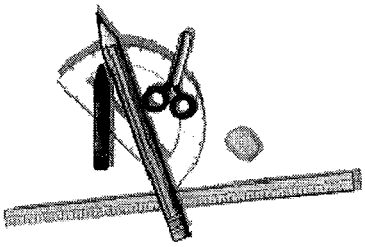
2014 2015

Mathematics Department - Prep. Stage

Second Term - Algebra

Unit (1) : Factorization

2 Prep.



1 Sheet No.

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اسم التلميذ : .....

Revision on factorization by taking out (H.C.F)

Definition :

Factorizing the algebraic expression means to write it as a product of two factors or more.

How to factorize an expression by taking out the (H.C.F.) :

1. Determine the H.C.F. of the terms of the algebraic expression.
2. Put the H.C.F. out of two arcs.
3. Divide each term of the algebraic expression by the H.C.F. and put the quotients inside the arcs.

Example 1 **Factorize each of the following by taking out the highest common factor :**

1. $5a + 15b$
2. $10xy - 8xz$
3. $12x^2 - 4xy$
4. $3x^2y + 2xy^2 - xy$
5. $2x(m+3) - 4y(m+3)$
6. $x(z-y) + \ell(y-z)$

Solution

1. $\because$ H.C.F. $= 5$

$\therefore 5a + 15b = 5(a + 3b)$

2. $\because$ H.C.F. $= 2x$

$\therefore 10xy - 8xz = 2x(5y - 4z)$

3. $\because$ H.C.F. $= 4x$

$\therefore 12x^2 - 4xy = 4x(3x - y)$

4. $\because$ H.C.F. $= xy$

$\therefore 3x^2y + 2xy^2 - xy = xy(3x + 2y - 1)$

5. $\because$ H.C.F. $= 2(m+3)$

$\therefore 2x(m+3) - 4y(m+3)$

$= 2(m+3)(x - 2y)$

Notice that :

The H.C.F. may be an algebraic expression.

6. $\because y - z = -(z - y)$

, $\because$ H.C.F. $= (z - y)$

$\therefore x(z-y) + \ell(y-z)$

$= x(z-y) - \ell(z-y)$

$= (z-y)(x - \ell)$

Lesson (1) : Factorizing quadratic trinomial Part (1)

From the previous , we deduce that :

The trinomial which is in the form : $x^2 + b\,x + c$ is factorized to two factors :

- The first term in each factor is x
- The two other terms in the two factors are two numbers whose product is c (the last term in the trinomial), and their sum is b (the coefficient of x in the trinomial).

From the previous example , we notice that :

When we factorize the trinomial : $x^2 + b\,x + c$ in the form $(x + \ell)\,(x + m)$, then :

1. If c is positive (***i.e.*** The product of the two numbers is positive) , then ℓ and m have the same sign as b
2. If c is negative (***i.e.*** The product of the two numbers is negative) , then ℓ and m have different signs such that the great one (numerically) has the same sign as b

Remarks

Before factorizing the trinomial , we must do the following :

- Arrange the terms of the expression descendingly or ascendingly according to the indices (exponents) of one of the given algebraic symbols. It is better to be descending.
- Taking out the H.C.F. of the terms of the expression.
- Performing operations included in arcs and simplifying the algebraic expression.

Example 1

Factorize each of the following :

1. $x^2 + 56 - 15\,x$
2. $x^2 + x\,y - 12\,y^2$
3. $3\,a^3 + 9\,a^2 - 120\,a$
4. $m\,(m + 7) - 18$
5. $x^4 - 3\,x^2\,y - 10\,y^2$

You can check the truth of your solution by multiplying the two factors by inspection to get the main expression.

Solution

1. $x^2 + 56 - 15\,x = x^2 - 15\,x + 56$
$= (x - 7)\,(x - 8)$

2. $x^2 + x\,y - 12\,y^2$
$= (x - 3\,y)\,(x + 4\,y)$

Notice that :

You must arrange the expression descendingly according to the powers of x before factorizing.

3 We notice that there is H.C.F. can be taken out from the terms of the expression (H.C.F. = 3 a)

$\therefore 3a^3 + 9a^2 - 120a = 3a(a^2 + 3a - 40)$

$= 3a(a + 8)(a - 5)$

Notice that :
You must take out the H.C.F. before factorizing to be perfect.

4 $m(m + 7) - 18$

$= m^2 + 7m - 18$

$= (m + 9)(m - 2)$

Notice that :
You must remove arcs before factorizing.

5 $x^4 - 3x^2y - 10y^2$

$= (x^2 - 5y)(x^2 + 2y)$

Notice that :
x^4 is factorized to be $x^2 \times x^2$

Exercises on Lesson (1)

[A] Choose the correct answer from the gives ones :

1 If $(x + 2)$ is one of the factors of : $x^2 + 5x + 6$, then the other factor is .....

(a) $x + 3$ (b) $x - 3$ (c) $2x + 3$ (d) $2x - 3$

2 If $(x - 3)$ is a factor of the expression : $x^2 + 3x - 18$, then the second factor is .....

(a) $x - 6$ (b) $x - 15$ (c) $x - 18$ (d) $x + 6$

[B] Complete the following :

1 If $x = y + 1$, $x + y = 2$, then $3x^2 - 3y^2 =$ .....

2 $x(a + b) - y(a + b) = (.....)(x - y)$

3 $15x^3 + 5x^2 =$ ..... $(3x +$ .....$)$

4 $3x^2y + 6xy =$ ..... $(x + 2)$

5 $x^2 - xy - 2y^2 = (.....)(.....)$

6 If $(x - 5)$ is one factor of $x^2 - 2x - 15$, then the other factor is .....

4

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7 If $(x + 2)$ is a factor of the expression $x^2 + 3x + 2$, then the other factor is .....

