Sets

In our daily life, we often talk of a collection of objects. For instance, your class, your baseball team, your family. They all denote collection.

A class is a collection of students.

A team is a collection of players.

A family is a collection of persons.

Now consider the following examples.

1. a, e, i, o, u
2. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$
3. New York, London, Paris, New Delhi
4. Beautiful women of your area.
5. Rich people of New Jersey.

All these are collections of certain objects such as, vowels, numbers (fractions), women, persons, metropolitan cities etc.

In examples 1, 2 and 3 we know the objects precisely. But in examples 4 and 5 the terms, beautiful women or rich people are relative terms. Hary may be clever by Pat's standard but may not be so clever by Liza's. So we can not decide whether to include Hary in our collection. Similarly ' Rich people' is also a relative term.

In Mathematics, a collection of well defined and distinct objects is called a set.

Methods of writing a set

We denote sets by capital (bold face) letters A, B, X, Y etc. and its elements by small case letters a, b, c etc.

There are two ways to describe a set

1) listing (Roster) method

2) Rule or set builder method

Listing method

In this method, we describe a set by listing all its elements enclosed in braces, the elements are separated by commas without any repetition.

For example

A = { a, e, i, o, u } , B = { 1, 2, 3, . . . 100 }

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N = { 1, 2, 3 . . . } , I = { ... -3, -2, -1, 0, 1, 2, 3, . . . }

Property method

In this method, we describe a set by specifying the properties which determines its elements uniquely.

For example

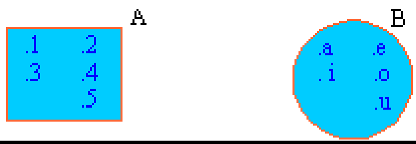
1. A is a set of all the days in a week and is written as A={x : x is a day in a week }
2. x = { x : x ∈ R and 5 < x <10 }
3. B = { x : x ∈ N and x is divisor of 70 = { 1, 2, 5, 7, 10, 14, 35, 70 }

Note that

1. The stroke (|) or colon (:) stands for 'such that'.
2. The symbol ∈ means belongs to . If x is an elements of the set A, then we write x ∈ A and read it as 'x belongs to A' or 'x is member of the set A' or 'x is an element of the set A'. Similarly the symbol ∉ means does not belong to.

Venn diagram

It is a way of pictorially describing sets. The concept was first introduced by Leonard Euler and then by Logician John Venn. We use rectangles, and circles to represent sets and one elements of the set are denoted by dots or points. For example,



Equal sets

Two sets are said to be equal if they contain exactly the same elements.

For example, if X = { x : X is letter in the word ' tea' }

and Y = { y : y is a letter in the 'eat' }

Then X = Y

Now consider A = { a, b, c } and B = { 1, 2, 3 } ⇒ A ≠ B

The empty set

Consider a set A = { a : a ∈ I, 5 < x < 6 } Now note that there is no integer between 5 and 6. Therefore A has no element. We call such a set as the empty, null or void set. Thus a set which has no member is an empty set. It is denoted by ϕ or { }. But a set { 0 } is not an empty set.

Finite and infinite set

Finite sets are those whose elements are countable. They have an end.

For example A = { 1, 2, 3, 4 }.

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Infinite sets are those whose elements are uncountable, they continue forever.

For example, N = { 1, 2, 3, . . . }

Subsets: A set obtained by taking some or all the elements of the given set is called a subset of the given set. If set A and B are two sets such that every element of set B is the element of the set A, then B is said to be the subset of set A. Symbolically it is written as $B \subset A$.

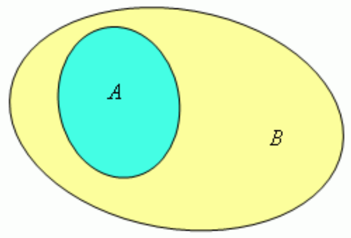


Fig.1

Now if B is a part of A i.e. B contains at least one element less than A then B is called the proper subset of A and show by $B \subset A$. But if A and B have exactly the same elements i.e. A = B then B is called the improper subset of A and it is written as $B \subseteq A$ or $A \subseteq B$

1. Thus every set is a subset of itself .
2. Every set has an empty set as its subset.

For example

1) A = { 1, 2, 3, 4, 5, 6, 7 } and B = { 1, 4, 5, 7 }
   Then $B \subset A$

Universal set Suppose A, B and C are given sets. Then any set of which A, B and C are subsets is called the universal set.

