3^{rd} year secondary

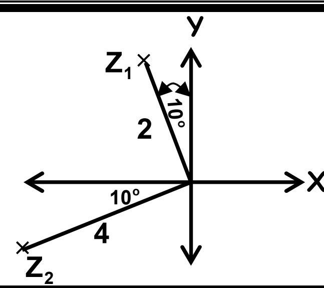
last look 2017

Mr./Mohamed Ali

Algebra and solid geomtry

Q(1) In the opposite figure:

find the exponetel form of Of $\frac{Z_1}{Z_2}$



$$\frac{2\left(\cos 100^\circ + i\sin 100^\circ\right)}{4\left(\cos -170^\circ + i\sin -170^\circ\right)} = \frac{1}{2}\left(\cos 270^\circ + i\sin 270^\circ\right) = -\frac{1}{2}i$$

Q(2) If Z_1, Z_2 are two complex numbers the amplitude of $(Z_1 Z_2) = \frac{5\pi}{18}$

And the amplitude of $\left(\frac{Z_1}{Z_2}\right) = \frac{\pi}{9}$ then the amplitude of $Z_1 =$.........

$\theta_1 + \theta_2 = \frac{5\pi}{18}$ and $\theta_1 - \theta_2 = \frac{\pi}{9}$ $\quad \therefore \theta_1 = \frac{7\pi}{36}$

Q(3) 4 non collinear and coplanar points. Find the number of line segments joining each two of them?

${}^4C_2 = 6$

Q(4) $1 + 3\omega + 3\omega^2 =$......

$1 + 3\left(\omega + \omega^2\right) = 1 + 3 \times -1 = -2$

Q(5) If ${}^{X+Y}P_4 = 360$, $\underline{\lfloor 2X + Y} = 5040$ then ${}^{Y}C_{2X} =$......

$X + Y = 6$, $2X + Y = 7$ $\therefore x = 1$, $y = 5$ $\quad {}^5c_2 = 10 \therefore$

Q(6) If ${}^{n+2}P_r = 2 \times {}^{n+2}C_r$, $\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{5}{3}$ find the value of ${}^{2n}C_{n-r} + {}^{n+3}P_{r-1}$

$\because {}^{n+2}P_r = 2 \times {}^{n+2}C_r \quad \therefore {}^{n+2}P_r = 2 \times \frac{{}^{n+2}P_r}{\lfloor r} \quad \therefore \lfloor r = 2 \quad \therefore r = 2$

$\frac{{}^nC_3}{{}^nC_2} = \frac{n-3+1}{3} = \frac{5}{3} \quad \therefore n-2 = 5 \qquad \therefore n = 7$

${}^{2n}C_{n-r} + {}^{n+3}P_{r-1} = {}^{14}C_5 + {}^{10}P_1 = 2002 + 10 = 2012$

Q(7) $\omega^2\left(1 - \frac{1}{\omega^2} + \omega^2\right) =$.........

$\omega^2\left(1 - \omega + \omega^2\right) = \omega^2 \times -2\omega = -2\omega^3 = -2$

Q(8) Find the multiplicative inverse $A=\begin{pmatrix}1 & -1 & 2\\ 2 & -3 & 1\\ 1 & 5 & 21\end{pmatrix}$ $\quad |A|=-1$

$$Adj(A)=\begin{pmatrix}\begin{vmatrix}-3 & 1\\ 5 & 21\end{vmatrix} & -\begin{vmatrix}2 & 1\\ 1 & 21\end{vmatrix} & \begin{vmatrix}2 & -3\\ 1 & 5\end{vmatrix}\\ -\begin{vmatrix}-1 & 1\\ 5 & 21\end{vmatrix} & \begin{vmatrix}1 & 2\\ 1 & 21\end{vmatrix} & -\begin{vmatrix}-1 & 1\\ 5 & 1\end{vmatrix}\\ \begin{vmatrix}-1 & 2\\ -3 & 1\end{vmatrix} & -\begin{vmatrix}1 & 2\\ 2 & 1\end{vmatrix} & \begin{vmatrix}1 & -1\\ 2 & -3\end{vmatrix}\end{pmatrix}=\begin{pmatrix}-68 & -41 & 13\\ 31 & 19 & -6\\ 5 & 3 & -1\end{pmatrix}\therefore A^{-1}=\frac{1}{|A|}\begin{pmatrix}-68 & 31 & 5\\ -41 & 19 & 3\\ 13 & -6 & -1\end{pmatrix}$$

$$A^{-1}=-1\begin{pmatrix}-68 & 31 & 5\\ -41 & 19 & 3\\ 13 & -6 & -1\end{pmatrix}=\begin{pmatrix}68 & -31 & -5\\ 41 & -19 & -3\\ -13 & 6 & 1\end{pmatrix}$$

Q(9) Find $Z=\dfrac{-8}{1+\sqrt{3}i}$ where $i^2=-1$ in the trigonometric form then find the two square roots of the number Z in the exponential form

$$Z=\frac{-8}{1+\sqrt{3}i}\times\frac{1-\sqrt{3}i}{1-\sqrt{3}i}=\frac{-8(1+\sqrt{3}i)}{1-(\sqrt{3}i)^2}=\frac{-8(1-\sqrt{3}i)}{1+3}=-2(1-\sqrt{3}i)=-2+2\sqrt{3}i$$

$$X=-2\ ,\ Y=2\sqrt{3}\ ,\ r=\sqrt{X^2+Y^2}=\sqrt{2^2+(2\sqrt{3})^2}=4\ \tan\theta=\frac{Y}{X}=\sqrt{3}\ (-,+)$$

θ in the 2nd $\therefore \theta=180^\circ-60^\circ=120^\circ$

$$Z=4(\cos 120^\circ+i\sin 120^\circ)$$

$$\sqrt{Z}=2\left(\cos\frac{120^\circ+2K\pi}{2}+i\sin\frac{120^\circ+2K\pi}{2}\right)$$

When $K=0\ \therefore Z_1=2\left(\cos\frac{120^\circ}{2}+i\sin\frac{120^\circ}{2}\right)=2e^{\frac{\pi}{3}i}$

When K=-1 $2(\cos -120^\circ+i\sin -120^\circ)$

Q(10) If the X axis cut the sphere which center (3,-4,12) and its radius length 13cm at the two points A and B then AB equals

$(X-3)^2+16+144=13^2\quad\therefore (X-3)^2=9\ \therefore X=0$ or $X=6$

$\therefore$ the two points A(0,0,0) and B(6,0,0) $\quad\therefore AB=6$

Q(11) The second , third and fourth terms in the expansion of $(X+a)^n$ according to the descending power of X are : 16,112,448 find the value of X , a , n

$$\frac{T_3}{T_2}=\frac{112}{16}=7 \quad \therefore \frac{n-2+1}{2}\times\frac{a}{X}=7 \quad \therefore \frac{n-1}{2}\times\frac{a}{X}=7\rightarrow(1)$$

$$\frac{T_4}{T_3}=\frac{448}{112}=4 \quad \therefore \frac{n-3+1}{3}\times\frac{a}{X}=4 \quad \therefore \frac{n-2}{3}\times\frac{a}{X}=4\rightarrow(2)$$

$$\frac{n-1}{2}\times\frac{3}{n-2}=\frac{7}{4} \quad \therefore 12(n-1)=14(n-2) \qquad \therefore 2n=16 \quad \therefore n=8$$

$$\frac{7}{2}\times\frac{a}{X}=7 \quad \therefore \frac{a}{X}=2 \quad \therefore a=2X \quad , T_2=16={}^{8}C_1(a)(X)^7$$

$$16=8\times 2X\times X^7 \qquad \therefore X^8=1 \quad \therefore X=\pm 1$$

Q(12) prove that: $\begin{vmatrix} 1 & 1 & 1 \\ L & m & n \\ L^2 & m^2 & n^2 \end{vmatrix}=(L-m)(m-n)(n-L)$