8 If $x - 3$ is one of factors of the expression : $x^2 - 7x + 12$, then the other factor is .....

9 $x^2 - ..... - 10 = (..... + 2)(x - 5)$

10 $x^2 - ..... - 10 = (..... + 2)(x - .....)$

11 $(x .....)(2x + 5) = 2x^2 + ... + 15$

12 If $x^2 - m x + 12 = (x - 3)(x - 4)$, then $m = ...$

[C] Essay Problems :

1 Factorize each of the following :

1. $x^2 + 8x + 15$
2. $x^2 + 11x + 10$
3. $x^2 - 7x + 12$
4. $x^2 - 17x + 30$
5. $x^2 + 5x - 14$
6. $x^2 + 4x - 12$
7. $x^2 - 6x - 16$
8. $x^2 - 3x - 10$

2 Factorize each of the following :

1. $x^2 + 5xy + 6y^2$
2. $a^2 + 11ab + 30b^2$
3. $b^2 + 3bc - 10c^2$
4. $a^2 + 22ab - 48b^2$
5. $x^2 - 15xy + 36y^2$
6. $a^2 - 13ab + 42b^2$
7. $x^2 - 5xy - 24y^2$
8. $x^2 - 7xy - 18y^2$

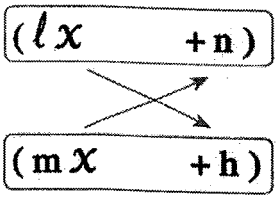
3 Factorize each of the following :

1. $5x^2 - 10x - 15$
2. $2a^2 + 28a + 96$
3. $y^3 + y^2 - 6y$
4. $x^3 - 3x^2 - 28x$
5. $3x^2 - 42 - 15x$
6. $18x - 15x^2 + 3x^3$
7. $-2x^2 - 2x + 40$
8. $-x^2 + 2x + 63$
9. $a^2b^2 - 24ab^2 + 143b^2$
10. $2a^4 - 24a^2b^2 - 26b^4$

Lesson (2) : Factorizing quadratic trinomial Part (2)

To factorize the trinomial : $a x^2 + b x + c$ where $(a \neq \pm 1)$, we do as follows :

1 Factorize : $a x^2$ into two factors : « $l x$, $m x$ » and write them inside two parentheses as shown in the opposite figure.



2 Factorize the last term in the trinomial (c) into two factors : « n and h » and write them as shown in the previous parentheses.

3 Find : «The product of extremes (outer terms) + the product of means (inner terms) » If the sum equals the middle term in the trinomial , then the factorization is true. If not , then the factorization is false hence , we should try again to get the true factorization.

Remark

- If the sign of the last term in the trinomial is positive , then the sign of the second term in each of the parentheses is the same as the sign of the middle term in the trinomial.
- If the sign of the last term of the trinomial is negative , then the two signs of the second term in each of the parentheses are different.

Example 2

Factorize each of the following expressions :

1 $2x^2 - x - 6$

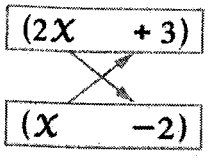
2 $6a - 27 + 5a^2$

3 $14x^2 - 11xy - 15y^2$

4 $48x^3 - 112x^2 - 20x$

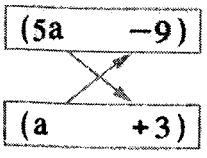
Solution

1 $2x^2 - x - 6 = (2x + 3)(x - 2)$



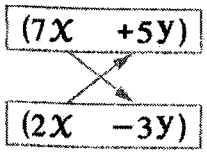
2 $6a - 27 + 5a^2 = 5a^2 + 6a - 27$

$= (5a - 9)(a + 3)$



3 $14x^2 - 11xy - 15y^2$

$= (7x + 5y)(2x - 3y)$

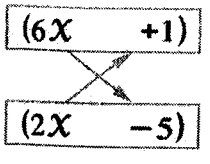


4 We notice that there is H.C.F. among the terms of the trinomial , then we should take it out « H.C.F. is $4x$ »

$\therefore 48x^3 - 112x^2 - 20x$

$= 4x(12x^2 - 28x - 5)$

$= 4x(6x + 1)(2x - 5)$



Exercises on Lesson (2)

[A] Choose the correct answer from the gives ones :

(1) If $(2x + 7)$ is a factor of the expression : $2x^2 - 3x - 35$, then the other factor is .....

(a) $x - 5$ (b) $x + 9$ (c) $x + 1$ (d) $x - 3$

[B] Complete the following :

1 $2x^2 - x - 6 = (2x + ...)(..... - 2)$

2 $3x^2 - 7x + 2 = (x - .....)(.....)$

3 $6x^2 - 11x + 3 = (.....)(.....)$

4 $5x^2 - 2x - 7 = (5x - .....)(x .....)$

5 If $(x - 1)$ is a factor of the expression : $7x^2 - 2x - 5$, then the other factor is .....

[C] Essay Problems :

1 Factorize each of the following expressions :

1 $2x^2 + 3x + 1$
2 $3a^2 + 7a + 2$
3 $5z^2 - 7z + 2$
4 $3x^2 - 10x + 7$
5 $3x^2 - 14x - 5$
6 $5x^2 + 4x - 12$
7 $3x^2 + 10x + 8$
8 $8x^2 + 14x + 5$
9 $6x^2 - 11x + 3$
10 $5a^2 - 18a + 16$
11 $3y^2 + 7y - 6$
12 $8z^2 + 2z - 3$
13 $4y^2 + 5y - 21$
14 $12a^2 - a - 6$

2 Factorize each of the following expressions :

1 $2x^2 - 5xy + 2y^2$
2 $3x^2 - 20xy - 7y^2$
3 $6a^2 + 5ab + b^2$
4 $2y^2 + yx - x^2$
5 $10a^2 + 11ab - 18b^2$
6 $6x^2 - 47xy - 63y^2$
7 $7x^4 + 23x^2y - 30y^2$

Lesson (3) : Part (3) : The perfect square

The perfect square trinomial has the following properties :

1. The first term is a perfect square and it is always positive.
2. The third term is a perfect square and it is positive also.
3. The middle term $= \pm 2\sqrt{1^{st}\text{term}} \times \sqrt{3^{rd}\text{term}}$

Remark

If the trinomial is a perfect square , then :

1. The middle term $= \pm 2 \times \sqrt{\text{the first term}} \times \sqrt{\text{the third term}}$
2. The first term $= \dfrac{(\text{the middle term})^2}{4 \times \text{the third term}}$
3. The third term $= \dfrac{(\text{the middle term})^2}{4 \times \text{the first term}}$

Example 2

Complete by the missing term in each of the following trinomials to be a perfect square :