Note that an universal set is the main set or a set of totality of elements of all sets under consideration under a particular discussion. In the universal set once fixed cannot be changed during that discussion. Thus the universal set cannot be unique but changes from problem to problem. The universal set is usually denoted by $\cup$ or X.

For example ,

A = { All books of Algebra in your library }

B = { All books of Geometry in your library }

C = { All books in your library }

Here, the set C is taken as the universal set .

Union of sets

If two sets are given, a set can be formed by using all the elements of the two sets. Such a collection is called the union of the given sets.

Definition:

Union of two sets A and B is the set of all elements which are in A, or in B, or both in A and B.

In symbols we write, $A \cup B$ for the union of two sets A and B.

'$A \cup B$' is read as 'A union B' or 'A cup B'.

In Set builder form,

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

Examples:

(1) Let A = {3, 4, 5, 6}, B = {6, 7, 8} and C = {8, 9, 7}.

Then

$A \cup B = \{3, 4, 5, 6, 7, 8\}$

$B \cup C = \{6, 7, 8, 9\}$

$A \cup C = \{3, 4, 5, 6, 7, 8, 9\}$

(2) Let $A = \{x : x \in Z \quad x \geq 10\}$

$B = \{x : x \in Z \quad x \geq 20\}$

$A \cup B = \{x : x \in Z \quad x \geq 10\}$

Intersection of sets

A set can be formed by using all the common elements of two given sets. Such a set is called the intersection of the given sets.

Definition:

Intersection of two sets A and B is a set whose elements belong to both A and B.

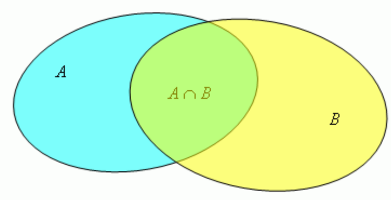


Fig 2

In symbols we write, $A \cap B$ for the intersection of two sets A and B.

$A \cap B$ is read as A intersection B.

In Set-builder form: $A \cap B = \{x : x \in A \text{ and } x \in B\}$

Examples:

(1) Let A = {3, 4, 5, 6}, B = {5, 6, 7}, C = {7, 8, 9}

then

$A \cap B = \{5, 6\} \qquad \because 5 \in A, 5 \in B, 6 \in A, 6 \in B$

$B \cap C = \{7\} \qquad \because 7 \in B, 7 \in A$

$A \cap C = \phi \qquad \because$ no elements are common

(2) Let $A = \{x : x \in Z, \; x \geq 10\} \qquad B = \{x : x \in Z, x \geq 20\}$

$A \cap B = \{x : x \in Z, x \geq 20\}$

Disjoint sets

Two sets A and B are said to be disjoint if their intersection is a null set.

i.e. $A \cap B = \phi \Rightarrow$ A and B are disjoint.

Example:

$A = \{1, 2, 3\}$ $B = \{4, 5, 6\}$

$A \cap B = \phi$

$\therefore$ A and B are disjoint sets.

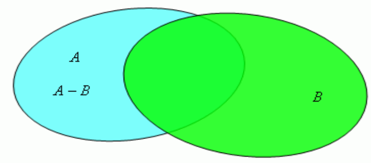


Fig 3

Difference of two sets (Relative complement)

The relative complement of set B in set A is the complement of B in A.

Definition:

If A and B are any two sets then the relative complement of B in A is the set of all elements in A which are not in B. It is denoted by A - B.

In sub-builder form, $A - B = \{x : x \in A \text{ and } x \notin B\}$

Alternate Definition:

Difference of two sets A and B, A - B is a set whose elements belong to A but not to B. A - B is called the relative complement of B w.r.t. A.

