The set of rational number(Q)

We studied :

1) ***The set of count number*** = {1,2,3,4,5,6,… }
2) ***The set of natural number(N)***={0,1,2,3,4,…… }
3) ***The set of integer number(Z)*** = {.,-2,-1,0,1,2,….}

But *: what about the percentage , fractions , decimal, ratio ?*
for any set the belong?
So : we have a new set which is
(the set of rational number)

The rational number: it *is number we can express at the form of* $\frac{a}{b}$ *and a , b are two integer and* $b \neq 0$

Which of the following is rational number?

1) $\frac{3}{5}$ ➢➢ *IS rational number because* 3 , 5 ∈ (z)

2) $\frac{-5}{4}$ ➢➢ *IS rational number because*- 5 , 4 ∈ (z)

3) 9 ➢➢ $9=\frac{9}{1}$ *IS rational number because* 9 , 1 ∈ (z)

4) 0.3 ➢➢ $0.3=\frac{3}{10}$ *IS rational number because* 9 , 1 ∈ (z)

5) zero ➢➢ 0 = $\frac{0}{1}$ *IS rational number because* 9 , 1 ∈ (z)×

6)15 % ➢➢15% =$\frac{15}{100}$ *IS rational number because* 15 , 100 ∈ (z)

7) $1\frac{1}{2}$ ➢➢ $1\frac{1}{2}=\frac{3}{2}$ IS rational number because 3 , 2 ∈ (z)

8) $\frac{1}{0}$ IS not rational number because the denominator=0

9) $\frac{6}{3-3}$ ➢➢➢ $\frac{6}{0}$ IS not rational number because the denominator=0

the set of rational number(Q) $=\{x: x=\frac{a}{b}$ and $a, b \in z$ and $b \neq 0\}$

We know that : $N \subset Z$

And $z=\{\ldots\ldots,-2,-1,0,1,2\}$ also

$Z=\{\ldots\ldots,\frac{-2}{1},\frac{-1}{1},\frac{0}{1},\frac{1}{1},\frac{2}{1}\ldots\ldots\}$ then $z \subset Q$

Then $N \subset Z \subset Q$

Remark(1)

If $\frac{a}{b}$ is rational number then $b \neq 0$

Example: if x is an integer , write the required condition to make each of the following a rational number:

1) $\frac{3}{2x}$ sol ➢➢➢ $2x \neq 0$ then $x \neq 0$

2) $\frac{7}{x-3}$ sol ➢➢➢ $x-3 \neq 0$ then $x \neq 3$

3) $\frac{5}{x+9}$ sol ➢➢➢ $x+9 \neq 0$ then $x \neq -9$

4) $\frac{7+x}{4-x}$ sol ⋙ $4-x \neq 0$ then $-x \neq -4$ then $x \neq 4$

5) $\frac{2x-3}{3x-5}$ sol ⋙ $3x-5 \neq 0$ then $3x \neq 5$ then $x \neq \frac{5}{3}$

6) $\frac{3}{|x|-3}$ sol ⋙ $|x| \neq 3$ then $x \neq \pm 3$

Complete

1) if $\frac{2x-3}{4-9x} \in \mathbb{Q}$ then $x \neq$ ………….

$4-9x \neq 0 \qquad -9x \neq -4 \qquad x \neq \frac{4}{9}$

2) if $\frac{2x-3}{2x+4}$ is rational number then $x \neq$ ……

$2x+4 \neq 0 \qquad 2x \neq -4 \qquad x \neq \frac{-4}{2} \neq$ 2-

Remark(2) If $\frac{a}{b} = 0$ then $a = 0$

Example: write the required condition to make each of the following =0:

1) $\frac{x-3}{x+3}$ sol ⋙ $x-3 = 0$ then $x = 3$

2) $\frac{2x}{x-3}$ sol ⋙ $2x = 0$ then $x = 0$

3) $\frac{2x-4}{x+9}$ sol ⋙ $2x-4 = 0$ then $2x = 4$ then $x = 2$

Different forms for the rational number

1) *writting the rational number in the simplest form*

Example

write each of the following in the simplest form:

1) $\frac{8}{12} = \frac{2}{3}$ 2) $\frac{-12}{36} = \frac{-1}{3}$

3) $\frac{-9}{6} = \frac{-3}{2}$ 4) $\frac{5}{25} = \frac{1}{5}$

You can use calculator

2) *writting the rational number in the percentage form*

Example

write each of the following in the percentage form

1) $5\frac{12}{125} = 509.6\%$ 2) $3.2 = 320\%$

3) $\frac{5}{16} = 31.25\ \%$ 4) $\frac{9}{20} = 45\%$

You can use calculator

3) *writting the rational number in the terminating decimal form:*

You can use calculator

Example

write each of the following in terminating decimal form:

1) $\frac{2}{5} = 0.4$ 2) $-2\frac{7}{25} = -2.28$ 3) $\left|\frac{-3}{8}\right| = 0.375$

4)*writting the rational number in the recurring decimal form:*

Example

write each of the following in recurring decimal form:

1) $\frac{2}{3} = 0.666666667 = 0.\dot{6}$

2) $\frac{2}{11} = 0.18181818 = 0.\dot{1}\dot{8}$

3) $\frac{71}{333} = 0.213231213213 = 0.\dot{2}1\dot{3}$

4) $5\frac{71}{333} = 5.213213213213 = 5.\dot{2}1\dot{3}$

اول حاجه ندخل الرقم علي الاله ونضغط علي =
تاني حاجه نكتب العلامة العشرية ونأخذ من كل رقم
مكرر رقم واحد وبعدين نضع فوق كل رقم نقطه

Example

write each of the following in the rational number form:

1) $0.25 = \frac{25}{100} = \frac{1}{4}$

2) $0.\dot{6} = \frac{2}{3}$ (*write at the calculator0.666666666 then press =*)

3) $0.\dot{1}\dot{6} = \frac{16}{99}$ (*write at the calculator0.1616161616 then press =*

4) $1.9 = \frac{19}{10}$

5) $35\% = \frac{35}{100} = \frac{7}{4}$

6) $0.2\dot{4}\dot{5} = \frac{27}{110}$

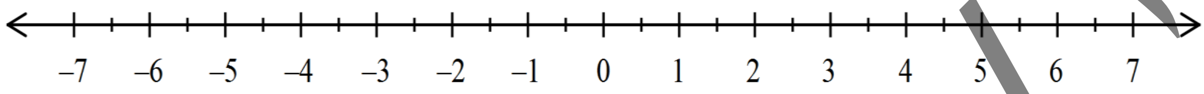
7) $4.\dot{1}2\dot{3} = 4\,\frac{41}{333}$

ندخل العدد علي الاله (اللي عليه نقطه يتكرر
ونملي الشاشة بيه او اكمل شويه بعد ما الشاشة
تتملي
لو كانت النقطة علي الاول والاخر يبقي اللي
في النص عليه نقطه

Comparing and ordering rational number

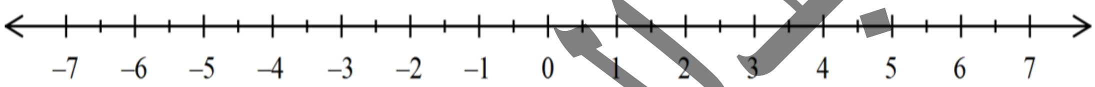
Example

Represent the rational number $\frac{3}{4}$ *on the number line*



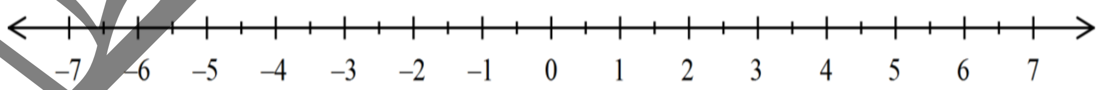
Example

Represent the rational number $\frac{7}{5}$ *on the number line*



Example

Represent the rational number $\frac{-24}{9}$ *on the number line*



Comparing between tow rational numbers

Frist:Comparing between tow rational numbers having the same denominator:

Example

Put the suitable sing(<,=,>)

1) $\frac{1}{3}$ ...<... $\frac{6}{3}$ 2) $\frac{-1}{4}$ ...>... $\frac{-5}{4}$

3) $\frac{4}{9}$ ...=... $\frac{8}{18}$ $(\frac{4}{9})$ 4) $\frac{6}{2}$ ...>... $\frac{-6}{2}$

Second :Comparing between two rational numbers having two different denominators:

Example

Put the suitable sing(<,=,>)

1) $\frac{6}{9}$ ...>... $\frac{4}{7}$ 2) $\frac{-1}{2}$ ...<... zero

3) $-4\frac{1}{2}$ ...>... -5 4) $\frac{3}{4}$ ...=... $\frac{6}{8}$

The density of rational number

Example(1)

Find two rational numbers are lying between : $\frac{4}{5}, \frac{1}{5}$

Sol:

$\frac{4}{5} < \frac{3}{5} < \frac{2}{5} < \frac{1}{5}$

The two rational numbers are $\frac{3}{5}, \frac{2}{5}$

Example(2)

Find three rational numbers are lying between : $\frac{4}{9}, \frac{5}{6}$

Sol:

Frist : we should convert their denominators .

$\frac{4}{9} = \frac{4 \times 6}{9 \times 6}$ *and* $\frac{5}{6} = \frac{5 \times 9}{6 \times 9}$

$\frac{24}{54}$ *and* $\frac{45}{54}$

$\frac{24}{54} < \frac{25}{54} < \frac{26}{54} < \frac{27}{54} < \frac{45}{54}$

The three rational numbers are $\frac{25}{54}, \frac{26}{54}, \frac{27}{54}$

Example(3)

Find three rational numbers are lying between : $\frac{1}{3}, \frac{1}{2}$

Sol:

Frist : we should convert their denominators .

