3rd year secondary

Static

booklets 2017

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مع خاص التمنيات للطلبه بالنجاح و التوفيف

Guide Answers

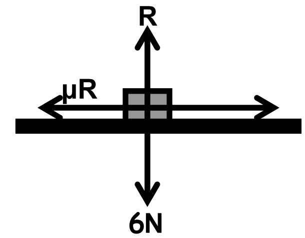
Answer the following questions | 20 questions

From 1to 12 choose the correct answer

Q(1) A body of weight 6 Newton's is placed on a rough horizontal plane whose coefficient of friction is $\frac{\sqrt{3}}{3}$ the friction force $\in$ ……..

① $[2\sqrt{3}, 3\sqrt{3}]$ ② $[0, 2\sqrt{3}]$

③ $[0,1]$ ④ $[0, 3\sqrt{3}]$

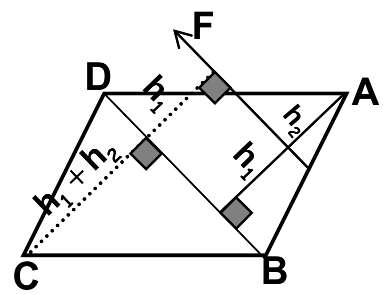


$$[0,\mu R] = \left[0, \frac{\sqrt{3}}{3} \times 6\right] = [0, 2\sqrt{3}]$$

Q(2) If the force F in the plane of the parallelogram ABCD and $M_A = -18$, $M_B = M_D = 32$ then $M_C =$….. moment unit

① 50 ② 82

③ 46 ④ 14



Line action of the force is // to $\overline{BD}$

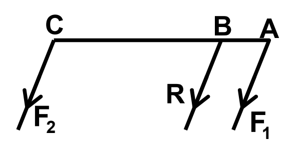
$Fh_2 = 18$, $Fh_1 = 32$

$M_C = F(h_1 + h_2 + h_1) = 2Fh_1 + Fh_2 = 2 \times 32 + 18 = 82$

Q(3) In the opposite figure : R is the resultant of the two parallel forces F_1 , F_2 if F_1=36Newton , F_2=24Newton then AB:BC= …………

① 2:3 ② 1:2

③ 3:2 ④ 2

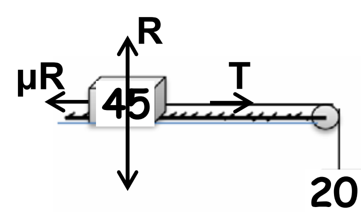


$$36 \times AB = 24 \times CB \quad \therefore \frac{AB}{BC} = \frac{24}{36} = 2:3$$

Q(4) If the system is about to move then The coefficent of friction equal………

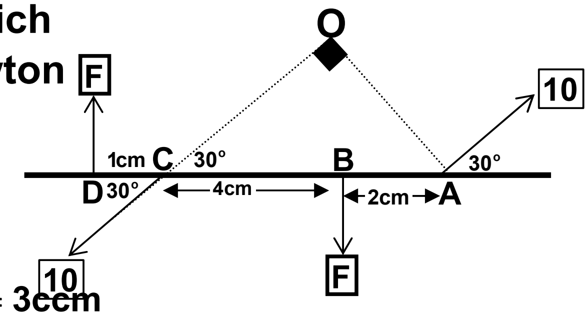
① $\frac{4}{9}$ ② $\frac{1}{3}$

③ $\frac{9}{4}$ ④ $\frac{2}{3}$



$$20 = 45\mu \quad \therefore \mu = \frac{20}{45} = \frac{4}{9}$$

Q(5) Four forces form two couples which are in equilibrium Then F=…..Newton



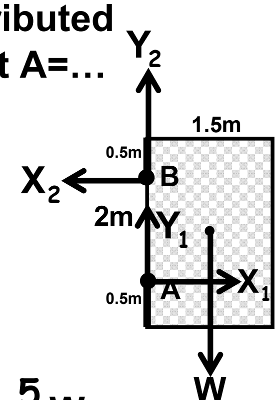
① 6 ② 9

③ 8 ④ 10

$10\times AO = F\times BD \quad \because Ao = AC\sin 30^{o} = 6\times\frac{1}{2} = 3cm$

$10\times 3 = F\times 5 \qquad \therefore F = 6$

Q(6) If the door is in equilibrium Its weight is equally distributed On the two hinges at A and B then The the reaction at A=…



① $\frac{5}{8}W$ ② $\frac{1}{2}W$

③ $\frac{1}{8}W$ ④ $\frac{3}{8}W$

$Y_1 + Y_2 = W \quad \because Y_1 = Y_2 \quad \therefore Y_1 = \frac{W}{2}$

$M_B = 0 \ \therefore -W\times\frac{3}{4} + X_1\times 2 = 0 \quad \therefore X_1 = \frac{3}{8}W \ \therefore R = \sqrt{X_1{}^2 + Y_1{}^2} = \frac{5}{8}W$

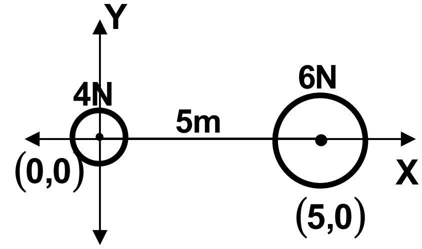
Q(7) If the two forces $\overrightarrow{F_1}$=ai+3j , $\overrightarrow{F_2}$=5i+bj form a couple then a + b=

① 8 ② -8

③ 10 ④ -3

$a = -5 \ , \ b = -3 \quad \therefore a + b = -8$

Q(8) The center of gravity of two physical particles of weights 4 Newton's and, 6 Newton's and the distance between them is 5 meters. it is at distance……………from the first particle



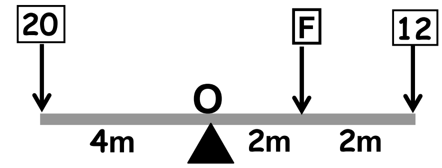
① 1cm ② 2cm

③ 3cm ④ 4cm

$X = \frac{4\times 0 + 6\times 5}{4 + 6} = 3cm$ 3cm from (4Newtons)

Q(9) In the opposite figure : if the rod is in equilibrium
Then the value of F is ……..Newton's

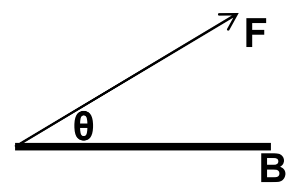
① 28
② 2
③ 16
④ 4



$M_O = 0 \quad 12 \times 4 + F \times 2 = 20 \times 4 \quad \therefore 48 + 2F = 80 \quad \therefore F = 16$

Q(10) The measure of the angle θ which generate the maximum Moment about (B) is ……….

① 0°
② 45°
③ 90°
④ 30°



90°

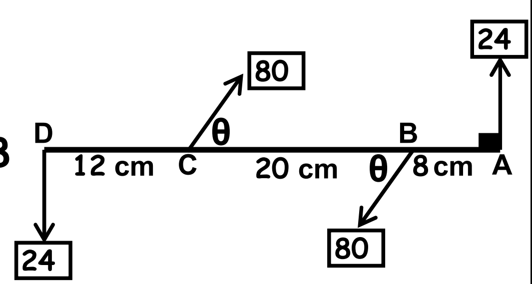
Q(11) If the coefficient of friction between the body and the plane =2sin30° then the measure of angle of friction =……….

① 30°
② 60°
③ 45°
④ 90°

$\tan\lambda = 2 \times \frac{1}{2} = 1 \quad \therefore \lambda = 45^\circ$

Q(12) In the opposite figure The rod is equilibrium
The value of sin Θ =….

① 0.4
② 0.6
③ 0.5
④ 0.8



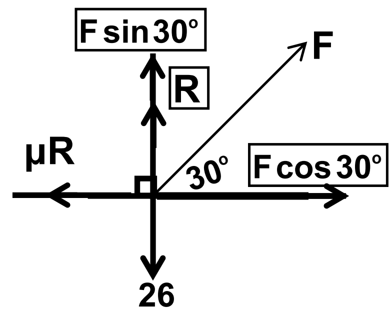
$80 \times 20 \sin\theta = 24 \times 40 \quad \therefore \sin\theta = 0.6$

Q(13) A body of weight 26N is about to slide it is placed on a rough inclined plane with angle whose tangent $= \frac{1}{4\sqrt{3}}$ .If the same body is placed on a horizontal rough plane having the same friction of the inclined plane find the magnitude of the force acting on the body and incline to the horizontal with 30° and make it about to move

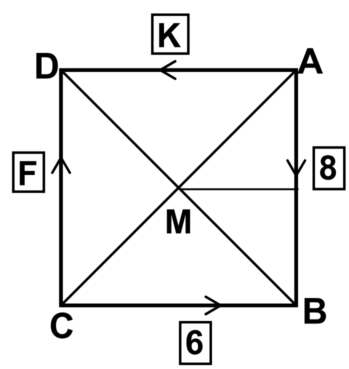
$\mu = \frac{1}{4\sqrt{3}}$, $R + F\sin 30^\circ = 26 \quad \therefore R + \frac{1}{2}F = 26$

$F\cos 30^\circ = \mu R \quad \therefore F \times \frac{\sqrt{3}}{2} = \frac{1}{4\sqrt{3}} R \quad \therefore R = 6F$

$6F + \frac{1}{2}F = 26 \quad \therefore \frac{13}{2}F = 26 \quad \therefore F = 4$



Q(14) ABCD is a square , its diagonals intesect at M , forces of magnitudes 8,6,F,K Newtons act along $\overrightarrow{AB}, \overrightarrow{CB}, \overrightarrow{CD}, \overrightarrow{AD}$ respectively if the algebraic sum of moments of these forces vanishes about each of M and A find the values of F , K



Let the length side of the square equal L

$M_A = 0 \quad \therefore 6 \times L - F \times L = 0 \quad \therefore F = 6$

$M_M = 0 \quad \therefore 6 \times \frac{1}{2}L - 8 \times \frac{1}{2}L + K \times \frac{1}{2}L - 6 \times \frac{1}{2}L = 0$

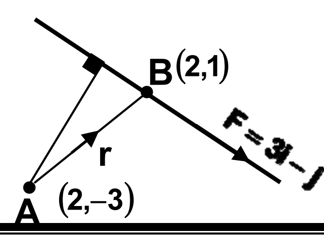
$\therefore 6 - 8 + K - 6 = 0 \quad \therefore K = 8$

Q(15) Force $\overrightarrow{F} = 3i - j$ acts at the point (2,1) find the moment of this force with respect to the point (2,-3) then find the length of the perpendicular from (2,-3) to the line action of the force

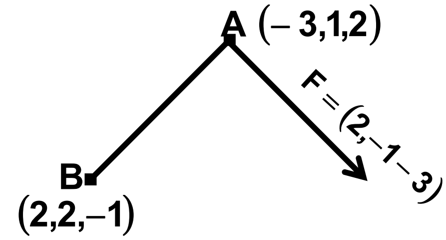
$M_A = [(2,1) - (2,-3)] \times (3,-1) \quad \therefore M_A = (0,4) \times (3,-1) = 0 \times -1 - 4 \times 3 = -12\hat{K}$

The perpendicular distance from A to the line action of the force

$= \frac{\|M\|}{\|F\|} = \frac{12}{\sqrt{3^2 + 1^2}} = \frac{12}{\sqrt{10}}$



Q(16) The force $\overrightarrow{F} = 2\hat{i} - \hat{j} + 3\hat{K}$ ats at the point (-3,1,2) find the moment o the force about the pont B(2,2-1) then find the length of the perpendicular from the point B to the line action of the force



$r = \overrightarrow{BA} = A - B$

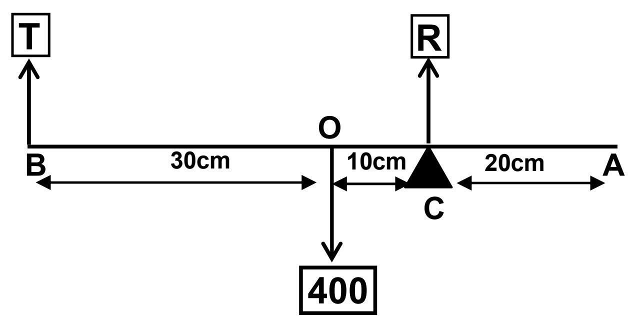
$= (-3,2,-1) - (2,2-1) = (-5,-1,3)$

$$M_B = \begin{vmatrix} i & j & k \\ -5 & -1 & 3 \\ 2 & -1 & 3 \end{vmatrix}$$

$= (-1\times 3 - 3\times -1)i - (-5\times 3 - 3\times 2)j + (-5\times -1 - 2\times -1)K$

$= 21J + 7K \qquad \therefore L = \dfrac{\sqrt{0^2 + 21^2 + 7^2}}{\sqrt{2^2 + (-1)^2 + 3^2}} = \sqrt{35}$

Q(17) AB is a rod of length 60cm and weight 400gm.wt (acting at its mid point) rests on support 20cm distant from A The rod kept horizontally in equilibrium by a light vertical string connected to B find (i) The magnitude o each of the tension in the string and the reaction of the support (ii)The magnitude of the weight that should be suspended from A suspended from A to make the tension in the string is about to vanish



$R + T = 400$

$M_C = 0 \quad \therefore 400\times 10 - T\times 40 = 0$

$\therefore 40T = 4000 \quad \therefore T = 100$

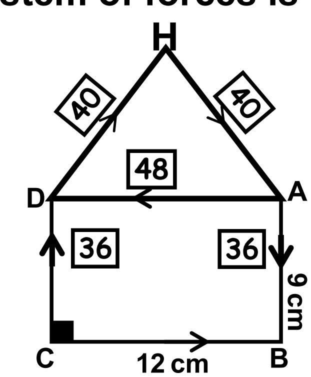
$\therefore R = 300$ gm.wt

Second : $T = 0$, $M_C = 0$

$400\times 10 - W\times 20 = 0$

$\therefore W = 200$ gm.wt

Q(18) ABCD is a rectangle in which AB=9cm , BC=12cm an isosceles triangle HAD is drawn outside this rectangle such that AD is its base and its height =8cm forces of magnitudes 36,48,36,40,40gm.wt act along $\overrightarrow{AB}, \overrightarrow{AD}, \overrightarrow{CD}, \overrightarrow{DH}, \overrightarrow{HA}$ Respectively prove that this system of forces is equivalent to a couple then find its moment norm



$AH = DH = 10cm \qquad \dfrac{40}{10} = \dfrac{48}{12} = \dfrac{40}{10} = 4$

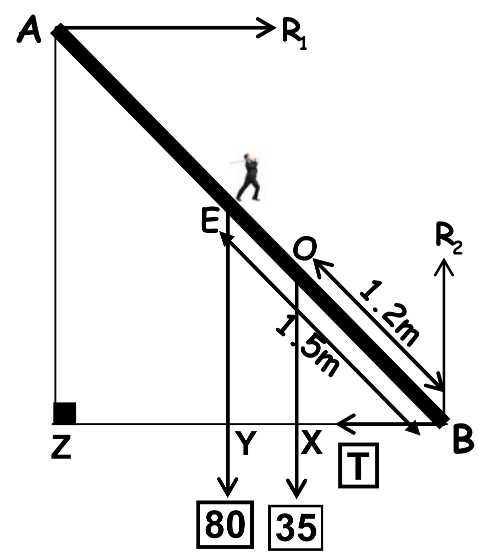
$\text{moment} = -4\times 2\times \frac{1}{2}\times 12\times 8 = -384$

36,36 form another couple its moment $=$

$-36\times 12 = -432$ gmwt.cm

norm of the couple $= -816$

Q(19) A ladder AB of weight 35 Kg.wt and length 3m. rests in a vertical plane with the end B on a smooth floor and A against a smooth vertical wall. The lower end B is attached by a string to a point on the floor vertically below A. Given that B is 1.8 m. away from the wall and the weight of the ladder is acting at a point on the ladder 1.2 m. away from B, find the tension in the string. Also find the tension in the string when a man of weight 80 kg. stands at the mid-point of the ladder.



$AZ = \sqrt{3^2 - 1.8^2} = 2.4m$

$R_2 = 35 \to (1)$ $\qquad T = R_1 \to (2)$

$M_B = 35 \times 1.2\cos\theta - R_1 \times 3\sin\theta = 0$

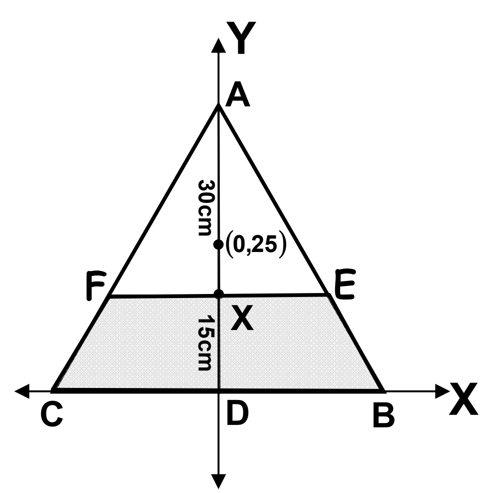
$35 \times 1.2 \times \frac{1.8}{3} = R_1 \times 3 \times \frac{2.4}{3} \qquad \therefore R_1 = 10.5Kg.wt$

After ascending of the man

$35 \times 1.2 \times \frac{1.8}{3} + 80 \times 1.5 \times \frac{1.8}{3} - R_1 \times 2.4 = 0$

$\therefore R_1 = 40.5Kg.wt \quad , \; T = 40.5Kg.wt$

Q(20) A uniform fine lamina in the form of an isosceles triangle A B C in which AB = AC and AD is the height of the triangle of length 45 cm. A straight line is constructed parallel to the base BC and passes through the center of gravity of the lamina to intersect $\overline{AB}$ and $\overline{AC}$ at E and F respectively. Prove that the center of gravity of the quadrilateral E B C F lies on AD and is distant 7 cm from point D.



$\frac{A(AEF)}{A(ABC)} = \left(\frac{30}{45}\right)^2 = \frac{4}{9}$

Mass $AEF = 4K$

mass of $ABC = 9K$

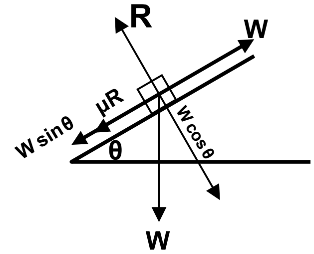
$Y = \left(\frac{9K \times 15 - 4K \times 25}{9K - 4K}\right) = 7$

$\therefore$ 7cm from D

Answer the following questions 20 questions

From 1to 12 choose the correct answer

Q(1) A body of weight (W) is placed on a rough inclined plane incline angle θ to the horizontal , a force of magnitude (W) acted on it in the direction of the line of greatest slope , if the body is about to move upwards then $\mu = ...$



① $\sec\theta - \tan\theta$ ② $1+\tan\theta$

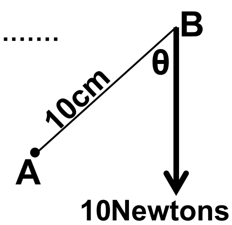
③ $\sec\theta + \tan\theta$ ④ $1+\cot\theta$

$R = W\cos\theta$, $W = W\sin\theta + \mu R$

$\therefore W = W\sin\theta + \mu W\cos\theta \div W$

$\therefore 1 = \sin\theta + \mu\cos\theta \quad \therefore \mu = \dfrac{1-\sin\theta}{\cos\theta} = \sec\theta - \tan\theta$

Q(2) The maximum moment of the force About A equals.........

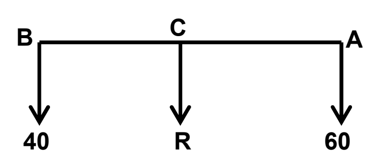


① 100N.cm ② 300N.cm

③ 200N.cm ④ 50N.cm

$10 \times 10 = 100$N.cm

Q(3) In the opposite figure the resultant of the two parallel Forces F_1 , F_2 is R if $F_1=60$, $F_2=40$ then $AC:BC=$.......



① 2:3 ② 1:2

③ 3:2 ④ 4:5

$60 \times AC = 40 \times BC \quad \therefore \dfrac{AC}{AB} = \dfrac{40}{60} = \dfrac{2}{3}$

Q(4) If $4F_1$, $3F_2$ two forces form a couple if $F_1 = (6,-9)$ then $\|F_2\| =$.........

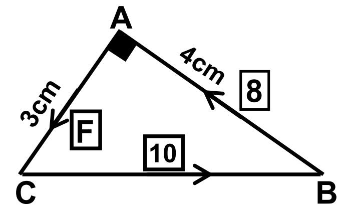
① $4\sqrt{13}$ ② $\sqrt{13}$

③ $5\sqrt{13}$ ④ $7\sqrt{13}$

$4(6,-9) = 3(l,m) \quad \therefore 3L = 24 \quad \therefore L = 8$, $3m = -36 \quad \therefore m = -12$

$\|F_2\| = \sqrt{8^2 + (-12)^2} = 4\sqrt{13}$

Q(5) ABC is a right angled triangle at A and the system of forces form a couple then F=…and the magnitude of moment of this couple=



① 12N.cm ② 16N.cm

③ 24N.cm ④ 26N.cm

$\frac{F}{3} = \frac{8}{4} = \frac{10}{5} \quad \therefore F = 6 \qquad \therefore M = 2 \times 2 \times \frac{1}{2} \times 3 \times 4 = 24\text{N.cm}$

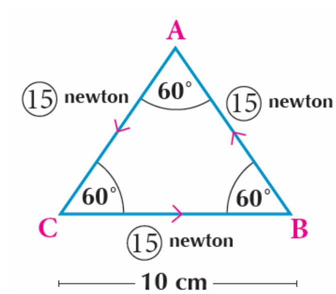
Q(6) If θ is the mesure of the angle between the fianl limiting forcfe and the resultant reaction then the coefficent of the static friction is equal to

① $\tan\theta$ ② $\cos\theta$

③ $\sin\theta$ ④ $\cot\theta$

$\tan(90° - \theta) = \cot\theta$

Q(7) The moment of the couple equals ….

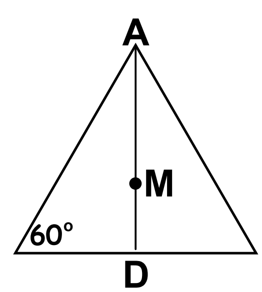


① 150 ② 75

③ $150\sqrt{3}$ ④ $75\sqrt{3}$

$\frac{15}{10} \times 2 \times \frac{1}{2} \times 10 \times 10 \times \sin 60° = 75\sqrt{3}\text{N.cm}$

Q(8) The center of gravity of a uniform fine lamina in the form of an equilateral triangle of side length 12 cm is distant.......... from one of the vertices of the triangle:.