Example:

Let $A = \{a, b, c, d\}$ $B = \{c, a, e, f\}$

Then $A - B = \{b, d\}$ $\quad b, d \in A \quad b, d \notin B$

$B - A = \{e, f\}$ $\quad e, f \in B \quad e, f \notin A$

Observe that $(A - B) \cap (B - A) = \phi$

$A - B$ and $B - A$ are disjoint sets.

Symmetric Difference of two sets

If A and B are two sets, we define their symmetric difference as the set of all elements that belong to A or to B, but not both A and B, and we denote it by A D B. Thus

$A \Delta B = x \in A \text{ and } x \notin B \text{ or } x \in B \text{ and } x \notin A$

$= (A - B) \cup (B - A)$

Complement of a set

Let U be a universal set, A be any subset of U, then the elements of U which are not in A i.e., U - A is the complement of A w.r.t. U is written as $A' = U - A = A^c$.

$A' = \{x : x \in U \text{ and } x \notin A\} = A^c$

Examples:

Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

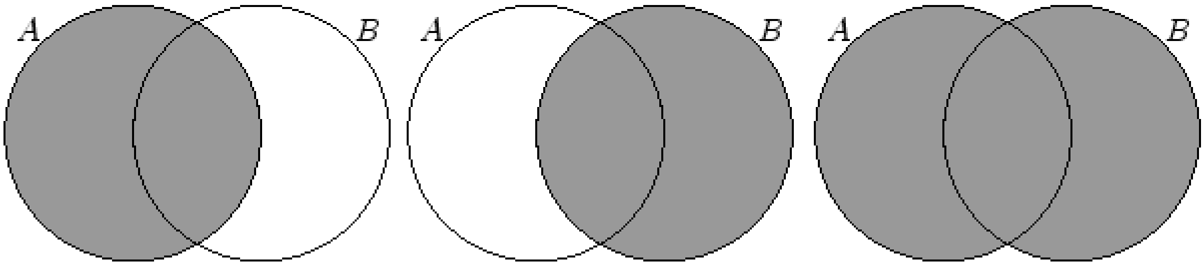
$A = \{1, 2, 4, 6, 9\}$

Then $A' = U - A = \{0, 3, 5, 7, 8\}$

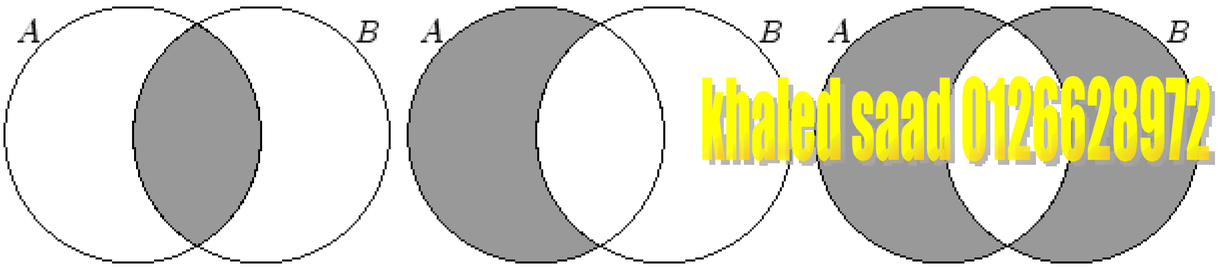
Properties of operations with sets:

E x :

The so-called Venn-diagrams illustrate the set operations.