1. $49\,x^2 \ldots\ldots + 25$
2. $16\,a^2 \ldots\ldots + 1$
3. $25\,x^2 - 60\,x + \ldots\ldots$
4. $\ldots\ldots + 12\,x\,y + 9\,y^2$

Solution

1. The middle term $= \pm 2 \times \sqrt{1^{st}\text{term}} \times \sqrt{3^{rd}\text{term}}$
$= \pm 2 \times \sqrt{49\,x^2} \times \sqrt{25} = \pm 2 \times 7\,x \times 5 = \pm 70\,x$

2. The middle term $= \pm 2 \times \sqrt{1^{st}\text{term}} \times \sqrt{3^{rd}\text{term}}$
$= \pm 2 \times \sqrt{16\,a^2} \times \sqrt{1} = \pm 2 \times 4\,a \times 1 = \pm 8\,a$

3. The third term $= \dfrac{(\text{the middle term})^2}{4 \times \text{the first term}} = \dfrac{(-60\,x)^2}{4 \times 25\,x^2} = \dfrac{3600\,x^2}{100\,x^2} = 36$

4. The first term $= \dfrac{(\text{the middle term})^2}{4 \times \text{the third term}} = \dfrac{(12\,x\,y)^2}{4 \times 9\,y^2} = \dfrac{144\,x^2\,y^2}{36\,y^2} = 4\,x^2$

Example 3

Find the positive value of c which makes each of the following trinomials a perfect square :

1. $x^2 + c\,x + 25$
2. $c^2\,y^2 - 90\,y + 81$

Solution

1. $\because c\,x = \pm 2\sqrt{x^2} \times \sqrt{25} = \pm 2 \times x \times 5 = \pm 10\,x$

$\therefore c = \pm 10$, $\because$ c is positive $\therefore c = 10$

2 $\because$ The 1st term $= \dfrac{(\text{the middle term})^2}{4 \times \text{the third term}}$ $\therefore c^2 y^2 = \dfrac{(-90\,y)^2}{4 \times 81}$ $\therefore c^2y^2 = \dfrac{8100\,y^2}{4 \times 81}$

$\therefore c^2 y^2 = 25 y^2$ $\therefore c^2 = 25$

$\therefore c = \pm 5$, $\because$ c is positive $\therefore c = 5$

Factorizing the perfect square trinomial

If the trinomial is a perfect square , then we can factorize it to be in the form :

$\left(\sqrt{\text{The first term}} \pm \sqrt{\text{The third term}}\right)^2$

Notice that : The sign between the two terms inside the parentheses is the same sign of the middle term in the trinomial after ordering its terms descendingly or ascendingly according to the exponents of one of its symbols.

The following example shows that.

Example 4

Factorize each of the following trinomials :

1 $25 a^2 + 20 a + 4$ **2** $16 x^2 - 24 x + 9$

3 $25 a^4 - 90 a^2 b + 81 b^2$ **4** $\frac{1}{9} x^2 + \frac{1}{3} x + \frac{1}{4}$

5 $18 x^2 - 48 x + 32$ **6** $28 x - 49 x^2 - 4$

Solution

After checking that each of the trinomials is a perfect square , we can factorize directly as follows :

1 $25 a^2 + 20 a + 4 = \left(\sqrt{25 a^2} + \sqrt{4}\right)^2 = (5 a + 2)^2$

2 $16 x^2 - 24 x + 9 = \left(\sqrt{16 x^2} - \sqrt{9}\right)^2 = (4 x - 3)^2$

3 $25 a^4 - 90 a^2 b + 81 b^2 = \left(\sqrt{25 a^4} - \sqrt{81 b^2}\right)^2 = (5 a^2 - 9 b)^2$

4 $\frac{1}{9} x^2 + \frac{1}{3} x + \frac{1}{4} = \left(\sqrt{\frac{1}{9} x^2} + \sqrt{\frac{1}{4}}\right)^2 = \left(\frac{1}{3} x + \frac{1}{2}\right)^2$

5 $18 x^2 - 48 x + 32 = 2 (9 x^2 - 24 x + 16)$

$= 2 \left(\sqrt{9 x^2} - \sqrt{16}\right)^2$

$= 2 (3 x - 4)^2$

Notice that :

The H.C.F. should be taken out before factorization.

6 $28x - 49x^2 - 4$

$= -49x^2 + 28x - 4$

$= -(49x^2 - 28x + 4)$

$= -(7x - 2)^2$

Notice that :

$-49x^2 + 28x - 4$ is not a perfect square while $49x^2 - 28x + 4$ is a perfect square.

Example 5

Use factorization to facilitate getting the value of each of the following :

1 $(55)^2 + 2 \times 55 \times 45 + (45)^2$ **2** $(312)^2 - 2 \times 312 \times 311 + (311)^2$

Solution

1 $(55)^2 + 2 \times 55 \times 45 + (45)^2 = \left(\sqrt{(55)^2} + \sqrt{(45)^2}\right)^2$

$= (55 + 45)^2 = (100)^2 = 10000$

2 $(312)^2 - 2 \times 312 \times 311 + (311)^2 = \left(\sqrt{(312)^2} - \sqrt{(311)^2}\right)^2$

$= (312 - 311)^2 = 1^2 = 1$

Exercises on Lesson (3)

[A] Choose the correct answer from the gives ones :

1 The expression : $x^2 - 6x + k$ is a perfect square when k = ...

(a) 36 (b) 12 (c) 3 (d) 9

2 The expression : $x^2 - 8x + k$ is a perfect square when k = ...

(a) -16 (b) 16 (c) -8 (d) 8

3 If the expression : $9x^2 + 24xy - ky^2$ is a perfect square , then k = ...

(a) 4 (b) 16 (c) ± 16 (d) -16

4 $x^2 + kx + 1$ is a perfect square when k = ...