$\frac{1}{3} = \frac{1\times2}{3\times2} = \frac{2}{6} = \frac{20}{60}$ *and* $\frac{1}{2} = \frac{1\times3}{3\times2} = \frac{30}{60}$

$\frac{20}{60}$ *and* $\frac{30}{60}$

$\frac{20}{60} < \frac{21}{60} < \frac{22}{60} < \frac{23}{60} < \frac{30}{60}$

The three rational numbers are $\frac{21}{60}, \frac{22}{60}, \frac{23}{60}$

<u>Example</u>(4)

Find three rational numbers are lying between : $\frac{5}{7}, \frac{1}{2}$

Sol:

Frist : we should convert their denominators .

$\frac{5}{7} = \frac{5\times2}{7\times2} = \frac{10}{14} = \frac{100}{140}$ *and* $\frac{1}{2} = \frac{1\times7}{7\times2} = \frac{7}{14} = \frac{70}{140}$

$\frac{100}{140}$ *and* $\frac{70}{140}$

$\frac{70}{140} < \frac{71}{140} < \frac{72}{140} < \frac{73}{140} < \frac{74}{140} < \frac{100}{140}$

The three rational numbers are $\frac{71}{140}, \frac{72}{140}, \frac{73}{140}, \frac{74}{140}$

Example(5)

Identify and write four rational numbers between $\frac{3}{2}, \frac{3}{4}$ *such that one of them is an integer and the other is a rational number*

Frist : we should convert their denominators .

$\frac{3}{2} = \frac{3\times4}{2\times4} = \frac{12}{8}$ *and* $\frac{3}{4} = \frac{3\times2}{4\times2} = \frac{6}{8}$

$\frac{6}{8}$ *and* $\frac{12}{8}$

$\frac{6}{8} < \frac{7}{8} < \frac{8}{8} < \frac{9}{8} < \frac{10}{8} < \frac{11}{8}$ $\frac{12}{8}$ *we see that the integer* $= \frac{8}{8}$

The four rational numbers are $\frac{7}{8}, \frac{8}{8}, \frac{9}{8}, \frac{10}{8},$

Adding and subtracting rational numbers

كيف كنا نجمع ونطرح الاعداد الصحيحة؟

1) لو الاشارات زي بعض (++ او - -)كنا نجمع

2) لو الاشارات مختلفة (+ - او - +) نطرح ونضع اشاره الكبير

We studied before :

$4 - 2 = 2$ and $-6 - 4 = -10$

$4 + 3 = 7$ and $-6 + 7 = 1$

$-9 + 8 = -1$

Adding and subtracting two rational numbers with The same denominator

Example(1)

1) $\frac{6}{8} + \frac{4}{8} = \frac{6+4}{8} = \frac{10}{8}$

2) $\frac{2}{7} - \frac{5}{7} = \frac{2-5}{7} = \frac{-3}{7}$

3) $-\frac{2}{5} + \frac{7}{5} = \frac{-2+7}{5} = \frac{5}{5} = 1$

4) $-\frac{4}{11} - \frac{18}{11} = \frac{-4-18}{11} = \frac{-22}{11} = -2$

Adding and subtracting two rational numbers with The two different denominator

Example(1)add:

two different denominator then We should find the common denominator (بطريقه المقص)

1) $\frac{3}{8} + \frac{1}{4}$ **sol**

$\frac{3}{8} + \frac{1}{4} = \frac{3\times 4}{8\times 4} + \frac{1\times 8}{8\times 4} = \frac{12}{32} + \frac{8}{32} = \frac{20}{32}$

Before adding or subtracting We should put the rational numbers in the simplest form

2) $\frac{4}{12}+(-\frac{10}{15})$ **sol**

$=\frac{1}{3}+(-\frac{2}{3})=\frac{1-2}{3}=-\frac{1}{3})\ \frac{4}{12}+(-\frac{10}{15})$

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3) $\frac{2}{5}+3$ **sol**

$\frac{2}{5}+3=\frac{2}{5}+\frac{3}{1}=\frac{2\times1}{1\times5}+\frac{3\times5}{1\times5}=\frac{2}{5}+\frac{15}{5}=\frac{17}{5}$

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4) $\frac{5}{7}-1$ **sol**

$\frac{5}{7}-1=\frac{5}{7}-\frac{7}{7}=\frac{5-7}{7}=\frac{-2}{7}$

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5) $\frac{3}{4}-\frac{5}{6}$ **sol**

$\frac{3}{4}-\frac{5}{6}=\frac{3\times6}{4\times6}-\frac{5\times4}{4\times6}=\frac{18}{24}-\frac{20}{24}=\frac{18-20}{24}=\frac{-2}{24}=\frac{--1}{12}$

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6) $7\frac{2}{5}-3\frac{1}{4}$ **sol**

$7\frac{2}{5}-3\frac{1}{4}=\frac{7\times5+2}{5}-\frac{3\times4+1}{4}=\frac{37}{5}-\frac{13}{4}=\frac{4\times37-5\times13}{20}$

$=\frac{83}{20}$

properties of Addition and subtraction operation in Q

(1)The closure property:

The sum of any two rational numbers is rational number

Example

$If: \frac{1}{3} \in Q$ and $\frac{1}{2} \in Q \Longrightarrow \frac{1}{3} + \frac{1}{2} = \frac{2+3}{2 \times 3} = \frac{5}{6} \in Q$

The subtraction of any two rational numbers is rational number

Example

$If: \frac{1}{3} \in Q$ and $\frac{1}{2} \in Q \Longrightarrow \frac{1}{3} - \frac{1}{2} = \frac{2-3}{2 \times 3} = \frac{-1}{6} \in Q$

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(2)The commutative property:

$If: \frac{a}{b} \in Q$ and $\frac{c}{d} \in Q \Longrightarrow \frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$

Example

$\frac{2}{3} + \frac{1}{4} = \frac{8+3}{12} = \frac{11}{12} \Longrightarrow \frac{1}{4} + \frac{2}{3} = \frac{3+8}{12} = \frac{11}{12}$

$$\frac{2}{3} + \frac{1}{4} = \frac{1}{4} + \frac{2}{3}$$

The subtraction operation in Q is not **commutative**

Example

$\frac{2}{3} - \frac{1}{4} = \frac{8-3}{12} = \frac{5}{12} \Longrightarrow \frac{1}{4} - \frac{2}{3} = \frac{3-8}{12} = \frac{-5}{12}$

$$\frac{2}{3} - \frac{1}{4} \neq \frac{1}{4} - \frac{2}{3}$$

(4)The associative property:

If : $\frac{a}{b} \in Q$, $\frac{c}{d} \in Q$ and $\frac{e}{f} \in Q \Rightarrow$

$$\frac{a}{b}+(\frac{c}{d}+\frac{e}{f}) =(\frac{a}{b}+\frac{c}{d})+\frac{e}{f}=\frac{a}{b}+\frac{c}{d}+\frac{e}{f}$$

Example

$$\frac{3}{7}+(\frac{2}{7}+\frac{1}{7})=\frac{3}{7}+\frac{3}{7}=\frac{6}{7} \text{ and } (\frac{3}{7}+\frac{2}{7})+\frac{1}{7}=\frac{5}{7}+\frac{1}{7}=\frac{6}{7}$$

then $\Rightarrow \frac{3}{7}+(\frac{2}{7}+\frac{1}{7})=(\frac{3}{7}+\frac{2}{7})+\frac{1}{7}$

The subtraction operation in Q is not associative

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(5)The existence of identity element property in Addition

Zero is the identity element in Addition operation in Q

Example

$$zero+\frac{1}{4}=\frac{1}{4}+zero=\frac{1}{4}$$

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(6)The existence of additive inverse property in Addition

If : $\frac{a}{b}$ is rational number then there exit additive inverse

$-\frac{a}{b}$ where $\frac{a}{b}+(-\frac{a}{b})=0$

Example

1) *the additive inverse of the number* $\frac{2}{3}$ *is* $-\frac{\mathbf{2}}{\mathbf{3}}$

2) *the additive inverse of the number* $\frac{5}{6}$ *is* $-\frac{\mathbf{5}}{\mathbf{6}}$

3) *the additive inverse of the number* $\frac{-4}{7}$ *is* $\frac{\mathbf{4}}{\mathbf{7}}$

4) *the additive inverse of the number* $\frac{-2}{5}$ *is* $\frac{\mathbf{2}}{\mathbf{5}}$

5) *the additive inverse of the number -1 is* ***1***

6) *the additive inverse of the number zero is* ***zero***

7) *the additive inverse of the number* $\frac{-6}{-7}$ *is* $\frac{\mathbf{6}}{\mathbf{-7}}$

8) *the additive inverse of the number* $\left|\frac{-5}{7}\right|$ *is* $\frac{\mathbf{-5}}{\mathbf{7}}$

9) *the additive inverse of the number* $\left(\frac{-2}{5}\right)^0$ *is* ***-1***

10) *the additive inverse of the number 1 is* ***-1***

Example

Put in the simplest form: $0.\dot{1}\dot{8} - 30\%$

sol

$0.\dot{1}\dot{8} = \frac{\mathbf{2}}{\mathbf{11}}$ *and* $30\ \% = \frac{\mathbf{3}}{\mathbf{10}}$

$\frac{2}{11} - \frac{3}{10} = \frac{2x10-3x11}{110} = \frac{20-33}{110} = \frac{-13}{110}$

You can use calculator

Example

If : $a=\frac{3}{4}$, $b=\frac{-5}{2}$ *and* $c=\frac{1}{2}$ *find the numerical value of*

(1) $a-b$ (2) $(a+b)-c$

sol

$$a-b=\frac{3}{4}-\left(\frac{-5}{2}\right)=\frac{3}{4}+\frac{5}{2}=\frac{3\times2+4\times5}{4\times2}=\frac{6+20}{8}=\frac{26}{8}=\frac{13}{4}$$

$$(a+b)-c=\left(\frac{3}{4}+\left(\frac{-5}{2}\right)\right)-\frac{1}{2}=\left(\frac{3}{4}-\frac{5}{2}\right)-\frac{1}{2}=\frac{3\times2-4\times5}{4\times2}-\frac{1}{2}$$

$$=\frac{6-20}{8}-\frac{1}{2}=\frac{-14}{8}-\frac{1}{2}=\frac{-7}{4}-\frac{1}{2}=\frac{-9}{4}$$