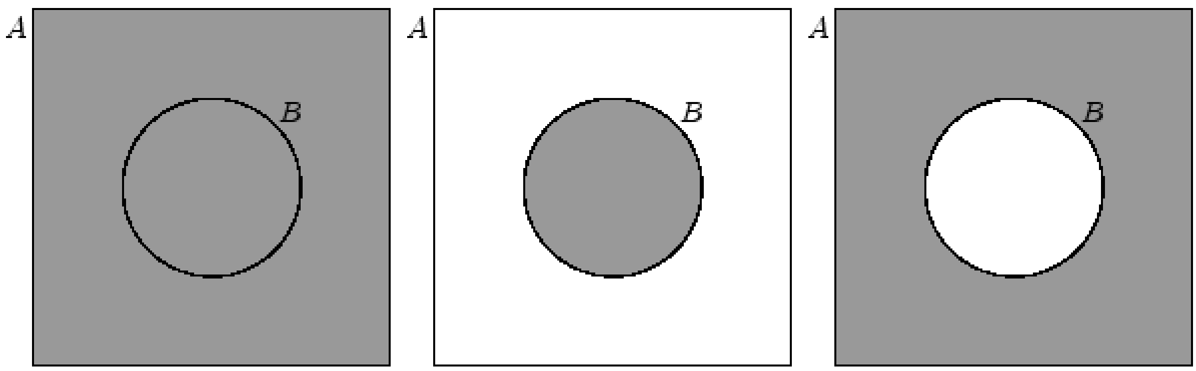


A B union: $A \cup B$



intersection: $A \cap B$ difference: $A \setminus B$ symmetric difference: $A \Delta B$

If $B \subset A$, some of the above diagrams are identical to one another:



union: $A = A \cup B$ intersection: $B = A \cap B$ complement set : $B^{\backslash}$

For example

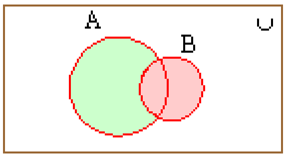
Let A = { 3, 4, 6, 7 } and B = { 4, 7, 9, 10 }

then $A \cup B$ = { 3, 4, 6, 7, 9, 10 }

Read $A \cup B$ as A union B

Thus $A \cup B = \{ x : x \in A$ or $x \in B$ or both $\}$ or

$A \cup B = \{ x : x$ belong to at least one of the sets A and B $\}$

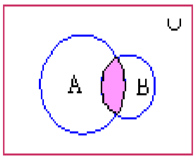


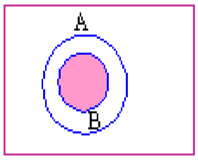
$A \cup B$

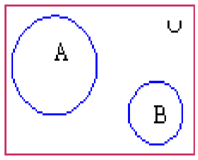
The shaded region in the above figure shows $A \cup B$.We can easily see that $A \cup B = B \cup A$. Obviously, for any set A, $A \cup A = A$ and $A \cup \phi = A$

For example $A = \{ 1, 2, 3, 4 \}$ and $B = \{ 2, 3, 4, 5, 6, 7 \}$

Then $A \cap B = \{ 2, 3, 4 \}$







We can easily see that for any two sets, A and B

$A \cap B = B \cap A$

Also note that for any set A, $A \cap A = A$

and $A \cap \phi = \phi$

The sets A and B are such that (see above figure) $A \cap B = \phi$, then A and B are called as disjoint sets.

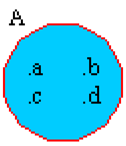
Example

Show the following sets with the help of Venn diagrams.

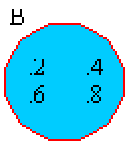
1. A is a set of first four alphabets.
2. $B = \{ x : x$ is an even number and $x < 10 \}$
3. $C = \{ c : c \in N$ and $3 < c < 10 \}$

Solution :

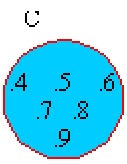
1) $A = \{ a, b, c, d \}$



2) $B = \{ 2, 4, 6, 8 \}$



3) $C = \{ 4, 5, 6, 7, 8, 9 \}$



For example

∪ = { 1, 2, 3, 4, . . . } and A = { 1, 3, 5, 7, . . . }
then A′ = { 2, 4, 6, 8, . . . }

.

Example

If U = {3, 6, 7, 12, 17, 35, 36 }, A = {odd numbers }, B = {Numbers divisible by 7 }, C = {prime numbers > 3 } List the elements of

1) A′ and B′

2) The subset of C

3) The power set of C

Solution :

1) A = { 3, 7, 17, 35 } ⇒ A′ = { 6, 12, 36 }

B = {7, 35 } ⇒ B′ = {3, 6, 12, 17, 36 }

2) C = {7,17 } ⇒ Subset of C = {7}, {17}, {7,17}, ϕ

3) The power set C = the set of all subsets of C =
{{7}, {17}, {7,17}, ϕ}

Example

If A = { 2, 4, 6, 8, 10, 12 }, B = {4, 8, 12 }.