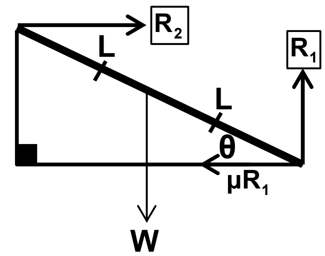
① $2\sqrt{3}$ ② $4\sqrt{3}$

③ 6 ④ $6\sqrt{3}$

The center of gravity is point of intersecting of its median
Divide the median ratio 1:2 from base

$AD = 12\sin 60° = 6\sqrt{3} \quad \therefore AM = \frac{2}{3} \times 6\sqrt{3} = 4\sqrt{3}$

Q(9) If α is the friction angle between the rod and the horizontal And the rod inclined angle of measure θ to the horizontal Then $\tan\theta \times \tan\alpha =$.....



① 0.5 ② 2

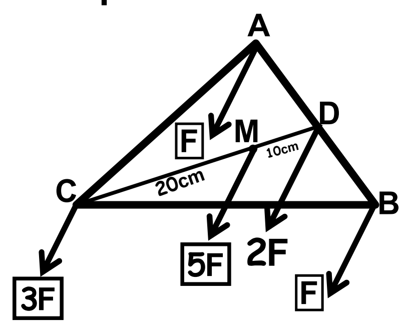
③ 1 ④ 3

$R_1 = W$, $R_2 = \mu R_1 = \mu W$

$M_A = 0 \quad \therefore W L\cos\theta = \mu W 2L \sin\theta \quad \therefore \cos\theta = \mu 2 \sin\theta$

$\therefore \tan\theta \times \tan\alpha = 0.5$

Q(10) ABC is a triangle M is the point of intersection of its medians Parallel force as shown acts in the plane of the triangle If CD=30cm then the resultant of these force acts at a point ..... distance from C



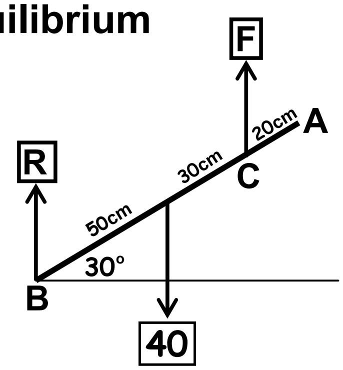
① 15 ② 5

③ 16 ④ 10

$M_C = 0 \;\therefore$

$5F \times 20 + 2F \times 30 = 10F \times X \quad \therefore X = 16$

Q(11) $\overline{AB}$ is a uniform rod and the system is in equilibrium Then F equal ......



① 15 ② 25

③ $\dfrac{20}{3}$ ④ $\dfrac{\sqrt{3}}{4}$

$M_B = 0 \;\therefore -40 \times 50\cos 30^\circ + F \times 80 \cos 30^\circ = 0 \quad \therefore F = 25$

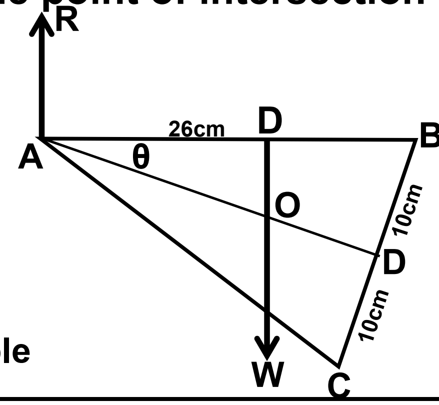
Q(12) If the force F acts about the point (3,5) and (-1,1) with moment 6K and -6K then the moment about .....=0

① (-1,-3) ② (2,6)

③ (2,2) ④ (1,3)

mid point of is $(1,3)$

ABC is a thin lamina in the form of an isoscles triangle in which AB=AB=26cm , BC=20cm The laminal is suspended by a thin horizonta pin from a small hole near the vertex A such that its plane is vertical . Acouple the magnitude of its moment equals 720Newtons .cm acts on the lamina in a diraction perpendicular to its plane such that it is in equilibrium in a position in which AB is horizontal find the magnitude o the weight of the lamina given that it acts at the point of intersection of its medians



(b) $AD = \sqrt{26^2 - 10^2} = 24cm$

$, AO = \frac{2}{3} AD \quad \therefore \quad AO = 16cm$

$\cos\theta = \frac{AD}{AB} = \frac{24}{26} = \frac{12}{13} = \frac{AD}{AO} = \frac{AD}{16} \quad \therefore AD = 16 \times \frac{12}{13}$

$\because$ The lamina is in equilibrium

$\therefore$ the weight and the reaction of the pin form a couple

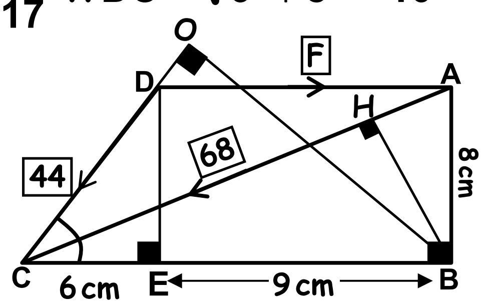
$\therefore R = W$

Q(14) ABCD is a trapezium in which $\overline{AD} // \overline{BC}$,$(\angle ABC) = 90^o$ AB=8cm , BC=15cm , AD=9cm forces of magnitudes F , 44, 68 Kg.wt act along $\overrightarrow{DA}, \overrightarrow{DC}, \overrightarrow{AC}$ respectively if the line of action of the resultant of these forces passes thought the point B find the value of F

Answer:

inΔABC $AC = \sqrt{8^2 + 15^2} = 17 \quad \therefore BH = \frac{8 \times 15}{17} = \frac{120}{17} \therefore DC = \sqrt{6^2 + 8^2} = 10$

Notes that $\angle C$ lies in $\Delta\Delta$ DCE , OCB



$\sin C = \frac{8}{10} = \frac{BO}{15} \quad \therefore BO = 12cm$

$M_B = 0 \quad \therefore \quad 68 \times \frac{120}{17} + 44 \times 12 - F \times 8 = 0$

$\therefore F = 126gm.wt$

Q(15) If $F = 3i - 4j$ act at $A = (0,2)$ if $B = (3,-2)$, $C = (2,3)$,$D = (-2,1)$,$H = (5,-1)$ Prove that: the line action of the force

(a) passing through B (b) bisect CD (c) parallel to CH

(b) $M_B = [(0,2) - (3,-2)] \times (3,-4) = (-3,4) \times (3,-4) = 0$

$M_C = [(0,2) - (2,3)] \times (3,-4) = (-2,-1) \times (3,-4) = 8 + 3 = 11$

$M_D = [(0,2) - (-2,1)] \times (3,-4) = (2,1) \times (3,-4) = -8 - 3 = -11$

$M_H = [(0,2) - (5,-1)] \times (3,-4) = (-5,3) \times (3,-4) = 20 - 9 = 11$

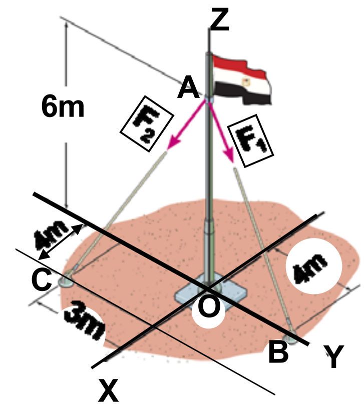
$\because M_B=0 \quad \therefore$ the line action of the force passing through B

$\because M_C=M_H \quad \therefore$ the line action of the force //CH

$\because M_C=-M_D \quad \therefore$ the line action of the force bisect CD

Q(16) In the given figure : $F_1 = 6\sqrt{13}$ N , $F_2 = \sqrt{61}$ N acts in direction of $\overrightarrow{AB}$, $\overrightarrow{AC}$ find (1) the sum of moments of the two forces about O
(2) the moment of the resultant about O

$A \to (0,0,6)$, $B \to (0,4,0)$, $C \to (4,-3,0)$



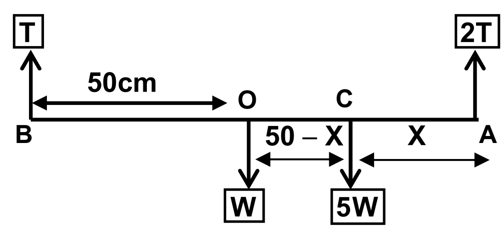
$$AB = B - A = (0,4,0) - (0,0,6) = (0,4,-6)$$

$$F_1 = \frac{6\sqrt{13}}{\sqrt{0^2 + 4^2 + 6^2}}(0,4,-6) = (0,12,-18)$$

$$M_O = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 6 \\ 0 & 12 & -18 \end{vmatrix} = -72\,i$$

$$M_o = -54\hat{i} - 24\hat{j}$$

Q(17) A rod of length 100cm and weight (W) acting at its mid point is suspended in a horizontal position by mean of two vertical strings at its end determine a point on the rod which a weight of (5W) is suspended such that the magnitude of the tension in one of the two strings equals twice its magnitude in the other string



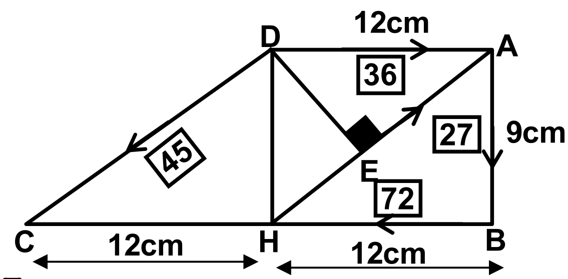
$$2T + T - 5W - W = 0 \quad \therefore \quad T = 2W$$

$$M_A = 0$$

$$5WX + 50W - T \times 10 = 0$$

$$5WX + 50W - 20W = 0 \div W \qquad \therefore X = 30cm$$

Q(18) ABCD is a trapezium right angled at B $\overline{AD} // \overline{BC}$,AB=9cm , BC=2AD =24cm , H is the mid point of $\overline{BC}$ forces of magnitudes 27,72,45 and 36 Newton acting along $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DA}$ respectively prove that this system of forces is equivalent to a couple and find the norm of its moment . find the magnitudes of each of the two forces acting along $\overrightarrow{HA}$ and $\overrightarrow{DC}$ for the system to be in equilibrium



$$(CD)^2 = \sqrt{12^2 + 9^2} = 15cm$$

$$\because \frac{27}{9} = \frac{72}{24} = \frac{45}{15} = \frac{36}{12} = 3$$

$$mom. = 3 \times 2 \times \frac{1}{2}(12 + 24) \times 9 = 972$$

$$DE = \frac{12 \times 9}{15} = \frac{36}{5} \to \; F \times \frac{36}{5} = 972 \Rightarrow F = 135$$

Q(19) A uniform rod rests with one end against a rough vertical plane, the coefficient of friction between them is $\frac{1}{2}$, and the other end on a rough horizontal floor, the coefficient of friction between them is $\frac{3}{4}$. Find the rod's inclination to the floor when being in limiting equilibrium.

$\underline{\text{Horizontal}}: R_1 = \frac{3}{4}R_2 \quad \underline{\text{Vertical}}: \frac{1}{2}R_1 + R_2 = W$

$\frac{1}{2}\left(\frac{3}{4}R_2\right) + R_2 = W \quad \therefore \frac{3}{8}R_2 + R_2 = W$

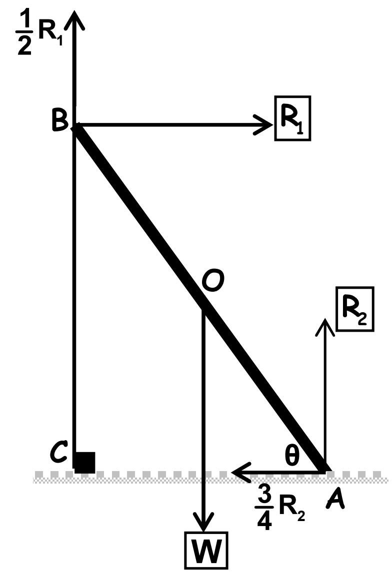
$\frac{11}{8}R_2 = W \quad \therefore R_2 = \frac{8}{11}W \quad \therefore R_1 = \frac{3}{4}R_2 = \frac{3}{4}\left(\frac{8}{11}W\right) = \frac{6}{11}W$

$M_A = 0 \therefore W(L\cos\theta) = R_1(2L\sin\theta) + \frac{1}{2}R_1(2L\cos\theta)$

$\therefore \cos\theta = \frac{12}{11}\sin\theta + \frac{6}{11}\cos\theta \therefore \cos\theta - \frac{6}{11}\cos\theta = \frac{12}{11}\sin\theta$

$\therefore \frac{5}{11}\cos\theta = \frac{12}{11}\sin\theta \therefore \frac{\sin\theta}{\cos\theta} = \frac{5}{12}$

$\therefore \tan\theta = \frac{5}{12}$



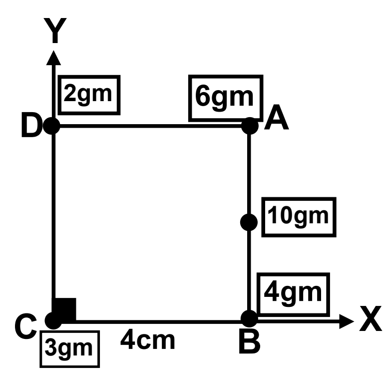
Q(20) ABCD is a square of side length 4 cm. The masses 6 , 4 , 3 and 2 gm are attached at A, B , C and D respectively. Another mass of magnitude 10 gm is attached at the midpoint of AB. identify the distance between the center of gravity of the system and both CD and CB

Mass	2	3	4	6	10
X	0	0	4	4	4
Y	4	0	0	4	2

$X = \left(\frac{2\times0 + 3\times0 + 4\times4 + 6\times4 + 10\times4}{2+3+4+6+10}\right) = 3.2$

$Y = \left(\frac{2\times4 + 3\times0 + 4\times0 + 6\times4 + 10\times2}{2+3+4+6+10}\right) = 2.08$

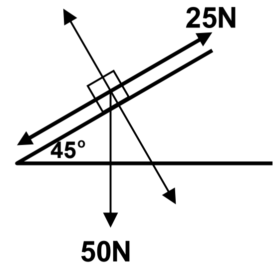
$\therefore G(3.2, 2.08)$



Answer the following questions	20 questions

From 1to 12 choose the correct answer

Q(1) A body of weight 50Newtons is placed on an inclined Rough plane incline 45° to the horizontal a force of magnitude 25Newtons acts on the body in the direction of the line of greatest slope make it about to slide then the coefficient of friction equal



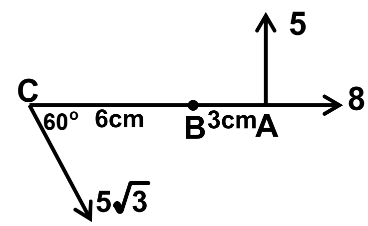
① $\frac{\sqrt{2}}{2}$ ② $\frac{\sqrt{2}-1}{2}$

③ $1-\frac{\sqrt{2}}{2}$ ④ $\sqrt{2}-1$

$R = 50\cos 45 = 25\sqrt{2}$ $F + \mu R = W\sin\theta$

$25\sqrt{2}\mu = 25\left(\sqrt{2}-1\right)$ $\therefore \mu = 1-\frac{1}{\sqrt{2}}$

Q(2) The moment of the force about A equals……..

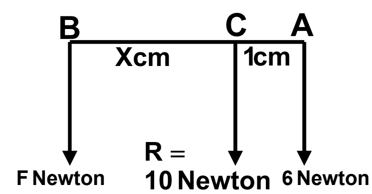


① 67.5 ② $9\sqrt{3}$

③ $45\sqrt{3}$ ④ 12

$M_A = 5\sqrt{3}\sin 60^\circ \times 9 = 67.5$

Q(3) <u>In the opposite figure :</u> the value of X equals



① 1 ② 1.5

③ 2 ④ 4

$F = 10-6 = 4$ $\therefore 6\times 1 = 4X$ $\therefore X = 1.5cm$

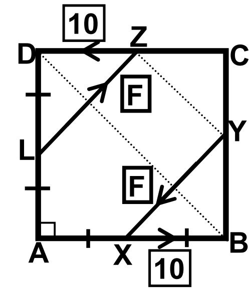
Q(4) a body is placed on an inclined plane inclines 45° to the horizontal if the body is about to slide then thecoefficient of friction ….

① 1 ② $\sqrt{2}$

③ $\sqrt{3}$ ④ 0

$\tan 45^\circ = 1$

Q(5) In the opposite figure : ABCD is a square X , Y , Z and L are mid points $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{AD}$ given that the forces are in equilibrium then F =…..



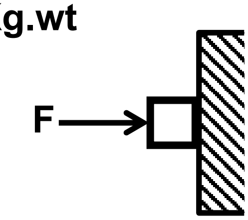
① 10 ② 20

③ $10\sqrt{2}$ ④ $20\sqrt{2}$

$BD = \sqrt{L^2 + L^2} = \sqrt{2}\,L \therefore\ ZY = \frac{1}{2}\sqrt{2}\,L$

$10 \times L = F \times \frac{1}{2}\sqrt{2}\,L \quad \therefore\ F = 10\sqrt{2}$

Q(6) The magnitude of the smallest horizontal force that makes a body of mass 10Kg in equilibrium on a vertical rough wall where the coefficent of friction between them equals $\frac{1}{5}$ equals ….Kg.wt



① 50 ② 25

③ 2.5 ④ 5

$F = R \quad , \therefore \mu R = W \quad \therefore \frac{1}{5}R = 10 \quad \therefore R = 50\text{Kg.wt}$

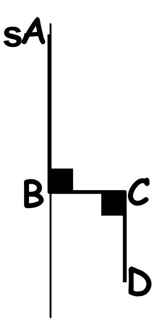
Q(7) If a driver of a car wants to ascend an plane inclined to the horizontal by an angle of measure 30° ,then the static friction coefficient between the wheels of the care and the inclined plane must be not less

① $\sqrt{3}$ ② 1

③ $\frac{\sqrt{3}}{3}$ ④ 2

$\frac{\sqrt{3}}{3}$

Q(8) In the opposite figure, ABCD is a wire of length 32 cm in which AB = 2 BC = 2 CD = 16 cm ,then the distance between the center of gravity of the wire and both $\overleftrightarrow{BC}$ and $\overleftrightarrow{BA}$ respectively is

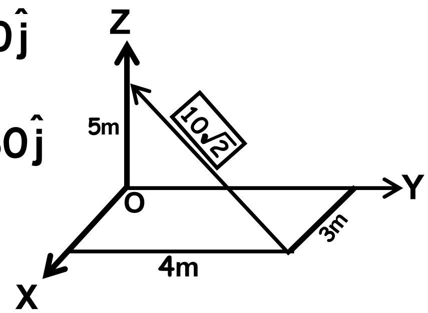


① (3,3) ② (3,5)

③ (4,4) ④ (4,8)

(3,3)

Q(9) The moment of the force $10\sqrt{2}$ about the origin point equals



① $-40\hat{i}+30\hat{j}$ ② $30\hat{i}+40\hat{j}$

③ $40\hat{i}-30\hat{j}$ ④ $30\hat{i}-30\hat{j}$

$-40\hat{i}-30\hat{j}$

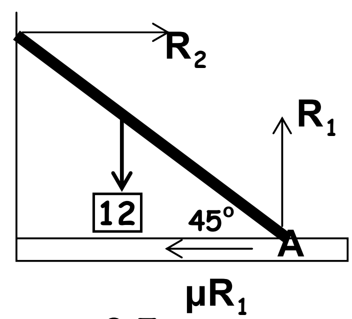
Q(10) If the normal reaction equals $2\sqrt{5}$ Newton and the coefficient of friction equals $\frac{1}{2}$ then the value of the resultant reaction equals

① 1 ② 4

③ 5 ④ 6

$R' = R\sqrt{1+\mu^2} = 2\sqrt{5}\times\sqrt{1+\left(\frac{1}{2}\right)^2} = 5$

Q(11) In the opposite figure :A ladder of weight 12 Acts at the mid point rest on rough ground and smooth wall In state of equilibrium the ladder makes angle 45° with the horizontal then the coefficient of friction equals



① 0.3 ② 0.4

③ 0.5 ④ 1

$R_1 = 12$, $R_2 = \mu R$ $\therefore R_2 = 12\mu$

$M_A = 0$ $\therefore 12\times\frac{1}{2}L\cos 45^\circ = R_2\times L\sin 45^\circ = 0$ $\therefore R_2 = 6$ $\therefore \mu = 0.5$

Q(12) If the limiting friction force =40newtons and the coefficient Of friction =0.25 then the reaction force =…..Newton

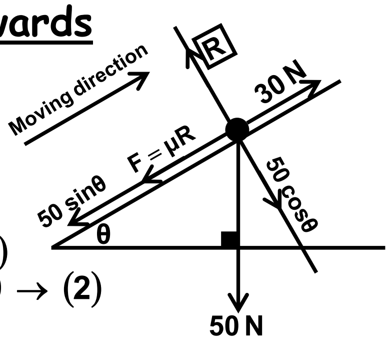
① 40.25 ② 10

③ 39.75 ④ 160

$\mu R = 40$ $\therefore \frac{1}{4}R = 40$ $\therefore R = 160$

Q(13) A body of weight 50 Newton's is placed on a rough inclined plane and is acted on by a force (P) along the line of the greatest slope upwards given that the body is about to move upwards when $P = 30$ Newtons and is about to move downwards when $P = 20$ Newtons find the coefficient of friction between the body and the plane

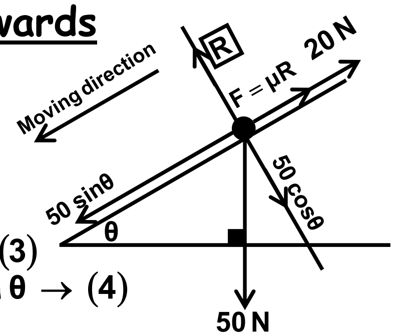
(b) Moving upwards



$R = 50\cos\theta \rightarrow (1)$

$50\sin\theta + \mu R = 30 \rightarrow (2)$

Moving downwards



$R = 50\cos\theta \rightarrow (3)$

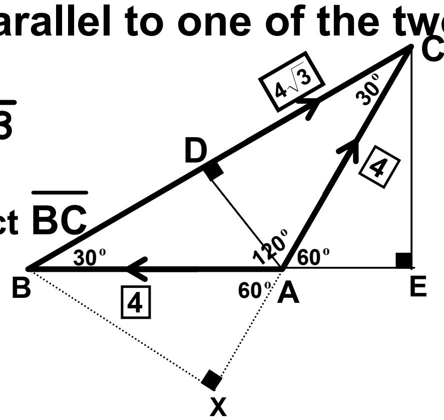
$\mu R + 20 = 50\sin\theta \rightarrow (4)$

$\mu R = 50\sin\theta - 20$

$50\sin\theta + 50\sin\theta - 20 = 30 \quad \therefore \sin\theta = \frac{1}{2} \quad \therefore \quad \theta = 30^\circ$

$R = 50\cos 30^\circ \quad \therefore \ R = 25\sqrt{3} \qquad \therefore \ \mu \times 25\sqrt{3} + 20 = 50 \times \frac{1}{2} \therefore \mu = \frac{\sqrt{3}}{15}$

Q(14) ABC is an isosceles triangle in which $m(\angle A) = 120^\circ$ and AB=10cm forces of magnitudes $4, 4, 4\sqrt{3}$ Kg.wt acting along $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$ respectively prove that the line of action of the resultant of these forces bisect BC and parallel to one of the two equal sides



$M_B = 4 \times \underbrace{5\sqrt{3}}_{BX} = 20\sqrt{3} \qquad M_C = -4 \times \underbrace{5\sqrt{3}}_{CE} = -20\sqrt{3}$

$\because M_B = -M_C$ $\therefore$ The line action of the resultant bisect $\overline{BC}$

$M_A = -4\sqrt{3} \times \underbrace{5}_{AD} = -20\sqrt{3} \qquad \therefore M_A = M_B$

$\therefore$ The line action of the resultant parallel to $\overline{AB}$

Q(15) The force $F = Li + mj$ acts at the point $A = (1, -3)$ and the algebraic measure of moment vector of F about a point $B = (-5, 0)$ equals -21 units and vanishes about a point $C = (2, -7)$ find the magnitide of F and the equation of its line of action

$[(1,-3) - (-5,0)] \times (L, M) = -21 \qquad \therefore \quad (6,-3) \times (L, m) = -21$

$\therefore \ 6m + 3L = -21 \quad \therefore 2m + L = -7$

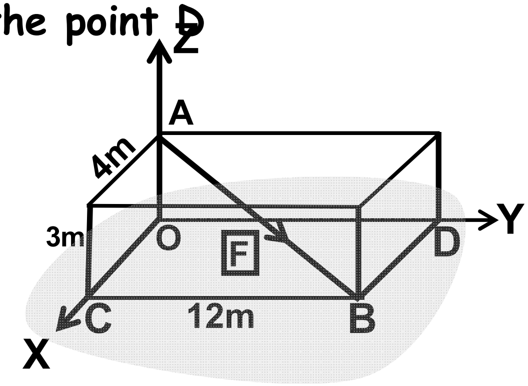
$[(1,-3) - (2,-7)] \times (L, M) = 0 \qquad \therefore \ (-1, 4) \times (L, m) = 0$

$\therefore -m - 4L = 0 \qquad \therefore L = 1, \quad m = -4$

$\therefore \ F = i - 4j \quad \therefore \quad \|F\| = \sqrt{1 + 16} = \sqrt{17} \quad \tan\theta = \frac{-4}{1}$

$\therefore$ equation is $\frac{Y+3}{X-1} = -4 \quad \therefore \quad 4X + Y - 1 = 0$

Q(16) A force of magnitude 130Newtons acts along the diagonal $\overline{AB}$ Find the moment of the force about the point D



$A \to (0,0,3)$, $B \to (4,12,0)$

$AB = B - A = (4,12,0) - (0,0,3) = (4,12,-3)$

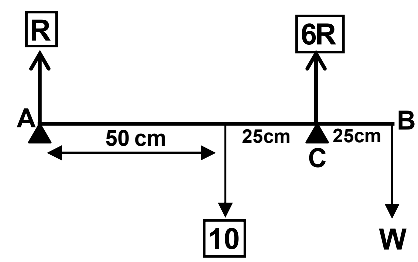
$AB = \sqrt{3^2 + 12^2 + 3^2} = 13$

$F = \frac{130}{13}(4,12,-3) = 40i + 120j - 30K$

$r = DA = A - D = (0,0,3) - (0,12,0) = (0,-12,3)$

$$M_D = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -12 & 3 \\ 40 & 120 & -30 \end{vmatrix} = 120\hat{j} + 480\hat{k}$$

Q(17) AB is a rod of length 100cm and weight 10N acting at its mid point .the rod rest in a horizontal position on two supports one of them at A and the other at a point 25cm from B find the weight that should be suspended at B so that the reaction at the support nearer to B will be six times as much as the reaction at the support at A also find the reaction at each of the two supports



$R + 6R = 10 + W \quad \therefore \quad 7R = 10 + W$

$M_B = 0$

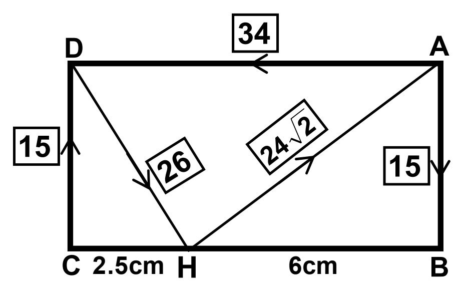
$R \times 100 + 6R \times 25 - 10 \times 50 = 0$

$250R = 500 \qquad \therefore R = 2$

$\therefore w = 4$

reaction at $A = 2$, at $C = 6 \times 2 = 12$

Q(18) ABCD is a rectangle in which AB=6cm , BC=8.5cm the point $H \in \overline{BC}$ where CH=2.5cm forces of magnitudes $15,34,15,26,24\sqrt{2}$ act along $\overrightarrow{AB}, \overrightarrow{AD}, \overrightarrow{CD}, \overrightarrow{DH}, \overrightarrow{HA}$ respectively prove that these forces are equivalent to a resultant couple 75.5gm.wt cm



$AH = 6\sqrt{2}$

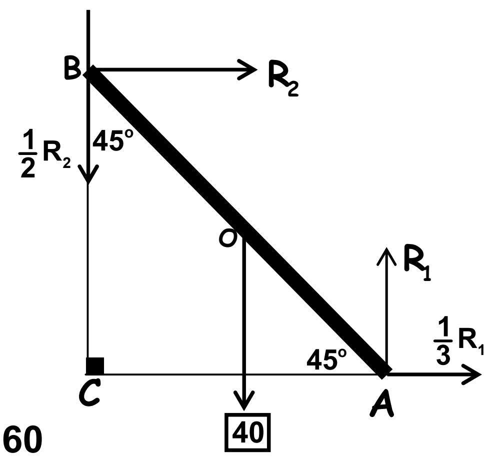
$DH = \sqrt{2.5^2 + 6^2} = 6.5 \because \frac{34}{2.5} = \frac{26}{6.5} = \frac{24\sqrt{2}}{6\sqrt{2}} = 4$

$C_1 = 8 \times \frac{1}{2} \times 8.5 \times 6 = 204$

15,15 form a couple $\therefore C_2 = -15 \times 8.5 = -127.5$

$\therefore C = 204 - 127.5 = 75.5$

Q(19) A uniform rod of weight 40 Newton's rests in a vertical plane with one end against a rough vertical wall, the coefficient of friction between them is $\frac{1}{2}$, and the other end on a rough horizontal floor, the coefficient of friction between them is $\frac{1}{3}$. The rod is inclined at 45o to the floor. Find the least horizontal force that will make the lower end of the rod about to move towards the wall.



Vertival : $R_1 = 40 + \frac{1}{2}R_2$

Horizontal : $\frac{1}{3}R_1 + R_2 = F$

$M_A = 0$

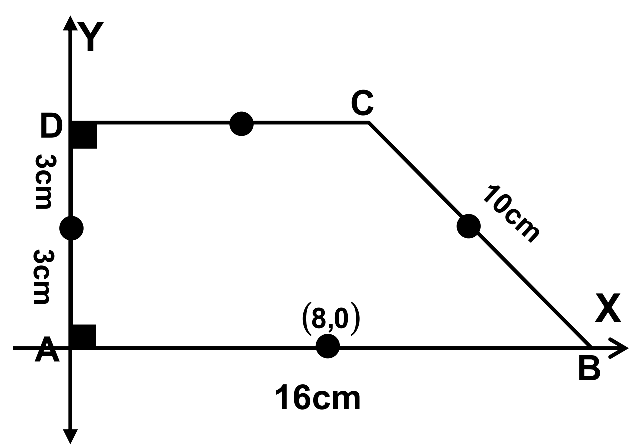
$40(L\cos 45^\circ) + \frac{1}{2}R_2(2L\cos 45^\circ)$

$= R_2(2L\sin 45^\circ)$

$40L + R_2L = R_2(2L)$

$R_2 = 40 \qquad R_1 = 40 + \frac{1}{2}(40) = 60 \qquad \therefore F = 60$

Q(20) A fine wire of uniform thickness and density and length 40 cm. If the wir is bent in the form of a trapezium ABCD in which AB = 16 cm, CD = 8 cm, DA = 6 cm, m(∠DAB) = m(∠CDA) = 90°. Find the distance between the center of gravity of the wire and the two sides AD and AB . If the wire is freely suspended from A, Find the tangent of the angle which AB makes with the vertical in the equilibrium position



$$X = \frac{10\times12+8\times4+6\times0+16\times8}{10+8+16+6} = 7$$

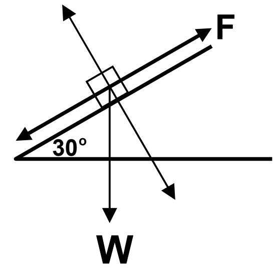
$$Y = \frac{10\times3+8\times6+3\times6+16\times0}{10+8+6+6} = 2.4$$

$$\tan\theta = \frac{12}{35}$$

Answer the following questions 20 questions

From 1to 12 choose the correct answer

Q(1) A body of weight (W) is placed on a rough inclined plane incline at an angle of 30° to the horizontal it was about to slide and if a force of magnitude (F) acts on the body the body became about to move up the plane then F=

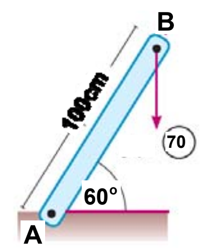


① $\frac{\sqrt{3}}{2}W$ ② $\frac{1}{\sqrt{3}}W$

③ $\frac{1}{2}W$ ④ $\underline{W}$

$$F = \mu R + W\sin\theta = \frac{\sqrt{3}}{3}W\cos 30^\circ + W\sin 30^\circ = W$$

Q(2) The opposite figure: shows a rod fixed by a hinge at A. If a vertical force of a magnitude 70 Newton acts on the end B downward, then the norm of the moment of the force about A is equal to .... Newton's.



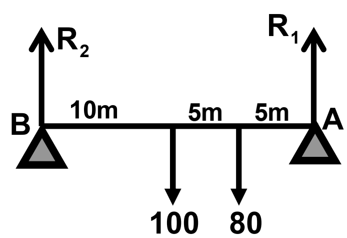
① 3500 ② 100

③ 400 ④ 7000

$$70 \times 100\cos 60^\circ = 3500$$

Q(3) In the opposite figure :

If the rod in equilibrium then the reaction at A equal



① 110Kg.wt ② 80Kg.wt

③ 20Kg.wt ④ 180Kg.wt

$$M_B = 0 \therefore -100\times 10 - 80\times 15 + R_1\times 20 = 0 \quad \therefore R_1 = 110$$

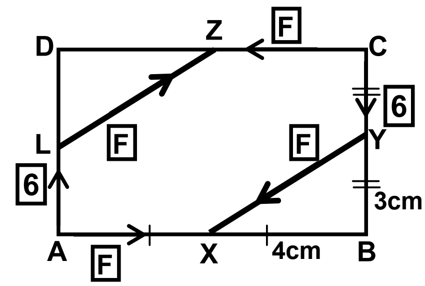
Q(4) If the two forces $\overrightarrow{F_1}$=ai-4j and $\overrightarrow{F_2}$=3i+bj form a couple then a-b=....

① 7 ② -12

③ -1 ④ -7

$$a = -3, b = 4 \therefore a - b = -7$$

Q(5) In the opposite figure :ABCD is a rectangle X , Y , Z and L are mid points $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{AD}$ given that the forces are in equilibrium then F =.....



① 20 ② 25

③ 30 ④ 40

$-6\times 8+F\times 6-F\times 4.8=0 \qquad \therefore F=40$

Q(6) A rod is attached a hinge in a vertical wall from one of its end A X and Y are two algebraic components of the resultant If X=3Kg.w , Y=4Kg.wt then the resultant reaction =……. Kg.wt

① 5 ② 7

③ 1 ④ $\sqrt{7}$

$R=\sqrt{X^2+Y^2}=\sqrt{9+16}=5$

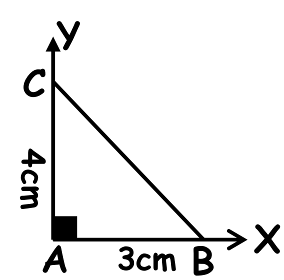
Q(7) A body of weight 12Kg.wt placed on horizontal rough ground And the static confection friction between the body and the plane is 0.25 a force of magnitude 2Kg.wt act in the body to generate a friction force (F) then

① $0<R<2$ ② $2<R<3$

③ $2\le R\le 3$ ④ $R>3$

$2\le R\le 3$

Q(8) The center of gravity of three equal masses each of 2 kg placed at the vertices of a right- angled triangle whose length of the two legs of the right angle are 3 cm and 4 cm is:



① $\left(1,\frac{4}{3}\right)$ ② $\left(\frac{7}{6},\frac{4}{3}\right)$

③ $\left(\frac{4}{3},1\right)$ ④ $(1,2)$

$c=(0,4)$, $A(0,0)$, $B=(3,0)$

$\left(\frac{0+0+3}{3},\frac{4+0+0}{3}\right)=\left(1,\frac{4}{3}\right)$

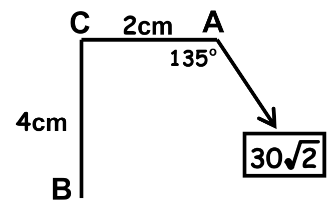
Q(9) The Force F=Ki+4j-K acts at the point A(1,2,2) if the component of the moment about Y-axis equals 7 then K=…..

① 3 ② 6

③ 7 ④ 4

$ZF_x - XF_z = 7 \quad \therefore 7 = 2K + 1 \quad \therefore K = 3$

Q(10) The moment of the force $30\sqrt{2}$ about the point B equals……..



① 180 ② -180

③ $60\sqrt{2}$ ④ $-60\sqrt{2}$

Resolve the force $30\sqrt{2}$ into two components

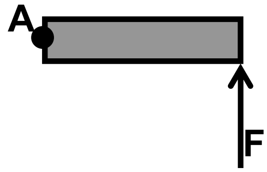
$M_B = -30\sqrt{2}\cos 45^\circ \times 2 = -180$

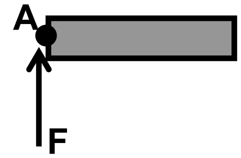
Q(11) If μ_S and μ_K are static and kinetic coefficient of friction Between two bodes then

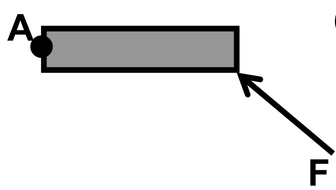
① $\mu_S = \mu_K$ ② $\mu_S < \mu_K$

③ $\mu_S > \mu_K$ ④ no relation

Q(12) In the following figure represent a door with a hinge A if a force acts on the door which of the following figures has greatest moment about A

①


②


③


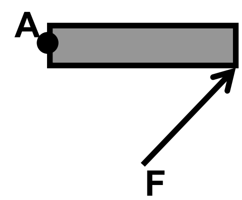
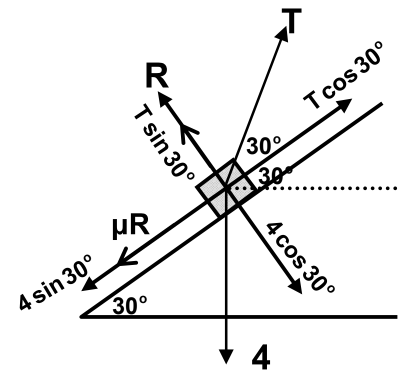
④


Figure (1)

Q(13) If a body of weight 4Kg.wt is placed on arough inclined plane with angle 30° then the body is about to slide . If the body is tied by a string making 60° with the horizontal makingthe body about to move up to the plane then Find: (1) the tension in the string (2) the force of friction



$$\mu = \tan 30^\circ = \frac{\sqrt{3}}{3}$$

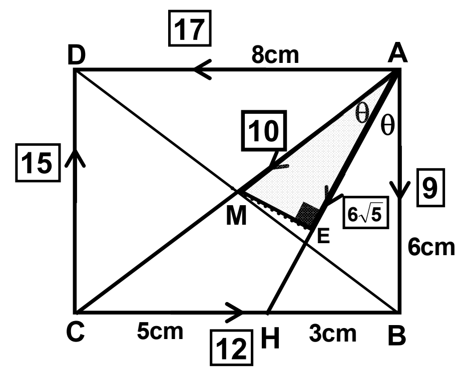
$$R + T\sin 30^\circ = 4\cos 30^\circ \quad \therefore\ R = 2\sqrt{3} - \frac{1}{2}T$$

$$T\cos 30^\circ = \mu R + 4\sin 30^\circ$$

$$\frac{\sqrt{3}}{2}T = \frac{\sqrt{3}}{3}\left(2\sqrt{3} - \frac{1}{2}T\right) + 2 \qquad \therefore\ T = 2\sqrt{3}$$

$$R = \sqrt{3} \qquad \therefore\ \mu R = 1\text{Kg.wt}$$

Q(14) ABCD is a rectangle AB=6cm , BC=8cm $H \in \overline{BC}$ such that BH=3cm forces of magnitudes 9,12,15,17,10,$6\sqrt{5}$ Newton acts along $\overrightarrow{AB}, \overrightarrow{CB}, \overrightarrow{CD}, \overrightarrow{AD}, \overrightarrow{AC}, \overrightarrow{AH}$ respectively find the algebraic sum of the resultant of thes forces about (i) A (ii) the intersection point of the diagonals



$$AC = \sqrt{6^2 + 8^2} = 10 \text{in } \Delta\ ABC \ \because \frac{AB}{AC} = \frac{6}{10} = \frac{BH}{HC} = \frac{3}{5}$$

$$\therefore m(\angle BAH) = m(\angle HAC)$$

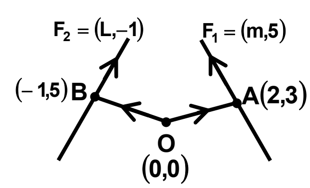
$$AH = \sqrt{6^2 + 3^2} = 3\sqrt{5}$$

$$M_A = 12 \times 6 - 15 \times 8 = -48\text{N.cm}$$

$$M_M = 17 \times 3 + 12 \times 3 - 15 \times 4 - 9 \times 4 - 6\sqrt{5}\sin(\angle\theta)$$

$$= -39\text{N.cm}$$

Q(15) Two forces $\overrightarrow{F_1} = m\hat{i} + 5\hat{j}$, $\overrightarrow{F_2} = L\hat{i} - \hat{j}$ act at the points $A = (2,3), B = (-1,5)$ respectively find the constants L , m given that the sum o moments of these forces vanishes about the origin and this sum equals 7k about the point D(-1,1)



About the origin

$$[(2,3) - (0,0)] \times (m,5) + [(-1,5) - (0,0)] \times (L,-1)$$

$$(2,3) \times (m,5) + (-1,5) \times (L,-1) = 0$$

$$\therefore 10 - 3m + 1 - 5L = 0 \qquad \therefore 3m + 5L = 11 \rightarrow (1)$$

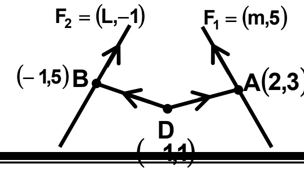
About the point D

$$[(2,3) - (-1,1)] \times (m,5) + [(-1,5) - (-1,1)] \times (L,-1)$$

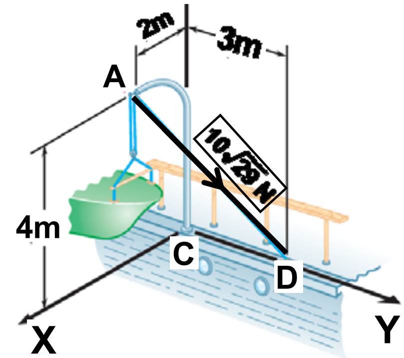
$$\therefore (3,2) \times (m,5) + (0,4) \times (L,-1) = 7$$

$$15 - 2m - 4L = 7 \qquad \therefore m + 2L = 4 \rightarrow (2)$$

$$\therefore L = 1 \ , \ m = 2$$



Q(16) In the given figure Find M_C



$A \to (2,0,4)$, $D \to (0,3,0)$

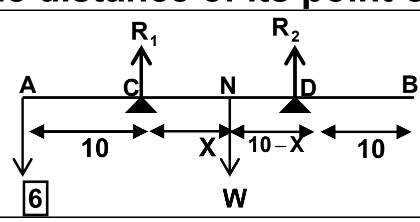
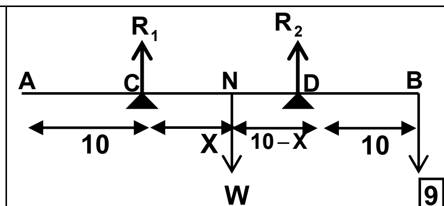
$\overrightarrow{AD} = D - A = (0,3,0) - (2,0,4) = (-2,3,-4)$

$F = \dfrac{10\sqrt{29}}{\sqrt{2^2+3^2+4^2}}(-2,3,-4) = (-20,30,-40)$

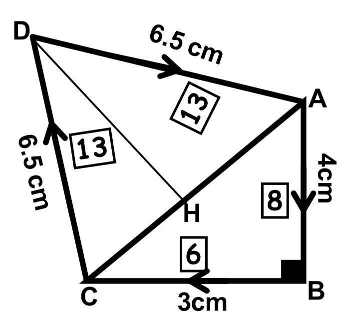
$r = (2,0,4)$

$$M_C = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 4 \\ -20 & 30 & -40 \end{vmatrix} = -120\,\hat{i} + 60\,\hat{k}$$

Q(17) AB is a <u>non –uniform</u> bar of length 30cm rests in a horizontal position on two supports at C and D such that AC=CD=DB . The bar tends to rotate around C when we hang a weight of 6 Kg.wt at A whereas it tends to rotate around D when we hang a weight of 9Kg.wt at B find the weight of the bar and the distance of its point of application from the end A

	
When 6 the rod is tend to rotate around C $\therefore R_2 = 0 , M_C = 0$ $M_C = 0 \quad \therefore \; 6\times 10 - W \times X = 0 \quad \therefore WX = 60$	When 9 the rod is tend to rotate around D $\therefore R_1 = 0 , M_D = 0$ $\therefore \quad W\times(10 - X) - 9\times 10 = 0$ $10W - WX = 90$ $\therefore 10W = 150 \quad \therefore \quad W = 15Kg \quad X = 4cm$

Q(18) ABCD is a quadrilateral in which AB=4cm , BC=3cm , CD=DA =6.5cm and $m\angle ABC = 90^\circ$ forces of magnitudes 8,6,13 and 13 Newton act along $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CD}$ and $\overrightarrow{DA}$ respectively prove that the system of forces is equivalent to a couple and calculate the norm of its moment



$AC = \sqrt{3^2+4^2} = 5$

$DH = \sqrt{6.5^2 - 2.5^2} = 6cm$

$\dfrac{4}{8} = \dfrac{3}{6} = \dfrac{6.5}{13} = \dfrac{6.5}{13} = \dfrac{1}{2}$

<u>Moment norm</u> $= 2\times 2\left(\dfrac{1}{2}\times 3\times 4 + \dfrac{1}{2}\times 5\times 6\right) = 84$

Q(19) A uniform ladder of length one meter and weight of magnitude 20Kg.wt rests with its end A on a rough horizontal floor and with its end B on a smooth vertical wall (the ladder is in equilibrium in a vertical plane perpendicular to the line of intersection of the wall with the floor) by a string whose one of its ends is attached at a point C that lies vertically below B and on the line of intersection of the wall with the floor such that the string is perpendicular to the ladder if the end A is about to slide away from the wall when the magnitude of the tension in the string is 5Kg.wt find the coefficient of friction between the ladder and the floor

$(AC)^2 = AE \times AB \quad \therefore (AC)^2 = 36 \times 100 \quad \therefore AC = 60cm$

$(BC)^2 = (AB)^2 - (AC)^2 = 100^2 - 60^o \quad \therefore BC = 80cm$

$\sin\theta = \frac{80}{100} = \frac{4}{5} \quad \cos\theta = \frac{60}{100} = \frac{3}{5}$

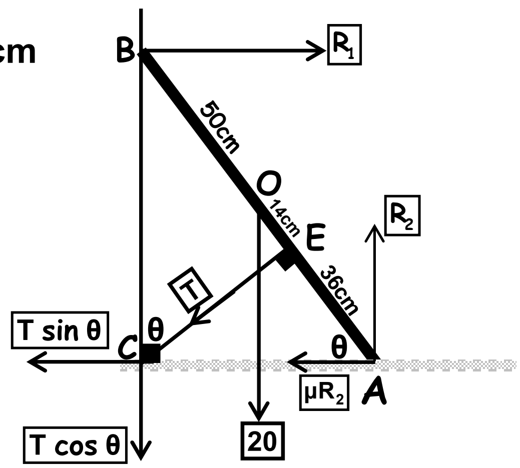
$R_1 = T\sin\theta + \mu R_2 \quad \therefore R_1 = \frac{4}{5} \times 5 + \mu R_2$

$R_2 = 20 + T\cos\theta \quad \therefore R_2 = 20 + \frac{3}{5} \times 5 \quad \therefore R_2 = 23$

$M_A = 0$

$5 \times 36 + 20 \times 50\cos\theta - R_1 \times 100\sin\theta = 0$

$180 + 600 - R_1 \times 80 \quad \therefore R_1 = \frac{39}{4} \quad \therefore \mu = \frac{1}{4}$



Q(20) fine lamina of uniform thickness and density in the form of a trapezium ABCD in which m(∠A) = m (∠D) = 90c, CD = 40 cm , AD = 60 cm and AB = 120 cm. identify the distance between the center of gravity of the lamina and both AD , AB .

Rectangle AHCD $(20,30)$

Triangle HBC

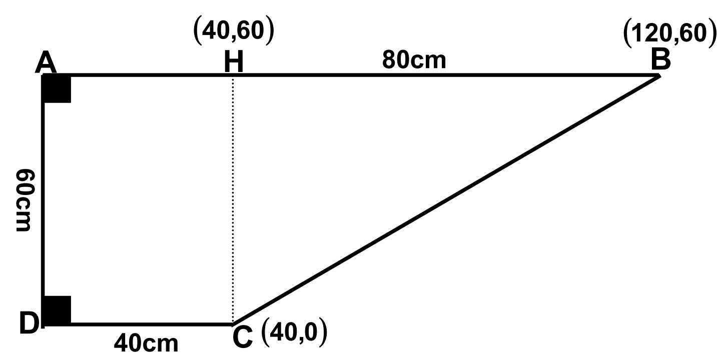
$\left(\frac{120+40+40}{3}, \frac{60+60+0}{3}\right)$

$= \left(\frac{200}{3}, 40\right)$

Area of reactangle $= 40 \times 60 = 2400$

Area of triangle $= \frac{1}{2} \times 80 \times 60 = 2400$

$\left(\frac{K \times 20 + K \times \frac{200}{3}}{K+K}, \frac{K \times 30 + K \times 40}{K+K}\right) = \left(\frac{130}{3}, 35\right)$



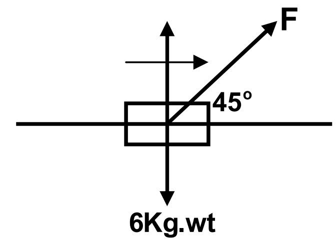
Answer the following questions 20 questions

From 1to 12 choose the correct answer

Q(1) A body of weight 6Kg.wt placed on a rough horizontal plane its coefficient of friction is $\frac{1}{2}$ a force of magnitude (F) acts on the body and incline 45° to the horizontal then the value of (F) which make the body about to move equals

① $3\sqrt{2}$ ② $\frac{3\sqrt{2}}{2}$

③ $2\sqrt{2}$ ④ $\frac{3}{\sqrt{2}}$

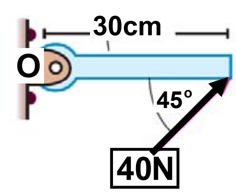


$R + F\sin 45^\circ = 6$, $F\cos 45^\circ = \mu R$ $\therefore F = 2\sqrt{2}$

Q(2) In the opposite figure: the norm of the moment of the force about the origin point (O) equals ....

① $600\sqrt{2}$ N.cm ② 1200N.cm

③ $-600\sqrt{2}$ N.cm ④ -1200N.cm

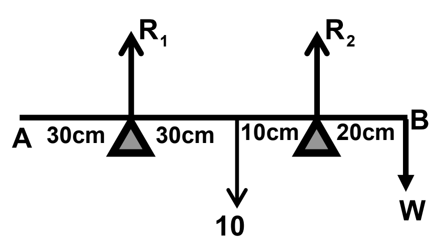


$M_O = 40 \times 30 \sin 45^\circ = 600\sqrt{2}$N.cm

Q(3) In the opposite figure : The least weight can be suspended at D makes the rod about to rotate about D is

① 20Kg.wt ② 25Kg.wt

③ 5Kg.wt ④ 2.5Kg.wt



$W \times 20 = 10 \times 10 \ \therefore W = 5$

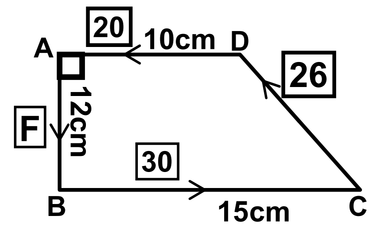
Q(4) If a driver of a car wants to ascend an plane inclined to the horizontal by an angle of measure 45° ,then the static friction coefficient between the wheels of the care and the inclined plane must be not less than ....

① 0.5 ② 1

③ 2 ④ 0

$\tan 45^\circ = 1$

Q(5) In the opposite figure :
If the system are equivalent to a couple then F=…….

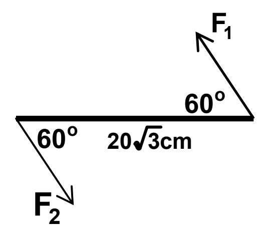


① 20N ② 24N

③ 30N ④ 40N

$\frac{F}{12} = \frac{20}{10} \quad \therefore F = 24$

Q(6) F_1=5Newtons The two forces F_1 , F_2 form a couple its moments =............

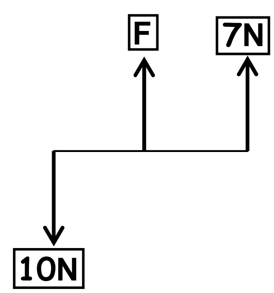


① 100 ② 120

③ 150 ④ 200

$M = 20\sqrt{3}\sin 60^{o} \times 5 = 150$

Q(7) If the system of forces is equivalent to a couple, then F = ……… Newton



① 3 ② 10

③ 7 ④ 17

$F + 7 = 10 \quad \therefore F = 3$

Q(8) The center of gravity of two physical bodies of masses 3 Newton's and 6 Newton's and the distance between them is 15 cm is at distance .... cm from the 3 Newton's body

① 5 ② 10

③ 7.5 ④ 9

$\frac{0 + 6 \times 15}{9} = 10$

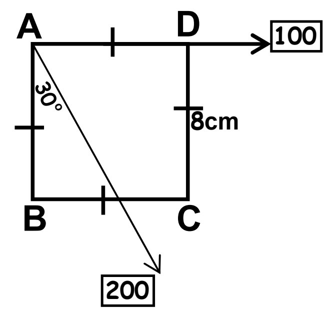
Q(9) If the two forces $F_1 = 3\hat{i} + b\hat{j} - 9\hat{K}$ and $F_2 = a\hat{i} + 5\hat{j} - c\hat{K}$ form a couple then a+b+c=.............

① -1 ② -17

③ 1 ④ 17

$-3-5-9=-17$

Q(10) ABCD is a square the sum of the moment of the forces about C Equals ….

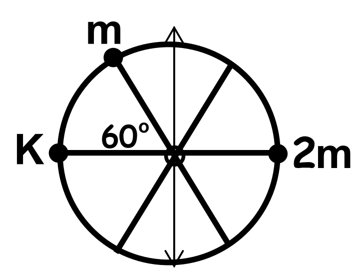


① $800(\sqrt{3}-3)$ ② $400(\sqrt{3}+3)$

③ $400(\sqrt{3}-2)$ ④ $800(\sqrt{3}-2)$

$-200\times 8+100\sqrt{3}\times 8=800(\sqrt{3}-2)$

Q(11) The following figure represents a wheel of negligible mass with radius . the wheel can spin in a vertical plane about a smooth horizontal column, three masses of magnitudes m,2m and K , and are fastened on the wheel such that the wheel is in equilibrium as in the figure, then the value K of in terms m of is …..



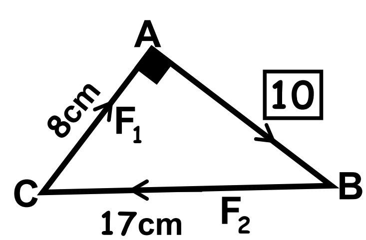
① m ② 2m

③ $\frac{1}{2}$m ④ $\frac{3}{2}$m

$2m \rightarrow (r,0)$, $m \rightarrow \left(-\frac{1}{2}r, \frac{\sqrt{3}}{2}r\right)$, $K \rightarrow (-r,0)$

$\therefore \frac{2mr - \frac{1}{2}mr - Kr}{3m+K} = 0 \therefore 2mr - \frac{1}{2}mr - Kr = 0 \therefore \frac{3}{2}m = K$

Q(12) The system is equivalent to a couple its moment equal …….



① -80 ② -200

③ -120 ④ -300

$\frac{10}{2}\times 2\times \frac{1}{2}\times 8\times 15=-80$

Q(13) A body of weight 10Kg.wt is placed on a rough inclined plane with angle 30^o then the body is tied by a string which passes over a smooth pulley at the top of the plane and carries a weight 8Kg.wt from its free second end . given that the body is about to move up the plane . then find the coefficient of friction .And also find the weight which if it replaced the weight 8Kg.wt the body is about to move downwards the plane

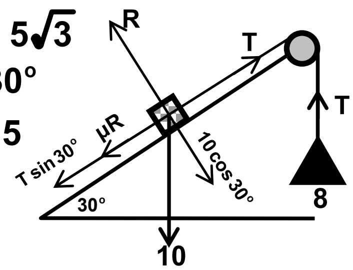
First:

$R = 10\cos 30^o = 5\sqrt{3}$

$T = \mu R + 10\sin 30^o$

$\therefore 8 = \mu \times 5\sqrt{3} + 5$

$\therefore \mu = \frac{\sqrt{3}}{5}$



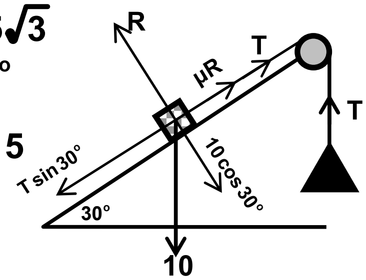
second:

$R = 10\cos 30^o = 5\sqrt{3}$

$T + \mu R = 10\sin 30^o$

$\therefore W + \frac{\sqrt{3}}{5} \times 5\sqrt{3} = 5$

$\therefore W = 2Kg.wt$



Q(14) ABCD is trapezium in which $\overline{AD} // \overline{BC}$, $m(\angle A) = 90^o$, AB=AD=10cm , BC=20cm the forces K , 5 , F , 5 , $20\sqrt{2}$ acts along $\overrightarrow{BA}, \overrightarrow{CB}, \overrightarrow{DC}, \overrightarrow{DA}, \overrightarrow{BD}$ if the algebraic sum of the moments of these forces about A vanishes ,the algebraic sum of the moments of these forces about B equals sum of the moments of these forces about D Find the values of K , F

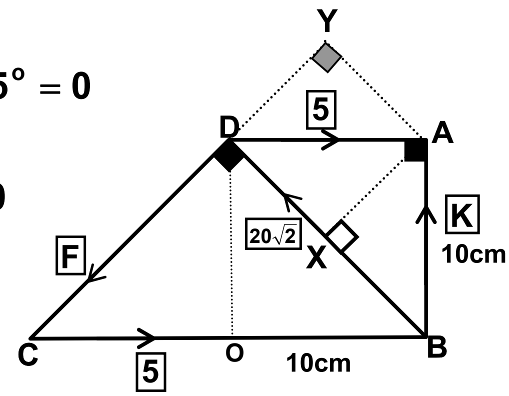
$M_A = 0 \quad \therefore -20\sqrt{2} \times 10\sin 45^o + 5 \times 10 + F \times 10\sin 45^o = 0$

$-200 + 50 + 5\sqrt{2}F = 0 \quad \therefore F = 15\sqrt{2}$

$M_B = -5 \times 10 + F \times 10\sqrt{2} = -50 + 10\sqrt{2} \times 15\sqrt{2} = 250$

$M_D = K \times 10 + 5 \times 10 = 10K + 50$

$\therefore 10K + 50 = 250 \quad \therefore 10K = 200 \quad \therefore K = 20$



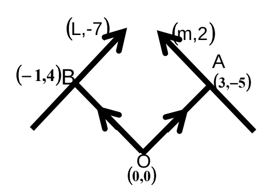
Q(15) The forces $F_1 = m\hat{i} + 2\hat{j}$, $F_2 = L\hat{i} - 7\hat{j}$ acts at the two points $A = (3,-5), B = (-1,4)$ determine the values of L , M if the sum of moments of the two forces F_1, F_2 About each of the origin and the point $D = (2,-1)$ vanishes

About the origin

$[(3,-5) - (0,0)] \times (m,2) + [(-1,4) - (0,0)] \times (L,-7) = 0$

$(3,-5) \times (m,2) + (-1,4) \times (L,-7) = 0$

$6 + 5m + 7 - 4L = 0 \quad \Rightarrow \quad 5m - 4L = -13 \rightarrow (1)$



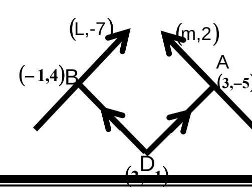
About the point D

$[(3,-5) - (2,-1)] \times (m,2) + [(-1,4) - (2,-1)] \times (L,-7) = 0$

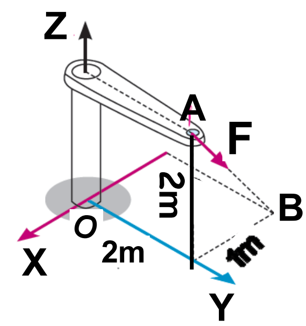
$(1,-4) \times (m,2) + (-3,5) \times (L,-7) = 0$

$4m - 5L = -23 \rightarrow (2)$

$L = 7 \quad , m = 3$



Q(16)In the opposite figure Find the moment of the force F=$14\sqrt{5}$ about the point O

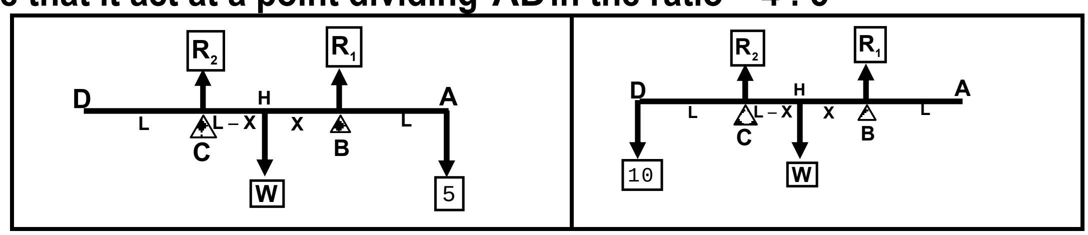


$$AB = B - A = (-1, 0 - 2)$$

$$F = \frac{14\sqrt{5}}{\sqrt{5}}\left(-1\hat{i} - 2\hat{j}\right)$$

$$M_O = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 2 \\ -14 & 0 & -28 \end{vmatrix} = -56\hat{i} - 28\hat{j} + 28\hat{k}$$

Q(17) A non –uniform rod $\overline{AD}$ rests in a horizontal position on two supports at B and C such that AB=BC=CD if a weight of magnitude 5Kg.wt is suspended from A the rod will be about to rotate about B , and if a weight of magnitude 10Kgwt is suspended from D the rod will be about to rotate about c find the weight of the rod and prove that it act at a point dividing $\overline{AD}$ in the ratio 4 : 5



$\therefore R_2 = 0 , M_B = 0$

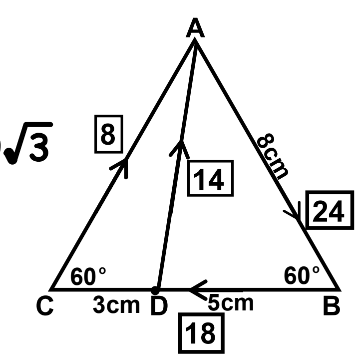
$-5 \times L + wX = 0 \quad \therefore 5L = WX \rightarrow (1)$

$\therefore R_1 = 0 , M_C = 0$

$10L = W(L - X) \quad \therefore W = 15$

$$\therefore WX = 5L \quad \therefore 15X = 5L \quad \therefore X = \frac{5L}{15} = \frac{1}{3}L \quad \therefore \frac{AH}{HD} = \frac{X + L}{2L - X} = \frac{\frac{1}{3}L + L}{2L - \frac{1}{3}L} = \frac{4}{5}$$

Q(18) ABC is an equilateral triangle its side length 8cm $D \in \overline{BC}$ where $BD = 5cm$ forces of magnitudes 24,18,14,8 acts in the directions $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{DA}, \overrightarrow{CA}$ prove that the system tends to a couple and find the magnitude of its moment



$$AD = \sqrt{8^2 + 5^2 - 2 \times 8 \times 5 \times \cos 60^\circ} = 7$$

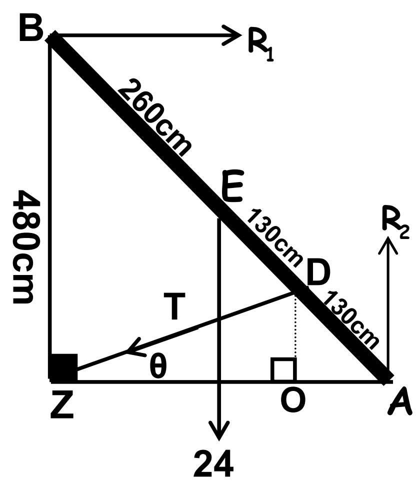
$$\frac{24}{8} = \frac{15}{5} = \frac{21}{7} = 3 \quad \text{area of } \Delta ABD = \frac{1}{2} \times 8 \times 5 \times \sin 60^\circ = 10\sqrt{3}$$

$$C_1 = 2 \times 3 \times 10\sqrt{3} = 60\sqrt{3}$$

$$\frac{8}{8} = \frac{7}{7} = \frac{3}{3} = 1, \text{area } \Delta ADC = \frac{1}{2} \times 8 \times 3 \times \sin 60^\circ = 6\sqrt{3}$$

$$C_2 = 2 \times 1 \times 6\sqrt{3} = -12\sqrt{3}$$

Q(19)AB is a uniform ladder of length 520 cm and weight 24 kg .wt rests with its end B against a smooth vertical wall and with its end A on a smooth horizontal plane . The ladder is kept from slipping by a string , one end of string fixed vertically below B and the other end fixed to one stair at a distance 130 cm from A . If the end B is at a distance 480 cm from the horizontal plane . find the reaction of the ground at its ends A , B and find the magnitude of the tension in the string



$ZA = \sqrt{520^2 - 480^2} = 200 \because \overline{OD} // \overline{BZ} \quad \therefore \frac{AD}{AB} = \frac{AO}{AZ} = \frac{OD}{BZ}$

$\frac{130}{520} = \frac{AO}{200} = \frac{OD}{480} \quad \therefore AO = 50cm , OD = 120cm$

$ZO = 150cm$

$\therefore ZD = \sqrt{150^2 + 120^2} = 30\sqrt{41}$

$\cos\theta = \frac{150}{30\sqrt{40}} = \frac{5}{\sqrt{41}} \quad , \sin\theta = \frac{4}{\sqrt{41}}$

$R_2 = 24 + T\sin\theta \quad , \quad R_1 = T\cos\theta \therefore R_2 = 24 + \frac{4}{\sqrt{41}}T \quad , R_1 = \frac{5}{\sqrt{41}}T$

$M_Z = 0$

$-24 \times 100 + R_2 \times 200 - R_1 \times 480 = 0$

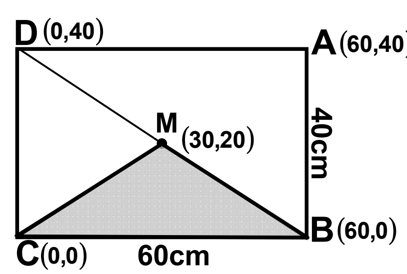
$-2400 + 200\left(24 + \frac{4}{\sqrt{41}}T\right) - 480 \times \frac{5}{\sqrt{41}}T = 0$

$-2400 + 4800 + \frac{800}{\sqrt{41}}T - \frac{2400}{\sqrt{41}}T = 0 \quad \therefore \frac{1600}{\sqrt{41}}T = 2400 \quad \therefore T = 1.5\sqrt{41}$

$R_1 = 1.5\sqrt{41} \times \frac{5}{\sqrt{41}} = 7.5 \quad , R_2 = 30$

Q(20) ABCD is a uniform fine lamina in the form of a rectangle in which AB = 40 cm, BC = 60 cm, and its diagonals intersect at M. The triangle BCM is cut off and the remaining part is freely suspended from vertex C. Identify the tangent of the angle of inclination of CB to the vertical in the equilibrium position.

Rectangle - ΔMBC

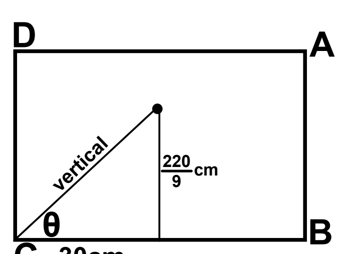


Triangle $\left(\frac{0+30+60}{3}, \frac{0+20+0}{3}\right) = \left(30, \frac{20}{3}\right)$

Mass	rect	Tri.
	4K	K
X	30	30
Y	20	$\frac{20}{3}$

$X = \frac{4K \times 30 - K \times 30}{4K - K} = 30$

$Y = \frac{4K \times 20 - K \times \frac{20}{3}}{4K - K} = \frac{220}{9}$

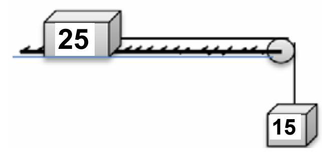


$\tan\theta = \frac{\frac{220}{9}}{30} = \frac{22}{27}$

Answer the following questions 20 questions

From 1to 12 choose the correct answer

Q(1) If the system is about to move then
The coefficent of friction equal



① $\frac{3}{5}$ ② 5/3

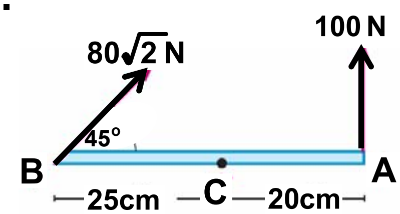
③ $\frac{1}{2}$ ④1/3

$\mu R = 15 \quad \therefore 25\mu = 15 \quad \therefore \mu = \frac{3}{5}$

Q(2) In the opposite figure: the norm of the sum of moment of the Forces about the origin point (C) equals…..

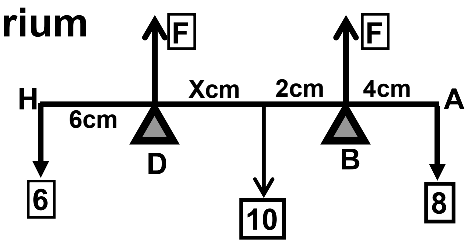
① 0 ② -4000

③ 4000 ④ 300



$100 \times 20 - 80\sqrt{2} \times 25 \times \frac{1}{\sqrt{2}} = 0$

Q(3) In the opposite figure : The rod in equilibrium
Then F=……… , X=……….



① 12Newton ,8cm ② 12Newton ,2cm

③ 12Newton ,6cm ④ 12Newton ,4cm

$F + F = 6 + 10 + 8 \quad \therefore 2F = 24 \quad \therefore F = 12$

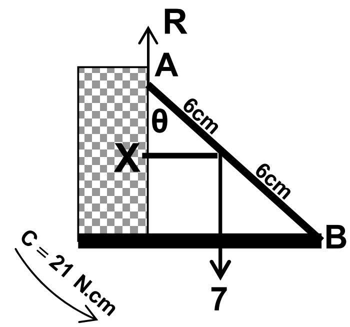
$M_B = 0 \quad \therefore -8 \times 4 + 10 \times \times 2 - 12 \times (X + 2) + 6(8 + X) = 0 \therefore X = 2cm$

Q(4) A mass of 5 kg acts at point (2 , -1) and a mass of 7 kg acts at point (1 , 2) then the center of gravity of the two masses acts at point

① (17,9) ② $(\frac{17}{12}, \frac{3}{4})$

③ (19,13) ④ $(\frac{19}{12}, \frac{1}{4})$

Q(5) In the opposite figure : $\overline{AB}$ is a uniform rod is in equilibrium under the action of a couple 21N.cm then the reaction of the henge = and $m(\angle\theta) = ...$

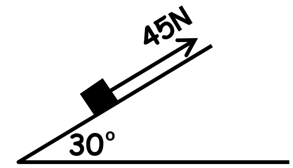


① 45° ② 30°

③ 60° ④ 75°

$R = 7$, $7 \times MX = 21 \therefore MX = 3 \quad \therefore \sin\theta = \frac{3}{6} = \frac{1}{2} \therefore \theta = 30^\circ$

Q(6) A body of weight 60N placed on inclined rough plane
The force 45N makes the body about to move up
Then the coefficient of friction =....

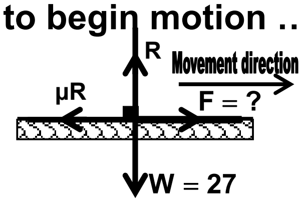


① $\frac{1}{\sqrt{3}}$ ② $\frac{1}{\sqrt{2}}$

③ $\frac{\sqrt{3}}{6}$ ④ $\frac{1}{2}$

$60 \times \frac{1}{2} + \mu \times 60 \times \frac{\sqrt{3}}{2} = 45 \quad \therefore \mu = \frac{\sqrt{3}}{6}$

Q(7) A body of weight 27Kg.wt is placed on a horizontal rough plane , the coeffiecent of friction between the body and the plane is $\frac{1}{3}$ then the magnitude of the force parallel to the plane which makes the body about to begin motion ......



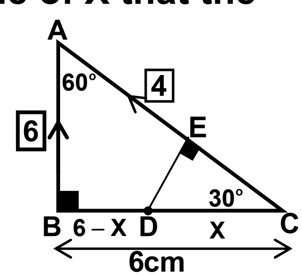
① 9Kg.wt ② 27Kg.wt

③ 18Kg.wt ④ 81Kg.wt

$R = W \quad \therefore R = 27 \rightarrow (1)$

$F = \mu R \quad \therefore F = \frac{1}{3} \times 27 = 9Kg.wt$

Q(8) ABC is a right angled triangle at B where BC=6cm , $m(\angle A) = 60^\circ$ two forces 6,4 newtons act along $\overrightarrow{BA}, \overrightarrow{CA}$ respectively the value of X that the algebraic sum of moments about D equals 0



① 4.5cm ② 4

③ 3 ④ 5

$ED = \frac{1}{2}(X) M_D = 0 \quad \therefore -6(6 - X) + 4 \times \left(\frac{1}{2}X\right) = 0$

$-36 + 6X + 2X = 0 \quad \therefore 8X = 36 \quad \therefore X = 4.5cm$

Q(9)) If the ratio between the resultant reaction force and the normal reaction force Equals $2\sqrt{3} : 3$ then the angle of friction is equal

① 30° ② 60°

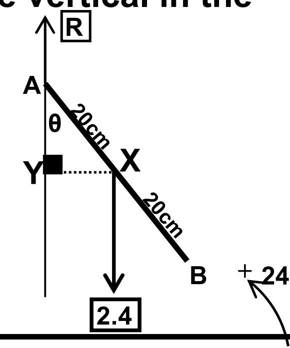
③ 90° ④ 45°

$$\cos\theta = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2} \quad \therefore \theta = 30^\circ$$

Q(10) A rod of length 40cm and weight 2.4 Kg.wt acting at its midpoint can rotate easily in a vertical plane about a fixed hinge at its end . A couple of magnitude 24Kg.wt.cm acts on the rod then the inclination of the rod to the vertical in the position of equilibrium

① 30° ② 60°

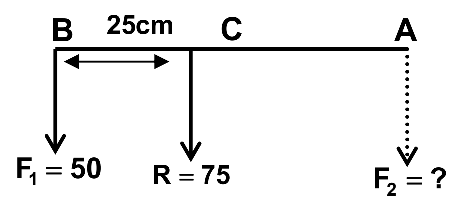
③ 90° ④ 45°



$2.4 \times 20 \sin\theta = 24$

$48 \sin\theta = 24 \quad \therefore \quad \sin\theta = \frac{1}{2} \quad \therefore \theta = 30^\circ$

Q(11) In the opposite figure :



① $F_2 = 25$, $AC = 50cm$

③ $F_2 = 25$, $AC = 25cm$ ④ $F_2 = 125$, $AC = 25cm$

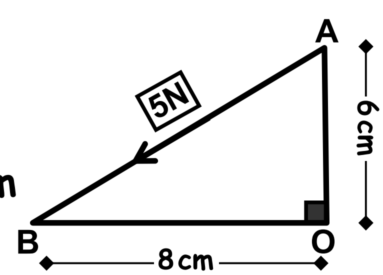
$R = F_1 + F_2 \quad \therefore F_2 = R - F_2 = 75 - 50 = 25$

$50 \times 25 = 25 \times AC \quad \therefore AC = 50cm$

Q(12) In the opposite figure the moment of the force f Newton About O equals

① 30N.cm ② 24N.cm

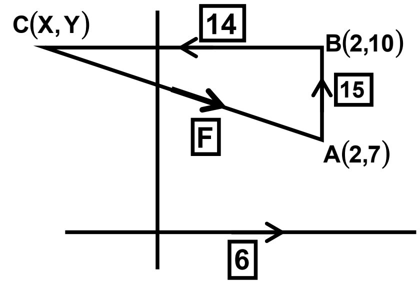
③ 40N.cm ④ 48N.cm



$$5 \times \frac{6 \times 8}{10} = 24$$

Q(13) ΔABC is right –angled triangle at B where A(2,7) , B(2,10) , C(X,Y) the forces of magnitude 15,14,F Kg.wt act along $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CA}$ respectively .if the resultant force equals 6Kg.wt and acts of the positive direction of X-axis find
(1) the coordinates of the point C (2) the magnitde of F

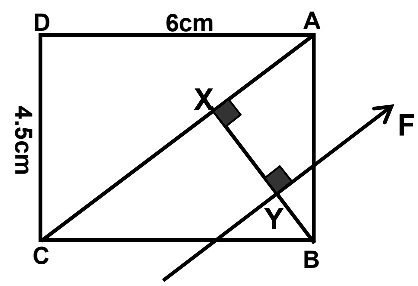
$\because$ ABCD is right angle triangle $\therefore Y = 10$
The algebraic sum of the moment about C
=the moment of the resultant about C
$15(2 - X) = 6 \times 10 \quad \therefore X = -2 \quad \therefore C = (-2,10)$
The algebraic sum of the moment about B
=the moment of the resultant about B
$\therefore F \times \frac{4 \times 3}{6} = 6 \times 10 \quad \therefore F = 25\text{Kg.wt}$



Q(14) ABCD is a rectangle , AB=4.5cm ,BC=6cm a force F acts in the plane of the triangle ABC if the algebraic sum of the moments of this forces about B equals algebraic sum of the moments of this forces about C equals 720 and about B equals -240Find the resultant of these forces

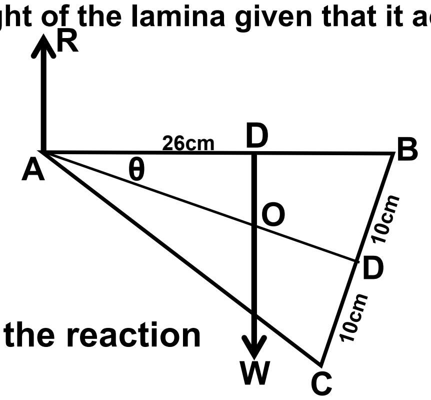
Line action of the resultant //AC

$AC = \sqrt{4.5^2 + 6^2} = 7.5 \quad , BX = \frac{4.5 \times 6}{7.5} = 3.6$
$M_A = F \times XY \quad \therefore F \times XY = 720 \rightarrow (1)$
$M_B = F \times BY \quad \therefore F \times BY = 240 \rightarrow (2)$
$\therefore \frac{XY}{BY} = \frac{720}{240} = 3 \quad \therefore BY = \frac{3.6}{4} = 0.9\text{cm}$
$\therefore 0.9 \times F = 240 \quad \therefore F = \frac{800}{3}$



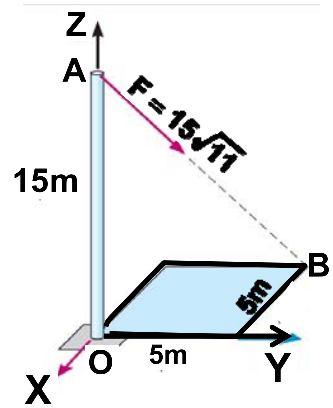
Q(15) ABC is a thin lamina in the form of an isosceles triangle in which AB=AB=26cm , BC=20cm The lamina is suspended by a thin horizontal pin from a small hole near the vertex A such that its plane is vertical . Couple the magnitude of its moment equals 720Newtons .cm act on the lamina in a direction perpendicular to its plane such that it is in equilibrium in a position in which AB is horizontal find the magnitude of the weight of the lamina given that it acts at the point of intersection of its medians

$AD = \sqrt{26^2 - 10^2} = 24\text{cm}$
$,AO = \frac{2}{3}AD \quad \therefore AO = 16\text{cm}$
$\cos\theta = \frac{AD}{AB} = \frac{24}{26} = \frac{12}{13} = \frac{AD}{AO} = \frac{AD}{16}$
$\therefore AD = 16 \times \frac{12}{13}$



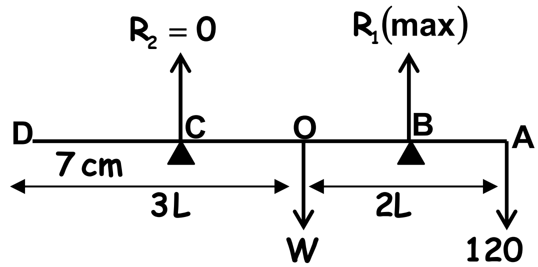
$\because$The lamina is in equilibrium $\therefore$ the weight and the reaction of the pin form a couple
$\therefore R = W$
$\therefore -W \times AD = -720 \quad \therefore W = 48.75$

Q(16) In the opposite figure Find the momeoment of the force about (O)



$$A = (0,015)\ ,\ B = (-5,5,0)$$
$$AB = B - A = (-5,5,0) - (0,015) = (-5,5,-15)$$
$$= (-1,1,-3)$$
$$F = \frac{15\sqrt{11}}{5\sqrt{11}}(-1,1,-3) = -3\hat{i} + 3\hat{j} - 6\hat{K}$$
$$M_O = \begin{vmatrix} \hat{i} & \hat{j} & \hat{K} \\ 0 & 0 & 15 \\ -3 & 3 & -6 \end{vmatrix} = 45\hat{i} - 45\hat{j}$$

Q(17) A no-uniform rod ABCD rests in equilibrium horizontally on two smooth supports at B and C such that AB=6cm , CD=7cm and the point of application of the rod weight divide it in the ratio 2:3 from the end A ,it was found that if a weight of magnitude 120gm.wt is suspended from the end A or if a weight of magnitude 180gm.wt is suspended from the end D then the rod is either cases is about to rotate find the rod weight as well as the distance between both supports



in case of 120

$M_B = 0$

$W(2L-6) = 120 \times 6 = 720 \rightarrow (1)$

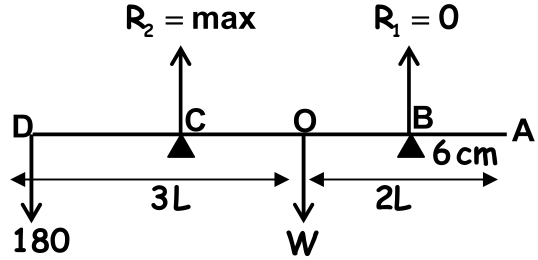
in case of 120

$M_C = 0$

$W(3L-7) = 180 \times 7 = 1260 \rightarrow (2)$

by dividing 1,2 $\therefore \frac{3L-7}{2L-6} = \frac{1260}{720} = \frac{7}{4}$

$L = 7 \qquad \therefore\ BC = 22cm \qquad \therefore\ W = 90$



Q(18) ABCD is an isosceles trapezium in which AD=BC=10cm , AB=21cm , CD=9cm forces of magnitudes 18,20,68,20 ,42 kg .wt along $\overrightarrow{CD}$, $\overrightarrow{DA}$, $\overrightarrow{AC}$, $\overrightarrow{CB}$, $\overrightarrow{BA}$ respectively prove that the system is equivalent to a couple then find the norm of its moment

$CX = \sqrt{10^2 - 6^2} = 8 \quad , \quad AC = \sqrt{15^2 + 8^2} = 17$

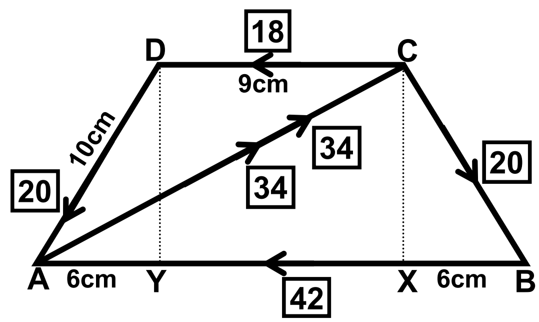
In Δ ACB : $\frac{20}{10} = \frac{34}{17} = \frac{42}{21} = 2$

20,42,34 in cyclic order they form a couple

$M_1 = 2 \times 2 \times \left(\frac{1}{2} \times 21 \times 8\right) = -336$

In Δ ACD : $\frac{18}{9} = \frac{20}{10} = \frac{34}{17} = 2$

18,20,34 in cyclic order they form a couple



Q(19) A fine lamina of uniform thickness and density in the form of an isosceles triangle ABC where AB = AC = 26 cm, B C = 20 cm. $\overline{AD}$ is drawn to intersect BC at D , If H is the midpoint of AD and the triangle EBC is cut off, find the distance between of the center of gravity of the remaining part and point E.

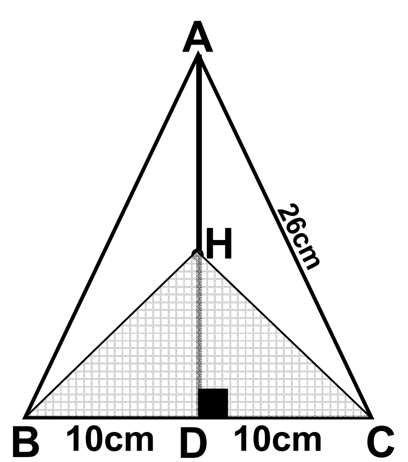
$AD = \sqrt{26^2 - 10^2} = 24cm$

Area Δ ABC $= \frac{1}{2} \times 20 \times 24 = 240cm^2$

Area Δ HBC $= \frac{1}{2} \times 20 \times 12 = 120cm^2$

mass of ABC $= 2K$, mass of HBC $= K$

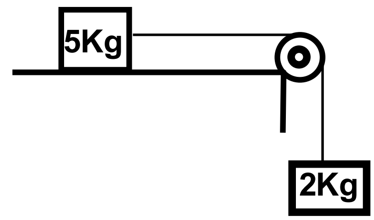
$Y = \left(\frac{2K \times 8 - K \times 4}{2K - K}\right) = 12cm$



Answer the following questions 20 questions

From 1to 12 choose the correct answer

Q(1) In the opposite figure : If the system about to move then $\mu =$



① 1

② $\frac{2}{5}$

③ 10

④ $\frac{5}{2}$

$2 = 5\mu \qquad \therefore \mu = \frac{2}{5}$

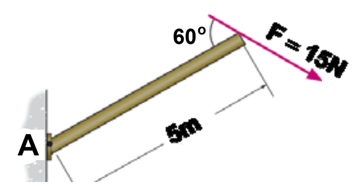
Q(2) The moment of the force about A equals........................

① $-37.5\sqrt{3}$

② $37.5\sqrt{3}$

③ 75

④ $75\sqrt{3}$



$-15 \times 5 \sin 60^\circ = -37.5\sqrt{3}$

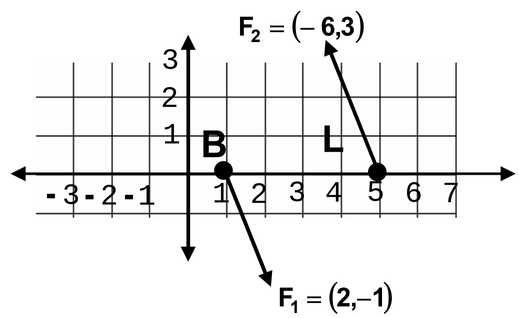
Q(3) If the two forces $F_1 = (2,m)$, $F_2 = (-6,3)$ are parallel find m and if the two forces acts at the two points (1,0) , (5,0) respectively then the coordinate of the intersection of their resultant with the X axis

① (7,0)

② (9,0)

③ (6,0)

④ (0,0)



$\because F_1 \,//\, F_2 \quad \therefore \frac{m}{2} = \frac{3}{-6} \quad \therefore a = -1$

$\|F_1\| = \sqrt{2^2 + (-1)^2} = \sqrt{5} \qquad \|F_2\| = \sqrt{3^2 + (-6)^2} = 3\sqrt{5}$

$\because M_D = 0 \quad \therefore \sqrt{5} \times L = 3\sqrt{5}(L - 4)$

$\therefore L = 3(L - 4) \quad \therefore L = 3L - 12 \quad \therefore 2L = 12 \quad \therefore L = 6$

$\therefore$ Point D is (7,0)

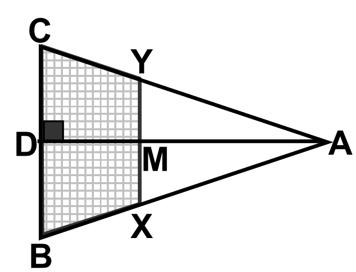
Q(4) Thin lamina in the form of isosceles triangle AB=AC AD=45cm $\overline{XY} \,//\, \overline{CB}$ the centre of gravity of BXYC Is distance …..cm from D

① 15

② 10

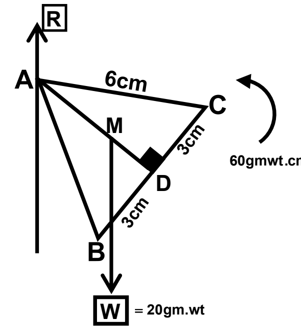
③ 12

④ 7



$\frac{9K \times 15 - 4K \times 25}{9K - 4K} = 7$

Q(5) In the opposite figure : ABC if a fine lamina shaped in equilateral triangle Of side length 6cm and weight 20gm.wt is equilibrium Under the action of a couple of magnitude =60gm.wt .cm then $\overline{AM}$ inclined to the vertical with angle



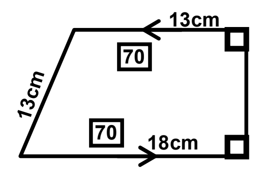
① ②

③ ④

$AD = 3\sqrt{3}$, $AM = \frac{2}{3} \times 3\sqrt{3} = 2\sqrt{3}$

$60 = 20 \times MX \quad \therefore MX = 3 \quad \therefore \sin\theta = \frac{3}{2\sqrt{3}} \quad \therefore \theta = 60^\circ$

Q(6) The moment of the couple formed By the two forces 70 ,70 equals ....