$c_2-c_1\ ,c_3-c_1$

$$\begin{vmatrix} 1 & 0 & 0 \\ L & m-L & n-L \\ L^2 & m^2-L^2 & n^2-L^2 \end{vmatrix}=(m-L)(n-L)\begin{vmatrix} 1 & 0 & 0 \\ L & 1 & 1 \\ L^2 & m+L & n+L \end{vmatrix}$$

c_3-c_2

$$\therefore (m-L)(n-L)\begin{vmatrix} 1 & 0 & 0 \\ L & 1 & 0 \\ L^2 & m+L & n-m \end{vmatrix}=(m-L)(n-L)(n-m)$$

Q(13) How many ways we can put(distribute) 10 identical balls into 6 distinct bins

$${}^{6+10-1}C_{10}=3003 \qquad {}^{n+r-1}C_r \quad n=6 \quad ,r=10$$

Q(144) The least positive integer n which makes

${}^{n-1}C_5+{}^{n-1}C_6<{}^{n}C_7$ Is ....

$$\because {}^{n-1}C_5+{}^{n-1}C_6<{}^{n}C_7 \qquad \therefore {}^{n}C_6<{}^{n}C_7 \quad \therefore \frac{\underline{|n}}{\underline{|6}\,\underline{|n-6}}<\frac{\underline{|n}}{\underline{|7}\,\underline{|n-7}}$$

$$\therefore \frac{1}{n-6}<\frac{1}{7} \quad \therefore 7<n-6 \quad \therefore n>13 \quad \therefore n=14$$

Q(15) If ${}^{n+1}P_{r+1} : {}^{n+1}C_{r+1} = 720$, ${}^{n}C_{r-2} + {}^{n}C_{r-3} = 56$ find the value of n , r

$${}^{n+1}P_{r+1} \div \frac{{}^{n+1}P_{r+1}}{\underline{|r+1}} = 720 \quad \therefore \underline{|r+1} = 720 \quad \therefore r+1 = 6 \quad \therefore r = 5$$

$${}^{n}C_{r-2} + {}^{n}C_{r-3} = {}^{n}C_3 + {}^{n}C_2 = {}^{n+1}C_3 = 56 = {}^{8}C_3 \quad \therefore \; n+1 = 8 \quad \therefore n = 7$$

Q(16) In the expansion: $\left(X^2 - \frac{1}{X}\right)^9$ Find ❶general term

❷The term free of X

To get the General term:

$$T_{r+1} = {}^{9}C_r \left(-X^{-1}\right)^r \times \left(X^2\right)^{9-r} = {}^{9}C_r(-1)^r X^{-r} \times X^{18-2r} = {}^{9}C_r(-1)^r \times X^{18-3r}$$

<u>To get the term free of X</u>

$$18 - 3r = 0 \quad \therefore \; 3r = 18 \quad \therefore r = 6 \qquad \therefore T_7 = \underline{{}^{9}C_6(-1)^6} \times X^{18-3\times 6} = 84$$

Q(17) If ${}^{n}C_r : {}^{n}C_{r+1} : {}^{n}C_{r+2} = 5 : 10 : 14$ find the values of n and r

$$\frac{{}^{n}C_{r+1}}{{}^{n}C_r} = \frac{n-(r+1)+1}{r+1} = \frac{10}{5} = 2 \quad \therefore \frac{n-r}{r+1} = 2 \quad \therefore \; n - 3r - 2 = 0 \quad \therefore n - 3r = 2$$

$$\frac{{}^{n}C_{r+2}}{{}^{n}C_{r+1}} = \frac{n-(r+2)+1}{r+2} = \frac{14}{10} = \quad \therefore \frac{n-r-1}{r+2} = \frac{7}{5} \qquad \therefore \; 5n - 12r - 19 = 0$$

$r = 3$, $n = 11$

Q(18) Using the properties of the determinant prove that

$$\begin{vmatrix} Y+Z & X & X \\ Y & Z+X & Y \\ Z & Z & X+Y \end{vmatrix} = 2\begin{vmatrix} 0 & Z & Y \\ Z & 0 & X \\ Y & X & 0 \end{vmatrix}$$

$$r_1 - (r_2 + r_3)\begin{vmatrix} 0 & -2Z & -2Y \\ Y & Z+X & Y \\ Z & Z & X+Y \end{vmatrix} = -2\begin{vmatrix} 0 & Z & Y \\ Y & Z+X & Y \\ Z & Z & X+Y \end{vmatrix} r_2 - r_1, r_3 - r_1$$

Q(19) Find in the trigonometric and exponential forms the roots of the equation $Z^4 = 8\left(1 - \sqrt{3}\, i\right)$ then write the solution set

$X = 8$, $Y = -8\sqrt{3}$ $\therefore r = \sqrt{8^2 + \left(-8\sqrt{3}\right)^2} = 16$

, $\tan\theta = \sqrt{3}$ in the 4^{th} quadrant $\therefore \theta = -\frac{\pi}{3}$

$$\therefore Z^4 = 16\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$$

$$\therefore Z = 2\left(\cos\frac{-\frac{\pi}{3} + 2\pi r}{4} + i\sin\frac{-\frac{\pi}{3} + 2\pi r}{4}\right)$$

When r=0 then $Z_1 = 2\left(\cos\frac{-\pi}{12} + i\sin\frac{-\pi}{12}\right) = 2e^{\frac{-\pi}{12}i}$

When r=1 then $Z_2 = 2\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right) = 2e^{\frac{5\pi}{12}i}$

When r=-1 then $Z_3 = 2\left(\cos\frac{-7\pi}{12} + i\sin\frac{-7\pi}{12}\right) = 2e^{\frac{-7\pi}{12}i}$

When r=2 then $Z_4 = 2\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right) = 2e^{\frac{11\pi}{12}i}$

S.S={ $2e^{\frac{-\pi}{12}i}$, $2e^{\frac{5\pi}{12}i}$, $2e^{\frac{-7\pi}{12}i}$, $2e^{\frac{11\pi}{12}i}$ }

Q(20) $\left(\frac{a - d\omega}{a\omega^2 - d} - \omega^2\right)^2 =$ ....

$$\frac{a\omega^3 - d\omega}{a\omega^2 - d} - \omega^2 = \frac{\omega^2\left(a\omega^2 - d\right)}{\left(a\omega^2 - d\right)} - \omega^2 = \left(\omega^2 - \omega\right)^2 = \left(\pm\sqrt{3}i\right)^2 = -3$$

Q(21) Find the number of diagonals of a decagon? ${}^{10}C_2 - 10 = 35$

Q(22) If Z_1, Z_2 are two complex numbers the amplitude of $\left(Z_1Z_2\right) = \frac{5\pi}{18}$

And the amplitude of $\left(\frac{Z_1}{Z_2}\right) = \frac{\pi}{9}$ then the amplitude of $Z_1 =$

$\theta_1 + \theta_2 = \frac{5\pi}{18}$ and $\theta_1 - \theta_2 = \frac{\pi}{9}$ $\therefore \theta_1 = 35^\circ = \frac{7\pi}{36}$

(23)**Find in the trigonometric and exponential forms the roots of the equation $Z^4 = 8(1-\sqrt{3}\,i)$ then write the solution set**