(a) 1 (b) 2 (c) 3 (d) 4

5 The expression : $x^2 + kx + 9$ is a perfect square if k = ...

(a) 8 (b) 10 (c) ± 6 (d) 3

6 If $4x^2 + kx + 9$ is a perfect square , then k = ...

(a) 72 (b) 6 (c) 12 (d) 36

7 If the expression : $25x^2 + k + 49$ is a perfect square , then k = ...

(a) $\pm 35x$ (b) $\pm 70x$ (c) $\pm 24x$ (d) $\pm 14x$

8 If $4x^2 + kxy + 25y^2$ is a perfect square , then k = ...

(a) 25 (b) 10 (c) ± 20 (d) 30

9 The expression : $kx^2 + 24x + 16$ is a perfect square if k = ...

(a) 2 (b) 3 (c) 4 (d) 9

10 If $(3x - 2)(2x - 5) = 6x^2 + kx + 10$, then k = ...

(a) 19 (b) 4 (c) – 19 (d) 15

11 If the expression : $kx^2 + 4x + 1$ is a perfect square trinomial , then k = ...

(a) 1 (b) 2 (c) 3 (d) 4

12 The expression : $25x^2 +$ ..... $+ 4$ is a perfect square.

(a) $4x$ (b) $10x$ (c) $5x$ (d) $20x$

13 If $x^2 + y^2 = 17$, $xy = 5$, then $(x - y)^2 =$ .....

(a) 7 (b) 17 (c) 12 (d) 5

14 If $(x + y)^2 = 64$ and $xy = 15$, then $x^2 + y^2 =$ .....

(a) 8 (b) – 34 (c) 34 (d) 49

15 If $xy = 16$ $x^2 + y^2 = 4$, then the value of : $(4x + y)^2 =$ .....

(a) 25 (b) 36 (c) 5 (d) 4

[B] Complete the following :

1 If $(x - y)^2 = 36$ and $xy = 10$, then $x^2 + y^2 =$ .....

2 If $(x + y)^2 = 64$, $xy = 15$, then $x^2 + y^2 =$ .....

3 If $ab = -2$, $a^2 + b^2 = 5$, then the value of : $(a + b)^2 =$ .....

4 $4x^2 - 20xy + 25y^2 = ($ ..... $)^2$

5 If $x^2 + m x + 9$ is a perfect square , then m = ...

[C] Essay Problems :

1 Factorize each of the following :

1. $m^2 - 2m + 1$
2. $x^2 + 2xy + y^2$
3. $9x^2 + 12x + 4$
4. $25b^2 - 10b + 1$
5. $9a^2 + 6ab + b^2$
6. $4x^2 - 4xy + y^2$
7. $16a^2 - 40ab + 25b^2$
8. $1 + 14x + 49x^2$
9. $36 - 60k + 25k^2$
10. $1 - 10a^2 + 25a^4$

2 Factorize each of the following :

1. $18y^2 - 12y + 2$
2. $2x^2 + 8xy + 8y^2$
3. $12x^2 + 36xy + 27y^2$
4. $24a^4 + 24a^2 + 6$
5. $6a^4 - 12a^2b^2 + 6b^4$
6. $20ay^2 - 60ay + 45a$
7. $24x + 24x^2 + 6x^3$
8. $3z + 42z^4 + 147z^7$
9. $4b^2c + bc^2 + 4b^3$
10. $60ab - 36a^2 - 25b^2$

3 Find the positive value of k which makes each of the following trinomial expressions a perfect square :

1. $36x^2 + kx + 1$
2. $16y^2 + ky + 100$
3. $4x^2 + x + k$
4. $9\ell^2 + 12\ell + k$
5. $kx^2 - 6x + 1$
6. $ka^2b^2 - 12ab + 9$
7. $k^2x^2 - 4x + 16$
8. $1 + 14y + k^2y^2$

4 Use factorization to get the value of each of the following easily :

1. $(87)^2 + 2 \times 13 \times 87 + (13)^2$
2. $(99)^2 - 2 \times 99 \times 98 + (98)^2$
3. $(7.3)^2 + 2 \times 7.3 \times 2.7 + (2.7)^2$
4. $(20.7)^2 - 1.4 \times 20.7 + (0.7)^2$
5. $(997)^2 + 6 \times 997 + 9$
6. $(99)^2 + 2 \times 99 + 1$
7. $25 - 2 \times 45 + 81$

Lesson (4) : Factorizing The Difference Of Two Squares

We know that : $(a + b)(a - b) = a^2 - b^2$ **i.e.** $a^2 - b^2 = (a + b)(a - b)$

The expression : $a^2 - b^2$ is the difference of two squares of the two quantities a and b , therefore it is called the difference of two squares.

Then we deduce that :

The difference of two squares of two quantities
= (the sum of the two quantities) × (the difference of the two quantities)

Example 1

Factorize each of the following :

1. $x^2 - 25$
2. $x^2 - 9y^2$
3. $49x^4 - 1$
4. $\frac{1}{9}a^2 - \frac{1}{4}$

Solution

1. $x^2 - 25 = (x + 5)(x - 5)$
2. $x^2 - 9y^2 = (x + 3y)(x - 3y)$
3. $49x^4 - 1 = (7x^2 + 1)(7x^2 - 1)$
4. $\frac{1}{9}a^2 - \frac{1}{4} = (\frac{1}{3}a + \frac{1}{2})(\frac{1}{3}a - \frac{1}{2})$

Example 2

Factorize each of the following :

1. $2x^2 - 18$
2. $x^3 - 64x$
3. $\frac{1}{2}x^2 - 2$
4. $16x^4 - 81$

Solution

1. $2x^2 - 18 = 2(x^2 - 9)$ (Taking out the H.C.F.)
   $= 2(x - 3)(x + 3)$
2. $x^3 - 64x = x(x^2 - 64)$ (Taking out the H.C.F.)
   $= x(x - 8)(x + 8)$
3. $\frac{1}{2}x^2 - 2 = \frac{1}{2}(x^2 - 4) = \frac{1}{2}(x - 2)(x + 2)$
4. $16x^4 - 81 = (4x^2 + 9)(4x^2 - 9) = (4x^2 + 9)(2x - 3)(2x + 3)$

Example 3

Use factorization to get the value of each of the following easily :

1 $(25)^2 - (15)^2$

2 $(1.6)^2 - (1.4)^2$

3 $(99)^2 - 1$

4 52×48

Solution

1 $(25)^2 - (15)^2 = (25 - 15)(25 + 15) = 10 \times 40 = 400$

2 $(1.6)^2 - (1.4)^2 = (1.6 - 1.4)(1.6 + 1.4) = 0.2 \times 3 = 0.6$

3 $(99)^2 - 1 = (99 + 1)(99 - 1) = 100 \times 98 = 9800$

4 $52 \times 48 = (50 + 2)(50 - 2) = (50)^2 - (2)^2 = 2500 - 4 = 2496$

Exercises on Lesson (4)