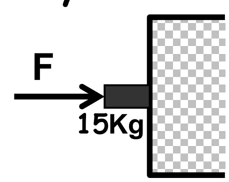


① 70 ② 350

③ 910 ④ 840

$70 \times 12 = 840$

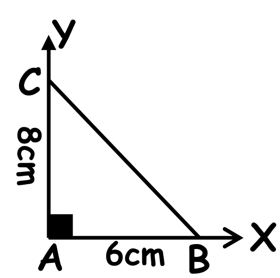
Q(7) The magnitude of the lest horizontal force needed to Equilibrate a body of weight 15Kg.wt on a rough vertical wall If the coefficient of friction between the wall and the body is 0.2



① 75Kg.wt ② 15 Kg.wt

③ 45 Kg.wt ④ 7.5 Kg.wt

Q(8) The center of gravity of the shaded lamina in the opposite figure is:



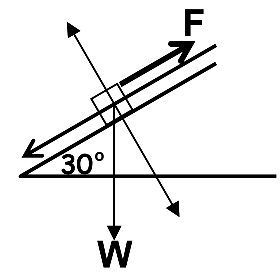
① (3,4) ② (4,3)

③ $\left(2, \frac{8}{3}\right)$ ④ (8,6)

Q(9) In the given figure the coefficient of friction equal $\frac{1}{\sqrt{3}}$ and the body about to move up then F =

① $\frac{\sqrt{3}}{2}W$ ② $\frac{1}{\sqrt{3}}W$

③ $\frac{1}{2}W$ ④ W

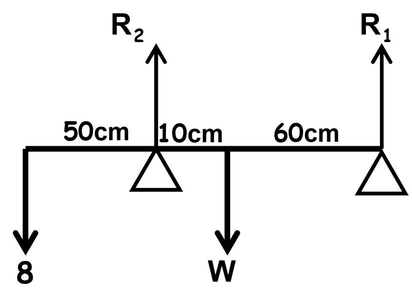


$$F = \mu R + W\sin 30^\circ = \frac{1}{\sqrt{3}} \times W \times \frac{\sqrt{3}}{2} + \frac{1}{2}W = W$$

Q(10) In the given figure If the maximum weight which hanged from B and the rod still equilibrium is 8N then the weight of the rod =

① 16 ② 40

③ 8 ④ 20

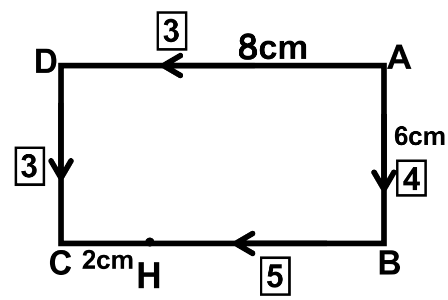


$$W \times 10 = 8 \times 50 \quad \therefore W = 40$$

Q(11) Sum of moment of the forces about H equals

① 12 ② 20

③ 0 ④ 13

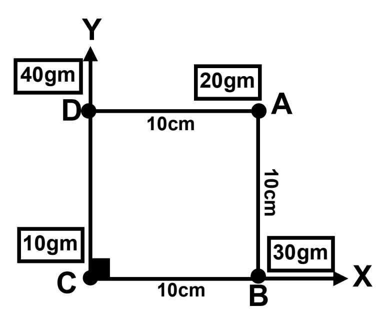


$$M_H = 3 \times 2 + 3 \times 6 - 4 \times 6 = 0$$

Q(12) Center of gravity of the system is

① (5,6) ② (5,5)

③ (6,6) ④ (2,4)



$$X = \frac{10 \times 0 + 30 \times 10 + 20 \times 10 + 40 \times 0}{10 + 30 + 20 + 40} = 5$$

$$Y = \frac{10 \times 0 + 30 \times 0 + 20 \times 10 + 40 \times 10}{10 + 30 + 20 + 40} = 6$$

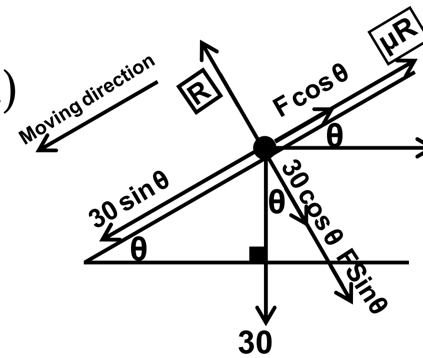
Q(13) A body of weight 30 Newton's is placed on a rough plane .inclined to the horizontal with an angle whose tangent is $\frac{12}{5}$ and the coefficient of friction between the body and the plane is $\frac{2}{3}$ find the magnitude of the horizontal force acts on the body and lies in the vertical plane containing the line of the greatest slope of the plane to prevent sliding the body

$$\mu R + F\cos\theta = 30\sin\theta \therefore \frac{2}{3}R + \frac{5}{13}F = 30\times\frac{12}{13} \rightarrow (1)$$

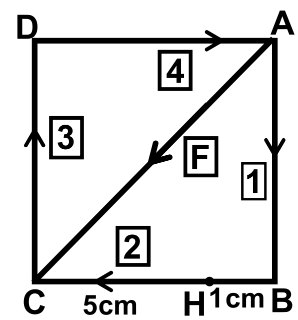
$$R = F\sin\theta + 30\cos\theta \therefore R = \frac{12}{13}F + 30\times\frac{5}{13} \rightarrow (2)$$

$$\frac{2}{3}\left(\frac{12}{13}F + 30\times\frac{5}{13}\right) + \frac{5}{13}F = 30\times\frac{12}{13}$$

$$\frac{24}{39}F + \frac{100}{13} + \frac{5}{13}F = \frac{360}{13} \qquad \therefore \quad F = 20$$



Q(14) ABCD is a square of side length 6cm $H \in \overline{BC}$ such that BH=1cm forces of magnitudes 1,2,3,4and F acts along $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CD}, \overrightarrow{DA}$ and $\overrightarrow{AC}$ if the line action of these forces passing through the point H find (F)



$$M_H = 0$$

$$-1\times1 + 3\times5 - -4\times6 + F\times5\times\frac{\sqrt{2}}{2} = 0$$

$$F = 2\sqrt{2}$$

Q(15) The force F acts in the plane of $\triangle$ ABC where A(3,2) B(1,-4) , C(-1,0) If $M_A = M_B = 60$ and $M_C = -60$ find F

$\because M_A = M_B \therefore \rightarrow$ line of action of the force // to $\overline{AB}$

$\because M_A = -M_C \therefore \rightarrow$ line of action of the force bisect $\overline{AC}$

$\because M_B = -M_C \therefore \rightarrow$ line of action of the force bisect $\overline{BC}$

$$X = \left(\frac{3+-1}{2}, \frac{0+2}{2}\right) = (1,1)$$

$$Y = \left(\frac{1+-1}{2}, \frac{-4+0}{2}\right) = (0,2)$$

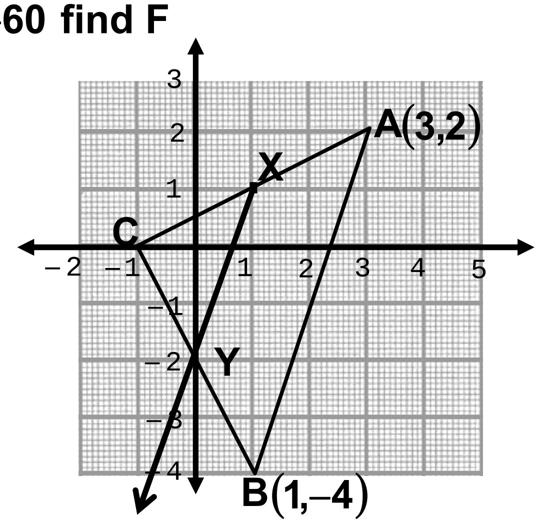
$$XY = (0-1, -2-1) = (-1,3) \qquad \therefore F = K(-1,-3)$$

$$M_A = [(1,1)-(3,2)]\times(-K,-3K) = 60$$

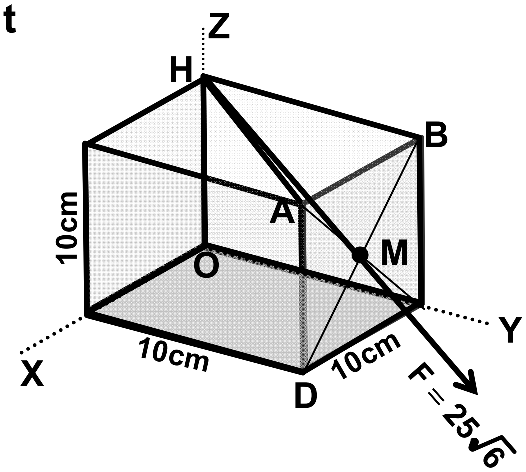
$$6K - K = 60 \qquad \therefore K = 12$$

$$F = K(-1,-3) = 12(-1,-3) = (-12,-36)$$

$$\|F\| = \sqrt{(-12)^2 + (-36)^2} = 12\sqrt{10}$$



Q(16) In the given figure find the moment Of the force about (O)



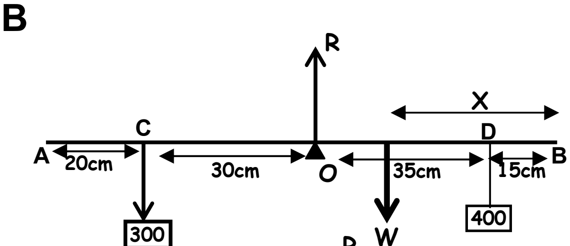
$HM = (5,10,-5)$

$F = \frac{25\sqrt{6}}{\sqrt{25+25+100}}(5,10,-5)$

$= 25\hat{i} + 50\hat{j} - 25\hat{K}$

$$M_O = \begin{vmatrix} \hat{i} & \hat{j} & \hat{K} \\ 25 & 50 & -25 \\ 0 & 0 & 10 \end{vmatrix} = 500i - 250j$$

Q(17) AB is a non uniform rod of length 100cm resting in a horizontal position by supporting it on a peg below its mid point and suspending a weight of magnitude 300gm.wt from a point C on the rod where AC=20cm and a weight of magnitude 400gm.wt from a point D on the rod where BD=15cm if the weight at D is increased to 880gm.wt the rod will be equilibrium by supporting it on a peg at 40cm distant from B find magnitude of the rod weight and its position from B

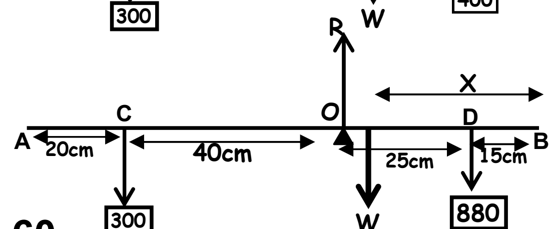


$M_O = 0$

$-400 \times 35 - W(50 - X) + 300 \times 30 = 0$

$-W(50 - X) - 5000 = 0$

$W(X - 50) = 5000$

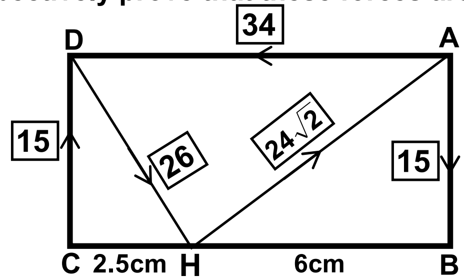


$-880 \times 25 - W(40 - X) + 300 \times 40 = 0$

$-W(40 - X) = 10000$

$\frac{X - 40}{X - 50} = 2 \quad \therefore 2X - 100 = X - 40 \quad \therefore X = 60$

Q(18) ABCD is a rectangle in which AB=6cm , BC=8.5cm the point $H \in \overline{BC}$ where CH=2.5cm forces of magnitudes 15,34,15,26,$24\sqrt{2}$ act along $\overrightarrow{AB}, \overrightarrow{AD}, \overrightarrow{CD}, \overrightarrow{DH}, \overrightarrow{HA}$ respectively prove that these forces are equivalent to a resultant couple 75.5gm.wt cm



$AH = 6\sqrt{2} \qquad DH = \sqrt{2.5^2 + 6^2} = 6.5$

$\because \frac{34}{2.5} = \frac{26}{6.5} = \frac{24\sqrt{2}}{6\sqrt{2}} = 4$

The set of forces is equivalent to a couple

Its moment

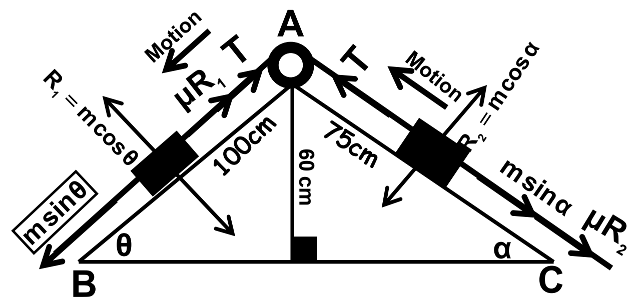
$C_1 = 8 \times \frac{1}{2} \times 8.5 \times 6 = 204$

Q(19) Two inclined planes from the same material their common height 60cm and their lengths 75cm , 100cm two bodies of same mass placed each on a plane and they connected together by a string passing through a pulley fixed on the top of the two planes if the system is about to move find the friction coefficient

$\because AC > AB \quad \therefore \theta > \alpha$

$\because m\sin\alpha > m\sin\theta$

The direction of motion as shown



$\sin\theta = \frac{60}{75} = \frac{4}{5}$

$\cos\theta = \frac{45}{75} = \frac{3}{5}$

190ddd7575

$\overline{\text{Along}AB}$

$m\sin\theta = \mu R_1 + T$

$m\sin\theta = \mu(m\cos\theta) + T$

$\frac{4}{5}m = \frac{3}{5}\mu m + T \rightarrow (1)$

$\overline{\text{Along}AC}$

$T = m\sin\alpha + \mu R_2$

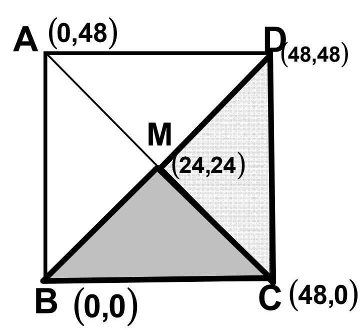
$T = \frac{3}{5}m + \mu(m\cos\alpha)$

$T = \frac{3}{5}m + \frac{4}{5}\mu m \rightarrow (2)$

$\frac{4}{5}m = \frac{3}{5}\mu m + \frac{3}{5}m + \frac{4}{5}\mu m \qquad \therefore \frac{4}{5} = \frac{3}{5}\mu + \frac{3}{5} + \frac{4}{5}\mu$

$\therefore \frac{1}{5} = \frac{7}{5}\mu \qquad \therefore \mu = \frac{1}{7}$

Q(20) A fine lamina of uniform thickness and density in the form of a square ABCD of side length 48 cm and M is the intersection point of its diagonals. The triangle CMD is cut off, then stuck on the triangle CMB such that MD is coincident to MB. Find the distance between the center of gravity of the lamina and both BA and BC.



$ABCD \rightarrow= (24,24)$

$DMC \rightarrow \left(\frac{48+24+48}{3}, \frac{48+24+0}{3}\right) = (40,24)$

$MBC \rightarrow \left(\frac{0+24+48}{3}, \frac{0+24+0}{3}\right) = (24,8)$

Mass	ABC	MBC	MCD
mass	4K	K	K
X	24	24	40
Y	24	8	24

$X = \left(\frac{4K\times24 + K\times24 - K\times40}{4K+K-K}\right) = 20 \quad , \quad Y = \left(\frac{4K\times24 + K\times8 - K\times24}{4K+K-K}\right) = 20$

Answer the following questions 20 questions

From 1to 12 choose the correct answer

Q(1) If θ is the mesure of the angle between the fianl limiting forcfe and the resultant reaction then the coefficent of the static friction is equal to

① tanθ ② cosθ

③ sinθ ④ cotθ

cotθ

Q(2) If the two forces F_1=ai+3j , F_2=-2i-bj form a couple then a + b=

① -1 ② 1

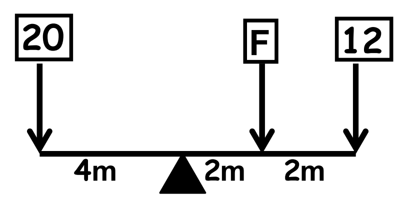
③ 5 ④ 6

$a = 2 \ , b = 3 \quad \therefore a + b = 5$

Q(3) The opposite figure represent a rod in equilibrium then F=.....

① 28N ② 16N

③ 2Newtons ④ 4N

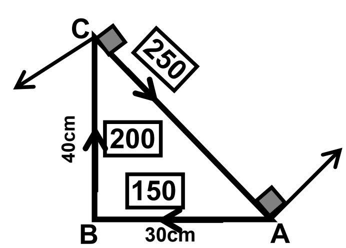


$-2F - 4 \times 12 + 20 \times 4 = 0 \quad \therefore F = 16$

Q(4) two forces of equal magnitude acting at A , C perpendicular to AC such that they are in equilibrium with the three given forces

① 120N ② 90N

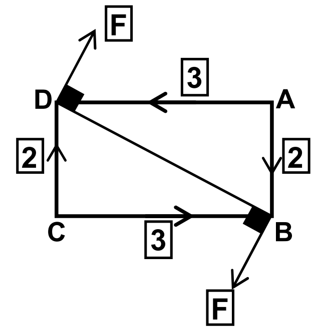
③ 100 ④ 200



$= 10 \times \frac{1}{2} \times 30 \times 40 = -600 gm.wt$

$F \times AC = 600 \Rightarrow F = \frac{6000}{50} = 120$

Q(5) In the opposite figure ABCD is a rectangle AB=3cm , BC=4cm
The system form a couple its moment =35N.cm
in direction of ABCD then F=……..N



① 5 ② 6

③ 7 ④ 8

$3 \times 3 - 2 \times 4 - F \times 5 = -34$

$\therefore F = 7$

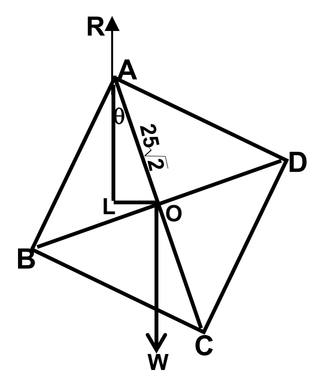
Q(6) If the two forces $F_1 = 5\hat{i} + a\hat{j} + 3\hat{K}$ and $F_2 = b\hat{i} - 9\hat{j} + c\hat{K}$ form a couple
a+b+c

① -1 ② 0

③ 1 ④ 7

$-5 + 9 - 3 = 1$

Q(7) ABCD is a thin uniform lamina in the form of a square of side length 50cm and of weight 300gm.wt acting at the centre of the square the lamina is suspended from a horizontal pin passing thought a small hole near the vertex A such that its plane is vertical A couple with a moment of magnitude 7500gm.wt .cm acts on the lamina in its plane find the inclination of the diagonal AC to the vertical in the equilibrium position



① 45° or 135° ② 60° or 120°

③ 30° or 150° ④ 20° or 160°

$\therefore 7500 = 300 \times OL \Rightarrow OL = 25 \because AC = 50\sqrt{2} \quad \therefore AO = 25\sqrt{2}$

$\sin\theta = \dfrac{25}{25\sqrt{2}} = \dfrac{1}{\sqrt{2}} \therefore \theta = 45^\circ or 135^\circ$

Q(8) two forces form a couple the magnitude of one of them is 15Newton and the moment of the couple is 45 Newton .cm then the perpendicular distance between them is equal ………

① 675cm ② 60cm

③ 3cm ④ 30cm

$L = 45 \div 15 = 3cm$

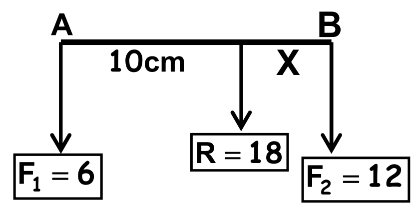
Q(9) A force $F = 3\hat{i} + \hat{j} - 2\hat{K}$ act at a point A(1,-1,2) then the moment of F about B(3,-2,1)

① $-3\hat{i} - \hat{j} - 5\hat{K}$ ② $5\hat{i} + \hat{j} + 3\hat{K}$

③ $4\hat{i} + 2\hat{j} + \hat{K}$ ④ $3\hat{i} + \hat{j} + 5\hat{K}$

$$M_B = \begin{vmatrix} i & j & K \\ -2 & 1 & 1 \\ 3 & 1 & -2 \end{vmatrix} = -3\hat{i} - \hat{j} - 5\hat{k}$$

Q(10) In the opposite figure : the value of X equal ....cm

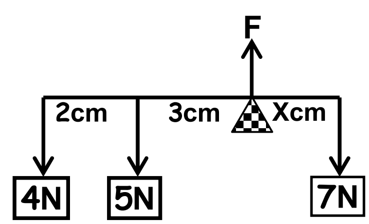


① 10 ② 7

③ 5 ④ 6

$12X = 60 \quad \therefore X = 12$

Q(11) In the opposite figure The rod in equilibrium then X=..............cm



① 5 ② 6

③ 7 ④ 4

$7X = 15 + 20 \quad \therefore X = 5$

Q(12) If F=(2,-3,4) act at the point (1,1,1) then the component of the moment of F about X-axis is equal to

① 7 ② -2

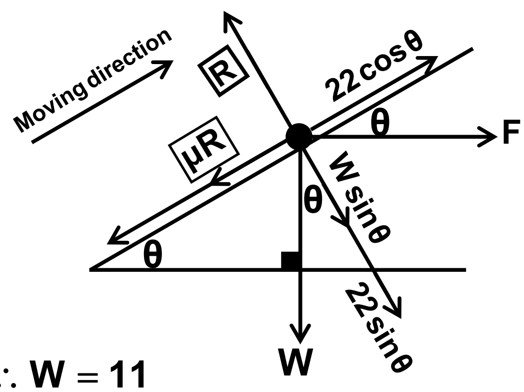
③ -5 ④ 2

$YF_Z - ZF_Y = 1 \times 4 - 1 \times -3 = 7$

Q(13) A body of weight (w) is placed on a rough inclined plane makes an angle of sine $\frac{5}{13}$ with the horizontal the body is attached by a horizontal force of magnitude 22 Newton's lies in the vertical plane which passes through the line of the greatest slope makes the body is about to move upwards the plane, if the static coefficient friction between the body and the plane is $\frac{1}{2}$, then find the magnitude of the weight (w).