Multiplying and dividing rational numbers

.we studied :

(1) $3 \times 4 = 12$

(2) $6 \times (-3) = -18$

(3) $(-4) \times 6 = -24$

(4) $(-5) \times (-7) = 35$

The sign rule of Multiplication

$+ \times + = +$

$- \times - = +$

$+ \times - = -$

$- \times + = -$

Multiplying two rational numbers

If : $\frac{a}{b} \in Q$, $\frac{c}{d} \in Q$ then $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

Example

(1) $\frac{3}{6} \times \frac{2}{5}$ **sol**

$\frac{3}{6} \times \frac{2}{5} = \frac{3 \times 2}{6 \times 5} = \frac{6}{30} = \frac{1}{5}$

(2) $-\frac{3}{4} \times \frac{2}{5}$ **sol**

$-\frac{3}{4} \times \frac{2}{5} = \frac{-3 \times 2}{4 \times 5} = \frac{-6}{20} = \frac{-3}{10}$

(3) $\frac{1}{2} \times (-2)$ **sol**

$\frac{1}{2} \times (-2) = \frac{1 \times -2}{2 \times 1} = \frac{-2}{2} = -1$

(4) $\frac{8}{5} \times \frac{-4}{9}$ ***sol***

$\frac{8}{5} \times \frac{-4}{9} = \frac{8 \times -4}{9 \times 5} = \frac{-32}{45}$

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(5) $-0.5 \times \frac{1}{2}$ ***sol***

$-0.5 \times \frac{1}{2} = =- \frac{1}{2} \times \frac{1}{2} = \frac{-1}{4}$

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(6) $-4\frac{2}{7} \times (-3\frac{1}{6})$ ***sol***

$-4\frac{2}{7} \times (-3\frac{1}{6}) = -\frac{7\times4+2}{7} \times -\frac{3\times6+1}{6} = \frac{-30}{7} \times \frac{-19}{6} =$

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(7) $2\frac{1}{2} \times 0.8$ ***sol***

$2\frac{1}{2} \times 0.8 = \frac{2\times2+1}{2} \times \frac{4}{5} = \frac{5}{2} \times \frac{4}{5} = \frac{20}{10} = 2$

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(8) $\frac{1}{2} \times |-12|$ ***sol***

$\frac{1}{2} \times |-12| = \frac{1}{2} \times 12 = 6$

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(9) $\frac{2}{3} \times |-1\frac{1}{2}|$ ***sol***

$\frac{2}{3} \times |-1\frac{1}{2}| = \frac{2}{3} \times |-1\frac{1}{2}| = \frac{2}{3} \times \frac{-3}{2} = -1$

$(10)\ 1\frac{2}{3} \times -0.\dot{1}\dot{8}$ **sol**

$$1\frac{2}{3} \times -0.\dot{1}\dot{8} = \frac{1\times 3+2}{3} \times \frac{2}{11} = \frac{2}{11} \times \frac{5}{2} = \frac{5}{11}$$

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dividing two rational numbers

If : $\frac{a}{b} \in Q$, $\frac{c}{d} \in Q$ then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c}$

we studied :

$6 \div 2 = 3$

$9 \div (-3) = -3$

$(-12) \div 2 = -6$

$(-72) \div (-9) = 8$

The sign rule of division

$+ \div + = +$

$- \div - = +$

$+ \div - = -$

$- \div - = +$

Example

(1) $\frac{2}{3} \div \frac{7}{5}$ **sol**

$$\frac{2}{3} \div \frac{7}{5} = \frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}$$

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(2) $\frac{-2}{5} \div \frac{6}{5}$ **sol**

$$\frac{-2}{5} \div \frac{6}{5} = \frac{-2}{5} \times \frac{5}{6} = \frac{-2 \times 5}{5 \times 6} = \frac{-10}{30} = \frac{-1}{3}$$

(3) $\frac{2}{3} \div (-2)$ **sol**

$$\frac{2}{3} \div (-2) = \frac{2}{3} \times \frac{-1}{2} = \frac{-2 \times 1}{3 \times 2} = \frac{-1}{3}$$

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(4) $(-3\frac{1}{3}) \div 1\frac{2}{3}$ **sol**

$$(-3\frac{1}{3}) \div 1\frac{2}{3} = -\frac{3\times3+1}{3} \div \frac{1\times3+2}{3} = \frac{10}{3} \div \frac{5}{3} = \frac{10}{3} \times \frac{3}{5} = \frac{10 \times 3}{3 \times 5}$$
$$=2$$

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(5) $0.5 \div 5\frac{1}{2}$ **sol**

$$0.5 \div 5\frac{1}{2} = \frac{1}{2} \div \frac{5\times2+1}{2} = \frac{1}{2} \div \frac{11}{2} = \frac{1}{2} \times \frac{2}{11} = \frac{10 \times 3}{3 \times 5} = \frac{1}{11}$$

(6) $(\frac{2}{7} + \frac{3}{7}) \div \frac{10}{7}$ **sol**

$$(\frac{2}{7} + \frac{3}{7}) \div \frac{10}{7} = \frac{2+3}{7} \div \frac{10}{7} = \frac{5}{7} \times \frac{7}{10} = \frac{5 \times 7}{7 \times 10} = \frac{1}{2}$$

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(7) $(\frac{5}{6} - \frac{3}{4}) \div (\frac{7}{12} - \frac{5}{9})$ **sol**

$$(\frac{5}{6} - \frac{3}{4}) \div (\frac{7}{12} - \frac{5}{9}) = (\frac{5\times4-6\times3}{6\times4}) \div (\frac{7\times3-5\times4}{36})$$
$$= \frac{20-18}{24} \div \frac{21-20}{36} = \frac{2}{24} \div \frac{1}{36} = \frac{2}{24} \times \frac{36}{1} = \frac{2 \times 36}{24 \times 1} = 3$$

(8) $0.\dot{3} \div \frac{2}{3}$ **sol**

$$0.3 \div \frac{2}{3} = \frac{1}{3} \div \frac{2}{3} = \frac{1}{3} \times \frac{3}{2} = \frac{1 \times 3}{3 \times 2} = \frac{1}{2}$$

---

(9) $30\% \div (-3\frac{1}{2})$ **sol**

$$30\% \div (-3\frac{1}{2}) = \frac{3}{10} \div (-\frac{3 \times 2 + 1}{2}) = \frac{3}{10} \div \frac{-7}{2}$$

$$= \frac{3}{10} \times \frac{-2}{7} = \frac{3 \times -2}{10 \times 7} = -\frac{3}{35}$$

properties of Multiplication operation in Q

(1)The closure property:

$If: \frac{a}{b} \in Q \text{ and } \frac{c}{d} \in Q \Longrightarrow \frac{a}{b} \times \frac{c}{d} = \frac{axc}{bxb}$

Example

$If: \frac{1}{3} \in Q \text{ and } \frac{1}{2} \in Q \Longrightarrow \frac{1}{3} \times \frac{1}{2} = \frac{1 \times 1}{2 \times 3} = \frac{1}{6} \in Q$

---

(2)The commutative property:

$If: \frac{a}{b} \in Q \text{ and } \frac{c}{d} \in Q \Longrightarrow \frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$

Example

$\frac{2}{3} \times \frac{1}{4} = \frac{2x1}{3x4} = \frac{2}{12} \Longrightarrow \frac{1}{4} \times \frac{2}{3} = \frac{1x2}{12} = \frac{2}{12}$

$\Longrightarrow \frac{2}{3} \times \frac{1}{4} = \frac{1}{4} \times \frac{2}{3}$

(3)The associative property:

If : $\frac{a}{b} \in Q$, $\frac{c}{d} \in Q$ and $\frac{e}{f} \in Q$

$$\frac{a}{b} \times (\frac{c}{d} \times \frac{e}{f}) = (\frac{a}{b} \times \frac{c}{d}) \times \frac{e}{f} = \frac{a}{b} \times \frac{c}{d} \times \frac{e}{f}$$

Example

$$\frac{1}{2} \times (\frac{1}{3} \times \frac{7}{5}) = \frac{1}{2} \times \frac{7}{15} = \frac{7}{30} \text{ and } (\frac{1}{2} \times \frac{1}{3}) \times \frac{7}{5} = \frac{1}{6} \times \frac{7}{5}$$

$$= \frac{7}{30}$$

then $\Longrightarrow$ $\frac{3}{7} \times (\frac{2}{7} \times \frac{1}{7}) = (\frac{3}{7} \times \frac{2}{7}) \times \frac{1}{7}$

---

(4)The existence of identity element property in Multiplication

1 is the identity element in Multiplication operation in Q

Example

$$1 \times \frac{1}{4} = \frac{1}{4} \times 1 = \frac{1}{4}$$

---

(5)The existence of Multiplicative inverse property in Multiplication

If : $\frac{a}{b}$ is rational number then there exit Multiplicative inverse

$\frac{b}{a}$ where $\frac{a}{b} \times \frac{b}{a} = 1$ **I .e $\frac{a}{b}$ Multiplicative inverse** $\Longrightarrow \frac{b}{a}$

Zero does not have identity element

Example

(1) the Multiplicative inverse of the number $\frac{2}{3}$ is $\frac{3}{2}$

(2) the Multiplicative inverse of the number $\frac{-3}{4}$ is $\frac{-4}{3}$

(3) the Multiplicative inverse of the number $\frac{1}{5}$ is **5**

(4) the Multiplicative inverse of the number 1 is **1**

(5) the Multiplicative inverse of the number -1 is **-1**

(6) the Multiplicative inverse of the number $\frac{-6}{-7}$ is $\frac{7}{6}$

(7) the Multiplicative inverse of the number $\left|\frac{-5}{7}\right|$ is $\frac{7}{5}$