$X = 8$, $Y = -8\sqrt{3}$ $\therefore r = \sqrt{8^2 + (-8\sqrt{3})^2} = 16$

, $\tan\theta = \sqrt{3}$ in the 4th quadrant $\therefore \theta = -\frac{\pi}{3}$

$\therefore Z^4 = 16\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$

$\therefore Z = 2\left(\cos\frac{-\frac{\pi}{3}+2\pi r}{4} + i\sin\frac{-\frac{\pi}{3}+2\pi r}{4}\right)$

When r=0 then $Z_1 = 2\left(\cos\frac{-\pi}{12} + i\sin\frac{-\pi}{12}\right) = 2e^{\frac{-\pi}{12}i}$

When r=1 then $Z_2 = 2\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right) = 2e^{\frac{5\pi}{12}i}$

When r=-1 then $Z_3 = 2\left(\cos\frac{-7\pi}{12} + i\sin\frac{-7\pi}{12}\right) = 2e^{\frac{-7\pi}{12}i}$

When r=2 then $Z_4 = 2\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right) = 2e^{\frac{11\pi}{12}i}$

S.S={ $2e^{\frac{-\pi}{12}i}, 2e^{\frac{5\pi}{12}i}, 2e^{\frac{-7\pi}{12}i}, 2e^{\frac{11\pi}{12}i}$ }

Q(24) Using the properties of the determinant prove that $\begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix} = (x+2)(x-1)^2$

$C_1 + C_2 + C_3 \ \therefore \begin{vmatrix} X+2 & 1 & 1 \\ X+2 & X & 1 \\ X+2 & 1 & X \end{vmatrix} = (X+2)\begin{vmatrix} 1 & 1 & 1 \\ 1 & X & 1 \\ 1 & 1 & X \end{vmatrix} \ r_2 - r_1 , r_3 - r_1$

$(X+2)\begin{vmatrix} 1 & 1 & 1 \\ 0 & X-1 & 0 \\ 0 & 0 & X-1 \end{vmatrix} = (X+2)(X-1)^2$

Q(25) If the number of terms in the expansion $(X+Y)^{2n-1}$ equals 12 terms then n=…..

$$2n-1+1=12 \quad \therefore 2n=12 \quad \therefore n=6$$

Q(26) If the side length of a triangle are $\frac{1}{2}\underline{\lfloor n}$, $\underline{\lfloor n-2}$ and $\underline{\lfloor 2-n}$ cm

Then the numerical value of the area of the triangle =….cm^2

$n-2 \geq 0 \quad ,2-n \geq 0 \quad \therefore \ n=2 \text{ or } 3$

if $n=2$ sides are $1,1,1$ area $=\frac{1}{2}\times 1\times 1\times \sin 60^\circ = \frac{\sqrt{3}}{4}$

Q(27) $\left(\dfrac{3+5\omega}{5+3\omega^2}+\dfrac{5+3\omega^2}{3+5\omega}\right)^8 =$

Q(28) Number of solution(natural) such that a + b + c =7

${}^{n+r-1}C_r = {}^{3+7-1}C_7 = 36$ (n) number of variables ,(r) their sum

<u>Similar example</u>

Number of ways to distribute 4 identical balls among 3 boxes

${}^{n+r-1}C_r = {}^{3+4-1}C_3$ (n) number of boxes ,(r) number of balls

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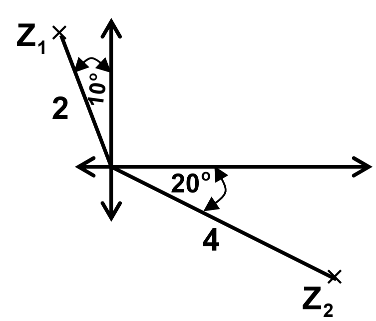
Q(29) If Z_1 , Z_2 , are two complex numbers represented On argend's plane as in the opposite figure Find the form of X+iY the number $\dfrac{Z_2}{Z_1}$



$$\frac{4(\cos 100^\circ + i\sin 100^\circ)}{2(\cos -20^\circ + i\sin -20^\circ)}$$

$$=2(\cos 120^\circ + i\sin 120^\circ)=2\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)=\sqrt{3}-i$$

Q(30) The principle amplitude of the number $Z=1-i$ is …..

Q(31) Solve the following system of linear equations using the inverse matrix where 4X+Y=0 , X+2Z=15 , Y-7Z=0

$$|A| = \begin{vmatrix} 4 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & -7 \end{vmatrix} = -1 \qquad \text{The cofactor matrix} \begin{pmatrix} -2 & 7 & 1 \\ 7 & -28 & -4 \\ 2 & -8 & -1 \end{pmatrix}$$

$$\therefore \text{Adj }(A) = \begin{pmatrix} -2 & 7 & 2 \\ 7 & -28 & -8 \\ 1 & -4 & -1 \end{pmatrix} \qquad \therefore A^{-1} = \frac{1}{|A|}\text{ Adj }(A) = \begin{pmatrix} 2 & -7 & -2 \\ -7 & 28 & 8 \\ --1 & 4 & 1 \end{pmatrix}$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 2 & -7 & -2 \\ -7 & 28 & 8 \\ --1 & 4 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 15 \\ 0 \end{pmatrix} = \begin{pmatrix} -105 \\ 480 \\ 60 \end{pmatrix}$$

Q(32)Prove that : $\begin{vmatrix} X & a & a \\ a & X & a \\ a & a & X \end{vmatrix} = (X+2a)(X-a)^2$

$C_1 + C_2 + C_3$

$$\begin{vmatrix} X+2a & a & a \\ X+2a & X & a \\ X+2a & a & X \end{vmatrix} = (X+2a)\begin{vmatrix} 1 & a & a \\ 1 & X & a \\ 1 & a & X \end{vmatrix} = (X+2a)\begin{vmatrix} 1 & a & a \\ 0 & X-a & 0 \\ 0 & 0 & X-a \end{vmatrix}$$

$$= (X+2a)(X-a)^2$$

Q(33)prove that $\begin{vmatrix} X+y+2 & X & Y \\ 1 & 2X+Y+1 & Y \\ 1 & X & X+2Y+1 \end{vmatrix}$

$= 2(X+Y+1)^3$

$C_1 + C_2 + C_3$

$$\begin{vmatrix} 2X+2Y+2 & X & Y \\ 2X+2Y+2 & 2X+Y+1 & Y \\ 2X+2Y+2 & X & X+2Y+1 \end{vmatrix} = (2X+2Y+2)\begin{vmatrix} 1 & X & Y \\ 1 & 2X+Y+1 & Y \\ 1 & X & X+2Y+1 \end{vmatrix}$$

$r_2 - r_1 , r_3 - r_1$

$$= 2(X+Y+1)\begin{vmatrix} 1 & X & Y \\ 0 & X+Y+1 & 0 \\ 0 & 0 & X+Y+1 \end{vmatrix} = 2(X+Y+1)^2$$

Q(34) In the expansion of $(X+Y)^n$ in descending power of X if T_2, T_3, T_4 are respectively 240 , 720 , 1080 evaluate the value of each of X , Y , n

Answer :

$$\frac{T_3}{T_2} = \frac{n-2+1}{2} \times \frac{Y}{X} = \frac{720}{240} = 3 \rightarrow (1) \quad , \frac{T_4}{T_3} = \frac{n-3+1}{3} \times \frac{Y}{X} = \frac{1080}{720} = \frac{3}{2} \rightarrow (2)$$

$$\text{dividing } 1,2 \therefore \frac{n-1}{2} \times \frac{3}{n-2} = 2 \quad \therefore \frac{3n-3}{2n-4} = 2 \therefore 4n-8 = 3n-3 \quad \therefore n = 5$$

$$\therefore \text{from } 1 \quad \therefore \frac{Y}{X} = \frac{3}{2} \quad \therefore Y = \frac{3X}{2}$$

$$T_2 = {}^5C_1 \times \left(\frac{3X}{2}\right)^1 X^4 = 240 \qquad \therefore \quad X^5 = 32 \therefore X = 2 \quad \therefore Y = 3$$

Q(35) If X={A,B,C,D,E,F} then The number of triangles whose vertices $\in X$ equals

① 15 ② 20

③ 17 ④ 21

$${}^6C_3 = 20$$

Q(36) $(2 + 7\omega + 2\omega^2)(2 + 7\omega^2 + 2\omega^4) = ...$

① 25ω ② 25

③ 125 ④ 20

$$(-2\omega + 7\omega)(-2\omega^2 + 7\omega^2) = 5\omega \times 5\omega^2 = 25\omega^3 = 25$$

Q(37) If $A = \begin{pmatrix} 1 & -2 & 3 \\ m & 0 & 1 \\ 3 & 2 & -1 \end{pmatrix}$ **and If rank(A) =2 then m=……..**

$$\begin{vmatrix} 1 & -2 & 3 \\ m & 0 & 1 \\ 3 & 2 & -1 \end{vmatrix} = 0 \quad \therefore 4m = 8 \therefore m = 2$$

Q(38) Put the number $Z=\frac{8}{1+\sqrt{3}i}$ in the trigonometric form
And hence find its two square roots in the exponential form

$$Z=\frac{8}{1+\sqrt{3}i}\times\frac{1-\sqrt{3}i}{1-\sqrt{3}i}=2-2\sqrt{3}i \quad \therefore r=4 \qquad 4^{th} \text{ quadrant}$$

$$\theta=-\tan^{-1}\sqrt{3}=-\frac{\pi}{3}$$

$$\sqrt{Z}=2e^{\frac{-\frac{\pi}{3}+2K\pi}{2}} \qquad \text{put } K=0 \ , K=1$$

Q(39) Using matrix solve : 3X+Y-2Z=-3 , 2X+7Y+3Z=9 , 4X-3Y-Z=7

$$|A|=\begin{vmatrix}3 & 1 & -2\\ 2 & 7 & 3\\ 4 & -3 & -1\end{vmatrix}=88$$

$$\begin{pmatrix}X\\Y\\Z\end{pmatrix}=\frac{1}{88}\begin{pmatrix}2 & 7 & 17\\ 14 & 5 & -13\\ -34 & 13 & 19\end{pmatrix}\begin{pmatrix}-3\\9\\7\end{pmatrix}=\begin{pmatrix}2\\-1\\4\end{pmatrix}$$

Q(40) The amplitude of $(Z_1Z_2)=\frac{\pi}{6}$ and The amplitude of $(Z_1Z_3)=\frac{2\pi}{9}$ and The amplitude of $(Z_2Z_3)=\frac{5\pi}{18}$ then amplitude$(Z_1Z_2Z_3)$

$$\theta_1+\theta_2=30^\circ\ ,\theta_1+\theta_3=40^\circ\ ,\theta_2+\theta_3=50^\circ$$

$$\therefore 2(\theta_1+\theta_2+\theta_3)=120^\circ \quad \therefore \theta_1+\theta_2+\theta_3=60^\circ=\frac{\pi}{3}$$

Q(41) If $\begin{pmatrix} X-1 & 4 \\ 2 & X+1 \end{pmatrix}$ is singular matrix then X=

$X^2 - 1 = 8 \quad \therefore X^2 = 9 \quad \therefore X = \pm 3$

Q(42) If $(X-2) \times {}^nC_3 = {}^nP_3$ then X=…….

$(X-2) = 6 \quad \therefore \quad X = 8$

Q(43) If 35 , 21 , 7 are the coefficient of three consecutive terms in the expansion $(1+X)^n$ find the value of n and the order of these terms

Let the terms are T_r , T_{r+1} , T_{r+2}

$$\frac{\text{coof}T_{r+1}}{\text{coof}T_r} = \frac{n-r+1}{r} \times \frac{1}{1} = \frac{21}{35} = \frac{3}{5} \to 5n - 8r + 5 = 0$$

$$\frac{\text{coof}T_{r+2}}{\text{coof}T_{r+1}} = \frac{n-(r+1)+1}{r+1} \times \frac{1}{1} = \frac{7}{21} = \frac{1}{3} \to 3n - 4r - 1 = 0$$

$\therefore n = 7, \; r = 5$

Q(44) Without expanding pro5ve that $\begin{vmatrix} X & a & b \\ a & x & b \\ a & b & x \end{vmatrix} = (x-a)(x-b)(x+a+b)$

$$C_1 + C_2 + C_3 \begin{vmatrix} X+a+b & a & b \\ X+a+b & x & b \\ X+a+b & b & x \end{vmatrix} = (x+a+b)\begin{vmatrix} 1 & a & b \\ 1 & x & b \\ 1 & b & x \end{vmatrix} = \; r_2 - r_1 , r_3 - r_1$$

$$(x+a+b)\begin{vmatrix} 1 & a & b \\ 0 & X-a & 0 \\ 0 & b-a & X-b \end{vmatrix} r_3 - r_2 \quad \therefore (x+a+b)\begin{vmatrix} 1 & a+b & b \\ 0 & X-a & 0 \\ 0 & 0 & X-b \end{vmatrix}$$

$= (x+a+b)(X-a)(X-b)$

Q(45) without expanding the determinant prove that

$$\begin{vmatrix} a+b+2 & a & b \\ 1 & 2a+b+1 & b \\ 1 & a & a+2b+1 \end{vmatrix} = 2(a+b+1)^3$$

$c_1 + c_2 + c_3$

$$\begin{vmatrix} 2a+2b+2 & a & b \\ 2a+2b+2 & 2a+b+1 & b \\ 2a+2b+21 & a & a+2b+1 \end{vmatrix} = (2a+2b+2)\begin{vmatrix} 1 & a & b \\ 1 & 2a+b+1 & b \\ 1 & a & a+2b+1 \end{vmatrix}$$

$$= 2(a+b+1)\begin{vmatrix} 1 & a & b \\ 0 & a+b+1 & 0 \\ 0 & 0 & a+b+1 \end{vmatrix} = 2(a+b+1)^3$$

Q(46) If $Z = \sin\frac{\pi}{9} + i\cos\frac{\pi}{9}$ find the cubic roots of $(\overline{Z})^9$

$Z = \cos 70^\circ + i\sin 70^\circ$

$\overline{Z} = \cos 70^\circ - i\sin 70^\circ \quad \therefore \overline{Z} = \cos - 70^\circ + i\sin - 70^\circ$

$(\overline{Z})^7 = \cos - 70^\circ \times 9 + i\sin - 70^\circ \times 9 = \cos - 630^\circ + i\sin - 630^\circ$

$= \cos 90^\circ + i\sin 90^\circ$

$\sqrt[3]{\;} = \cos\frac{90^\circ + 2\pi r}{3} + i\sin\frac{90^\circ + 2\pi r}{3}$

When r=0 $\quad Z_1 = \cos 30^\circ + i\sin 30^\circ \qquad e^{\frac{\pi}{6}i}$

When r=1 $\quad Z_2 = \cos 150^\circ + i\sin 150^\circ \qquad e^{\frac{5\pi}{6}i}$

When r=-1 $\quad Z_2 = \cos - 90^\circ + i\sin - 90^\circ \qquad e^{\frac{-\pi}{2}i}$

Q(47) The coefficient of X^5 in the expansion $(3-2X)^7$ equals

$T_{r+1} = {}^{n}C_r(-2X)^r(3)^{n-r} = {}^{7}C_r(-2X)^r(3)^{n-r} \quad \therefore r = 5$

$T_6 = {}^{7}C_5(-2)^5(3)^{7-5} = -6048$

Q(48) find the two square roots of the complex number :

$Z = 2 - 2\sqrt{3}i$ in the trigonometric form

Q(49) If $Z = \sin 60^\circ - i\cos 60^\circ$, then the principle amplitude of Z =

Q(50) $(3 + 7\omega + 3\omega^2)(3 - 7\omega^2 + 3\omega)$

Q(51) The greatest value of $(3+2X)^6$ at X=1 is

$\frac{T_{r+1}}{T_r} \geq 1 \quad \therefore \frac{6-r+1}{r} \geq 1 \quad \therefore 14-2r \geq 3r \quad \therefore r = 2$

$\therefore$ the greatest term is T_3

Q(1) The distance between the two planes
3X+2Y-6Z-14=0 and 3X+2Y-6Z+21=0 is ……..units

$$\frac{|3(0)+2(7)-6(0)+21|}{\sqrt{3^2+2^2+6^2}}=3\text{units}$$

Q(2) If the point (-2,4,m) lies on the sphere
$(X+2)^2+(Y-1)^2+(Z-3)^2=25$ then one value of m=….

$3^2+(m-3)^2=25 \quad \therefore m-3=4 \quad \therefore m=7$

Q(3) If $\overrightarrow{A}$ = (4, -k, 6) , $\overrightarrow{B}$ = (2, 2, m) and $\overrightarrow{A} \parallel \overrightarrow{B}$, then k + m = ……………………

$\frac{4}{2}=\frac{-K}{2}=\frac{6}{m} \quad \therefore m=3 \ , \ K=-4 \quad \therefore K+m=-1$

Q(4) Find the standard form and the general form of the equation of the plane passing through point (3 , -5 , 2) and the vector n = (2 , 1, 1) is normal to the plane.

$\boxed{n \bullet r = n \bullet A}$

$(2,1,1)\bullet(X,Y,Z)=(2,1,1)\bullet(3,-5,2)$

$2X+Y+Z=2\times3+1\times-5+1\times2=3 \quad \therefore 2X+Y+Z-3=0$

Q(5) If A = (-3 , 1 , 2) , B = (3 , 4 , -1), find the area of the parallelogram in which A and B are two adjacent sides.

$$\overrightarrow{A}\times\overrightarrow{B}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{K} \\ -3 & 1 & 2 \\ 3 & 4 & -1 \end{vmatrix}=-9\hat{i}+3\hat{j}-15\hat{K}$$

$\therefore \|A\times B\|=\sqrt{(-9)^2+(3)^2+(-15)^2}=3\sqrt{35}$ unit area

Q(6) If the X axis cut the sphere which center (3,-4,12) and its radius length 13cm at the two points A and B then AB equals

$(X-3)^2+16+144=13^2 \quad \therefore (X-3)^2=9 \therefore X=0 \text{ or } X=6$

$\therefore$ the two points A(0,0,0) and B(6,0,0) $\quad \therefore AB=6$

Q(7) Find the Cartesian equation of the plane
$(X,Y,Z)=(2,3,5)+t_1(-1,3,4)+t_2(6,1,-2)$ where t_1 and t_2 are parameters

$(-1,3,4)$, $(6,1,-2)$ are the direction vectors of two lines in the plane
To get the normal to this plane

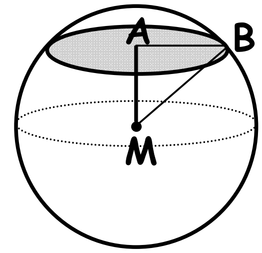
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{K} \\ -1 & 3 & 4 \\ 6 & 1 & -2 \end{vmatrix} = -10\hat{i}+22\hat{j}-19\hat{K}$$

$$\therefore (X,Y,Z)\bullet(-10,22,-19)=(2,3,5)\bullet(-10,22,-19)$$
$$10X-22Y+19Z=49$$

Q(8) If the vector A makes angles of measure α , β , θ with X , Y and Z axes then $\sin^2\alpha+\sin^2\beta+\sin^2\theta=\ldots$

$$1-\cos^2\alpha+1-\cos^2\beta+1-\cos^2\theta = 3-\left(\cos^2\alpha+\cos^2\beta+\cos^2\theta\right)=2$$

Q(9) If the plane $2x-2y+Z=5$ intersect the sphere $(x-2)^2+(y-3)^2+(z+2)^2=25$, then the area of the cross section (trace)



$$MA=\frac{|2(2)-2(3)-2-5|}{\sqrt{4+4+1}}=3$$
$$\therefore AB=\sqrt{5^2-3^2}=4 \quad \therefore Area=16\pi$$

Q(10)
If θ is the measure of the angle included between $A=(2,0,2)$, $B=(0,0,4)$, then $\theta=$ ....

$$\cos\theta=\frac{(2,0,2)\bullet(0,0,4)}{\sqrt{2^2+0^2+2^2}\times\sqrt{0^2+0^2+4^2}}=\frac{8}{8\sqrt{2}}= \quad \therefore \theta=\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)=45^\circ$$

Q(11) If the vector A makes angles of measure α , β , θ with X , Y and Z axes then $\cos 2\alpha+\cos 2\beta+\cos 2\theta=\ldots$

$$2\cos^2\alpha-1+2\cos^2\beta-1+2\cos^2\theta-1=2\left(\cos^2\alpha+2\cos^2\beta+2\cos^2\theta\right)-3=$$
$$=2\times1-3=-1$$

Q(12) A line passing through the origin with the direction cosines $\frac{2}{7}, \frac{-3}{7}, \frac{6}{7}$ intersect the plane 3X+5Y+2Z-6=0 at the point B find the length of OB

$r = (0,0,0) + t(2,-3,6) = (2t,-3t,6t)$

$\therefore 3(2t) + 5(-3t) + 2(6t) - 6 = 0$

$6t - 15t + 12t - 6 = 0 \quad \therefore 3t = 6 \quad \therefore t = 2$

point of intersection(B) $= (4,-6,12)$

$OB = \sqrt{16 + 36 + 144} = \sqrt{196} = 14$

Q(13) Find the equation of the line of intersection of the two planes X + 2Y – 2Z = 1 , 2 X + Y – 3Z = 5

First method:

$$\begin{array}{l} -2X - 4Y + 4Z = -2 \\ \underline{\;2X + \;\; Y - 3Z = 5\;} \\ -3Y + Z = 3 \end{array}$$

put $Y = 0 \quad \therefore Z = 3 \quad \therefore X = 7 \quad \therefore P_1 = (7,0,3)$

Put $Z = 0 \quad \therefore Y = -1 \quad \therefore X = 3 \quad \therefore P_2 = (3,-1,0)$

$r = (7,0,3) - (3,-1,0) = (4,1,3)$

Equation : $(7,0,3) + t(4,1,3) \qquad \therefore \frac{X-7}{4} = \frac{Y}{1} = \frac{Z-3}{3}$

Q(14) If $L_1 : \frac{X+2}{-1} = \frac{Y+3}{3} = \frac{Z+5}{2}$ is perpendicular to the line $L_2 : \frac{X}{2} = \frac{Y-5}{K} = \frac{Z-6}{m}$ then the value of 3K+2m=….

$(-1,3,2) \bullet (2,K,m) = 0 \quad \therefore 3K + 2m = 2$

Q(15) If $\overrightarrow{A} = (1, -2, 1)$, $\overrightarrow{B} = (-2, 1, 2)$,then the component of $\overrightarrow{A}$ in the direction of B =…………………

$$\frac{A \bullet B}{\|B\|} = \frac{(1,-2,1) \bullet (-2,1,2)}{\sqrt{(-2)^2 + 1^2 + 2^2}} = -\frac{2}{3}$$

Q(16) If The two sphere $(X-3)^2+Y^2+(Z-3)^2=16$, $(X+1)^2+(Y-4)^2+(Z-K)^2=25$ Touch each other find the value of K

$C_1=(3,0,3)$ $C_2=(-1,4,K)$ $r_1=\sqrt{16}=4$ $r_1=\sqrt{25}=5$

$\therefore \sqrt{4^2+4^2+(K-3)^2}=5+4=9$

$\therefore \sqrt{32+(K-3)^2}=9$

$(K-3)^2+32=81$ $\therefore (K-3)^2=49$

$K-3=7 \rightarrow K=10$ or $K-3=-7$ $\therefore K=-4$

Q(17) Find the equation of the sphere has its center at the point (5,-2,3) and touch the plane 3X+2Y+Z=0

Radius $=\dfrac{|3\times5+2\times-2+3|}{\sqrt{3^2+(2)^2+1^2}}=\sqrt{14}$

Equation $(X-5)^2+(Y+2)^2+(Z-3)^2=14$

Q(18) Find the equation of a plane which bisects perpendiculary the line joining the points (2,3,4) and (4,5,8)

Mid point $\left(\dfrac{2+4}{2},\dfrac{3+5}{2},\dfrac{4+8}{2}\right)=(3,4,6)$

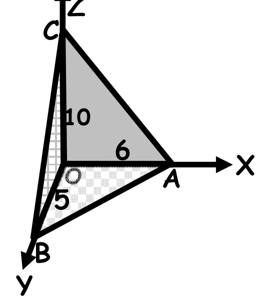
$d=(4-2,5-3,8-4)=(2,2,4)=(1,1,2)$ is normal to the required plane

$n \bullet r=n \bullet A$ $\therefore (1,1,2)\bullet(X,Y,Z)=(1,1,2)\bullet(3,4,6)$

$X+Y+2Z=3+4+12=19$

Q(19) If the plane $10X+12Y+6Z=60$ intersected with the axes X,Y and Z at the points A ,B and C respectively then the volume of the solid ABCO where O is the origin point equals …..cube unit

Volume of pyramid $=\dfrac{1}{3}$ base area $\times$ height



$=\dfrac{1}{3}\left(\dfrac{1}{2}\times5\times6\right)\times10=50$

Q(20) If A=$\left(2\cos\theta, \log_5 X, \sin\theta\right)$ and A=$\left(\cos\theta, \log_3 27, 2\sin\theta\right)$ and $A \bullet B = 11$

Then X=

$2\cos^2\theta + \log_5 X \times \log_3 27 + 2\sin^2\theta = 11$

$\therefore 2\cos^2\theta + 2\sin^2\theta + \log_5 X \times \log_3 27 = 11 \quad \therefore 2 + \log_5 X \times \log_3 27 = 11$

$\log_5 X \times \log_3 27 = 9 \quad \therefore \log_5 X = 3 \quad \therefore X = 5^3 = 125$

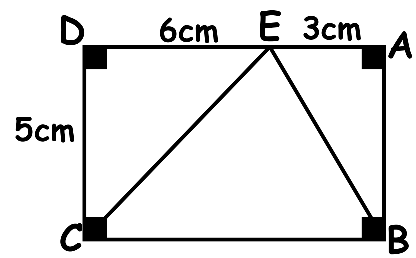
Q(21) The radius length of the sphere

$X^2 + Y^2 + Z^2 - 2X - 6Y + 10Z - 1 = 0$ equals .... length unit

Center =(1,3,-5) radius $= \sqrt{1^2 + 3^2 + (-5)^2 + 1} = 6$

Q(22) In the opposite figure ABCD is a rectangle

$E \in \overline{AD}$ then $\overrightarrow{EB} \bullet \overrightarrow{EC} =$



$C = (0,0)$, $B = (9,0)$ $E = (6,5)$

$EB = B - E = (3,-5) \qquad EC = C - E = (-6,-5)$

$EB \bullet EC = (3,-5) \bullet (-6,-5) = -18 + 25 = 7$

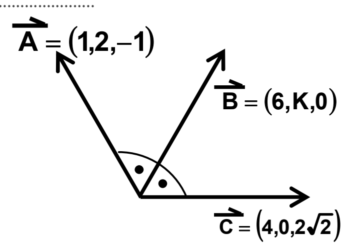
Q(23)

If $\overrightarrow{A}$=(2,3,-4) and $\overrightarrow{B}$=(4,2,m) and $\overrightarrow{A} \perp \overrightarrow{B}$ then m=...

$A \bullet B = 0 \quad \therefore 2 \times 4 + 3 \times 2 - 4m = 0 \quad \therefore 8 + 6 - 4m = 0$

$\therefore 14 - 4m = 0 \qquad \therefore m = \dfrac{7}{2}$

Q(24) In the opposite figure, the value of k = ...



$(6,K,0) \bullet \left(4,0,2\sqrt{2}\right) = (6,K,0) \bullet (1,2,-1)$

$24 = 6 + 2K \quad \therefore 2K = 18 \quad \therefore K = 9$

Q(25) Find the standard form and the general form of the equation of the plane passing through point (3 , -5 , 2) and the vector n = (2 , 1, 1) is normal to the plane.

$\boxed{n \bullet r = n \bullet A}$

$(2,1,1) \bullet [(X,Y,Z) - (3,-5,2)] = 0 \quad \therefore (2,1,1) \bullet (X-3, Y+5, Z-2) = 0$

$2(X-3) + 1(Y+5) + 1(Z-2) = 0 \quad \therefore 2X - 6 + Y + 5 + Z - 2 = 0$

$2X + Y + Z - 3 = 0$

Q(26)

Find the volume of the parallelepiped which three of its adjacent sides are represented by the vectors $-12i - 3k$, $3j - K$ and $2i + j - 15K$

$$V = \begin{vmatrix} -12 & 0 & -3 \\ 0 & 3 & -1 \\ 2 & 1 & -15 \end{vmatrix} = 546$$

Q(27) Find the vector form of the equation of the straight line passing through point (3, -1, 0) and the vector (-2, 4 , 3) is a direction vector for it.

$\boxed{\vec{r} = \text{point} + t\ (\text{direction vector})}$

❶ The vector equation:

$$\vec{r} = (3, -1, 0) + t(-2, 4, 3)$$

❷ Parametric equations:

$(X, Y, Z) = (3, -1, 0) + t(-2, 4, 3)$

$\therefore X = 3 - 2t \quad , \quad Y = -1 + 4t \quad , \quad Z = 0 + 3t$

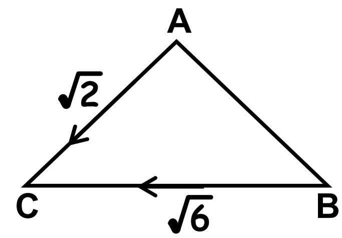
❸ The Cartesian equation:

$\therefore t = \dfrac{X-3}{-2} \quad , \quad t = \dfrac{Y+1}{4} \quad , \quad t = \dfrac{Z}{3} \qquad \therefore \dfrac{X-3}{-2} = \dfrac{Y+1}{4} = \dfrac{Z}{3}$

Q(28) Find the volume of the parallelepiped which three of its adjacent sides are represented by the vectors
$-12\hat{i}-3\hat{k}$, $3\hat{j}-\hat{K}$ and $2\hat{i}+\hat{j}-15\hat{K}$

$$V=\begin{vmatrix}-12 & 0 & -3\\ 0 & 3 & -1\\ 2 & 1 & -15\end{vmatrix}=546$$

Q(29) In the given figure $\|BC\|=\sqrt{6}$ and $\|AC\|=\sqrt{2}$
$BA=(-1,0,1)$ then $BA\bullet BC=$….



① 1 ② 2
③ 4 ④ 3

$\because \|BA\|=\sqrt{1+0+1}=\sqrt{2}$ $\therefore m(\angle A)=m(\angle B)$

$\cos(\angle B)=\cos(\angle C)=\dfrac{2+6-2}{2\sqrt{2}\times\sqrt{6}}=\dfrac{\sqrt{3}}{2}$ $\therefore BA\bullet BC=\sqrt{2}\times\sqrt{6}\times\dfrac{\sqrt{3}}{2}=3$

Q(30) If $(X-2)^2+(Y+4)^2+(Z-2)^2=1$, $(X+4)^2+(Y-4)^2+(Z-2)^2=4$ Are the equations of two spheres then the distance between their centers

$$\sqrt{(2+4)^2+(-4-4)^2+(2-2)^2}=10$$

Q(31) Find the equation of the line passing through the point (1,2,3) Perpendicular to the plane $2X-3Y+Z+1=0$

$r=(1,2,3)+t(2,-3,1)$

Q(32) If The Y axis cut the circle which center (3,-4,12) and its radius 13cm at the points A and B then AB=…..

$(X-3)^2+(Y+4)^2+(Z-12)^2=13^2$ $\therefore (0-3)^2+(Y+4)^2+(0-12)^2=13^2$
$\therefore (Y+4)^2=16$ $\therefore Y+4=\pm4$ $\therefore Y=0$ or $Y=-8$
$A=(0,0,0)$ and $B=(0,-8,0)$ $\therefore AB=\sqrt{0+8^2+0}=8$

Q(33) The equation of the plane passing through the points (1,-2,5) and vector (2,1,3) is perpendicular to it is ……

$n \bullet r = n \bullet A \quad \therefore (2,1,3) \bullet (X,Y,Z) = (2,1,3) \bullet (1,-2,5)$

$\therefore 2X + Y + 3Z = 15$

Q(34) The principle amplitude of the number $Z = 1 - i$ is …..

The number in 4^{th} quadrant $\therefore \theta = -\tan^{-1} 1 = -\frac{\pi}{4}$

Q(35) The equation of the plane intercepting from coordinates axes(X,Y,Z) parts a,b,c respectively such that ab=2 , ac=3 ,bc=6

$a^2b^2c^2 = 36 \quad \therefore abc = 6 \quad \therefore \frac{abc}{ab} = c = 3 \ , \ \frac{abc}{ac} = b = 2 \ , \ \frac{abc}{bc} = a = 1$

$\frac{X}{1} + \frac{Y}{2} + \frac{Z}{3} = 1 \quad \therefore 6X + 3Y + 2Z - 6 = 0$

Q(36) The general equation of the sphere whose centre (3,4,-5) and touch the YZ plane is …

Q(37) If the two straight lines L_1: X=2t-1 , Y=t+1 , Z=t-1
L_2:X=at-1 ,Y=2t+1, Z=bt-2 are parallel then a+b=….

$\frac{a}{2} = \frac{1}{2} = \frac{1}{b} \qquad \therefore a = 4 \ , b = 2 \qquad a + b = 6$

Q(39) If the two planes X-3Y+mZ=5 and 3X+KY+6Z=10 are parallel then Km=……

$\frac{1}{3} = \frac{-3}{K} = \frac{m}{6} \quad \therefore K = -9 \ , \ m = 2 \quad \therefore mK = -18$

Q(39)Find the equation of the plane passing through the point (2,1,4) and perpendicular to each of the two planes 7X+Y+2Z=6 , 3X+5Y-6Z=8

$\because$ required plane $\perp$ to the planes

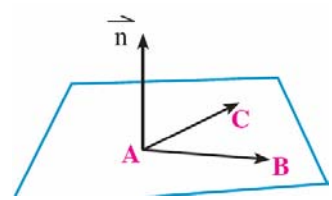
$\therefore (7,1,2)$, $(3,5,-6)$ are direction vectors lies in the required plane

$$\therefore \begin{vmatrix} \hat{i} & \hat{j} & \hat{K} \\ 7 & 1 & 2 \\ 3 & 5 & -6 \end{vmatrix} = -16\hat{i} + 48\hat{j} + 32\hat{K} \qquad \div 16 \qquad \therefore \text{required } (n) \text{ is } (1,-3,-2)$$

$$r \bullet (1,-3,-2) = (1,-3,-2) \bullet (2,1,4)$$

Q(40) Find the equation of the plane passing through points (3 , -1, 0) , (2 , 1 , 4) and (0 , 3 , 3).

$$n = AB \times AC = \begin{vmatrix} \hat{i} & \hat{j} & \hat{K} \\ -1 & 2 & 4 \\ -3 & 4 & 3 \end{vmatrix} = -10\hat{i} - 9\hat{j} + 2\hat{K}$$



$$n \bullet r = n \bullet A$$

$$(-10,-9,2) \bullet r = (-10,-9,2) \bullet (3,-1,0) = -21$$

$$-10X - 9Y + 2Z + 21 = 0$$

Q(41) Find the equation of the line of intersection of the two planes X + 2Y – 2Z = 1 , 2 X + Y – 3Z = 5

$$\begin{array}{l} -2X - 4Y + 4Z = -2 \\ \underline{\ \ 2X + \ \ Y - 3Z = 5} \\ -3Y + Z = 3 \end{array}$$

put $Y = 0$ $\quad \therefore Z = 3 \quad \therefore X = 7 \quad \therefore P_1 = (7,0,3)$

Put $Z = 0$ $\quad \therefore Y = -1 \quad \therefore X = 3 \quad \therefore P_2 = (3,-1,0)$

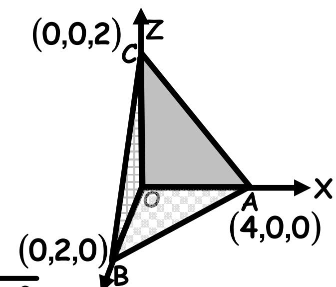
$r = (7,0,3) - (3,-1,0) = (4,1,3)$

Equation : $(7,0,3) + t(4,1,3)$ $\qquad \therefore \dfrac{X-7}{4} = \dfrac{Y}{1} = \dfrac{Z-3}{3}$

Q(42) If the plane $20X + 15Y + 12Z = 60$ intersected with the axes X,Y and Z at the points A ,B and C respectively then the volume of the solid ABCO where O is the origin point equals …..cube unit

$$\frac{1}{3} \times \left(\frac{1}{2} \times 3 \times 4\right) \times 5 = 10$$

Q(43) If the plane $\frac{X}{4} + \frac{Y}{2} + \frac{Z}{2} = 1$ cut the coordinates axes at A and B and C then the area of ΔAbc equal



① 12 ② 10

③ 6 ④ 4

$AC = C - A = (-4,0,2)$, $AB = B - A = (-4,2,0)$

$$A = \frac{1}{2} AC \times AB = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{K} \\ -4 & 0 & 2 \\ -4 & 2 & 0 \end{vmatrix} = -4\hat{i} + 8\hat{j} - 8\hat{K} \therefore \text{Area} = \frac{1}{2}\sqrt{4^2 + 8^2 + 8^2} = 6$$

Q(44) A sphere of center (2,-1,-2) and radius 3units placed on the plane 2X+6Y-3Z+K=0 then K=……..

$$\frac{2(2) + 6(-1) - 3(-2) + K}{\sqrt{4 + 36 + 9}} = 3 \therefore \frac{4 - 6 + 6 + K}{7} = 3 \therefore 4 + K = 21 \therefore K = 21$$

Q(45) Measure of the angle between the two planes :X+Y-1=0 and Y+Z-1=0 equals ………….°

Angle between the normal $\cos\theta = \frac{(1,1,0) \bullet (0,1,1)}{\sqrt{1^2 + 1^2 + 0}\sqrt{0^2 + 1^2 + 1^2}} = \frac{1}{2} \therefore \theta = 60°$

Q(46) The equation of the sphere its center M(1,-2,-5) and touch the XY plane …..

$(X - 1)^2 + (Y + 2)^2 + (Z + 5)^2 = 25$

Q(47) Find the measure of the angle between the two straight lines :
$r_1 = (2,-1,3) + t_1(-2,0,2)$ and $r_2 :\ X = 1\ ,, \frac{Y-4}{3} = \frac{Z+5}{-3}$

$d_1 = (-2,0,2)$, $d_2 = (0,3,-3)$

$$\cos\theta = \frac{d_1 \bullet d_2}{\|d_1\| \times \|d_2\|} = \frac{(-2,,0,2)\bullet(0,3,-3)}{\sqrt{(-2)^2 + 0^2 + 2^2}\sqrt{0^2 + 3^2 + (-3)^2}} = \frac{1}{2}$$

$$\therefore \theta = \cos^{-1}\frac{1}{2} = 60^\circ$$

Q(48) A sphere of centre (1,2,1) touches the plane
X+Y+Z=1 find the equation of the sphere

Radius of the sphere is the perpendicular length from the centre
To the plane

$$= \frac{|1+2+1-1|}{\sqrt{1+1+1}} = \frac{3}{\sqrt{3}}$$

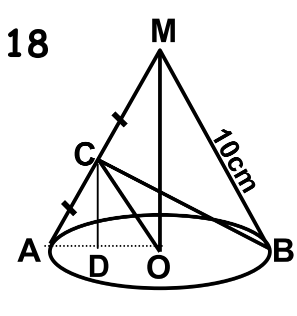
Equation of the sphere

$$(X-1)^2 + (Y-2)^2 + (Z-1)^2 = 3$$

Q(49) In the opposite figure, a right circular cone ,
the perimeter of its base =12πcm C is the midpoint of
AM then $OC \bullet OA =$

$2\pi r = 12\pi \ \therefore r = 6\ \ OA = 6\ \therefore MO = 8cm\ , CO = 5\ \ \therefore DO = 3cm$

$CD = 4cm\ \ \therefore OC \bullet OA = \|OC\| \times \|OA\| \cos(\angle COA) = 5 \times 6 \times \frac{3}{5} = 18$



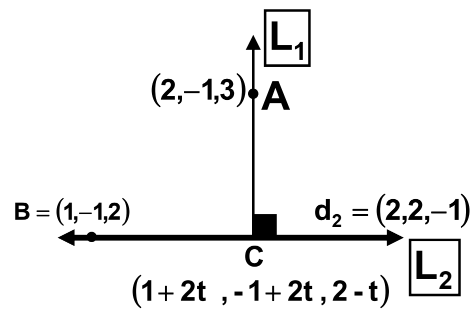
Q(50) If $\|A\| = 2$, $\|B\| = 3$, $\|C\| = 12$ and A , B , C are mutually orthogonal then $\|A + B + C\| =$

$$(A + B + C)^2 = A^2 + B^2 + C^2 + 2AB + 2AC + 2BC$$
$$= 4 + 9 + 144 = 157 + 0 + 0 + 0 = 157 \quad \therefore \|A + B + C\| = \sqrt{157}$$

Q(51) If $X = \frac{-1 - \sqrt{3}i}{2}$, $i^2 = -1$, then the numerical value of

$X^8 + X^4 + 5 =$ (4)

Q(52) Find the equation of the straight line passing through the point (2 , -1 , 3) and intersects the straight line r_1 = (1 , -1 , 2) + t (2 , 2 , -1) orthogonally.



$$d_1 = AC = C - A = (\ 2t - 1\ ,2t\ \ ,-1-t\ )$$
$$\because L_1 \perp L_2 \ \therefore d_1 \bullet d_2 = 0$$
$$(\ 2t - 1\ ,2t\ \ ,-1-t\ ) \bullet (2,2,-1) = 0$$
$$4t - 2 + 4t + 1 + t = 0 \quad \therefore 9t - 1 = 0 \quad \therefore\ t = \frac{1}{9}$$
$$\therefore d_1 = \left(-\frac{7}{9}, \frac{2}{9}, \frac{-10}{9}\right) = (\ -7, 2, -10\ )$$

Equitation of the line is $r = (2,-1,3) + t(-7,2,-10)$

Q(53)Find the equation of the straight line passing through the point (3,-1,0) and intersect the straight line r=(2,1,1) +t(1,2,-1) orthogonally

$$(1,2,-1) \bullet (-1 + t, 2 + 2t, 1 - t) = 0$$
$$\therefore t = -\frac{1}{3} \qquad\qquad \therefore r = (3,-1,0) + t(-1,1,1)$$

Q(54) Use the multiplicative inverse to solve the set of following equations $\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}=1$, $\frac{1}{X}-\frac{1}{Y}+\frac{2}{Z}=\frac{1}{2}$, $\frac{2}{X}+\frac{3}{Y}-\frac{4}{Z}=\frac{4}{3}$
Where X and Y and Z not equals zeros

$$\begin{pmatrix}\frac{1}{X}\\ \frac{1}{Y}\\ \frac{1}{Z}\end{pmatrix}=\frac{1}{11}\begin{pmatrix}-2 & 7 & 3\\ 8 & -6 & -1\\ 5 & -1 & -2\end{pmatrix}\begin{pmatrix}1\\ \frac{1}{2}\\ \frac{4}{3}\end{pmatrix}=\begin{pmatrix}\frac{1}{2}\\ \frac{1}{3}\\ \frac{1}{6}\end{pmatrix} \quad \therefore \frac{1}{X}=\frac{1}{2} \quad \therefore X=2$$

Q(55) Find the equation of a sphere its diameter $\overline{AB}$ where
A=(-1,5,4) , B=(5,1,-2)

center $=\left(\frac{-1+5}{2},\frac{5+1}{2},\frac{4-2}{2}\right)=(2,3,1)$
Radius $=\sqrt{(2+1)^2+(3-5)^2+(1-4)^2}=\sqrt{22}$
The equation is $(X-2)^2+(Y-3)^2+(Z-1)^2=22$

Q(56) Find the measure of the angle between the two straight lines
$r_1=(2,-1,3)+t_1(-2,0,2)$ and $X=1$, $\frac{Y-4}{3}=\frac{Z+5}{-3}$

$d_1=(-2,0,2)$, $d_2=(0,3,-3)$
$$\therefore \cos\theta=\frac{d_1\bullet d_2}{\|d_1\|\times\|d_2\|}=\frac{(-2,0,2)\bullet(0,3,-3)}{\sqrt{(-2)^2+0^2+2^2}\times\sqrt{0^2+3^2+(-3)^2}}=\frac{1}{2} \quad \therefore \theta=60^{o}$$