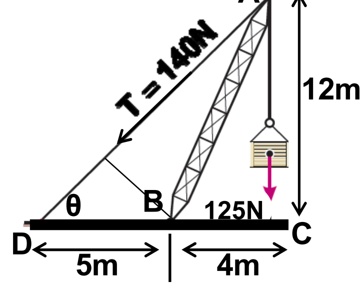
(17)

$$F\cos\theta = W\sin\theta + \mu R$$
$$22\times\frac{12}{13} = W\times\frac{5}{13}+\frac{1}{2}R \rightarrow (1)$$
$$R = F\sin\theta + W\cos\theta$$
$$R = 22\times\frac{5}{13}+W\times\frac{12}{13} \rightarrow (2)$$
$$22\times\frac{12}{13} = W\times\frac{5}{13}+\frac{1}{2}\left(22\times\frac{5}{13}+W\times\frac{12}{13}\right)$$
$$\frac{264}{13} = \frac{5}{13}W+\frac{22}{2}+\frac{6}{13}W \quad \therefore \frac{11}{13}W = \frac{121}{13} \quad \therefore W = 11$$

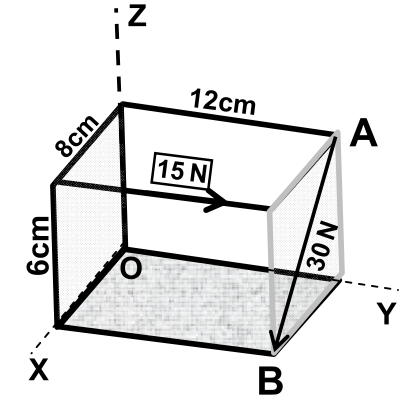


Q(15) In the opposite figure: A B represents a crane for lifting the goods. If the tension in the string is equal to 140 Newton's, and the weight of the box is 125 Newton's, find the sum of the two moments of the two forces about B.

$$AD = \sqrt{9^2+12^2} = 15$$
$$\sin\theta = \frac{BX}{5} = \frac{12}{15} \quad \therefore BX = 5m$$
$$M_B = -125\times 5 + 140\times 5 = 79N.m$$



Q(16) In the opposite figure: find the sum of moments of the forces about O



$15 = 15\hat{j}$

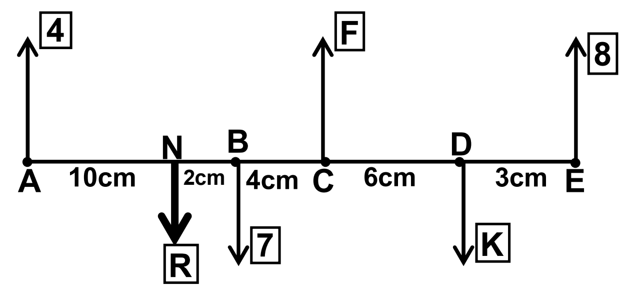
$A = (0,12,6)$, $B = (8,12,0)$

$AB = B - A = (8,0,-6)$

$30 = \frac{30}{10}(8,0,-6) = 24\hat{i} - 18\hat{j}$

$$M_O = \begin{vmatrix} \hat{i} & \hat{j} & \hat{K} \\ 8 & 0 & 6 \\ 0 & 15 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{K} \\ 0 & 12 & 6 \\ 24 & 0 & -18 \end{vmatrix} = -305\hat{i} + 144\hat{j} - 168\hat{k}$$

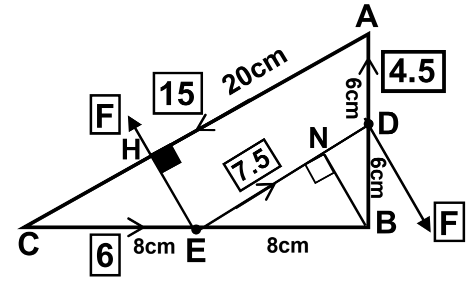
Q(17) A , B ,C , D and E are five collinear points AB=12cm , BC=4cm , CD=6cm , DE=3cm forces of magnitudes 4,F,8 N acts vertically downwards a the points A, C and E respectively and the two forces 7 and K act vertically upwards at B and D respectively if the resultant =7N and acts at the point $N \in \overline{AE}$ where AN=10cm and act vertically downwards find F and K



Sum of moments about A
= moment of R about A

F=15newtons , K=13Newtons

Q(18) ABC is a triangle :AB=12cm , BC=20cm , CA =16cm , D and H are the midpoint points of $\overline{AB}$ and $\overline{AC}$ the forces 15,6,7.5and 4.5N act along $\overrightarrow{BC}, \overrightarrow{CH}, \overrightarrow{HD}, \overrightarrow{DB}$ respectively if these forces are in equilibrium with another two forces act at D and H perpendicular to $\overline{DH}$ then find the magnitude and the direction of these two forces



$\frac{4.5}{6} = \frac{15}{20} = \frac{6}{8} = \frac{7.5}{10} = \frac{3}{4}$ and forces in cyclic coder

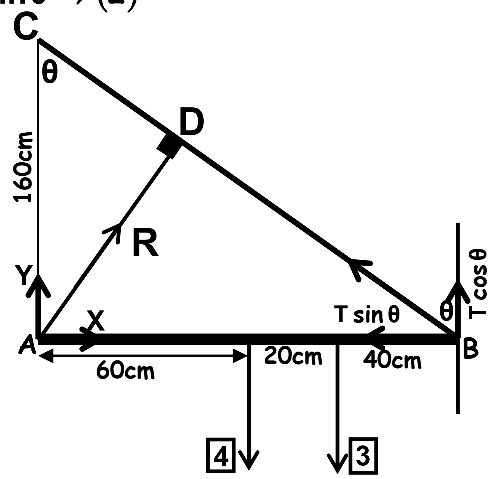
Moment of its couple =

$\frac{3}{4} \times 2 \times \frac{1}{2}(10 + 20) \times 4.8 = 108N.cm$

Q(19) A uniform bar $\overline{AB}$ hinged at A to a vertical wall, weighs 4 Newton's, and is 120 cm. long. A weight of 3 Newton's is hung from a point on the bar 80cm. From A and the bar is kept horizontal by a string attached with one end to B and is fixed to the wall 160 cm. above A. Find the tension of the string and the reaction at the hinge.

resolve the reaction at A into two components X , Y

Vertical : $Y + T\cos\theta = 7 \rightarrow (1)$ Horizontal : $X = T\sin\theta \rightarrow (2)$



$CB = \sqrt{160^2 + 120^2} = 200$

$\therefore AD = \frac{120 \times 160}{200} = 96$

$M_A = 0 \quad \therefore -3 \times 80 - 4 \times 60 + T \times \underbrace{96}_{AD} = 0$

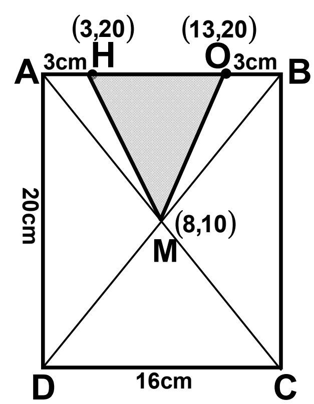
$96T = 480 \quad \therefore T = 5$

$X = T\sin\theta = 5 \times \frac{160}{200} = 4$

$Y + 5 \times \frac{120}{200} = 7 \quad \therefore Y = 4$

$R = \sqrt{4^2 + 4^2} = 4\sqrt{2}$

Q(20) A fine lamina of uniform thickness and density in the form of a rectangle ABCD whose center is M where AB = 16 cm and BC = 20 cm. The two points E and F are taken on AB where AE = BF = 3 cm , If the triangle MEF is cut off, Find the distance between the center of gravity of the remaining part and both CD and AD . If this part is freely suspended from D , Find the tangent of the angle which DA makes to the vertical i



Area of $\Delta MOH \rightarrow \frac{1}{2} \times 10 \times 10 = 50cm^2$

Area of rectangle $ABCD = 20 \times 16 = 320cm^2$

Mass of rectangle is K and mass of Δ is $\frac{5}{32}K$

$MHO \rightarrow \left(\frac{8+3+13}{3}, \frac{20+20+10}{3}\right) = \left(8, \frac{50}{3}\right)$

Mass	ABCD	MHO
mass	$\frac{32}{5}K$	K
X	8	8
Y	10	$\frac{50}{3}$

$$\left(\frac{\frac{32}{5}K \times 8 - K \times 8}{\frac{32}{5}K - K}, \frac{\frac{32}{5}K \times 10 - K \times \frac{50}{3}}{\frac{32}{5}K - K}\right) = \left(\underbrace{8}_{AD}, \frac{710}{81}\right)$$ n the equilibrium position

Answer the following questions 20 questions

From 1to 12 choose the correct answer

Q(1) If the friction force equals 25 nektons and the coefficient of friction equals $\frac{5}{12}$ then the resultant reaction equals

① 56 ② 65

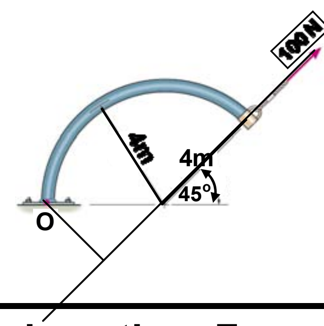
③ 13 ④ 75

$F = \mu R \quad \therefore 25 = \frac{5}{12}R \quad \therefore R = 60 \quad \therefore R' = R\sqrt{1+\mu^2} = 60 \times \sqrt{1+\left(\frac{5}{12}\right)^2} = 65$

Q(2) The moment of the force 100N about the point (O) N.m

① 100 ② $-100\sqrt{2}$

③ $-200\sqrt{2}$ ④ -200

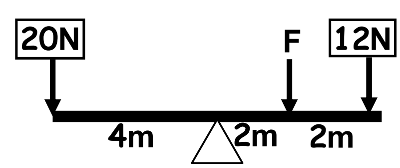


$-100 \times 4\sin 45^\circ = -200\sqrt{2}$

Q(3) The opposite figure represents a rod in equilibrium, then F =…..

① 28 Newton ② 2 Newton

③ 16 Newton ④ 4 Newton



$-12 \times 4 - F \times 2 + 20 \times 4 = 0 \quad \therefore -48 - 2F + 80 = 0 \quad \therefore F = 16$

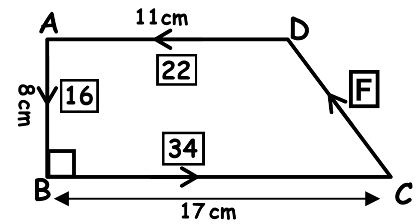
Q(4) Three forces are represented completely by the sides of an equilateral triangle ABC in the same cyclic order with scale 2cm for each 8 N. If the side length of the triangle is 5 cm ,then the norm of the moment of the couple equals ……… gm .wt . cm to ....

① $50\sqrt{3}$ ② $50\sqrt{2}$

③ 50 ④ $100\sqrt{2}$

$4 \times 2 \times \frac{1}{2} \times 5 \times 5 \times \sin 60^o = 50\sqrt{3}$

Q(5) ABCD is right trapezium The set of forces form a couple Then F=.....

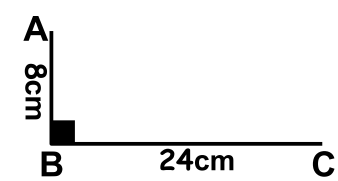


① 10N ② 30N

③ 20N ④ 12N

$$\frac{F}{10} = \frac{16}{8} \quad \therefore F = 20N$$

Q(6) Uniform density and thickness wire suspended freely from B Then $\overline{BC}$ inclined to the vertical angle of measure



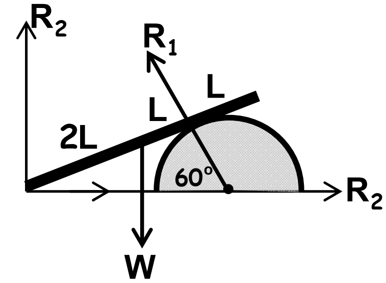
① $\tan^{-1}\frac{1}{3}$ ② $\tan^{-1}\frac{1}{9}$

③ $\tan^{-1}3$ ④ $\tan^{-1}\frac{1}{2}$

$\tan^{-1}\frac{1}{9}$

Q(7) In the opposite figure If the rod is about to slide , then

$(R_1, R_2) =$

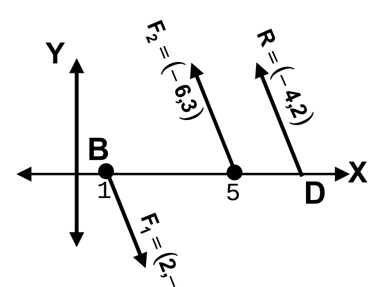


① $\left(W, \frac{1}{2}W\right)$ ② (W, W)

③ $\left(W, \frac{3}{2}W\right)$ ④ $(W, 2W)$

$\left(W, \frac{1}{2}W\right)$

Q(8) In the given figure F_1 and F_2 act at the points (1,0) and (5,0) are two parallel forces their resultant R acts the point D the point D is



① (7,0) ② (8,0)

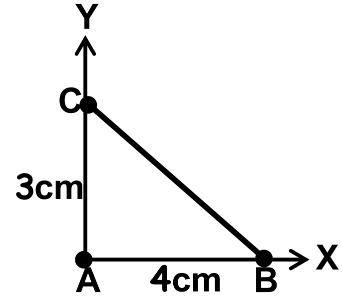
③ (8,0) ④ (2,0)

$\|F_1\| = \sqrt{2^2 + (-1)^2} = \sqrt{5} \quad \|F_2\| = \sqrt{3^2 + (-6)^2} = 3\sqrt{5}$

$\because M_D = 0 \quad \therefore \quad \sqrt{5} \times L = 3\sqrt{5}\,(L - 4) \therefore L = 3(L - 4) \quad \therefore \quad L = 3L - 12$

$\therefore 2L = 12 \; \therefore L = 6 \therefore$ Point D is $(7,0)$

Q(9) The center of gravity of three equal masses each of 2Kg placed at the vertices of a right –angled triangle whose length of two legs 3cm and 4cm

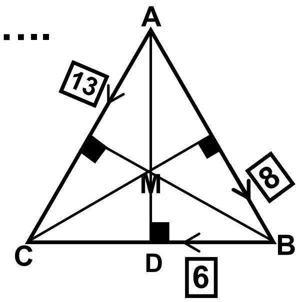


① $\left(1,\frac{4}{3}\right)$ ② $\left(\frac{4}{3},1\right)$

③ $\left(2,\frac{3}{2}\right)$ ④ $\left(\frac{3}{2},2\right)$

$$\left(\frac{0+4+0}{3},\frac{3+0+0}{3}\right)=\left(\frac{4}{3},1\right)$$

Q(10) ABC is an equilateral triangle the length of its side equals 18cm forces of magnitudes 8,6,13 Newtons act along $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{AC}$ the sum of algebraic moments of the resultant of thes forces about the centroid =…..



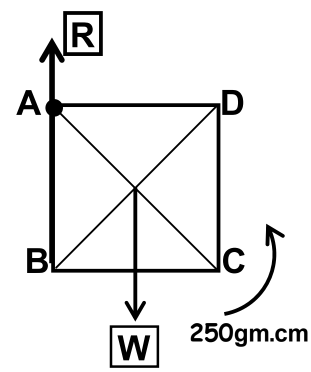
① $2\sqrt{3}$ N.cm ② $3\sqrt{3}$ N.cm

③ $\sqrt{3}$ N.cm ④ $4\sqrt{3}$ N.cm

$AD = 9\sqrt{3} \quad \therefore MD = \frac{1}{3}\times 9\sqrt{3} = 3\sqrt{3}$

$M_M = -8\times 3\sqrt{3} - 6\times 3\sqrt{3} + 13\times 3\sqrt{3} = \sqrt{3}$ N.cm

Q(11) ABCD is a uniform lamina of side length 50cm Suspended from nail at A an equilibrium under a couple Of magnitude 250gmwt.cm If $\overline{AB}$ is vertical then W=R= ……

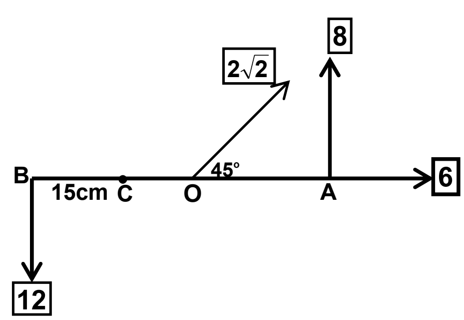


① 2 ② 10

③ 5 ④ 25

$W = R \quad \therefore W\times 25 = 250 \quad \therefore W = 10cm$

Q(12) The algebraic sum of the moments of this system of forces about C equals



① 570 ② 580

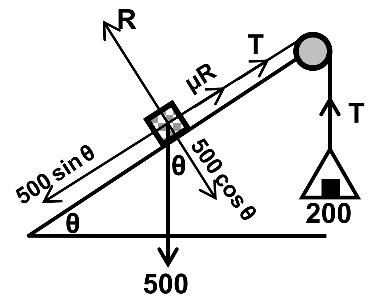
③ 500 ④ 600

$M_C = 12\times 15 + 2\sqrt{2}\times 15\sin 45^{o} + 8\times 45 + 6\times 0 = 570$Kg.wt.cm

Q(13) A body of mass 500gm.wt is placed on a rough plane which is inclined to the horizontal at an angle of measure θ where $\tan\theta = \frac{4}{3}$ the body is connected by a string passing over a smooth pulley at the top of the plane from the other end of the string a scale pan of weight 25gm.wt if the least weight should be put in the scale pan so that the body in equilibrium is 175gm find the coefficient of friction then prove that the greatest weight which should by in the scale pan with keeping equilibrium is 575gm.wt

In case of least force :

The body is about to move down $T = \underbrace{200}_{25+175}$



$R = W\cos\theta = 500 \times \frac{3}{5} = 300$

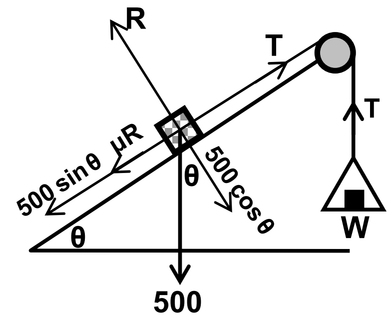
$T + \mu R = 500\sin\theta$

$\therefore\ 200 + \mu \times 300 = 500 \times \frac{4}{5}$

$\therefore 300\mu = 200 \qquad \therefore \mu = \frac{2}{3}$

In case of the greatest weight:

The body is about to move upward $T = W$



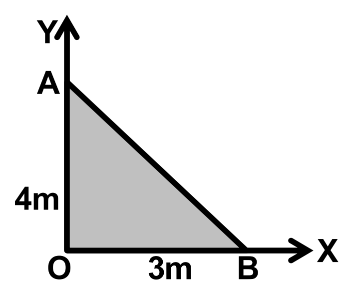
$R = 500\cos\theta = 500 \times 35 = 300$

$T = 500\sin\theta + \mu R$

$W = 500 \times \frac{4}{5} + \frac{2}{3} \times 300 = 600$

greatest weight: 600-25=75

Q(15) A force F at the xy-plane acts on the triangle A O B. If the algebraic measure of the moment of F about point O is equal to 84 Newton . m , the algebraic measure of its moment about point A is equal to - 100 Newton . m , and the algebraic measure of its moment at point B is equal to zero, determine F



Let the force (L,M)

$M_O = [(3,0) - (0,0)] \times (L,M) = 84$

$3M = 84 \quad \therefore M = 28$

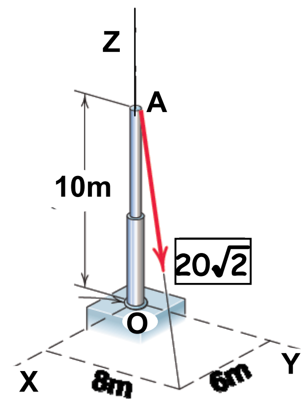
$M_A = [(3,0) - (0,4)] \times (L,M) = -100$

$3M + 4L = -100 \quad \therefore\ 4L = -184 \quad \therefore L = 46$

$F = \sqrt{28^2 + 46^2} = 10\sqrt{29}$

Q(16) Find the moment of the ofrce about (0)

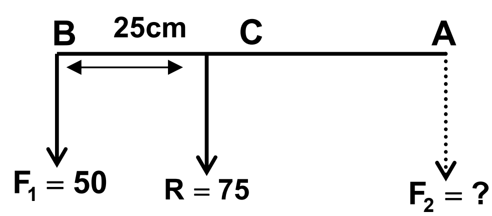
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 10 \\ 16 & 12 & -20 \end{vmatrix} = -120\hat{i} - 160\hat{j}$$



Q(17) (The resultant of two parallel forces is 75Newtons , one of the two forces equals 50 Newton's and acts at a distance 25 cm from the line action of the resultant find the second force and also the distance between the two forces if the given force and the resultant act in the same direction

$F_2 = R - F_2 = 75 - 50 = 25$

$50 \times 25 = 25 \times AC \quad \therefore AC = 50cm$



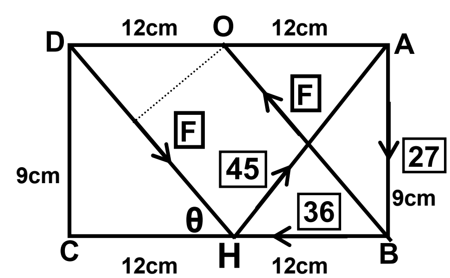
Q(18) ABCD is a rectangle , AB=9cm , BC=24cm , H and O are the mid-points of $\overline{BC}$ and $\overline{AD}$, the forces 27,36 and 45N act along $\overrightarrow{AB}, \overrightarrow{BH}$ and $\overrightarrow{HA}$ find the two forces act along $\overrightarrow{BO}$ and $\overrightarrow{DH}$ to be the system in equilibrium

The forces 27,36,45 in cyclic order

$\therefore \frac{27}{9} = \frac{36}{12} = \frac{45}{15} = 3$

Norm of couple momemt = $3 \times 2\left(\frac{1}{2} \times 9 \times 12\right) = -324$

$F \times 12 \sin\theta = 324 \qquad \therefore F = 45$

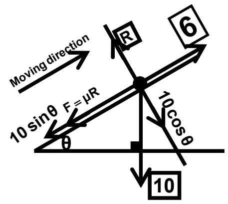


Q(19) A 10 Kg.wt weight is placed on a rough plane which is inclined to the horizontal. A force is applied to the weight parallel to the line of greatest slope upwards . If the limit force lies which the applied , so as to make the weight about to move upwards is 6Kg.wt ,the limit force lies which the applied , so as to make the weight about to move . downwards is 4Kg.wt
find (1) the angle of inclined of the horizontal plane
(2) the friction coefficient between the body and the plane

About to move upwards

$$10\sin\theta + \mu R = 6 \rightarrow (1)$$

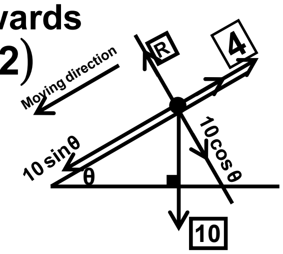
$$\mu R = 6 - 10\sin\theta \rightarrow (3)$$



$$R = 10\cos\theta \rightarrow (4)$$

About to move downwards

$$10\sin\theta = 4 + \mu R \rightarrow (2)$$



Substituting from 3 in 2

$$\therefore 10\sin\theta = 4 + 6 - 10\sin\theta \rightarrow \therefore \ 20\sin\theta = 10 \ \therefore \sin\theta = \frac{1}{2} \quad \therefore \theta = 30^o$$

$$\therefore R = 10 \times \cos 30^o = 5\sqrt{3} \quad \text{substituting in } (3) \therefore \mu \times 5\sqrt{3} = 6 - 10\sin 30^o$$

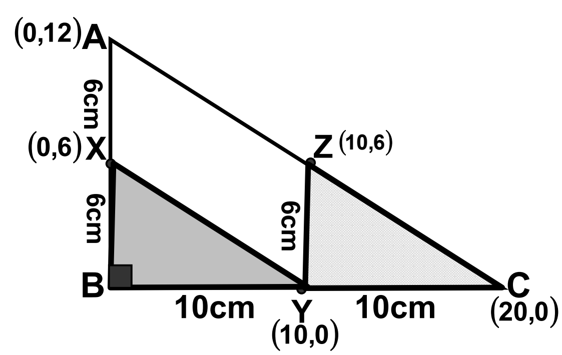
$$\therefore 5\sqrt{3}\mu = 1 \quad \therefore \mu = \frac{1}{5\sqrt{3}}$$

Q(20) ABC is a fine lamina of uniform thickness and density in the form of a right - angled triangle at B where AB = 12 cm, BC = 20 cm and X , Y and Z are the midpoints of AB , BC and CA respectively. The triangle CYZ is cut off and coincided with the triangle Y B X. If the system is freely suspended from point B. Find the tangent of the angle of inclination of BC to the vertical in the equilibrium position.

$$ABC \rightarrow \left(\frac{0+0+20}{3}, \frac{0+12+0}{3}\right) = \left(\frac{20}{3}, 4\right)$$

$$XYB \rightarrow \left(\frac{0+0+10}{3}, \frac{0+6+0}{3}\right) = \left(\frac{10}{3}, 2\right)$$

$$ZYC \rightarrow \left(\frac{10+10+20}{3}, \frac{0+6+0}{3}\right) = \left(\frac{40}{3}, 2\right)$$



Mass	ABC	XYB	ZYC
mass	4K	K	K
X	$\frac{20}{3}$	$\frac{10}{3}$	$\frac{40}{3}$
Y	4	2	2

$$X = \left(\frac{4K \times \frac{20}{3} + K \times \frac{10}{3} - K \times \frac{40}{3}}{4K + K - K}\right) = \frac{25}{6} \quad , \quad Y = \left(\frac{4K \times 4 + K \times 2 - K \times 2}{4K + K - K}\right) = 4$$

$$\tan\theta = 4 \div \frac{25}{6} = \frac{24}{25}$$

Answer the following questions	20 questions

From 1to 12 choose the correct answer

Q(1) If the ratio between the resultant reaction force and the normal reaction force Equals $2\sqrt{3}:3$ then the angle of friction is equal

① 45° ② 30°

③ 90° ④ 60°

$\cos\theta = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2} \quad \therefore \theta = 30^\circ$

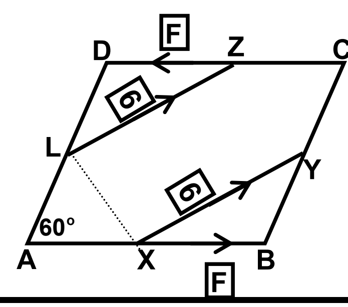
Q(2) A body of weight 3Newtons is placed on a horizontal rough plane whose coefficient with the body is $\frac{1}{3}$ then the friction force

① $\in\left[\frac{1}{3},3\right]$ ② $[1,\infty[$

③ $\in\left[0,\frac{1}{3}\right]$ ④ $[0,1]$

$[0,1]$

Q(3) In the opposite figure : ABCD is a parallelogram $m(\angle A)=60^\circ$ X , Y , Z and L are mid points $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{AD}$ given that the forces are in equilibrium then F =….. N



① 3 ② 5

③ 4 ④ 6

$-6\times\frac{\sqrt{3}}{4}L + F\times\frac{\sqrt{3}}{2}L = 0 \quad \therefore F = 3$

Q(4) If a set of forces are in equilibrium then :

① R=0 ② R=1

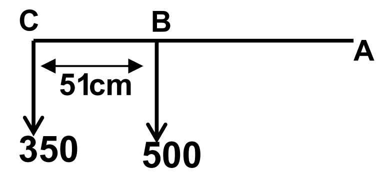
③ $R \neq 0$ ④ otherwise

R=0

Q(5) The resultant of two parallel forces is 350Newtons , one of the two forces equals 500 Newton's and acts at a distance 51cm then distance between the two force

① 30cm ② 150cm

③ 170cm ④ 51cm



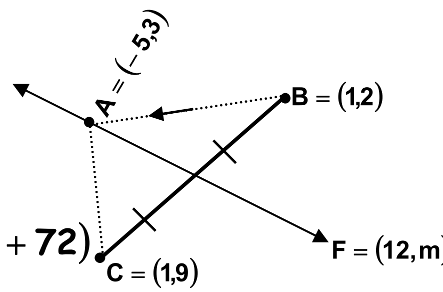
$R = F_1 - F_2 \quad \therefore F_2 = 500 - 350 = 150$

$500 \times 51 = 150 \times AC \quad \therefore AC = 170cm$

Q(6) The force $F = 12i + mj$ acts at the point $A = (-5,3)$ and its line of action bisect $\overline{BC}$ Where $B = (1,2), C = (1,9)$ use the moment to the value of m

① ②

③ ④



$\therefore (-6,1) \times (12,m) = -(-6,-6) \times (12,m) \therefore -6m - 12 = -(-6m + 72)$

$-6m - 12 = 6m - 72 \quad \therefore 12m = 60 \quad \therefore m = 5$

Q(7) If a driver of a car wants to ascend an plane inclined to the horizontal by an angle of measure $60°$,then the static friction coefficient between the wheels of the care and the inclined plane must be not less than ....

① 1 ② $\sqrt{3}$

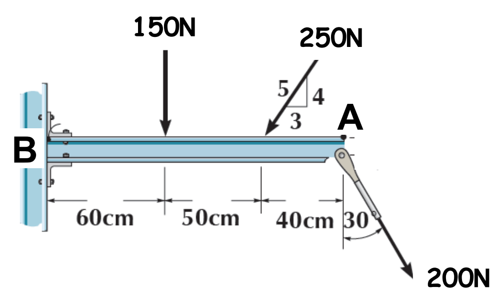
③ 0 ④ $\sqrt{2}$

$\tan 60° = \sqrt{3}$

Q(8) The sum of moment of the forces about B Equals

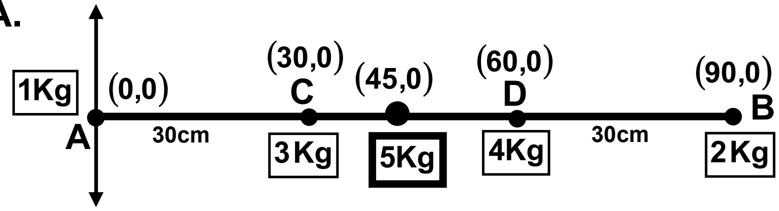
① ②

③ ④



$-250 \times \frac{4}{5} \times 110 + 200 \cos 30° \times 150 - 150 \times 60$

Q(9) AB is a uniform rod of length 90 cm and mass 5 kg. C and D are its two trisection points from end A. Masses of magnitudes 1 , 2 , 3 and 4 kg are placed at A , B , C and D respectively. Then distance from the center of gravity of the system to end A.

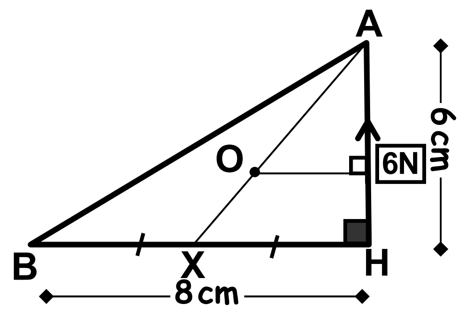


① 48cm ② 49cm

③ 7cm ④ 50cm

$$X = \left(\frac{1\times0+3\times30+4\times60+2\times90+5\times45}{1+3+4+2+5}\right) = 49\text{cm}$$

Q(10) The moment of the force 6N about the point O equals

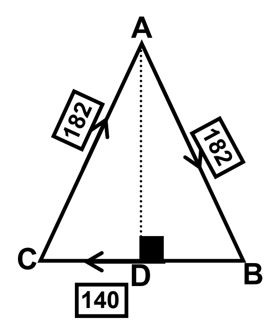


① 14 ② 16

③ 18 ④ 19

$$\frac{X}{4} = \frac{2}{3} \quad \therefore X = \frac{8}{3} \quad M_O = 6\times\frac{8}{3} = 16$$

Q(11) ABC is a triangle in which AB=AC =26cm , BC= this set of forces is equivalent to a couple the magnitude of its moment



① -3360 ② -1680

③ -3365 ④ -240

$$AD = \sqrt{26^2 - 10^2} = 24\text{cm} \therefore K = \frac{182}{26} = \frac{140}{20} = 7$$

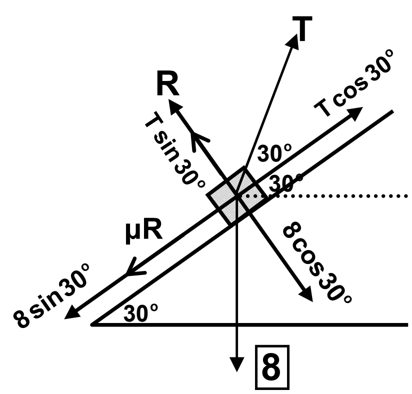
$$= 7\times2\times\left(\frac{1}{2}\times20\times24\right) = -3360$$

Q(13) A body of weight 8 kg .wt is placed on a horizontal rough plane, then the plane incline gradually until the body becomes about to move downwards the plane when the measure of the angle of inclination to the horizontal is 30c. Find the coefficient friction between the body and the plane, and if the body is tied by a string and the string is attached in the direction makes an angle of measure 30c with the plane until the body becomes about to move upwards the plane, find:
a the magnitude of the tension force b magnitude of the normal reaction

$$\mu = \tan 30^\circ = \frac{\sqrt{3}}{3}$$

$$R + T\sin 30^\circ = 8\cos 30^\circ$$

$$R + \frac{1}{2}T = 4\sqrt{3} \qquad \therefore R = 4\sqrt{3} - \frac{1}{2}T$$



$$T\cos 30^\circ = 8\sin 30^\circ + \mu R$$

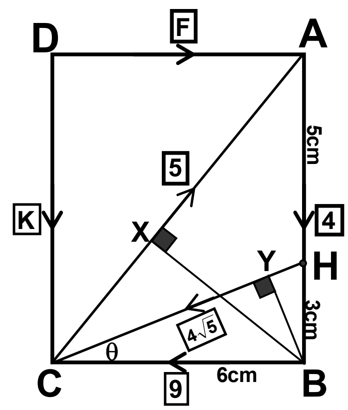
$$\frac{\sqrt{3}}{2}T = 4 + \frac{\sqrt{3}}{3}R$$

$$\frac{\sqrt{3}}{2}T = 4 + \frac{\sqrt{3}}{3}\left(4\sqrt{3} - \frac{1}{2}T\right)$$

$$\frac{\sqrt{3}}{2}T = 4 + 4 - \frac{\sqrt{3}}{6}T \qquad \therefore \frac{2\sqrt{3}}{3}T = 8 \quad \therefore T = 4\sqrt{3}$$

Q(15) ABCD is a rectangle in which AB=8cm , BC=6cm , $H \in \overline{AB}$ such that HB=3cm . forces of magnitudes F , 4 , 9 , K , 5 , $4\sqrt{5}$ N acts along $\overrightarrow{DA}, \overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{DC}, \overrightarrow{CA}, \overrightarrow{HC}$ respectively find the values of F and K given that the sum of algebraic measure of moments of these forces about C equals 72 in direction ABC and vanishes about B

$$AC = \sqrt{6^2 + 8^2} = 10 \ , \ CH = \sqrt{3^2 + 6^2} = 3\sqrt{5}$$



$$BY = BC\sin\theta = 6 \times \frac{3}{3\sqrt{5}} \quad , BX = \frac{6 \times 8}{10} = 4.8$$

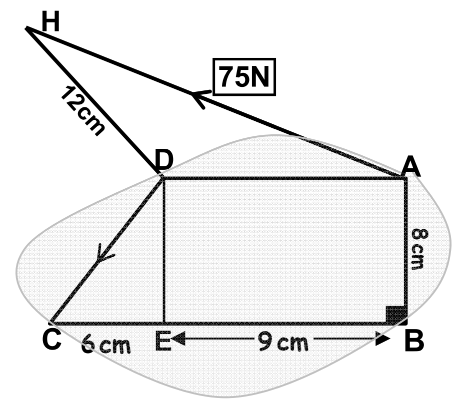
$$M_C = -72 = -4 \times 6 - F \times 8 = -72$$

$$\therefore \ 8F = 48 \quad \therefore F = 6$$

$$M_B = 0 \therefore 4\sqrt{5} \times BY - 5 \times BX - 6 \times 8 + K \times 6 = 0$$

$$24 - 24 - 48 + 6K \qquad \therefore K = 8$$

Q(16) ABCD is right trapezium at B , $\overline{AD} // \overline{BC}$, AB=8cm , BC=15cm , AD=9cm draw $\overline{DH} \perp$ plane of the trapezium where DH=12cm a force of magnitude 75N act along $\overrightarrow{AH}$ find the moment of this force about the point B



$H \to (0,0,12)$, $A = (0,9,0)$, $B = (8,9,0)$

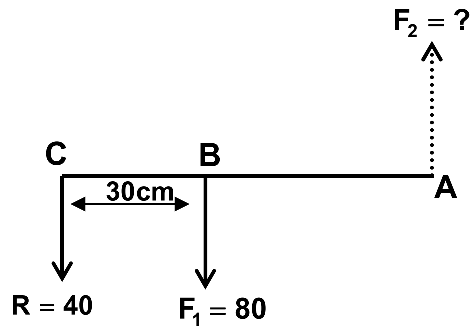
$AH = H - A = (0,0,12) - (0,9,0) = (0,-9,12)$

$F = \frac{75}{15}(0,-9,12) = -45j + 60K$

$BA = A - B = (0,9,0) - (8,9,0) = (-8,0,0)$

$$M_B = \begin{vmatrix} i & j & k \\ -8 & 0 & 0 \\ 0 & -45 & 60 \end{vmatrix} = 480j + 360j$$

Q(17) The resultant of two parallel forces is <u>40Newtons</u> , one of the two forces equals 80 Newton's and acts at a distance 30 cm from the line action of the resultant find the second force and also the distance between the two Forces if the <u>given force and the resultant act in the same direction</u>

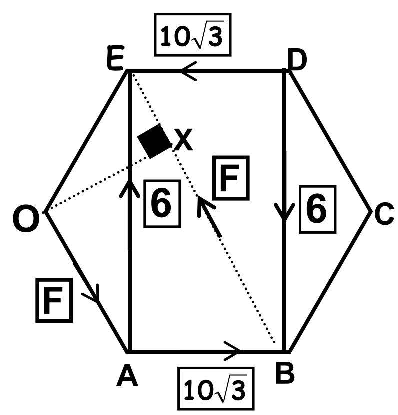


$\therefore F_2 = F_1 - R = 80 - 40 = 40$

$80 \times 30 = 40 \times AC \quad \therefore AC = 60cm$

$\therefore AB = 60 - 30 = 30cm$

Q(18) ABCDEO is a regular hexagon forces of magnitudes $10\sqrt{3}$,6 , $10\sqrt{3}$, 6 Newton act along $\overrightarrow{AB}, \overrightarrow{DB}, \overrightarrow{DE}, \overrightarrow{AE}$ respectively prove that the system tends to a couple and find the magnitude of its moment . find also the magnitude and the direction of two forces acting along $\overrightarrow{BE}, \overrightarrow{OA}$ such that the system will be in equilibrium



Let the length side of the hexagon side is L

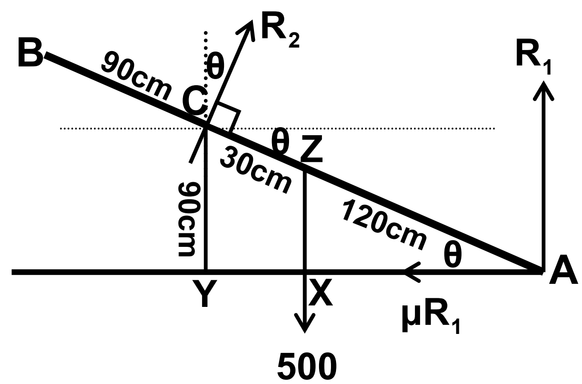
Magnitude of its moment =

$-6 \times L + 10\sqrt{3} \times L\sqrt{3} = 24L$

$24L = F \times OX$

$\therefore 24L = F \times \frac{1}{2} \times L\sqrt{3} \qquad \therefore F = 16\sqrt{3}$

Q(19) AB is a uniform rod of length 240cm and weight 500gm.wt rests with its end A on a rough horizontal plane . it rests with on of its points C on a smooth nail that's fixed at 90cm height from the plane when the rod was about to slip BC=90cm find the reaction o the nail at C and the coefficient of friction between the rod and the plane



$\sin\theta = \frac{90}{150} = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$

$M_A = 0$

$\therefore -R_2 \times 150 + 500 \times 120\cos\theta = 0$

$150R_2 = 500 \times 120 \times \frac{4}{5}$

$\therefore R_2 = 320\text{Newton}$

$Y = 0 \quad \therefore R_2\cos\theta + R_1 = 500$

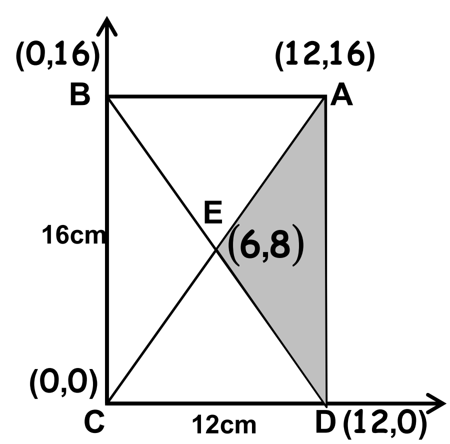
$\therefore 320 \times \frac{4}{5} + R_1 = 500$

$\therefore R_1 = 244\text{Newtons}$

$X = 0 \quad \therefore R_2\sin\theta = \mu R_1$

$\therefore 320 \times \frac{3}{5} = \mu \times 244 \quad \therefore \mu = \frac{48}{61}$

Q(20)ABCD is a lamina of a uniform thickness and density in the form of a rectangle in which AB = 12 cm , B C = 16 cm and point E is the intersection point of its diagonals AC and BD, the triangle A E D is separated and fixed above the triangle B E C. Find the center of gravity of the lamina in this case If the lamina is freely suspended from point C, find the tangent of the angle of CB to the vertical.



$\Delta CED \quad \text{mass}(2K)$

$,\text{point}\left(\frac{0+6+12}{3}, \frac{0+8+0}{3}\right) = \left(6, \frac{8}{3}\right)$

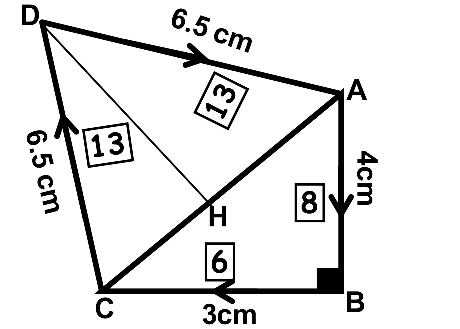
$\Delta ABE \quad \text{mass}(K)$

$\text{point}\left(\frac{0+6+12}{3}, \frac{16+16+8}{3}\right) = \left(6, \frac{40}{3}\right)$

Centre of gravity is

$$\left(\frac{6K+6K+4K}{4K}, \frac{\frac{40}{3}K + \frac{8}{3}K + 16K}{4K}\right) = (4,8)$$

Q(21) ABCD is a quadrilateral in which AB=4cm , BC=3cm , CD=DA =6.5cm and $m\angle ABC = 90^\circ$ forces of magnitudes 8,6,13 and 13 Newton act along $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CD}$ and $\overrightarrow{DA}$ respectively prove that the system of forces is equivalent to a couple and calculate the norm of its moment



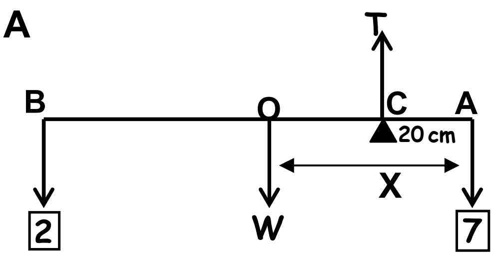
$AC = \sqrt{3^2 + 4^2} = 5$

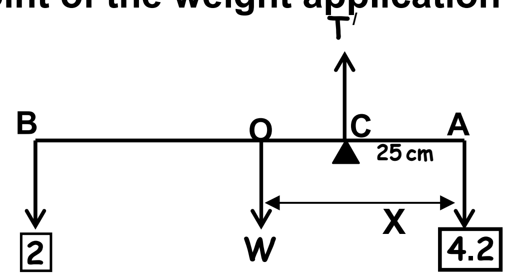
$DH = \sqrt{6.5^2 - 2.5^2} = 6cm$

$\frac{4}{8} = \frac{3}{6} = \frac{6.5}{13} = \frac{6.5}{13} = \frac{1}{2}$

$\text{Moment norm} = 2 \times 2\left(\frac{1}{2} \times 3 \times 4 + \frac{1}{2} \times 5 \times 6\right) = 84$

Q(22) AB is a <u>non- uniform</u> rod of length 65cm . if a weight of magnitude 2newtonls fixed at the end B and a weight of 7Newton is suspended at the end A the rod is in equilibrium horizontally at a point at a distance 20cm fror A . if the weight at A is decreased to become 4.2 Newton then the rod will be in equilibrium horizontally at point of distance 25cm from A find the weight of the rod and the distance between the point of the weight application and the end A





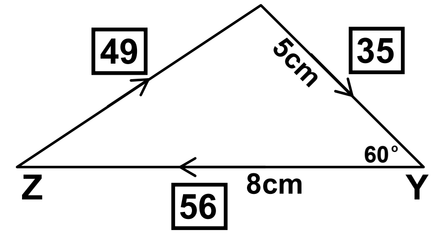
Let the weight acts at a point O distance X from A `

(i) $M_C = 0 \quad \therefore 2 \times 45 + W(X - 20) - 7 \times 20 = 0 \quad \therefore WX - 20W = 50 \quad \rightarrow (1)$

(ii) $M_C = 0 \quad \therefore 2 \times 40 + W(X - 25) - 4.2 \times 25 = 0 \quad \therefore WX - 25W = 25 \quad \rightarrow (2)$

subtracting 1,2 $\therefore 5W = 25 \quad W = 5 \quad , X = 30$

ΔXYZ XY=5cm , YZ=8cm , $m(\angle Y) = 60^\circ$ forces of magnitude 35,56,49 dyne acts along XY , YZ , ZX respectively prove that the system tends to a couple and find the magnitude of its moment . find also the magnitude and the direction of two forces acting at X and Z perpendicular to XZ such that the system will be in equilibrium



$XZ = \sqrt{5^2 + 8^2 - 2 \times 5 \times 8 \times \cos 60^\circ} = 7$

$\frac{35}{5} = \frac{56}{8} = \frac{49}{7} = 7$

$\therefore C = 2 \times 7 \times \left(\frac{1}{2} \times 5 \times 8 \times \sin 60^\circ\right) = 140\sqrt{3}$

$140\sqrt{3} = F \times 7 \quad \therefore F = 20\sqrt{3}$