1) A - B

2) B - A

3) How many subsets can be formed from the set A ?

4) How many proper subsets can be formed from B ?

Solution :

1) A - B = { 2, 6, 10 }

2) B - A = { }

3) The set A contains 6 elements ⇒ The number of subsets = $2^6 = 64$

4) n (B) = 3 ⇒ $2^3 = 8$

∴ The number of proper subsets = 23 - 1 = 8 - 1 = 7

Example

If $A = \{2, 4, 8, 12\}$, $B = \{3, 4, 5, 8\}$ and $U = \{x \mid x \in N \text{ and } x < 13\}$,

find A' ; B' , $(A \cap B)'$ and verify $(A \cap B)' = A' \cup B'$

Solution :

$A' = U - A = \{1, 3, 5, 6, 7, 9, 10, 11\}$

$B' = U - B = \{1, 2, 6, 7, 9, 10, 11, 12\}$

Now $A \cap B = \{4, 8\}$

$\therefore (A \cap B)' = U - (A \cap B) = \{1, 2, 3, 5, 6, 7, 9, 10, 11, 12\}$

Also $A' \text{ U } B' = \{1, 2, 3, 5, 6, 7, 9, 10, 11, 12\}$ . . . (II)

$\therefore (A \cap B)' = A' \cup B'$ . . . (from (I) and (II))

Example

Given $U = \{x \mid x \leq 10 , x \in N\}$,

$A = \{2, 4, 6\}$, $B = \{1, 3, 5, 7\}$ and $C = \{3, 4, 5\}$

Prove that (a) $(A \cup B) \cap B' = A$ if and only if $A \cap B = \phi$

Solution :

(a) $A \cup B = \{2, 4, 6\} \cup \{1, 3, 5, 7\} = \{1, 2, 3, 4, 5, 6, 7\}$

and $B' = U - B = \{2, 4, 6, 8, 9, 10\}$

Therefore, $(A \cup B) \cap B' = \{1, 2, 3, 4, 5, 6, 7\} \cap \{2, 4, 6, 8, 9, 10\} = \{2, 4, 6\} = A$

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1. Commutative Laws

a. For addition or union
$A \cup B = B \cup A$
b. For multiplication or intersection
$A \cap B = B \cap A$

2. Associative Laws

a. For addition or union
$(A \cup B) \cup C = A \cup (B \cup C)$
b. For multiplication or intersection
$(A \cap B) \cap C = A \cap (B \cap C)$

3. Distributive Laws

a. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
b. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

4. Identity Laws

a. $A \cup \phi = A$ b. $A \cap \phi = \phi$

c. $A \cup U = U$ d. $A \cap U = A$

5. Idempotent Laws

a. $A \cap A = A$ b. $A \cap A = A$

6. Complement Laws

a. $A \cup A' = \cup$ b. $A \cap A = \phi$

c. $(A')' = A$ d. $\phi' = \cup$ and $\cup' \phi$

7. Absorption Laws

$A \cup (A \cap B) = A$

8. De Morgan's Laws

a. $(A \cup B)' = A' \cap B'$

b. $(A \cap B)' = A' \cup B'$

Example

A and B are two sets such that A has 12 elements, B has 17 elements and $A \cup B$ contains 21 elements. Find the number of elements in $A \cap B$.

Solution :

$n(A) = 12$, $n(B) = 17$ and $n(A \cup B) = 21$

Using the fact,

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$,

$21 = 12 + 17 - n(A \cap B)$

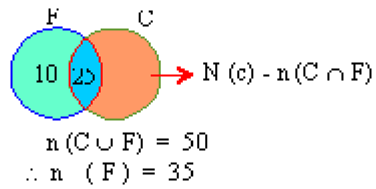
$\therefore n(A \cap B) = 29 - 21 = 8$

Example **FOR excellent student**

In a class of 50 students, 35 students play foot ball, 25 students play both football as well as base ball. All the students play at least one of the two games. How many students play base ball ?