[A] Choose the correct answer from the gives ones :

1 If $x^2 - y^2 = 16$, $x + y = 8$, then $x - y =$ .....

(a) 64 (b) 128 (c) 2 (d) 1

2 If $x^2 - y^2 = 12$, $y - x = 3$, then $x + y =$ .....

(a) -4 (b) 4 (c) 12 (d) -3

3 If $x^2 - y^2 = 28$, $x + y = 7$, then $x - y =$ .....

(a) 4 (b) -4 (c) 21 (d) -21

4 If $x^2 - 4y^2 = 24$ and $x - 2y = 8$, then $x + 2y =$ .....

(a) 3 (b) 16 (c) 36 (d) 6

5 If $x + y = 5$ and $x^2 - y^2 = 15$, then $x - y =$ .....

(a) 3 (b) 5 (c) 15 (d) 10

6 If $x - y = 5$ and $x + y = 6$, then $x^2 - y^2 =$ .....

(a) 11 (b) 20 (c) 30 (d) 40

7 If $x + y = 12$ and $x - y = 6$, then $x^2 - y^2 =$ .....

(a) 18 (b) 6 (c) 72 (d) -6

8 If $a + b = 3$, $a - b = 5$, then $a^2 - b^2 =$ .....

(a) 8 (b) 15 (c) 2 (d) $\frac{5}{3}$

9 $x^2 + a = (x - 4)(x + 4)$, then $a =$ .....

(a) 4 (b) -4 (c) 16 (d) -16

10 $(99)^2 - 1 =$ .....

(a) 890 (b) 990 (c) 9900 (d) 9800

[B] Complete the following :

1 If $x + y = 7$, $x - y = 1$, then $x^2 - y^2 =$ .....

2 If $x^2 + a = (x - 3)(x + 3)$, then $a =$ .....

3 If $x^2 - y^2 = 27$ and $x - y = 3$, then $x + y =$ .....

4 If $x^2 - y^2 = 24$ and $x + y = 8$, then $x - y =$ .....

5 $(75)^2 - (25)^2 = 100 \times$ .....

6 $(75)^2 - (25)^2 = 100 \times$ ..... = ...

7 $(73)^2 - (27)^2 =$ .....

8 $(101)^2 - 1 =$ ..... $\times$ ..... = ... (use factorizing)

[C] Essay Problems :

1 Factorize each of the following :

1. $x^2 - 4$
2. $a^2 - 25$
3. $16x^2 - 9$
4. $49y^2 - 1$
5. $x^2 - 4y^2$
6. $225x^2 - y^2$
7. $49x^2 - 64y^2$
8. $625a^2 - 81b^2$
9. $9 - y^2$
10. $-9x^2 + 25$
11. $a^2b^2 - 1$
12. $a^2 - b^2c^4$
13. $x^4 - 100$
14. $16a^6 - b^6$
15. $x^2 - \frac{1}{16}$
16. $\frac{1}{9}y^2 - 2\frac{1}{4}$
17. $\frac{a^2}{25} - \frac{4b^2}{49}$
18. $0.04x^2 - 0.25y^2$

2 **Factorize each of the following :**

1 $2x^2 - 32$	2 $3y^2 - 27$	3 $x^3 - 25x$
4 $x^4 - x^2$	5 📖 $8x^2 - 50$	6 $2x^3 - 72x$
7 $x^3y - xy^5$	8 📖 $27x^3 - 48xy^6$	9 $\frac{1}{3}x^2 - 3$

3 **Use factorization to get the value of each of the following easily :**

1 📖 $(77)^2 - (23)^2$	2 $(75)^2 - (25)^2$	3 $(78)^2 - (77)^2$
4 $(125)^2 - (25)^2$	5 $(11.6)^2 - (1.6)^2$	6 $(8.27)^2 - (1.73)^2$

Lesson (5) : Factorizing The Sum and difference Of Two Cubes

Hence we deduce that :

The sum of two cubes of two quantities =

(the first + the second) (the square of the first – the first × the second + the square of the second)

i.e. $a^3 \oplus b^3 = (a + b)(a^2 \ominus ab + b^2)$

For example :

$x^3 + 8 = x^3 + 2^3 = (x + 2)(x^2 - x \times 2 + 2^2)$

$= (x + 2)(x^2 - 2x + 4)$

Hence we deduce that :

The difference between two cubes of two quantities =

(the first – the second) (the square of the first + the first × the second + the square of the second)

i.e. $a^3 \ominus b^3 = (a - b)(a^2 \oplus ab + b^2)$

For example : $x^3 - 27 = x^3 - 3^3 = (x - 3)(x^2 + x \times 3 + 3^2) = (x - 3)(x^2 + 3x + 9)$

Example 1

Factorize each of the following perfectly :

1 $8x^3 + 125$ **2** $27a^3 - b^3$ **3** $8x^3 + \frac{1}{8}$

4 $a^6 - 64b^3$ **5** $40x^4 - 5x$ **6** $(x + y)^3 + x^3$

Solution

1 $8x^3 + 125 = (2x)^3 + (5)^3 = (2x + 5)\left((2x)^2 - 5 \times 2x + 5^2\right)$

$= (2x + 5)(4x^2 - 10x + 25)$

2 $27a^3 - b^3 = (3a)^3 - b^3 = (3a - b)\left((3a)^2 + 3a \times b + b^2\right)$

$= (3a - b)(9a^2 + 3ab + b^2)$

3 $8x^3 + \frac{1}{8} = (2x)^3 + \left(\frac{1}{2}\right)^3 = \left(2x + \frac{1}{2}\right)\left((2x)^2 - 2x \times \frac{1}{2} + \left(\frac{1}{2}\right)^2\right)$

$= \left(2x + \frac{1}{2}\right)\left(4x^2 - x + \frac{1}{4}\right)$

4 $a^6 - 64b^3 = (a^2)^3 - (4b)^3 = (a^2 - 4b)\left((a^2)^2 + a^2 \times 4b + (4b)^2\right)$

$= (a^2 - 4b)(a^4 + 4a^2b + 16b^2)$

5 $40x^4 - 5x = 5x(8x^3 - 1)$ (Taking out the H.C.F.)

$= 5x(2x - 1)(4x^2 + 2x + 1)$

Exercises on Lesson (5)

[A] Choose the correct answer from the gives ones :

1 If $x^3 + y^3 = 21$, $x^2 - xy + y^2 = 3$, then $x + y =$ .....

(a) 3 (b) 21 (c) -7 (d) 7

2 If $a^3 - b^3 = 18$ and $a^2 + ab + b^2 = 6$, then $a - b =$ .....

(a) 108 (b) 24 (c) 3 (d) 12

[B] Complete the following :

1 $x^3 + 8 = (x +$ .....$)(x^2 -$ ..... $+ 4)$

2 $x^3 - m^3 = (x - m)(x^2 + 4x + m^2)$, then $m =$ .....

3 $x^3 + 125 = (x + 5)(x^2$ ..... $)$

4 $8x^3 - y^3 = ($ ..... $- y)(4x^2 +$ ..... $+ y^2)$

5 $(a - 2)(a^2 + 2a + 4) =$ .....

6 If $(x - y)$ is a factor of $(x^3 - y^3)$, then the other factor is .....

[C] Essay Problems :

1 **Factorize each of the following :**

1 📖 $x^3 + 8$	2 $x^3 - 1$	3 $64x^3 + 27$
4 📖 $8x^3 - 125$	5 $125 + a^3$	6 $343 - 27m^3$
7 📖 $m^3 + 64n^3$	8 📖 $512x^3 - y^3$	9 $x^3y^3 + 27$
10 $27x^3y^3 - 64$	11 📖 $\frac{1}{8}a^3 - 8b^3$	12 $\ell^3 - \frac{1}{125}$
13 $8a^3 + 0.001$	14 📖 $0.027m^3 - n^3$	15 $1 + 125b^6$
16 📖 $8x^3 - 343y^6$	17 $x^6 + y^6$	18 $x^6 - 64$

2 **Factorize each of the following perfectly :**

1 $2x^3 + 16$	2 $3x^3 - 81$	3 $\ell^4 + 64\ell$
4 📖 $\ell^3 m - 27m^4$	5 $3x^4 + 3x$	6 $2x^5 - 54x^2$
7 $16x^3 + 250y^3$	8 📖 $16a^3b + 686b^4$	9 $54x^4y^2 - 16xy^5$

Lesson (6) : Factorizing By Grouping

Example 1

Factorize : $2a^2 - 2b + ab - 4a$

Solution

Let us divide the expression as follows : $2a^2 - 2b + ab - 4a = (2a^2 - 2b) + (ab - 4a)$

Taking out the H.C.F. between the terms of each of : $2a^2 - 2b$ and $ab - 4a$

, we find that the main expression $= 2(a^2 - b) + a(b - 4)$, then we notice that there is no common factors between $2(a^2 - b)$ and $a(b - 4)$, then we should regroup the main expression by another way as follows :

$2a^2 - 2b + ab - 4a = (2a^2 + ab) + (-2b - 4a)$ (Commutative and associative properties)

$= a(2a + b) - 2(b + 2a)$

$= a(2a + b) - 2(2a + b)$

Notice that : $b + 2a = 2a + b$

We notice here that there is a common factor which is $(2a + b)$, then we complete factorization by taking out the common factor to be $2a^2 - 2b + ab - 4a = (2a + b)(a - 2)$

Exercises on Lesson (6)

[A] Complete the following :

1) $ax + ay + bx + by = (x + y)$ (.....)

2) $ax + bx + 5a + 5b = (a + b)$ (.....)

[B] Essay Problems :

1 Factorize each of the following perfectly :

1. $ax + bx + ay + by$
2. $ab - bd + ah - dh$
3. $ax + yx + y + a$
4. $am - an + m - n$
5. $ax - cy - cx + ay$
6. $mx - my - nx + ny$
7. $xy + 5y + 7x + 35$
8. $7x - 28 + ax - 4a$
9. $5\ell - 10m - a\ell + 2am$
10. $3ax - a - 6bx + 2b$

2 Factorize each of the following perfectly :

1. $c^2 + cd + dh + ch$
2. $6m^2 - n + 2m - 3mn$
3. $8mn - 2m^2 + 12n\ell - 3m\ell$
4. $x^2 - 2xz - 2xy + 4yz$
5. $a^2 + 2ab + b^2 - c^2$
6. $25x^2 - 10x + 1 - y^2$

Lesson (7) : Factorizing By Completing the square

Example 1

Factorize each of the following expressions :

1 $4x^4 + y^4$ **2** $x^8 - 16$

Solution

1 Add to the given expression : $2 \times \sqrt{4x^4} \times \sqrt{y^4}$ ***i.e.*** $4x^2y^2$

, then we subtract it again in order not to change the main expression.

$$\therefore 4x^4 + y^4 = 4x^4 + y^4 + (4x^2y^2 - 4x^2y^2)$$

$$= \underbrace{(4x^4 + 4x^2y^2 + y^4)}_{\text{A perfect square trinomial}} - \underbrace{4x^2y^2}_{\text{A perfect square monomial}}$$ (Commutative and associative properties)

$$= (2x^2 + y^2)^2 - (2xy)^2$$

$$= (2x^2 + y^2 - 2xy)(2x^2 + y^2 + 2xy)$$

(Factorization of the difference between two squares)

$$= (2x^2 - 2xy + y^2)(2x^2 + 2xy + y^2)$$

(Ordering the terms of each expression)

2 $\because x^8 - 16 = (x^4 - 4)(x^4 + 4)$ (Difference between two squares) (1)

, $\because (x^4 - 4)$ Can be factorized as a difference between two squares as follows :

$$x^4 - 4 = (x^2 - 2)(x^2 + 2) \quad (2)$$

, $\because (x^4 + 4)$ Can be factorized by completing the perfect square as follows :

Add : $2 \times \sqrt{x^4} \times \sqrt{4}$ ***i.e.*** $4x^2$, then subtract it again

$$\therefore x^4 + 4 = x^4 + 4 + 4x^2 - 4x^2$$

$$= \underbrace{(x^4 + 4x^2 + 4)}_{\text{A perfect square trinomial}} - \underbrace{4x^2}_{\text{A perfect square monomial}}$$ (Commutative and associative properties)

$$= (x^2 + 2)^2 - (2x)^2 = (x^2 + 2 - 2x)(x^2 + 2 + 2x)$$

(The difference between two squares) (3)

From (1) , (2) and (3)

$$\therefore x^8 - 16 = (x^2 - 2)(x^2 + 2)(x^2 - 2x + 2)(x^2 + 2x + 2)$$

Exercises on Lesson (7)

[A] Complete the following :

1 $x^4 + 4 = (x^2 + 2)^2 -$ .....

[B] Essay Problems :

1 Factorize each of the following perfectly :

1. $x^4 + 4$
2. $x^4 + 64$
3. $x^4 + 4y^4$
4. $x^4 + 64y^4$
5. $a^4 + 2500b^4$
6. $81x^4 + 4z^4$
7. $4x^4 + 625z^4$
8. $64x^4 + 81y^4$

2 Factorize each of the following completely :

1. $9x^4 + 2x^2 + 1$
2. $x^4 - 28x^2 + 16$
3. $x^4 + 9x^2 + 81$
4. $9x^4 - 25x^2 + 16$
5. $x^4 + 3x^2y^2 + 4y^4$
6. $m^4 - 11m^2n^2 + n^4$

General Exercises on Factorization

1 **Factorize each of the following completely :**

(1) $3x^2 - 12$ (2) $b^3 + b^2 - b - 1$

(3) $x^2 + 5xy - 24y^2$ (4) $x^3 - 27$

---

2 **Factorize :**

(1) $5x - 10y - ax + 2ay$ (2) $27x^3 - 64$

(3) $5x^2 - 20$

---

3 **Factorize completely :**

(1) $a^2 - b^2 - a + b$ (2) $x^2 - 10x + 16$

---

4 **Factorize each of the following completely :**

(1) $x^6 + 1$ (2) $x^4 + 64$

(3) $99x^2 - x - 100$ (4) $2ab - 10a + 3b - 15$

---

5 **[a] Factorize :**

(1) $3x^2 + 13x + 12$ (2) $xy + 30 + 5y + 6x$

(3) $2x^4 - 16x$

[b] Use factorization to find the value of : $(77)^2 - (23)^2$

---

6 **Factorize each of the following expressions completely :**

(1) $2x^2 + 5x + 3$ (2) $x^2 + xz + xy + zy$

(3) $2x^2 - 50$ (4) $4a^4 + 1$ (by completing the square)

---

7 **Factorize the following expressions :**

(1) $\frac{4}{9}x^2 - 49$ (2) $x^3 - 3x^2 - 10x$

(3) $2ab - 10a + 3b - 15$ (4) $x^4 - 81$

---

8 **Factorize each of the following :**

(1) $x^4 + 4l^4$ (2) $x^3 + 27y^3$

(3) $x^2 - 6x + 5$ (4) $25x^2 - 49$

(5) $5ab + 10a + 3b + 6$

9 **Factorize :**

(1) $x^4 - 64$ (2) $3x^3 - 81$ (3) $6x^2 - 7x - 3$ (4) $2ab - 10a + 3b - 15$

---

10 **Factorize each of the following :**

(1) $2x^3 - 54$ (2) $ax + bx + ay + by$ (3) $x^2 - 7x + 12$

(4) $2x^2 + 7x - 4$ (5) $4x^2 - 9$

---

11 **Factorize each of the following expressions :**

(1) $3x^2 + 10x + 8$ (2) $x^2 - 81$

(3) $x^3 - 8$ (4) $x^4 + 64y^4$

---

12 **Use factorization to evaluate :** $(65)^2 + 2 \times 65 \times 35 + (35)^2$

---

13 **Factorize each of the following :**

(1) $x^2 - 5x - 14$ (2) $2x^2 - 5x + 3$

(3) $x^3 - 125$ (4) $ax + bx + ay + by$

---

14 **Factorize :** $y^4 + 4x^4$

---

15 **Factorize the following :**

(1) $x^3 - 125y^3$ (2) $2xy - 10x + 3y - 15$ (3) $x^8 - 16y^8$

---

16 **Evaluate using the factorization :**

(1) $(76)^2 + 2 \times 24 \times 76 + (24)^2$ (2) $(75)^2 - (25)^2$

---

17 **Factorize each of the following expressions :**

(1) $x^2 - x - 12$ (2) $2a^2 - 32$

(3) $2x^3 + 54$ (4) $x^3 - 9x + x^2 - 9$

---

18 **Factorize each of the following :**

(1) $x^2 - 10x + 25$ (2) $3x^2 + 7x - 6$ (3) $2x^2 - 8$ (4) $x^3 - 27$

Lesson (8) : Solving Quadratic Equations in one Variable

Fact

If a and b are two real numbers and if $a \times b = \text{zero}$, then $a = 0$ or $b = 0$

For example :

- If $x(x-3)=0$,
  then $x=0$
  or $x-3=0$, then $x=3$

- If $(x+2)(3x-5)=0$,
  then $x+2=0$ ***i.e.*** $x=-2$
  or $3x-5=0$ ***i.e.*** $x=\frac{5}{3}$

For solving the quadratic equation in one variable using factorization , we do as follows :

1. Make one of its sides equal zero , let it be the right hand side.
2. Simplify the expression if needed to put the equation in the form : $ax^2+bx+c=0$
3. Factorize the left side to two factors to get the values of x

The following examples show the steps which we mention previously.

Example 1

Find in $\mathbb{R}$ the solution set of each of the following equations :

1. $x^2-5x-6=0$
2. $2x^2+7x=0$
3. $x^2-6x=-9$
4. $(x+2)^2=25$

Solution

1 $\because x^2-5x-6=0$ $\therefore (x-6)(x+1)=0$ (Factorization of a trinomial)

$\therefore$ either $x-6=0$, then $x=6$ or $x+1=0$, then $x=-1$

$\therefore$ S.S. $=\{6,-1\}$

We can check the truth of solutions by substituting by each value of x in the main equation as follows :

- at $x=6$

  The left hand side $=6^2-5\times6-6=36-30-6=0=$ the right hand side

  $\therefore x=6$ satisfies the equation

- at $x=-1$

  The left hand side $=(-1)^2-5(-1)-6=1+5-6=0=$ the right hand side

  $\therefore x=-1$ satisfies the equation.

2 $\because 2x^2+7x=0$ $\therefore x(2x+7)=0$ (H.C.F.)

