62 + 18 = 80
Less than 145	80 + 13 = 93
Less than 150	93 + 7 = 100

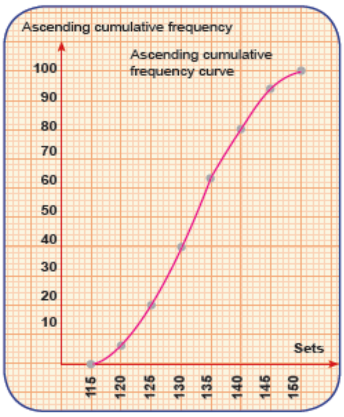
i.e.

ascending cumulative frequency table

Upper boundaries of sets	Ascending cumulative frequency
Less than 115	zero
Less than 120	8
Less than 125	20
Less than 130	39
Less than 135	62
Less than 140	80
Less than 145	93
Less than 150	100

To represent the ascending cumulative frequency table graphically:

1. Specify the horizontal axis to the sets and the vertical axis to the ascending cumulative frequency
2. Choose a drawing scale to draw the vertical axis such that the ascending cumulative frequency axis can hold the number of elements in a set
3. Represent the ascending cumulative frequency for each set and draw its line graph successively.



Second: The descending cumulative frequency table and its graphical representation. :

Of the previous frequency distribution which shows the heights of 100 students in a school in centimeters.

Find: The number of students with heights of 150cm and more..

The number of students with heights of 140cm and more..

The number of students with heights of 125cm and more..

Form the descending cumulative frequency table and represent it graphically..

Solution

There are no students with heights of 150cm and more .

The number of students with heights of 140cm and more is 7 + 13 = 20 students.

The number of students with heights of 125cm and more is

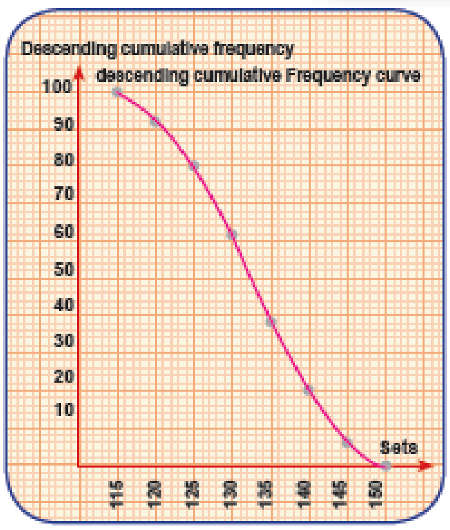
complete: 19 + ... + ... + ... + ... = ...

To answer these questions in an easier way, form the descending cumulative frequency table as follows :

Descending cumulative frequency table	
Lower limits of sets	Ascending cumulative frequency
115 and more	100
120 and more	92
125 and more	80
130 and more	61
135 and more	38
140 and more	20
145 and more	7
150 and more	zero

Lower limits of sets	descending cumulative frequency
115 and more	92 + 8 = 100
120 and more	80 + 12 = 92
125 and more	61 + 19 = 80
130 and more	38 + 23 = 61
135 and more	20 + 18 = 38
140 and more	7 + 13 = 20
145 and more	0 + 7 = 7
150 and more	0

To represent this table graphically, follow the steps of representing the ascending cumulative frequency to get the following graphical representation:



Lesson (3): Arithmetic Mean, Median & Node

First: the mean

You have learned to find the mean for a set of values and learned that:

$$\text{The arithmetic mean} = \frac{\text{The sum of values}}{\text{Number of values}}$$

Example: If the ages of 5 students are 13، 15، 16، 14، and 17 years old, then

The mean of their ages $= \frac{13+15+16+14+17}{5}$

$= \frac{75}{5} = 15$ years

Remark: $15 \times 5 = 13 + 15 + 16 + 14 + 17$

The mean: is the simplest and most commonly used type of averages, It's that value given to each item in a set, then the total of these new values is the same total of the original values. It can be calculated by adding up all values, then divide the sum by the number of values.