Solution :

Let F be the set of the students who play foot ball, and C be the set of students who play base ball.



Then we have $n(C \cup F) = 50$

$n(f) = 35$ and $n(C \cap F) = 25$

Now the problem can be visualized by means of a Venn diagram as in the adjoining figure.

Thus we have $n(C \cup F) = n(F) + n(C) - n(C \cap F)$

$\therefore$ The number of students who play only baseball

$= n(C) - n(C \cap F)$

$= n(C \cup F) - n(F)$

$= 50 - 35$

$= 15$

Example **FOR excellent student**

If $A \subset B$, $B \subset A$ and $B \subset C$, then find which of the sets A, B and C are equal.

Solution :

If $A \subset B$, $B \subset C$ $\quad \therefore A \subset C$

But given that $C \subset A$ $\quad \therefore A = C \ldots (1)$

Now, $A = C \Rightarrow A \subset B$ and $C \subset B \ldots (I)$

But given that $B \subset C \ldots (II)$

From (I) and (II) $B = C \ldots (2)$

From (1) and (2) $A = B = C$

[intersection ∩ and union ∪]

By using the opposite venn diagram to find

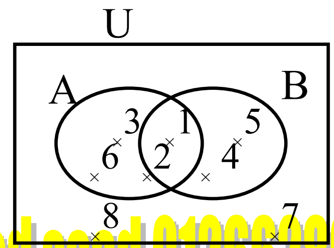
(1) A= { }

(2) B= { }

(3) U = { }

(4) A ∩ B = { }

(5) A ∪ B = { }



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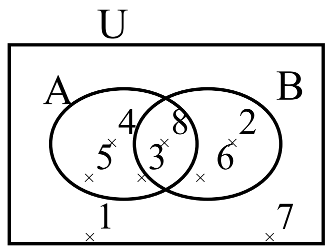

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(5) A ∩ B = { }

(6) A ∪ B = { }

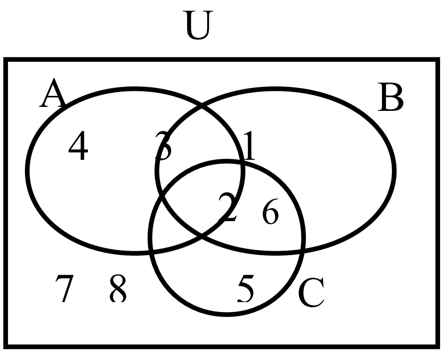
(7) A ∩ C = { }

(8) A ∪ C = { }

(9) B ∩ C = { }

(10) B ∪ C = { }

(11) A ∩ B ∩ C = { }



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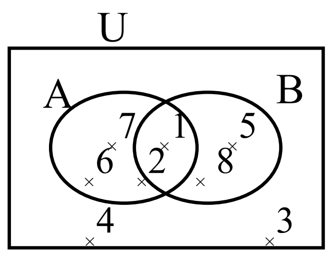

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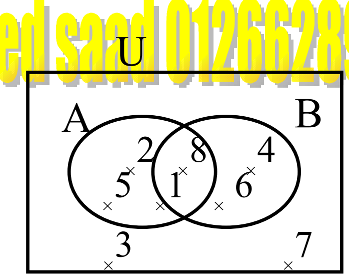

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(1) A= { }

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(3) C = { }

(4) U = { }

(5) A ∩ B = { }

(6) A ∪ B = { }

(7) A ∩ C = { }

(8) A ∪ C = { }

(9) B ∩ C = { }

(10) B ∪ C = { }

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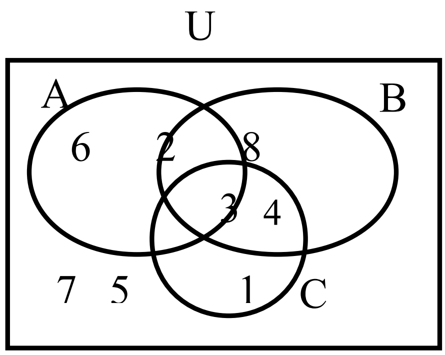

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By using ∪ and ∩ write down what the shaded part in each of the following figures represents: