3rd year secondary

Algebra &solid geometry

booklets 2017

إعداد فريق العمل

أ/جمال الشاهد	موجه اول المدارس الرسميه للغات بمحافظه الجيز ه
أ/محمد يس جمعه	موجه رياضيات
أ/محمد على السعيد	موجه رياضيات
أ/سيد محمد عبد العزيز	موجه رياضيات
أ/بسيونى ابراهيم	كبير معلمين (مدرسه الاورمان)
أ/محمد على قاسم	معلم اول .أ (مدرسه هضبه الاهرام)

مع خاص التمنيات للطلبه بالنجاح و التوفيف

Guide Answers

Answer the following questions 20 questions

From 1to 12 choose the correct answer

Q(1) The distance between the two planes
3X+2Y-6Z-14=0 and 3X+2Y-6Z+21=0 is ……..units

① 1 ② 2

③ 3 ④ 4

$$\frac{|3(0)+2(7)-6(0)+21|}{\sqrt{3^2+2^2+6^2}}=2\text{units}$$

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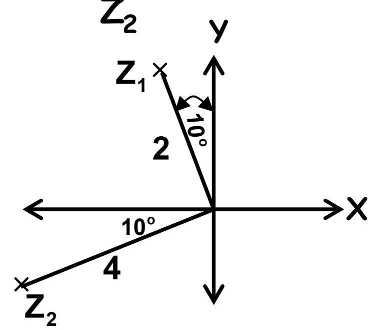
(2) In the opposite figure: find the exponetel form of Of $\frac{Z_1}{Z_2}$



① $\frac{1}{2}i$ ② $-\frac{1}{2}i$

③ $\frac{1}{2}$ ④ $-\frac{1}{2}$

$$\frac{2(\cos 100^\circ + i\sin 100^\circ)}{4(\cos -170^\circ + i\sin -170^\circ)}=-\frac{1}{2}i$$

Q(3) If Z_1, Z_2 are two complex numbers the amplitude of $(Z_1Z_2)=\frac{5\pi}{18}$

And the amplitude of $\left(\frac{Z_1}{Z_2}\right)=\frac{\pi}{9}$ then the amplitude of Z_1=………

① $\frac{7\pi}{36}$ ② $\frac{5\pi}{36}$

③ $\frac{\pi}{3}$ ④ $\frac{\pi}{4}$

$\theta_1+\theta_2=\frac{5\pi}{18}$ and $\theta_1-\theta_2=\frac{\pi}{9}$ $\therefore \theta_1=\frac{7\pi}{36}$

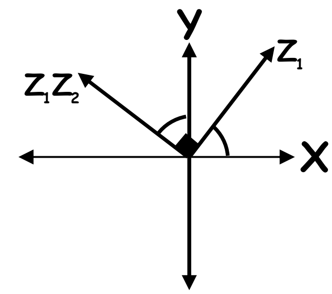
Q(4) In the expansion of $X^3(1+X)^7$ the coefficent of term containing X^4 is

① ${}^7C_1$ ② ${}^7C_3$

③ ${}^7C_4$ ④ 21

${}^7C_r(X)^r(1)^{7-r}$ $\therefore r=1$ ${}^7C_1$

Q(5) In the opposite figure: If Z_1 and Z_2 and Z_1Z_2 are complex numbers then $Z_2 =$



① -2i ② -i

③ i ④ 2i

$$Z_2 = \frac{Z_1Z_2}{Z_1} = \frac{re^{\left(\frac{1}{2}\pi+\theta\right)i}}{re^{\theta i}} = e^{\frac{1}{2}\pi i} = \cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right) = i$$

Q(6) If the point (-2,4,m) lies on the sphere $(X+2)^2 + (Y-1)^2 + (Z-3)^2 = 25$ then one value of m=….

① 6 ② 8

③ 7 ④ 9

$3^2 + (m-3)^2 = 25 \quad \therefore m - 3 = 4 \quad \therefore m = 7$

Q(7) If ω is an imaginary cube root of unity then $(1 + \omega - \omega^2)^7 =$……

① 128ω ② $128\omega^2$

③ -128ω ④ $-128\omega^2$

$(-2\omega^2)^7 = -128\omega^4 = -128\omega^2$

Q(8) If $\vec{A}$ = (1, 2, -4), $\vec{B}$ = (1, 1, k - 1) and $\|\vec{A} + \vec{B}\|$ = 7 unit of length then k = ……………

① -1,11 ② -11,-1

③ 1,11 ④ 1,12

{-1,11}

Q(9)
If $\overrightarrow{A} = (4, -k, 6)$, $\overrightarrow{B} = (2, 2, m)$ and $\overrightarrow{A} \,//\, \overrightarrow{B}$, then k + m = ………

① 12 ② 2

③ -1 ④ 3

$\frac{4}{2} = \frac{-K}{2} = \frac{6}{m} \quad \therefore m = 3 \ , \ K = -4 \quad \therefore K + m = -1$

Q(10) 4 non collinear and coplanar points. Find the number of line segments joining each two of them?

① 5 ② 7

③ 6 ④ 8

${}^{4}C_{2} = 6$

Q(11) $1 + 3\omega + 3\omega^{2} = \ldots\ldots$

① -2 ② -1

③ 0 ④ ω

$1 + 3 \times -1 = -2$

Q(12) If ${}^{X+Y}P_{4} = 360$, $\underline{|2X + Y} = 5040$ then ${}^{Y}C_{2X} = \ldots\ldots$

① 10 ② 30

③ 20 ④ 40

$X + Y = 6 \ , \ 2X + Y = 7 \quad \therefore x = 1 \ , y = 5 \quad {}^{5}c_{2} = 10 \therefore$

Q(13) Find the standard form and the general form of the equation of the plane passing through point (3 , -5 , 2) and the vector n = (2 , 1, 1) is normal to the plane.

$\boxed{n \bullet r = n \bullet A}$

$(2,1,1) \bullet (X,Y,Z) = (2,1,1) \bullet (3,-5,2)$

$2X + Y + Z = 2\times 3 + 1\times -5 + 1\times 2 = 3 \qquad \therefore 2X + Y + Z - 3 = 0$

Q(14) Use the multiplicative inverse to solve the set of following equations $\frac{1}{X} + \frac{1}{Y} + \frac{1}{Z} = 1$, $\frac{1}{X} - \frac{1}{Y} + \frac{2}{Z} = \frac{1}{2}$, $\frac{2}{X} + \frac{3}{Y} - \frac{4}{Z} = \frac{4}{3}$

Where X and Y and Z not equals zeros

$$\begin{pmatrix} \frac{1}{X} \\ \frac{1}{Y} \\ \frac{1}{Z} \end{pmatrix} = \frac{1}{11}\begin{pmatrix} -2 & 7 & 3 \\ 8 & -6 & -1 \\ 5 & -1 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ \frac{1}{2} \\ \frac{4}{3} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{6} \end{pmatrix} \qquad \therefore \frac{1}{X} = \frac{1}{2} \quad \therefore X = 2$$

Q(15) If A = (-3 , 1 , 2) , B = (3 , 4 , -1), find the area of the parallelogram in which A and B are two adjacent sides.

$$\overrightarrow{A} \times \overrightarrow{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{K} \\ -3 & 1 & 2 \\ 3 & 4 & -1 \end{vmatrix} = -9\hat{i} + 3\hat{j} - 15\hat{K}$$

$$\therefore \|A \times B\| = \sqrt{(-9)^2 + (3)^2 + (-15)^2} = 3\sqrt{35} \text{ unit area}$$

Q(16) Find the Cartesian equation of the plane
$(X,Y,Z)=(2,3,5)+t_1(-1,3,4)+t_2(6,1,-2)$ where t_1 and t_2 are parameters

$(-1,3,4)$, $(6,1,-2)$ are the direction vectors of two lines in the plane
To get the normal to this plane

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{K} \\ -1 & 3 & 4 \\ 6 & 1 & -2 \end{vmatrix} = -10\hat{i}+22\hat{j}-19\hat{K}$$

$$\therefore (X,Y,Z)\bullet(-10,22,-19)=(2,3,5)\bullet(-10,22,-19)$$

$$10X-22Y+19Z=49$$

Q(17) Find the multiplicative inverse $A=\begin{pmatrix} 1 & -1 & 2 \\ 2 & -3 & 1 \\ 1 & 5 & 21 \end{pmatrix}$ $\quad |A|=-1$

$$\text{Adj}(A)=\begin{pmatrix} \begin{vmatrix} -3 & 1 \\ 5 & 21 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 1 & 21 \end{vmatrix} & \begin{vmatrix} 2 & -3 \\ 1 & 5 \end{vmatrix} \\ -\begin{vmatrix} -1 & 1 \\ 5 & 21 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 1 & 21 \end{vmatrix} & -\begin{vmatrix} -1 & 1 \\ 5 & 1 \end{vmatrix} \\ \begin{vmatrix} -1 & 2 \\ -3 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 1 & -1 \\ 2 & -3 \end{vmatrix} \end{pmatrix}=\begin{pmatrix} -68 & -41 & 13 \\ 31 & 19 & -6 \\ 5 & 3 & -1 \end{pmatrix} \therefore A^{-1}=\frac{1}{|A|}\begin{pmatrix} -68 & 31 & 5 \\ -41 & 19 & 3 \\ 13 & -6 & -1 \end{pmatrix}$$

$$A^{-1}=-1\begin{pmatrix} -68 & 31 & 5 \\ -41 & 19 & 3 \\ 13 & -6 & -1 \end{pmatrix}=\begin{pmatrix} 68 & -31 & -5 \\ 41 & -19 & -3 \\ -13 & 6 & 1 \end{pmatrix}$$

Q(18) If ${}^{n+2}P_r=2\times{}^{n+2}C_r$, $\dfrac{{}^{n}C_{r+1}}{{}^{n}C_r}=\dfrac{5}{3}$ find the value of ${}^{2n}C_{n-r}+{}^{n+3}P_{r-1}$

$$\because {}^{n+2}P_r=2\times{}^{n+2}C_r \quad \therefore {}^{n+2}P_r=2\times\frac{{}^{n+2}P_r}{\lfloor r} \quad \therefore \lfloor r=2 \quad \therefore r=2$$

$$\frac{{}^{n}C_3}{{}^{n}C_2}=\frac{n-3+1}{3}=\frac{5}{3} \quad \therefore n-2=5 \qquad \therefore n=7$$

$${}^{2n}C_{n-r}+{}^{n+3}P_{r-1}={}^{14}C_5+{}^{10}P_1=2002+10=2012$$

Q(19) Find $Z=\frac{-8}{1+\sqrt{3}i}$ where $i^2=-1$ in the trigonometric form then find the two square roots of the number Z in the exponential form

$$Z=\frac{-8}{1+\sqrt{3}i}\times\frac{1-\sqrt{3}i}{1-\sqrt{3}i}=\frac{-8\left(1+\sqrt{3}i\right)}{1-\left(\sqrt{3}i\right)^2}=\frac{-8\left(1-\sqrt{3}i\right)}{1+3}=-2\left(1-\sqrt{3}i\right)=-2+2\sqrt{3}i$$

$X=-2$, $Y=2\sqrt{3}$, $r=\sqrt{X^2+Y^2}=\sqrt{2^2+\left(2\sqrt{3}\right)^2}=4$ $\tan\theta=\frac{Y}{X}=\sqrt{3}$ $(-,+)$

θ in the 2^{nd} $\therefore\theta=180^\circ-60^\circ=120^\circ$

$Z=4\left(\cos 120^\circ+i\sin 120^\circ\right)$

$$\sqrt{Z}=2\left(\cos\frac{120^\circ+2K\pi}{2}+i\sin\frac{120^\circ+2K\pi}{2}\right)$$

When $K=0$ $\therefore Z_1=2\left(\cos\frac{120^\circ}{2}+i\sin\frac{120^\circ}{2}\right)=2e^{\frac{\pi}{3}i}$

When K=-1 $2\left(\cos-120^\circ+i\sin-120^\circ\right)$

Q(20) The second , third and fourth terms in the expansion of $(X+a)^n$ according to the descending power of X are : 16,112,448 find the value of X , a , n

$\frac{T_3}{T_2}=\frac{112}{16}=7$ $\therefore$ $\frac{n-2+1}{2}\times\frac{a}{X}=7$ $\therefore\frac{n-1}{2}\times\frac{a}{X}=7\to(1)$

$\frac{T_4}{T_3}=\frac{448}{112}=4$ $\therefore$ $\frac{n-3+1}{3}\times\frac{a}{X}=4$ $\therefore\frac{n-2}{3}\times\frac{a}{X}=4\to(2)$

$\frac{n-1}{2}\times\frac{3}{n-2}=\frac{7}{4}$ $\therefore$ $12(n-1)=14(n-2)$ $\therefore 2n=16$ $\therefore$ $n=8$

$\frac{7}{2}\times\frac{a}{X}=7$ $\therefore\frac{a}{X}=2$ $\therefore$ $a=2X$, $T_2=16={}^8C_1\,(a)(X)^7$

$16=8\times 2X\times X^7$ $\therefore X^8=1$ $\therefore$ $X=\pm 1$

Answer the following questions 20 questions

From 1to 12 choose the correct answer

Q(1) If the X axis cut the sphere which center (3,-4,12) and its radius length 13cm at the two points A and B then AB equals

① 6units ② 8units

③ 24units ④ 26units

$(X-3)^2+16+144=13^2 \quad \therefore (X-3)^2=9 \therefore X=0 \text{ or } X=6$

$\therefore$ the two points A(0,0,0) and B(6,0,0) $\quad \therefore AB=6$

Q(2) $\omega^2\left(1-\frac{1}{\omega^2}+\omega^2\right)=\dots\dots\dots$

① -2 ② -2ω

③ 2 ④ 2ω

$\omega^2(1-\omega+\omega^2)=\omega^2\times -2\omega=-2\omega^3=-2$

Q(3) If $Z_1=2+2\sqrt{3}i$, $Z_2=-3-3\sqrt{3}\,i$ then arg $Z_1+Z_2=\dots\dots.$

① $-120°$ ② $135°$

③ $120°$ ④ $60°$

$Z_1+Z_2=-1-\sqrt{3}i$ 3rd quad $\theta=\tan^{-1}\sqrt{3}-\pi=-120°$

Q(4) If P and Q are the coefficients of X^n in the expansions of $(1+X)^{2n}$ and $(1+X)^{2n-1}$ then

① $P=Q$ ② $P=2Q$

③ $2P=Q$ ④ $P+Q=0$

$P={}^{2n}C_n$ and $Q={}^{2n-1}C_n \therefore \frac{P}{Q}=2$

Q(5) If the perpendicular distance between the two planes
$3X - 2Y + Z = 1$ and $6X - 4Y + 2Z = K$ is $\frac{3}{2\sqrt{14}}$ then K=

① 5,-1 ② 5,-1
③ -5,1 ④ -5,-1

Two planes are parallel $\frac{|2-K|}{\sqrt{36+16+4}} = \frac{3}{2\sqrt{14}}$ $\therefore |K-2| = 3$

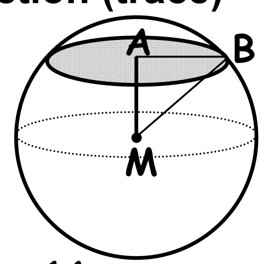
5,-1

Q(6) If C(-1,6,-5) mid point of $\overline{AB}$,A(K-2,-1,m+3) , B(2 , n-7 , -2)
then K+ m -n=...........