$\therefore$ either $x=0$ or $2x+7=0$, then $2x=-7$

$\therefore x = -\frac{7}{2}$ $\therefore$ S.S. $= \{0 , -\frac{7}{2}\}$

3 $\because x^2 - 6x = -9$ $\therefore x^2 - 6x + 9 = 0$

$\therefore (x-3)^2 = 0$ (Perfect square trinomial)

$\therefore x - 3 = 0$ $\therefore x = 3$

$\therefore$ S.S. $= \{3\}$

4 $\because (x+2)^2 = 25$ $\therefore x^2 + 4x + 4 = 25$

$\therefore x^2 + 4x + 4 - 25 = 0$ $\therefore x^2 + 4x - 21 = 0$

$\therefore (x+7)(x-3) = 0$ (Factorizing the trinomial)

$\therefore$ either $x + 7 = 0$, then $x = -7$ or $x - 3 = 0$, then $x = 3$

$\therefore$ S.S. $= \{-7 , 3\}$

Another solution :

$\because (x+2)^2 = 25$ $\therefore (x+2)^2 - 25 = 0$

Using factorization of the difference between two squares.

$\therefore (x+2-5)(x+2+5) = 0$ $\therefore (x-3)(x+7) = 0$

$\therefore$ either $x - 3 = 0$, then $x = 3$ or $x + 7 = 0$, then $x = -7$

$\therefore$ S.S. $= \{3 , -7\}$

Another solution :

$\because (x+2)^2 = 25$ $\therefore x + 2 = \pm 5$ either $x + 2 = 5$, then $x = 3$

or $x + 2 = -5$ $\therefore x = -7$ $\therefore$ S.S. $= \{3 , -7\}$

Remark

From the previous example, we observe that the quadratic equation has at most two solutions (two roots).

Exercises on Lesson (8)

[A] Choose the correct answer from the gives ones :

1 The solution set of the equation : $x^2 - 5x = 0$ in $\mathbb{R}$ is .....

(a) $\{0 , -5\}$ (b) $\{0 , 5\}$ (c) $\{2 , 3\}$ (d) $\{-2 , -3\}$

2 The solution set of the equation : $2x - x^2 = 0$ in $\mathbb{R}$ is .....

(a) $\{2\}$ (b) $\{0 , -2\}$ (c) $\{2 , -2\}$ (d) $\{0 , 2\}$

3 The S.S. of the equation : $x^2 - x = 0$ in $\mathbb{R}$ is .....

(a) $\{0\}$ (b) $\{1\}$ (c) $\{0 , 1\}$ (d) $\{-1 , 0\}$

4 The S.S. of the equation : $x^2 + 4 = 0$, $x \in \mathbb{Q}$ is .....

(a) $\{2\}$ (b) $\{-2\}$ (c) $\{-2 , 2\}$ (d) $\varnothing$

5 If $x \in \mathbb{R}$, then the solution set of the equation : $x^2 + 25 = 0$ is .....

(a) $\{5 , -5\}$ (b) $\{25\}$ (c) $\{5\}$ (d) $\varnothing$

6 The S.S. in $\mathbb{R}$ of the equation : $x^2 = -4$ is .....

(a) $\{2\}$ (b) $\{2 , -2\}$ (c) $\{-2\}$ (d) $\varnothing$

7 The solution set of the equation : $x^2 - 25 = 0$ in $\mathbb{R}$ is .....

(a) $\varnothing$ (b) $\{5\}$ (c) $\{5 , -5\}$ (d) $\{25\}$

8 The S.S. in $\mathbb{R}$ of the equation : $x^2 - 16 = 0$ is .....

(a) $\{4 , -4\}$ (b) {zero} (c) $\{6\}$ (d) $\{4\}$

9 The S.S. of the equation : $x^2 - 9 = 0$ in $\mathbb{R}$ is .....

(a) $\varnothing$ (b) $\{3\}$ (c) $\{-3\}$ (d) $\{-3 , 3\}$

10 If the age of Khaled now is x years , then the square of his age after 2 years is .....

(a) $x^2 + 4x + 4$ (b) $x^2 + 2$ (c) $x^2 + 4$ (d) $(x - 2)^2$

[B] Complete the following :

1 The S.S. in $\mathbb{R}$ of the equation : $x(x^2 + 4) = 0$ is .....

2 The solution set of the equation : $2x(x - 3) =$ zero in $\mathbb{R}$ is .....

3 The S.S. of the equation : $x^2 - 2x = 0$ in $\mathbb{R}$ is .....

4 The solution set of the equation : $x^2 - 3x = 0$ in $\mathbb{R}$ is .....

5 The solution set of the equation : $x(x^2 - 9) = 0$ in $\mathbb{R}$ is .....

6 The solution set of the equation : $x^2 - 8x + 15 = 0$, $x \in \mathbb{R}$ is .....

[B] Essay Problems :

1 **Find the S.S. of the following equation in $\mathbb{R}$:** $x^2 + 2x = 0$

2 **Find the solution set in $\mathbb{R}$ for :** $x^2 + 5x - 6 = 0$

3 * **Find the solution set of the following equation in $\mathbb{R}$:** $x^2 + 3x - 40 = 0$

4 **If $x \in \mathbb{N}$, find the S.S. of the equation :** $3x^2 + 14 = 13x$

5 **Find the S.S. of the equation :** $x^2 - 5x = 14$ **in** $\mathbb{Q}$

6 **Find the S.S. of the equation :** $3x^2 + x = 14$ **in** $\mathbb{R}$

7 **Solve the equation :** $x^2 - 3x = 10$

8 **Find the solution set of the equation :** $x(x-3) = 10$, $x \in \mathbb{R}$

9 **Find in $\mathbb{R}$ the S.S. of the equation :** $(x+2)^2 + 3(x+2) = 0$

10 What is the real number if it is added to its square , the result will be 6 ?

11 Find the real number if it is added to its square , the result will be 12

12 What is the positive real number if it is added to its square , the result will be 42 ?

13 Find the positive rational number whose square is more than 5 times the number by 24

14 Two positive numbers , one of them is less than the other by 4 and their product is 21. Find the two numbers.

15 A rectangle whose area is 8 cm^2 and the length is 7 cm. more than its width. Calculate its perimeter.