(8) the Multiplicative inverse of the number $|0.7|$ is

$0.7 = \frac{7}{9}$ then Multiplicative inverse $= \frac{9}{7}$

(9) the Multiplicative inverse of the number $|-2| = \frac{1}{2}$

(6)Property of distributing Multiplication over addition and subtraction

Example

Use the distributing property to find the value of each of the following

(1) $\frac{5}{11} \times \frac{6}{7} + \frac{5}{11} \times \frac{1}{7}$ **sol**

$\frac{5}{11} \times \frac{6}{7} + \frac{5}{11} \times \frac{1}{7} = \frac{5}{11} (\frac{6}{7} + \frac{1}{7}) = \frac{5}{11} \times \frac{7}{7} = \frac{5}{11} \times 1 = \frac{5}{11}$

---

(2) $\frac{9}{17} \times 21 - \frac{9}{17} \times 4$ **sol**

$\frac{9}{17} \times 21 - \frac{9}{17} \times 4 = \frac{9}{17} (21-4) = \frac{9}{17} \times 17 = 9$

(3) $\frac{22}{25} \times \frac{6}{11} + \frac{5}{11} \times \frac{22}{25} - \frac{22}{25}$ **sol**

$\frac{22}{25} \times \frac{6}{11} + \frac{5}{11} \times \frac{22}{25} - \frac{22}{25} = \frac{22}{25} (\frac{6}{11} + \frac{5}{11} - 1)$

$\frac{22}{25} (\frac{11}{11} - 1) = \frac{22}{25} (1-1) = \frac{22}{25} (0) = 0$

---

(4) $\frac{6}{37} \times 7 + \frac{6}{37} \times 5 + \frac{6}{37} \times (-11)$

Sol

$\frac{6}{37} \times 7 + \frac{6}{37} \times 5 + \frac{6}{37} \times (-11) = \frac{6}{37} (7+5-11)$

$\frac{6}{37} (1) = \frac{6}{37}$

(5) $\frac{-3}{7} \times 8 + 5 \times \frac{-3}{7} + \frac{-3}{7}$ sol

$$\frac{-3}{7} \times 8 + 5 \times \frac{-3}{7} + \frac{-3}{7} = \frac{-3}{7}(8+5+1) = \frac{-3}{7} \times 14 = -6$$

properties of division operation in Q

(1)The division by zero is impossible in Q then Q not closed under division operation

Example

$\frac{5}{11} \div 0$ = *impossible in Q*

---

(2) *Q is not commutative*

Example

$Zero \div \frac{5}{11} = 0$ *and* $\frac{5}{11} \div Zero$ = *impossible in Q*

---

(3) *Q is not associative*

Example

$(\frac{5}{11} \div Zero) \div \frac{6}{7}$ = *impossible in Q*

$(\frac{6}{7} \div \frac{5}{11}) \div Zero$ = *impossible in Q*

*(4)**There is not identity element in Q under division operation so there is not inverse numbers with respect to division operation***

Important examples

Complete

(1) $3 \times \frac{1}{3} = 1$

(2) $\frac{1}{4} \times 4 = 1$

(3) If $X + \frac{7}{11} = 0$ then $11x = -7$

(4) If: $\frac{a}{b} = 70$ then $\frac{a}{2b} = 35$

(5) If: $\frac{a}{b} = \frac{1}{2}$ then $\frac{2a}{b} = 1$

(6) If: $\frac{x}{y} = 1$ then $3x - 3y = 0$

(7) If $x + \frac{3}{x} = 4 + \frac{3}{4} = 4$

(8) If $x + \frac{5}{8} = \frac{5}{8}$ then $x = 0$

(9) $x \times \frac{5}{8} = \frac{5}{8}$ then $x = 1$

(10) $x \times \frac{5}{8} = 1$ then $x = \frac{8}{5}$

Example

If $a = 2$, $b = \frac{1}{2}$ and $c = \frac{3}{2}$

Find the numerical value of $(a - b) \div c$

Sol

$$(a-b) \div c = (2-\frac{1}{2}) \div \frac{3}{2} = \frac{3}{2} \div \frac{3}{2} = 1$$

Example

If $a = \frac{3}{4}$, $b = \frac{-5}{2}$ and

Find the numerical value of $\frac{(a-b)}{(a+b)}$

Sol

$$\frac{(a-b)}{(a+b)} = (a-b) \div (a+b) = (\frac{3}{4} - \frac{-5}{2}) \div (\frac{3}{4} + \frac{-5}{2})$$

$$= (\frac{3}{4} + \frac{5}{2}) \div (\frac{3}{4} - \frac{5}{2}) = \frac{5 \times 4 + 2 \times 3}{2 \times 4} \div \frac{5 \times 4 - 2 \times 3}{2 \times 4}$$

$$\frac{26}{8} \div \frac{-14}{8} = \frac{26}{8} \times \frac{-8}{14} = \frac{-13}{7}$$

Example

If $x = \frac{-1}{3}$, $y = \frac{3}{4}$ and $z = -3$

Find the numerical value of

(1) $\frac{y}{z}$ (2) $\frac{xy}{z}$ (3) $\frac{x}{y} - \frac{y}{z}$ (4) $\frac{1}{xyz}$ (5) $xy + yz$

Sol

(1) $\frac{y}{z} = y \div z = \frac{3}{4} \div (-3) = (\frac{3}{4} \times \frac{1}{3}) = \frac{1}{4}$

(2) $\frac{xy}{z} = (xy) \div z = (\frac{3}{4} \times \frac{-1}{3}) \times \frac{-1}{3} = \frac{1}{12}$

(3) $\frac{x}{y} - \frac{y}{z} = (x \div y) - (y \div z) = (\frac{-1}{3} \div \frac{3}{4}) - (\frac{3}{4} \div (-3))$

$= (\frac{-1}{3} \times \frac{4}{3}) - (\frac{3}{4} \times \frac{-1}{3}) = \frac{-4}{9} - \frac{-1}{4} = \frac{-4}{9} + \frac{1}{4} = \frac{1}{6}$

(4) $\frac{1}{xyz} = 1 \div (xyz) = 1 \div (\frac{-1}{3} \times (-3) \times \frac{3}{4})$

$1 \div \frac{3}{4} = 1 \times \frac{4}{3} = \frac{4}{3}$

(5) $xy + yz = \frac{-1}{3} \times \frac{3}{4} + \frac{3}{4} \times (-3) = \frac{-1}{4} + \frac{-9}{4} = \frac{-10}{4}$

Applications of the rational numbers

The distance between two numbers

The distance between x and y $= |x - y|$

Example

Find the distance between : $\frac{1}{2}$, $\frac{1}{4}$

Sol

the distance between : $\frac{1}{2}$, $\frac{1}{4} = \left|\frac{1}{2} - \frac{1}{4}\right| = \left|\frac{1}{2}\right| = \frac{1}{2}$

Example

Find the distance between : $\frac{-6}{5}$, $\frac{-7}{6}$

Sol

the distance between : $\frac{-6}{5}$, $\frac{-7}{6} = \left|\frac{-6}{5} - \frac{-7}{6}\right| = \left|\frac{-1}{36}\right| = \frac{1}{36}$

Example

Find the distance between : $\frac{-6}{5}$, $\frac{-7}{6}$

Sol

the distance between : $\frac{-6}{5}$, $\frac{-7}{6} = \left|\frac{-6}{5} - \frac{-7}{6}\right| = \left|\frac{-1}{36}\right| = \frac{1}{36}$

Example

Find the distance between : $\frac{6}{7}$, $\frac{-2}{9}$

Sol

the distance between : $\frac{6}{7}$, $\frac{-2}{9} = \left|\frac{6}{7} - \frac{-2}{9}\right| = \left|\frac{68}{63}\right| = \frac{68}{63}$

<u>the number that lies at the mid-point of the way between any two numbers</u>

the smallest number $+\frac{1}{2}$ *the distance*

<u>or</u>

the greatest number $-\frac{1}{2}$ *the distance*

Example

Find the rational number that lies at the mid-point of the way between $\frac{2}{5}$, $\frac{3}{7}$

<u>Sol</u>

the rational number=the smallest number $+\frac{1}{2}$ *the distance*

to know the smallest number we compare between $\frac{2}{5}$, $\frac{3}{7}$

$\frac{2}{5} = \frac{2 \times 7}{5 \times 7} = \frac{14}{35}$ $\quad \frac{3}{7} = \frac{3 \times 5}{7 \times 5} = \frac{15}{35}$ $\quad \frac{2}{5} < \frac{3}{7}$

the smallest number is $\frac{2}{5}$

the rational number$= \frac{2}{5} + \frac{1}{2}\left|\frac{2}{5} - \frac{3}{7}\right| = \frac{14}{35} + \frac{1}{2}\left|\frac{14}{35} - \frac{15}{35}\right|$

$\frac{14}{35} + \frac{1}{2}\left|\frac{-1}{35}\right| = \frac{14}{35} + \frac{1}{70} = \frac{29}{70}$

<u>the number that lies at the one third of the way</u>

from the smallest number =the smallest number $+\frac{1}{3}$ *the distance*

<u>or</u>

from the greatest number =the greatest number $-\frac{1}{3}$ *the distance*

Example

Find the rational number that lies ***<u>at one third of the way</u>*** *between* $\frac{2}{5}$ *, zero*

(1) *from the side of the smallest number*
(2) *from the side of the greatest number*

<u>Sol</u>

from the smallest number =the smallest number + $\frac{1}{3}$ *the distance*

to know the smallest number we compare between $\frac{2}{5}$ *, zero*

$zero < \frac{2}{5}$

the smallest number is zero

$$\text{the rational number} = zero + \frac{1}{3}\left|\frac{2}{5} - zero\right| = \frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$$

from the greatest number =the greatest number - $\frac{1}{3}$ *the distance*

$$\text{from the greatest number} = \frac{2}{5} - \frac{1}{3}\left|\frac{2}{5} - zero\right| = \frac{2}{5} - \frac{2}{15} = \frac{4}{15}$$

<u>the number that lies at the one fourth of the way</u>

from the side of the smallest number =the smallest number + $\frac{1}{4}$ *the distance*

<u>or</u>

from the side of the greatest number =the greatest number - $\frac{1}{4}$ *the distance*