Finding the mean of data from the frequency table with sets:

How can you find the mean of the following frequency distribution:

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	10	20	25	30	15	100

Remark: To find the mean for a frequency distribution with sets, follow the following steps:

1. **Determine the centers of sets:**

The center of the first set $= \frac{20+10}{2} = 15$. The center of the second set $= \frac{30+20}{2}$ $= 25$ ... and so on

Since the ranges of the subsets are equal and each = 10

We consider the upper limit of the last set = 60 and then :

its center $= \frac{50+60}{2} = 55$

2. **Form the following vertical table:**

Sets	Centre of the sets (X)	Frequency	Centre of the sets × frequency X × F
10 –	15	10	150
20 –	25	20	500
30 –	35	25	875
40 –	45	30	1350
50 –	55	15	825
Total		100	3700

3. **The mean** $= \frac{\text{The total of } (F \times X)}{\text{the total of } F}$

$= \frac{3700}{100} = 37$

Second: the median

The median is the middle value in a set of values after arranging it ascendingly or descendingy such that the number of values which are less than it is equal to the number of values which are greater than it.

Finding the median of a frequency distribution with sets graphically:

1. Draw the ascending or descending cumulative frequency table, then draw the cumulative frequency curve of it .
2. **Determine the order of the median** = $\frac{\text{The total of frequency}}{2}$.
3. Determine point A on the vertical axis (frequency) which represents the order of the median.
4. Draw a horizontal straight line from point A to intersect the curve at a point. Form this point, draw a vertical straight line on the horizontal axis to intersect it at a point that represants the median.

Example (2)

The following table shows the daily wages of 100 workers in a factory..

daily wages in LE (sets)	15–	20–	25–	30–	35–	40–	Total
Number of workers (frequency)	10	15	22	25	20	8	100

Required:

1. Graph the ascending and descending cumulative frequency curves on one figure.
2. Can you find the median wage from this curve?

Solution

Upper boundaries of sets	Cumulative frequency
Less than 15	zero
Less than 20	10
Less than 25	25
Less than 30	47
Less than 35	72
Less than 40	92
Less than 45	100

Lower boundaries of sets	Cumulative frequency
15 and more	100
15 and more	90
15 and more	75
15 and more	53
15 and more	28
15 and more	8
15 and more	zero

Remark:

The ascending cumulative frequency curve intersects with the descending cumulative frequency curve at one point which is m .

The y-coordinate for the point M = 50

$= \frac{100}{2}$

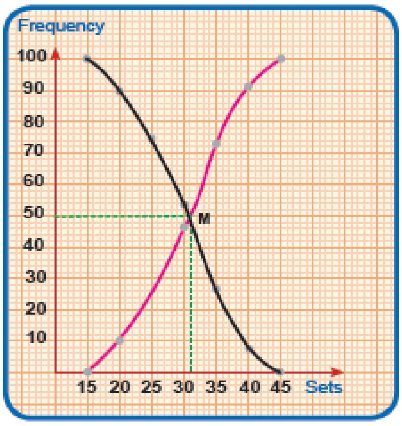
= the order of the median

$\therefore$ **The X-coordinate of the point M determines the median**

every 10 mm of the x coordinate represents L.E 5

Complete: 2 mm represents ....

The median wage = 30 + $\frac{2 \times 5}{10}$ = LE 31 .



practice

Draw **the descending cumulative**

Third: the mode

The mode is the most common value in the set or in other words, it is the value which is repeated more than any other values.

Example

The following table shows the frequency distribution for the scores of 40 students in an examination.

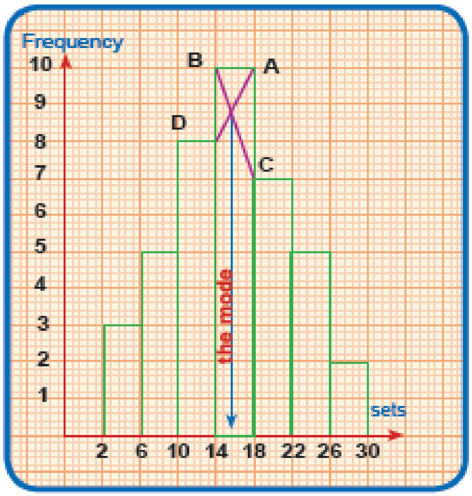
Sets	2–	6–	10–	14–	18–	22–	26–
Frequency	3	5	8	10	7	5	2

Find the mode of this distribution graphically

Solution

You can find the mode of this distribution graphically using the histogram as follows:
First: draw a histogram.

1. Draw two perpendicular axes: one horizontal to represent sets and the other vertical to represent the frequency of each set.
2. Divide the horizontal axis into a number of equal parts using a suitable drawing scale to represent sets.
3. Divide the vertical axis into a number of equal parts using a suitable drawing scale such that the greatest frequency among sets can be represented..
4. Draw a rectangle whose base is set (2-) and height is equal to the frequency (3).
5. Draw another rectangle adjacent to the first one whose base is set (6-) and height is equal to the frequency (5).
6. Repeat drawing the rest of adjacent rectangles till the last set (26-).



Second: Finding the mode from the histogram, to find the mode from the histogram, we observe that: the most repeated set is (14-), and it is called the mode set, why?

Define the intersection point of $\overline{AD}$, $\overline{BC}$ from the graph, and from this point, drop a vertical line on the horizontal axis to define the sequential value within that distribution.

From the graph, what's the mode value?

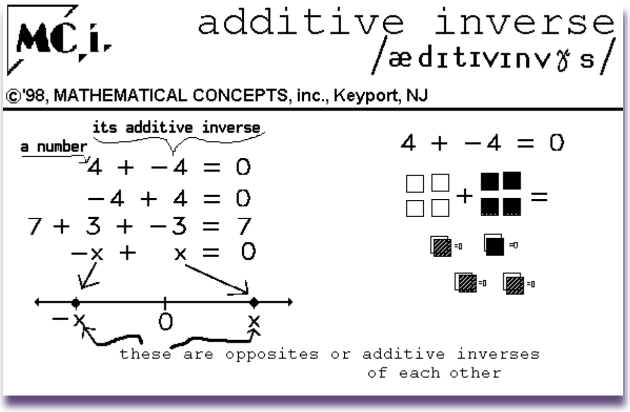
REMEMBER

$(a+b)(a-b) = a^2-b^2$ $(a+b)^2 = a^2 +2ab +b^2$ $(a-b)^2 = a^2 -2ab +b^2$

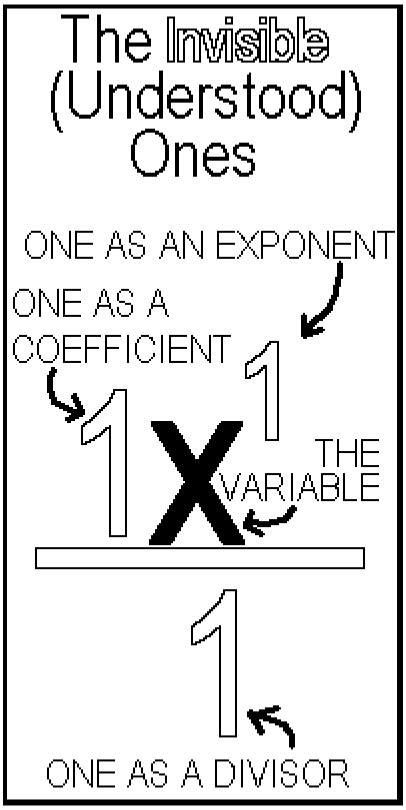
$a^2+ab+b^2 = (a+b)^2 – ab$ $\sqrt{\text{zero}} = \text{zero}$

It's meaningless to find $\sqrt{a}$ if a is a negative number.

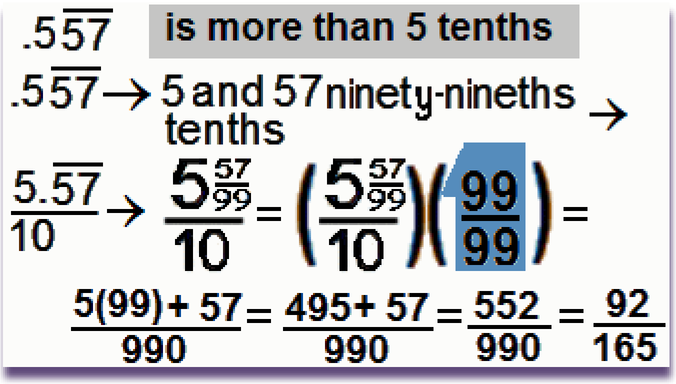
INDEX
radical sign with
inclusion bar
RADICAND
2
25



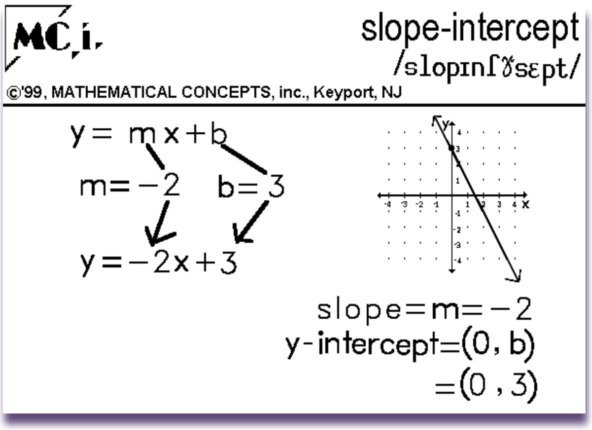
MC i.
additive inverse
/ædɪtɪvɪnvɤs/
©'98, MATHEMATICAL CONCEPTS, inc., Keyport, NJ
its additive inverse
a number
4 + −4 = 0
−4 + 4 = 0
7 + 3 + −3 = 7
−x + x = 0
4 + −4 = 0
these are opposites or additive inverses of each other



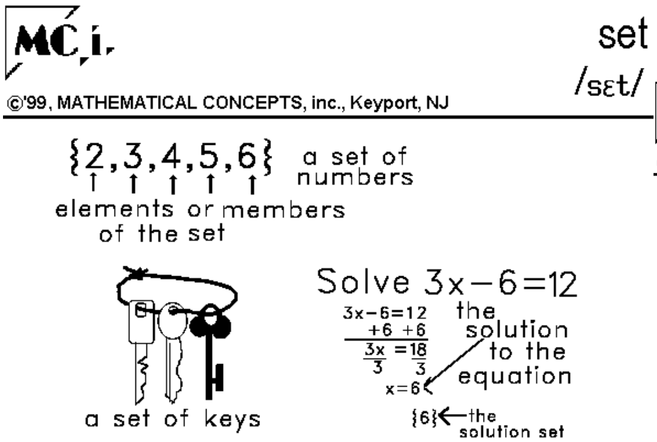
The Invisible (Understood) Ones
ONE AS AN EXPONENT
ONE AS A COEFFICIENT
THE VARIABLE
ONE AS A DIVISOR



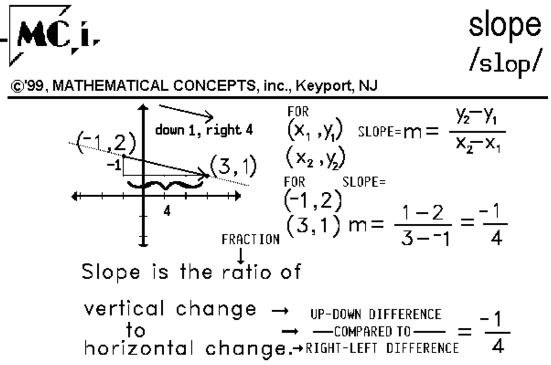
.557 is more than 5 tenths
.557 → 5 and 57 ninety-nineths tenths →
5.57/10 → (5 57/99)/10 = ((5 57/99)/10)(99/99) =
(5(99)+57)/990 = (495+57)/990 = 552/990 = 92/165



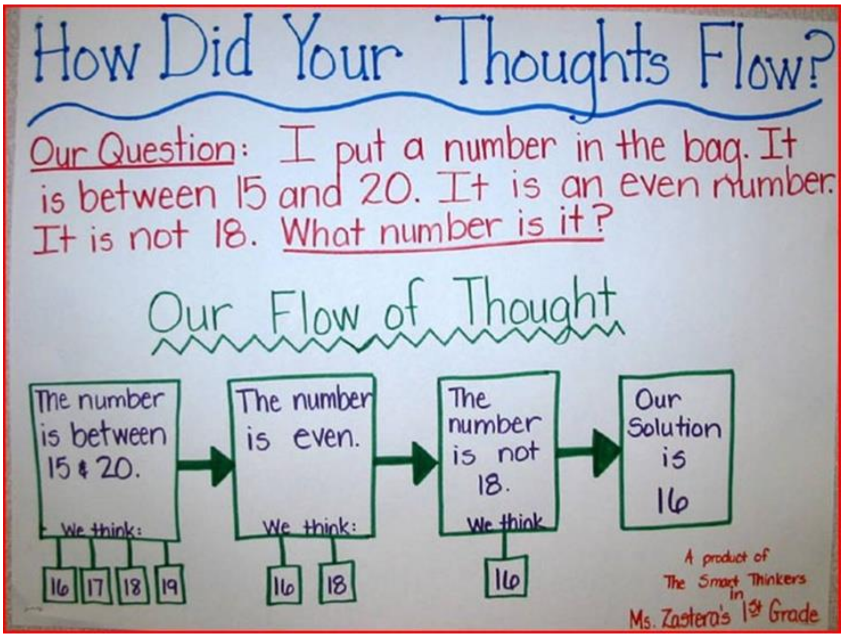
MC i.
slope-intercept
/slopɪnʃɤsɛpt/
©'99, MATHEMATICAL CONCEPTS, inc., Keyport, NJ
y = mx + b
m = −2
b = 3
y = −2x + 3
slope = m = −2
y-intercept = (0, b)
= (0, 3)

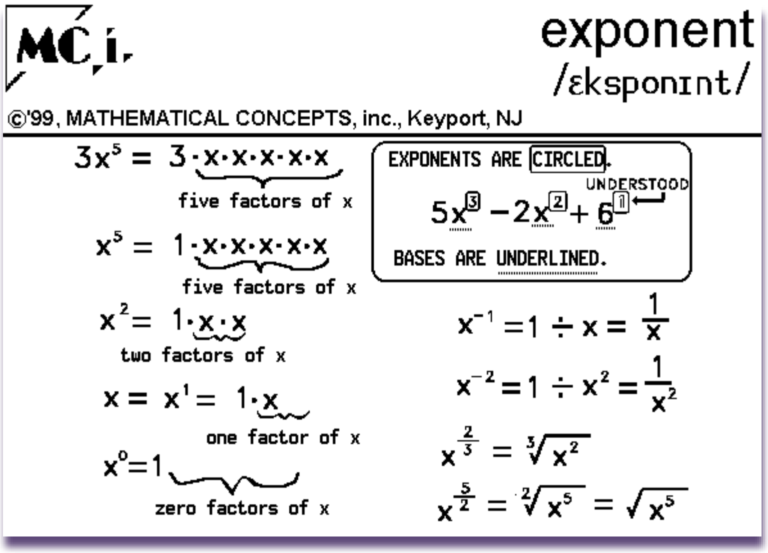


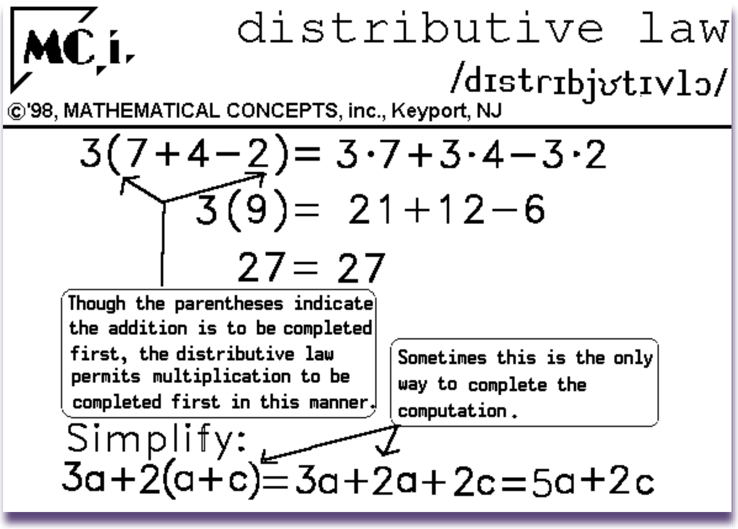
MC i.
set
/sɛt/
©'99, MATHEMATICAL CONCEPTS, inc., Keyport, NJ
{2,3,4,5,6} a set of numbers
elements or members of the set
a set of keys
Solve 3x − 6 = 12
3x−6=12
+6 +6
3x/3 = 18/3
x=6
the solution to the equation
{6} ← the solution set



MC i.
slope
/slop/
©'99, MATHEMATICAL CONCEPTS, inc., Keyport, NJ
down 1, right 4
(-1,2)
(3,1)
FOR (x1, y1) (x2, y2) SLOPE = m = (y2 − y1)/(x2 − x1)
FOR (-1,2) (3,1) SLOPE = m = (1 − 2)/(3 − −1) = -1/4
FRACTION
Slope is the ratio of
vertical change to horizontal change
UP-DOWN DIFFERENCE COMPARED TO RIGHT-LEFT DIFFERENCE = -1/4


How Did Your Thoughts Flow?
Our Question: I put a number in the bag. It is between 15 and 20. It is an even number. It is not 18. What number is it?
Our Flow of Thought
The number is between 15 & 20.
We think:
16 17 18 19
The number is even.
We think:
16 18
The number is not 18.
We think
16
Our Solution is 16
A product of The Smart Thinkers in Ms. Zastera's 1st Grade


MC,i.
exponent
/ɛksponɪnt/
©'99, MATHEMATICAL CONCEPTS, inc., Keyport, NJ
$3x^5 = 3 \cdot \underbrace{x \cdot x \cdot x \cdot x \cdot x}_{\text{five factors of } x}$
$x^5 = 1 \cdot \underbrace{x \cdot x \cdot x \cdot x \cdot x}_{\text{five factors of } x}$
$x^2 = 1 \cdot \underbrace{x \cdot x}_{\text{two factors of } x}$
$x = x^1 = 1 \cdot \underbrace{x}_{\text{one factor of } x}$
$x^0 = 1 \underbrace{\quad\quad}_{\text{zero factors of } x}$
EXPONENTS ARE CIRCLED.
UNDERSTOOD
$5x^{3} - 2x^{2} + 6^{1}$
BASES ARE UNDERLINED.
$x^{-1} = 1 \div x = \frac{1}{x}$
$x^{-2} = 1 \div x^2 = \frac{1}{x^2}$
$x^{\frac{2}{3}} = \sqrt[3]{x^2}$
$x^{\frac{5}{2}} = \sqrt[2]{x^5} = \sqrt{x^5}$


MC,i.
distributive law
/dɪstrɪbjʊtɪvlɔ/
©'98, MATHEMATICAL CONCEPTS, inc., Keyport, NJ
$3(7+4-2) = 3 \cdot 7 + 3 \cdot 4 - 3 \cdot 2$
$3(9) = 21 + 12 - 6$
$27 = 27$
Though the parentheses indicate the addition is to be completed first, the distributive law permits multiplication to be completed first in this manner.
Sometimes this is the only way to complete the computation.
Simplify:
$3a + 2(a+c) = 3a + 2a + 2c = 5a + 2c$