Example

Find the rational number that lies ***at one fourth of the way*** *between* $\frac{-1}{6}$, $\frac{-1}{3}$

(1) from the side of the smallest number

(2) from the side of the greatest number

Sol

(1) from the side of the smallest number

=*the smallest number* $+\frac{1}{4}$ *the distance*

to know the smallest number we compare between $\frac{-1}{6}$, $\frac{-1}{3}$

$\frac{-1}{3} = \frac{-1 \times 6}{3 \times 6} = \frac{-6}{18}$ and $\frac{-1}{6} = \frac{-1 \times 3}{6 \times 3} = \frac{-3}{18}$ then $\frac{-1}{3} < \frac{-1}{6}$

the smallest number is $\frac{-1}{3}$

the rational number $= \frac{-1}{6} + \frac{1}{4}\left|\frac{-1}{3} - \frac{-1}{6}\right|$

$\frac{-6}{18} + \frac{1}{4}\left|\frac{-6}{18} + \frac{3}{18}\right| = \frac{-6}{18} + \frac{1}{4}(\frac{1}{6}) = \frac{-3}{18} + \frac{1}{24} = \frac{-7}{24}$

from the greatest number =*the greatest number* $-\frac{1}{4}$ *the distance*

the rational number $= \frac{-1}{3} - \frac{1}{4}\left|\frac{-1}{3} - \frac{-1}{6}\right|$

$\frac{-3}{18} - \frac{1}{4}\left|\frac{-6}{18} + \frac{3}{18}\right| = \frac{-3}{18} - \frac{1}{4}(\frac{1}{6}) = \frac{-3}{18} - \frac{1}{24} = \frac{-5}{24}$

the number that lies at the one fifth of the way

from the side of the smallest number =the smallest number $+\frac{1}{5}$ *the distance*

or

from the side of the greatest number =the greatest number $-\frac{1}{5}$ *the distance*

Example

Find the rational number that lies ***at one fifth of the way*** *between* $\frac{2}{5}$, $\frac{4}{7}$

(3) *from the side of the smallest number*

(4) *from the side of the greatest number*

Sol

(2) *from the side of the smallest number*

=the smallest number $+\frac{1}{5}$ *the distance*

to know the smallest number we compare between $\frac{2}{5}$, $\frac{4}{7}$

$$\frac{2}{5}=\frac{2\times 7}{5\times 7}=\frac{14}{35} \quad \text{and} \quad \frac{4}{7}=\frac{4\times 5}{7\times 5}=\frac{20}{35} \quad \text{then } \frac{2}{5}<\frac{4}{7}$$

the smallest number is $\frac{2}{5}$

$$\text{the rational number}=\frac{2}{5}+\frac{1}{5}\left|\frac{2}{5}-\frac{4}{7}\right|$$

$$\frac{14}{35}+\frac{1}{5}\left|\frac{14}{35}-\frac{20}{35}\right|=\frac{14}{35}+\frac{1}{5}\left(\frac{6}{35}\right)=\frac{14}{35}+\frac{6}{175}=\frac{76}{175}$$

from the greatest number =the greatest number - $\frac{1}{5}$ *the distance*

the rational number$= \frac{2}{5} - \frac{1}{5} \left| \frac{2}{5} - \frac{4}{7} \right|$

you can complete the problem

Algebraic terms and Algebraic expression

Algebraic term *is formed from the product of two or more factors*

Examples

(1) $3\,x$ — Numerical factor: 3 ; Algebraic factor: x

(2) $-\,x\,y$ — Numerical factor: $-$; Algebraic factor: $x\,y$

(3) $7\,x^2$ — Numerical factor: 7 ; Algebraic factor: x^2

The degree of the Algebraic term

It is the sum of the indices of the algebraic factors in this term

Examples

(1) *the coefficient of the algebraic term* $5\,x$ *is* ***5***
and its degree is ***the first***(1^{st})

(2) *the coefficient of the algebraic term* $3\,yx$ *is* ***3***
and its degree is ***the second*** (2^{nd})

(3) *the coefficient of the algebraic term* $-5\,a^2$ *is* ***-5***
and its degree is ***the second*** (2^{nd})

(4) *the coefficient of the algebraic term* $4\,x^2y$ *is* ***4***
and its degree is ***the third***(3^{rd})

(5) *the coefficient of the algebraic term* $-2\,a^2b^2$ *is* ***-2***
and its degree is ***the fourth***(4^{th})

(6) *the coefficient of the algebraic term* $15\,a^3b$ *is* ***15***
and its degree is ***the fourth***(4^{th})

(7) *the coefficient of the algebraic term* x *is* ***1***
and its degree is ***the first***(1^{st})

(8) *the coefficient of the algebraic term* $-\,4$ *is* ***- 4***
and its degree is ***zero***

(9) *the coefficient of the algebraic term* $(-\,3)^2$ *is* ***9***
and its degree is ***zero***

Algebraic expression *consist of an algebraic term or more*

Examples

(1) $3x$

the Algebraic expression consist of one term **(monomial)**

(2) $5a + b$

the Algebraic expression consist of two terms **(binomial)**

(3) $5y^2 + 2xy - 3x$

the Algebraic expression consist of three terms **(trinomial)**

The degree of the Algebraic expression

It is the highest degree of the teams forming it

Examples

(1) The number of the terms of the Algebraic expression :- $2a^2b^3$ = **1**

And its name **(monomial** and its degree **the fifth(5th)**

(2) The number of the terms of the Algebraic expression $a^3 - 5a^2b^2 + 3b^2$ **=3**

And its name **(trinomial)** and its degree **the fourth(4th)**

(3) The number of the terms of the Algebraic expression $\frac{1}{2}a + \frac{1}{4}b - 5$ **=3**

And its name **(trinomial** and its degree **the first(1^{st})**

(4) The number of the terms of the Algebraic expression $2x^2y + 5xy + 4y$ =**3**

And its name **(trinomial** and its degree **the third(3^{rd})**

(5) The number of the terms of the Algebraic expression :$1 - 2a^2b$ =**2**

And its name **(binomial)** and its degree **the third(3^{rd})**

(6) *The number of the terms of the Algebraic expression :-*
$2a^2 + 2^4x =$ **2**

And its name ***(binomial)*** *and its degree* ***the second (2^{nd})***

Like algebraic terms

Add

(1) $5a$, $3a$, a , $6a$

They like terms so we can add

Then $(5a + 3a + a + 6a) = 15a$

---

(2) $7ab^2$, $-2ab^2$, $-4ab^2$ ، ab^2

They like terms so we can add

Then $(7ab^2 - 2ab^2 - 4ab^2 + ab^2) = 2ab^2$

---

(3) $7x^2$, $-3x^2$, $4x^2$, $9x^2$

They like terms so we can add

Then $(7x^2 - 3x^2 + 4x^2 + 9x^2) = 17x^2$

---

(4) $3x$, $-4x^2y$, z

We cannot add because they do not like terms

i.e. x *not like* x^2y *not like* z

---

(5) $3x$, $-4x^2$, x^3

We cannot add because they do not like terms

i.e. x *not like* x^2 *not like* x^3

---

(6) $3a$, $2b$, c

We cannot add because they do not like terms

i.e. a *not like* b^2 *not like* c

adding and subtracting algebraic expression

add the following expressions

(1) $3x - 2y + 5 \quad , x + 2y - 2$

sol

$\underline{3x} - 2y + 5 \quad + \quad \underline{x} + 2y - 2$

$(3x + x) + (-2y + 2y) + (5-2) = 4x + 0y + 3$

$= 4x + 3$

---

(2) $3N^2 + 5N - 6 \quad , -N^2 - 3N + 3$

sol

$\underline{3N^2} + 5N - 6 \quad + (\underline{-N^2} - 3N + 3)$

$(3N^2 - N^2) + (5N - 3N) + (-6+3)$

$2N^2 + 2N - 3$

---

(3) $3a^3 - 2a\,b^2 \quad , \quad a^3 - 4a\,b^2 - b^3$

sol

$\underline{3a^3} - 2a\,b^2 \quad + \quad \underline{a^3} - 4a\,b^2 - b^3$

$(3a^3 + a^3) + (-2a\,b^2 - 4a\,b^2) - b^3$

$4\,a^3 - 6\,a\,b^2 - b^3$

---

(4) $3a - 7b - 5c + 2 \quad , -a + 4b - 5\,c - 5 \, , \, 2a + 3c + 3$

sol

$\underline{3a} - 7b - \underline{5c} + 2 \quad + (\underline{-a} + 4b - \underline{5\,c} - 5) + (\underline{2a} + \underline{3c} + 3)$

$(3a - a + 2a) + (-7b + 4b) + (-5\,c - 5\,c + 3c) + (2 - 5 + 3)$

$4a - 3b - 7c + 0$

$4a - 3b - 7c$

Subtract *from*
Second - first

Subtract

(1) $x - 2$ from $2x - 5$

sol

$2x - 5 - (x - 2) = \underline{\underline{2x}} - 5 - \underline{\underline{x}} + 2$

$(2x - x) + (-5 + 2) = x - 3$

---

(2) $2x + 6y - 7$ from $2x - 5y + 2$

sol

$(2x - 5y + 2) - (2x + 6y - 7)$

$\underline{\underline{2x}} - 5y + 2 \ \underline{\underline{-2x}} - 6y + 7$

$(2x - 2x) + (-5y - 6y) + (2 + 7)$

$0 - 11y + 9 = -11y + 9$

$9 - 11y$

---

(3) $a + 2b + 3$ from $a - 3b + 5$

sol

$a - 3b + 5 - (a + 2b + 3) = \underline{\underline{a}} - 3b + 5 \ \underline{\underline{-a}} - 2b - 3$

$(a - a) + (-3b - 2b) + (5 - 3)$

$0 - 5b + 2$

$2 - 5b$

---

(4) $-x^2 - 4x + 7$ from $3x^2 - 4x - 2$

sol

$3x^2 - 4x - 2 - (-x^2 - 4x + 7) = \underline{\underline{3x^2}} - 4x - 2 \ \underline{\underline{-x^2}} + 4x - 7$

$(3x^2 - x^2) + (-4x + 4x) + (-2 - 7)$

$2x^2 + 0 - 9$

$2x^2 - 9$

What the increase of then first -Second

(1)What the increase of

$5x^2 - 5x - 1$ than $3x^2 + 2x - 3$

sol

$5x^2 - 5x - 1 - (3x^2 + 2x - 3) = 5x^2 - 5x - 1 - 3x^2 - 2x + 3$

$= (5x^2 - 3x^2) + (-5x - 2x) + (-1 + 3)$

$= 2x^2 - 7x + 2$

---

(2)What the increase of

$3x^2 + y^2 - xy$ than $4x^2 + y^2 - xy$

Sol

$3x^2 + y^2 - xy - (4x^2 + y^2 - xy) = 3x^2 + y^2 - xy - 4x^2 - y^2 + xy$

$= (3x^2 - 4x^2) + (y^2 - y^2) + (-xy + xy)$

$- x^2 + 0 + 0 = -x^2$

---

(3)What the increase of

$3x^2 - 5 + 2x$ than the sum of $x + 5x^2 + 1$ and , $2x^2 - 4 - 2x$

sol

the sum $= x + 5x^2 + 1 + 2x^2 - 4 - 2x$

$= (5x^2 + 2x^2) + (x - 2x) + (1 - 4)$

$= 7x^2 - x - 3$

the increase $= 3x^2 - 5 + 2x - (7x^2 - x - 3)$

$= 3x^2 - 5 + 2x - 7x^2 + x + 3$

$= (3x^2 - 7x^2) + (2x + x) + (-5 + 3)$

$= -4x^2 + 3x - 2$

What is the expression which should be added to
$8 - 3a^2 + 2a^3$ to get the result $5 + 4a^3 - 7a$)?

sol

Subtract from
Second - first

the expression $= 5 + 4a^3 - 7a - (8 - 3a^2 + 2a^3)$

$= 5 + 4a^3 - 7a - 8 + 3a^2 - 2a^3$

$= (4a^3 - 2a^3) + 3a^2 - 7a + (-8 + 5)$

$= 2a^3 + 3a^2 - 7a - 3$

---

(1)What the decrease of

What the decrease of about
second - first

$x^2 - 5x - 1$ about $3x^2 + 2x - 3$

sol

$3x^2 + 2x - 3 - (x^2 - 5x - 1) = 3x^2 + 2\underline{\underline{x}} - 3 - x^2 + 5\underline{\underline{x}} + 1$

$= (3x^2 - x^2) + (5x + 2x) + (1 - 3)$

$= 2x^2 + 7x - 2$

---

(4)***What is the expression which should be subtracted from***
$3 l2 - m2 - lm$ to get the result $7l2 - (m2 + lm)$?

Here we add

sol

the expression $= 7l^2 - (m^2 + lm) + (3l^2 - m^2 - l)$

$7l^2 - \underline{\underline{m^2}} - lm + 3l^2 - \underline{\underline{m^2}} - l$

$= (7l^2 + 3l^2) + (-m^2 - m^2) - lm - l$

$= 10l^2 - 2m^2 - lm - l$

$= 10l^2 - 2m^2 - lm - l$

Multiplying and dividing algebraic terms

When we Multiplying algebraic terms we follow the following :

(1) *Multiplying the coefficient using the sign rule*

(2) *Multiplying symbols taking care that the indices of the like bases should be added* $(a^m \times a^n = a^{m+n})$

Examples

(1) $2a \times 5b$

Sol

$(2 \times 5)(a \times b) = 10\,a\,b$

(2) $(5x2)\,x(3x)$

Sol

$(5 \times 3)(x^2 \times x) = 15\,x^{2+1} = 15\,x^3$

(3) $5a^3b \times 3ab$

Sol

$(5 \times 3)(a^2 \times a)(b \times b) = 15\,a^{2+1}\,b^{1+1} = 15\,a^3b^2$

(4) $\frac{3}{4}\,a2 \times \frac{4}{3}\,a$

Sol

$(\frac{3}{4} \times \frac{4}{3})(a^2 \times a) = 1 \times a^3 = a^3$

(5) $\frac{2}{5}\,x^2 \times (-15\,x^3)$

Sol

$(\frac{2}{5} \times -15)(x^2 \times x^3) = -6\,x^{3+2} = -6x^5$

(6) $-8\,y^5 \times -7\,y^4$

Sol

$(-8 \times -7)(y^5 \times y^4) = 56y^9$

(7) $-2ab \times 5a^2c$

Sol

$(-2 \times 5)(a \times a^2)(b \times c) = -10a^3bc$

When we dividing *algebraic terms we follow the following :*

(1) **dividing** *the coefficient using the sign rule*

(2) **dividing** *symbols taking care that the indices of the like bases should be subtracted* $(a^m \div a^n = a^{m-n})$

Find the quotient of each of the following

(1) $12a^3$ by $3a$

Sol

$(12 \div 3)(a^3 \div a) = 4a^{3-1} = 4a^2$

---

(2) $21x$ by (-3)

Sol

$(21 \div (-3))x = -7x$

---

(3) $-15x^2y^3$ by $5xy^2$

Sol

$(-15 \div 5)(x^{2-1})(y^{3-2}) = -3xy$

---

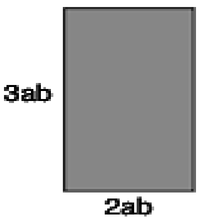
(4) $-24a^5b^3c^2$ by $(-8a^2b)$

Sol

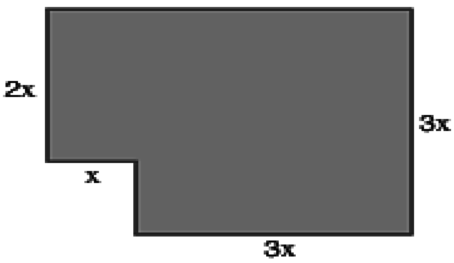
$(-24 \div (-8))(a^{5-2})(b^{3-1})c^2 = 3a^3b^2c^2$

5 Calculate the perimeter and the area of each shaded region:

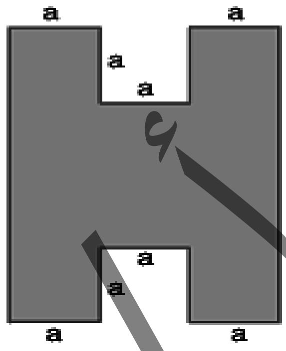
[a]



[b]



[c]



Multiplying a monomial by an algebraic expression

Examples

(1) $2x\ (2\,x + 5\,y)$

Sol

$2x\ (2\,x + 5\,y = (2x \times 2x) + (2x \times 5\,y) = 4\,x^{1+1} + 10\,x\,y$

$= 4\,x^{2} + 10\,x\,y$

---

(2) $b\ (-2a + a^2b)$

Sol

$b\ (-2a + a^2b) = (b \times -2a) + (b \times a^2b)$

$= (-2ab + a^2b^{1+1}) = (-2ab + a^2b^2)$

---

(3) $-3a\,b\ (5a - 2b + 3\ )$

Sol

$-3a\,b\ (5a - 2b + 3\ ) = (-3a\,b) \times 5a - (-3a\,b) \times 2\,b + (-3a\,b) \times 3$

$= -15a^{1+1}b + 6\,ab^{1+1} - 9ab$

$= -15a^2b + 6ab^2 - 9ab$

---

(4) $2b^{\,2}\ (\,a^2 - ab - 2b^{\,2})\ 4ab$

Sol

$4ab(\,a^2 - ab - 2b^{\,2}) = (4ab \times a^2) - (\,4ab \times ab) - (4ab \times 2b^{\,2})$

$= 4\,a^3b - 4a^2b^2 - 8ab^3$

---

(5) $\dfrac{1}{3}\,x^2(6x^2 - 9xy - 3y^2)$

Sol

$\dfrac{1}{3}\,x^2 \times (6x^2) + (\dfrac{1}{3}\,x^2 \times -9xy) + (\dfrac{1}{3}\,x^2 \times -3y^2)$

$2x^{2+2}-3x^{2+1}y + x^2y^2 = 2x^4-3x^3y + x^2y^2$

(6) $Lm^2(L^2-3Lm-4m^2)$

Sol

$Lm^2(L^2-3Lm-4m^2)\ (Lm^2 \times L^2) - (Lm^2 \times 3Lm) - (Lm^2 \times 4m^2)$

$= L^{1+2}\ m^2 - 3L^{1+1}m^{2+1} - 4Lm^{2+2}$

$= L^3M^2 - 3L^2m^3 - 4Lm^4$

Simplify

(1) ***$3(1-2x)-(x^2-5x+3)+2x(x+3)$ then find the numerical value when $x=-2$***

Sol

$3(1-2x)-(x^2-5x+3)+2x(x+3) = 3-6x-x^2+5x-3+2x^2+6x$

$3-6x-x^2+5x-3+2x^2+6x =$

Simplily: $5(2x-1)-3(x^2-1)+x(5x-1)$, then find the numerical value of the expression when $x=1$

Solution:

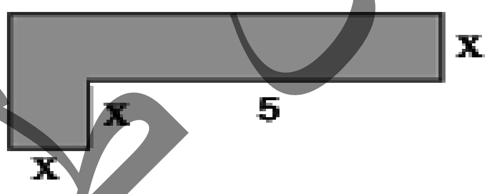
$5(2x-1)-3(x^2-1)+x(5x-1) = 10x-5-3x^2+3+5x^2-x$

$= 2x^2+9x-2$

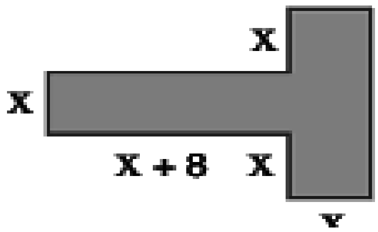
The numerical value $= 2(1)^2+(9\times 1)-2 = 2+9-2 = 9$

2 Find the area of each shaded region:

[a]



[b]



Multiplying a binomial by an algebraic expression

$(a+b)(c+d) = (a \times c)+(a \times d)+(b \times c)+(b \times d)$

First X First	First X second	second X First	second X second

Examples

(find the product of)

(1) $(x+2)(2x-3)$

Sol

$= (x \times 2x)+(x \times -3)+(2 \times 2x)+(2 \times -3)$

$= 2x^2 - 3x + 4x - 6$

$= 2x^2 + x - 6$

---

(2) $(4x+1)(2x+3)$

Sol

$= (4x \times 2x)+(4x \times 3)+(1 \times 2x)+(1 \times 3)$

$= 8x^2 + 12x + 2x + 3$

$= 8x^2 + 14x + 3$

---

(3) $(5a-2b)(7a-3b)$

Sol

$= (5a \times 7a)+(5a \times -3b)+(-2b \times 7a)+(-2b \times -3b)$

$= 35a^2 - 15ab - 14ab + 6b^2$

$= 35a^2 - 29ab + 6b^2$

(4) $(4x-3y)(3y+x)$

Sol

$= (4x \times 3y)+(4x \times x)+(-3y \times 3y)+(-3y \times x)$

$=12xy+4x^2-9y^2-3xy$

$= 4x^2-9y^2+12xy-3xy$

$= 4x^2-9y^2+9$

---

(5) $3(m-5)(m+2)$

Sol

$= 3(m \times m)+(m \times 2)+(-5 \times m)+(-5 \times 2)$

$=3(m^2+2m-5m-10)$

$= 3(m^2+2m-5m-10)$

$= 3(m^2-3m-10)$

$=3m^2-9m-30$

---

(6) $3a(2a-5b)(3a+b)$

Sol

$= 3a(2a \times 3a)+(2a \times b)+(-5b \times 3a)+(-5b \times b)$

$=3a(6a^2+2ab-15ab-5b^2)$

$= 3a(6a^2-13ab-5b^2)$

$=18a^3-39a^2b-15ab^2$

Two special cases

Expanding the square of an expression containing two terms

$(x+y)^2$ = square the first $+2\times$ the first $\times$ the second + square the second

$(x+y)^2 = x^2 + 2xy + y^2$

$(x-y)^2$ = square the first $-2\times$ the first $\times$ the second + square the second

$(x-y)^2 = x^2 - 2xy + y^2$

Find the expansion of each of the following

(1) $(3a+5)^2$

Sol

$$(3a+5)^2 = (3a)^2 + (2 \times 3a \times 5) + (5)^2$$
$$= 9a^2 + 30a + 25$$

---

(2) $(2x-3y)^2$

Sol

$$(2x-3y)^2 = (2x)^2 + (2 \times 2x \times 3y) + (3y)^2$$
$$= 4x^2 + 12xy + 9y^2$$

---

(3) $(x+4)^2 - (x+2)(x+6)$

Sol

$$(x+4)^2 = (x)^2 + (2 \times x \times 4) + (4)^2$$
$$= x^2 + 8x + 16$$

$(x+2)(x+6) =$

$= (x \times x) + (x \times 6) + (2 \times x) + (2 \times 6)$

$= x^2 + 6x + 2x + 12$

$= x^2 + 8x + 12$

$(x+4)^2 - (x+2)(x+6) = \cancel{x^2} + 8x + 16 \cancel{- x^2} - 8x - 12 = 16 - 12 = 4$

The product of the sum of two terms and the difference between them

$(a+b)(a-b) = first \times first - second \times second$
$(a+b)(a-b) = a^2 - b^2$

(1) $(2l-5)(2l+5)$

Sol

$(2l-5)(2l+5) = (2l)^2 - (5)^2$
$= 4l^2 - 25$

(2) $(x+5)(x-5)$

Sol

$(x+5)(x-5) = x^2 -$

(3) $(5x+3y)(5x-3y)$

Sol

$(5x+3y)(5x-3y) = (5x)^2 - (3y)^2$
$= 25x^2 - 9y^2$

(4) $(a^2+2b)(a^2-2b)$

Sol

$(a^2+2b)(a^2-2b) = (a^2)^2 - (2b)^2$
$= a^4 - 4b^2$

(5) $(\frac{1}{3}x+\frac{2}{5}y)(\frac{1}{3}x-\frac{2}{5}y)$

Sol

$$(\frac{1}{3}x+\frac{2}{5}y)(\frac{1}{3}x-\frac{2}{5}y) = (\frac{1}{3}x)^2-(\frac{2}{5}y)^2$$

$$=\frac{1}{9}x^2-\frac{4}{25}y^2$$

---

(6) $(x+5)(x-5)+(x-5)^2$

Sol

$$(x+5)(x-5) = x^2-25$$

$$(x-5)^2 = x^2 -10x +25$$

$$x+5)(x-5)+(x-5)^2 = x^2 -25 + x^2 -10x +25$$

$$=2x^2 -10x$$

Multiplying a binomial by an algebraic expression forming more than two terms

Examples(find the product of)

(1) $(x-3)(x^2+4x-7)$

Sol

$$= (x \times x^2)+(x\times 4x)+(x \times -7)+(-3\times x^2)+(-3\times 4x)+(-3\times -7)$$

$= x^3 +4x^2-7x -3x^2-12x +21$ (put the like terms with each other)

$$= x^3 +(4x^2-3x^2)+(-7x-12x) +21$$

$$= x^3 +x^2 -19x +21$$

(1) $(2y+1)(y2+y+7)$ and find the numerical value when $y=1$

Sol

$= (2y \times y^2)+(2y \times 2y)+(2y \times 7)+(1 \times y^2)+(1 \times y)+(1 \times 7)$

$= 2y^3 + 4y^2 + 14y + y^2 + y + 7$ (put the like terms with each other)

$= 2y^3 + (4y^2 + y^2) + (14y + y) + 7$

$= 2y^3 + 5y^2 + 15y + 7$

when $y = 1 >>> 2y^3 + 5y^2 + 15y + 7 = 2+5+15+7 = 29$

(2) $(3x-4)(x+2) - (2x-3)^2$ and find the numerical value when $x=0$

Sol

$(3x-4)(x+2) = (3x \times x)+(3x \times 2)+(-4 \times x)+(-4 \times 2)$

$= 3x^2 + 6x - 4x - 8$

$= 3x^2 + 2x - 8$

$(2x-3)^2 = (2x)^2 - (2 \times 2x \times 3) + (-3)^2$

$= 4x^2 - 12x + 9$

$(3x-4)(x+2) - (2x-3)^2 = 3x^2 + 2x - 8 - 4x^2 + 12x - 9$

$= -x^2 + 14x - 9$

when $x = 0 >>> -x^2 + 14x - 9 = -9$

use the multiplication by inspection to find the value of each of the following

(1) $(52)^2$ *(2)* $(195)^2$ *(3)* 502×498

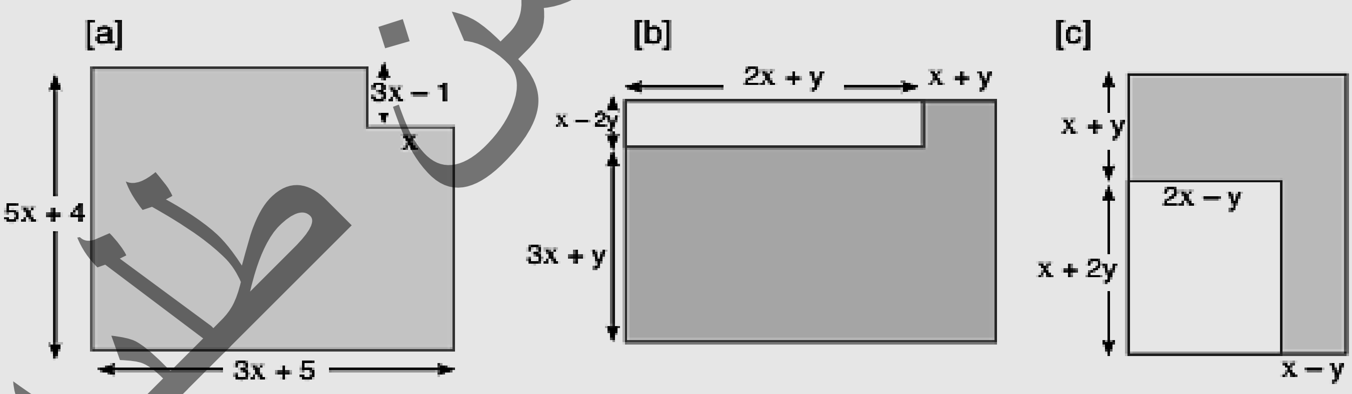
Sol

(1) $(52)^2 = (50+2)^2 = 2500 + 200 + 4 = 2704$

(2) $(195)^2 = (200-5)^2 = 40000 - 2000 + 25 = 38025$

(3) $502 \times 498 = (500+2)(500-2) = 250000 - 4 = 249996$

4 Write an expression for the perimeter and area of each shaded region:



Dividing an algebraic expression by a monomial

Find the quotient of dividing

(1) $21x^2+14x$ by $(7x)$

Sol

$$\frac{21x^2+14x}{7x}=\frac{21x^2}{7x}+\frac{14x}{7x}=3x+2$$

---

(2) $16x^3y+8x^2y^3-12x^2y$ by $(-4x^2y)$

Sol

$$\frac{16x^3y+8x^2y^3-12x^2y}{-4x^2y}=\frac{16x^3y}{-4x^2y}+\frac{8x^2y^3}{-4x^2y}-\frac{12x^2y}{-4x^2y}=-4x-2y^2+3$$

---

(3) $15n^3-9m^4n^2$ by $(-3n^2)$

Sol

$$\frac{15n^3-9m^4n^2}{-3n^2}=\frac{15n^3}{-3n^2}-\frac{9m^4n^2}{-3n^2}=-5n+3m^4$$

---

(4) $20a^3b^2+15\,a^2b^3+10ab$ by $(5ab)$

Sol

$$\frac{20a^3b^2+15\,a^2b^3+10ab}{5ab}=\frac{20a^3b^2}{5ab}+\frac{15\,a^2b^3}{5ab}+\frac{10ab}{5ab}$$

$$=4a^2b+3ab^2+2$$

---

(5) $x^3y^6-8x^2y^2+6xy^2$ by $(x\,y)$

Sol

$$\frac{x^3y^6-8x^2y^2+6xy^2}{xy}=\frac{x^3y^6}{xy}-\frac{8x^2y^2}{x\,y}+\frac{6xy^2}{x\,y}=x^2y^5-8xy+6y$$

(6) $3ab^2c-5a^2bc+2abc^2$ by $(a\,b\,c)$ when $a=1, b=-2, c=3$

Sol

$$\frac{3ab^2c-5a^2bc+2abc^2}{abc}=\frac{3ab^2c}{abc}-\frac{5a^2bc}{abc}+\frac{2abc^2}{abc}=3b-5a+2c$$

$$3b-5a+2c=3-5+2=0$$

Divide each of the following

[a] $\dfrac{26e^2+14e^4}{2e}$ [b] $\dfrac{9\ell^3m^4-18\ell m^4}{3\ell m^2}$

Solution:

[a] $\dfrac{26e^2+14e^4}{2e}=\dfrac{26e^2}{2e}+\dfrac{14e^4}{2e}=13e+7e^4$

[b] $\dfrac{9\ell^3m^4-18\ell m^4}{3\ell m^2}=3\ell^2m^2-6$

Dividing an algebraic expression by another one

[a] Rearrang the dividend $(x^2 + 5x + 6)$ and the divisor $(x + 2)$ according to the desendingl powers of x.

[b] Divide x^2 by x the result x

[c] Multiply x by the divisor ⟶

[d] Subtract $x^2 + 2x$ from x2 + 5x + 6 to get ⟶

[e] Repeat the steps 2, 3, 4 to be the final subtraction equals zero

$x^2 + 5x + 6$	$x + 2$
$^{-}x^2 \mp {}^{-}2x$	$x + 3$
$3x + 6$	
$^{-}3x \mp 6$	
0 0	

$\therefore$ The quotient $= x + 3$ the length of the rectangle

Example (1)

Find the quotient of $x^3 + 1$ by $x + 1$

Solution:

$x^3 +$ $+1$	$x + 1$
$-x^3 \mp x^2$	$x^2 - x + 1$
$-x^2$ $+1$	
$\mp x^2 \pm x$	
$x + 1$	
$-x \mp 1$	
0 0	

$\therefore$ The quotient $= x^2 - x + 1$

Find the quotient of dividing

(1) x^2+5x+6 by $x+2$ the quotient $= x+3$

x^2+5x+6	$x+2$
	$x+3$
$-x^2-2x$	
$3x+6$	
$-3x-6$	
0 0	

Find the quotient of dividing

(2) X^3+x+10 by $x+2$ *the quotient* $= x^2-2x+5$

x^3+x+10	$x+2$
	x^2-2x+5
$-x^3 \quad -2x^2$	
$-2x^2+x+10$	
$+2x^2+4x$	
$5x+10$	
$-5x-10$	
$0 \quad 0$	

Find the quotient of dividing

(3) $2X^3-5x^2-22x-15$ by $2x+3$ *the quotient* $= x^2-4x-5$

$2X^3-5x^2-22x-15$	$2x+3$
	x^2-4x-5
$-2x^3-3x^2$	
$-8x^2-22x-15$	
$+8x^2+12x$	
$-10x-15$	
$+10x+15$	
$0 \quad 0$	

Find the quotient of dividing

(4) $6x^2+13xy+6y^2$ by $2x+3y$ *the quotient* $= 3x+2y$

$6x^2+13xy+6y^2$	$2x+3y$
	$3x+2y$
$-6x^2-9xy$	
$4xy+6y^2$	
$-4xy-6y^2$	
$0 \quad 0$	

---

(5) $x^4+49-18x^2$ by $2x-7+x^2$ *the quotient* $= x^2-2x-14$

x^4-18x^2+49	x^2+2x-7
	$x^2-2x-14$
$-x^4-2x^3-7x$	
$-2x^3-18x^2-7x+49$	
$+2x^3+4x^2-14x$	
$-14x^2-21x+49$	
$+14x^2+21x-49$	
$0 \quad 0 \quad 0$	

Find the value of k which makes the expression

$2x^3 - x^2 - 5x + k$ is divisible by $2x - 5$

Solution:

$$
\begin{array}{r|l}
2x^3 - x^2 - 5x + k & 2x - 3 \\
\cline{2-2}
{}^{-}2x^3 \pm 3x^2 & x^2 + x - 1 \\
\hline
2x^2 - 5x + k & \\
{}^{-}2x^2 \pm 3x & \\
\hline
-2x + k & \\
\pm 2x \mp 3 &
\end{array}
$$

$\therefore k - 3 = 0 \longrightarrow k = 3$

If the area of rectangle is $(2x^2 + 7x - 15)$ and its length is $(x + 15)$. Find its width and its perimeter at $x = 3$cm

The length $= x + 15$ *and* $x = 3$

The length $= 18$ *cm*

The area $= 2x^2 + 7x - 15 = 2(9) + 7(3) - 15 = 18 + 21 - 15 = 24cm^2$

The area = the length X the width

$$\text{the width} = \frac{\text{The area}}{\text{The length}} = \frac{24}{18} = \frac{4}{3}\ cm$$

Factorization by Identifying the highest common factor (H.C.F.)

Note that

$4 \times (7+5) = (4 \times 7) + (4 \times 5)$ means that we used distributing multiplication on addition, while $(4 \times 7) + (4 \times 5) = 4 \times (7+5)$ means factorization by identifying the H.C.F. between the two terms (4×7) and (4×5), which is 4. Each of 4, $(7+5)$ is called a factor of the expression $4(7+5)$

Generally: $ab + ac = a(b+c)$

Factorize by identifying the H.C.F.:

(1) $35a + 7a^2$ ***Sol***	(2) $49b^2 - 7b^3$ ***Sol***	(3) $3x^2 + 12x - 6$ ***Sol***
H.C.F .of the numbers $= 7$ H.C.F .of the symbols $= a$ H.C.F $= 7a$ $7a\left(\frac{35a}{7a} + \frac{7a^2}{7a}\right) = 7a(5+a)$ $35a + 7a^2 = 7a(5+a)$	H.C.F .of the numbers $= 7$ H.C.F .of the symbols $= b^2$ H.C.F $= 7b^2$ $7b^2\left(\frac{49b^2}{7b^2} + \frac{7b^3}{7b^2}\right.$ $= 7b^2(7+b)$ $49b^2 - 7b^3 = 7b^2(7+b)$	H.C.F .of the numbers $= 3$ There is not H.C.F .of the symbols H.C.F $= 3$ $3\left(\frac{3x^2}{3} + \frac{12x}{3} - \frac{6}{3}\right.$ $= 3(x^2 + 4x - 2)$

(4) $12a^2b + 18a^3b^2$

Sol

H.C.F .of the numbers $= 6$ H.C.F .of the symbols $= a^2b$ H.C.F $= 6a^2b$

$12a^2b + 18a^3b^2 = 6a^2b\left(\frac{12a^2b}{6a^2b} + \frac{18a^3b^2}{6a^2b}\right) = 6a^2b(2 + 3ab)$

$12a^2b + 18a^3b^2 = 6a^2b(2 + 3ab)$

(5) $18a^2bc - 6abc + 30\ abc^2 - 24ab^2c^2$

<u>*Sol*</u>

H.C.F .of the numbers $= 6$ *H.C.F .of the symbols* $= abc$ *H.C.F* $= 6\ abc$

$$18a^2bc - 6abc + 30\ abc^2 - 24ab^2c^2 = 6\ abc\ \left(\frac{18a^2bc}{6\ abc} - \frac{6\ abc}{6\ abc} + \frac{30\ abc^2}{6\ abc} - \frac{24ab^2c^2}{6\ abc}\right)$$

$= 6abc(\ 3a - 1\ + 5c - 4bc)$

$= 6abc(3a + 5c - 4bc - 1)$

Factorize by identifying the H.C.F. of the expression:

$3x^2\,y^3 - 9x^3\,y^4 + 12x^3\,y^2$

Solution

The H.C.F. $= 3x^2\,y^2$

To find the other factor, we divide each term by the H.C.F.

$3x^2\,y^3 - 9x^3\,y^4 + 12x^3\,y^2$

$= 3x^2\,y^2\,(y - 3x\,y^2 + 4x)$

Factorize by identifying the H.C.F. of the expression:

$3a\,(4a + 5b) - 2b\,(4a + 5b)$

Solution

The H.C.F. $= (4a + 5b)$

$3a\,(4a + 5b) - 2b\,(4a + 5b)$

$= (4a + 5b)\,(3a - 2b)$

(8) $4m^2\ (2x + y) - 3m(2x + y) - 7(2x + y)$

<u>*Sol*</u>

There is not H.C.F .of the numbers

H.C.F .of the symbols $= (2x + y)$ *H.C.F* $= (2x + y)$

$4m^2\ (2x + y) - 3m(2x + y) - 7(2x + y) = (2x + y)\ (4m^2 - 3m - 7)$

[1] Factorize by identifying the H.C.F. : 3 a (a – 2b) – 6 (a – 2b) , then find the numerical value of the result when $a - 2b = \left| -\frac{1}{3} \right|$

(8) 3a (a–2b) —6(a–2b)

Sol

H.C.F .of the numbers = 3

H.C.F .of the symbols = (a–2b)   H.C.F =3 (a–2b)

3a (a–2b) —6(a–2b)= 3 (a–2b) (a–2)

$3\ (a-2b)\ (a-2) = 3\ (\frac{1}{3})\ (4-2) = 2$

Find the result by identifying the H.C.F.:

[a] 7 × 123 + 7 × 35 – 7 × 18

Sol

H.C.F =7

7x 123 + 7 x 35– 7 x 18 = 7(123+35–18) = 7 (140) =980