① 32 ② -33
③ -35 ④ -34

-2-11-20=-33

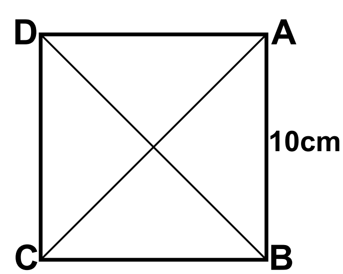
Q(7) If the plane 2x - 2y +Z =5 intersect the sphere
$(x-2)^2 + (y-3)^2 + (z+2)^2 = 25$, find the area of the cross section (trace)



① 4π ② 8π
③ 9π ④ 16π

$MA = \frac{|2(2) - 2(3) - 5|}{\sqrt{4+4+1}} = 3$ $\therefore AB = \sqrt{5^2 - 3^2} = 4$ $\therefore Area = 16\pi$

Q(8)ABCD is a square , the length of whose side is 10cm . the scalar product of the two vectors $\overrightarrow{BD}$, $\overrightarrow{BA}$



① 100 ② 200
③ $100\sqrt{2}$ ④ $50\sqrt{2}$

$BD \bullet BA = 10\sqrt{2} \times 10 \times \cos 45^\circ = 100$

Q(9) If the vector A makes angles of measure α , β , θ with X , Y and Z axes then $\sin^2\alpha + \sin^2\beta + \sin^2\theta = \dots$

① 1 ② 2

③ -1 ④ -2

$$1 - \cos^2\alpha + 1 - \cos^2\beta + 1 - \cos^2\theta = 3 - \left(\cos^2\alpha + \cos^2\beta + \cos^2\theta\right) = 2$$

Q(10) How many ways we can put(distribute) 10 identical balls into 6 distinct bins

① 151200 ② 3003

③ 210 ④ 3000

${}^{6+10-1}C_{10} = 3003$ ${}^{n+r-1}C_r$ $n = 6$, $r = 10$

Q(11) $\left(a + b\omega + a\omega^2\right)\left(a + b\omega^2 + a\omega^4\right)$

① 1 ② $b^2 - a^2$

③ a-b ④ $(a-b)^2$

$$(b\omega - a\omega)\left(b\omega^2 - a\omega^2\right) = (b-a)^2$$

Q(12) The least positive integer n which makes
${}^{n-1}C_5 + {}^{n-1}C_6 < {}^{n}C_7$ Is ....

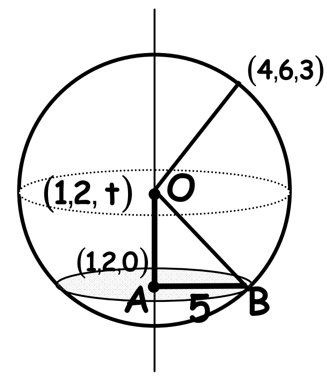
① 13 ② 10

③ 14 ④ 15

$\because {}^{n-1}C_5 + {}^{n-1}C_6 < {}^{n}C_7$ $\therefore {}^{n}C_6 < {}^{n}C_7$ $\therefore \dfrac{\lfloor n}{\lfloor 6 \lfloor n-6} < \dfrac{\lfloor n}{\lfloor 7 \lfloor n-7}$

$\therefore \dfrac{1}{n-6} < \dfrac{1}{7}$ $\therefore 7 < n - 6$ $\therefore n > 13$ $\therefore n = 14$

Q(13) A sphere goes through the point (4,6,3) and meets the (XY) plane in a circle whose centre is at the point (1,2,0) and radius is 5 find its equation



$(OA)^2 + (AB)^2 = (OB)^2 \quad \therefore t^2 + 5^2 = (4-1)^2 + (6-2)^2 + (t-3)^2$

$\therefore t^2 + 5^2 = 3^2 + 4^2 + (t-3)^2 \quad \therefore -6t + 9 = 0$

$\therefore t = \frac{3}{2} \quad \therefore \text{centre} = \left(1,2,\frac{3}{2}\right) \therefore r^2 = 3^2 + 4^2 + 2.25 = \frac{109}{4}$

$(X-1)^2 + (Y-2)^2 + \left(Z - \frac{3}{2}\right)^2 = \frac{109}{4}$

$X^2 - 2X + 1 + Y^2 - 4Y + 4 + Z^2 - 3Z + \frac{9}{4} = \frac{109}{4}$

$X^2 - 2X + Y^2 - 4Y + Z^2 - 3Z - 20 = 0$

Q(14) Use the multiplicative inverse to solve the set of following equations $X - 2Y + 2Z = 2$, $3X + 4Z = 10$, $6Z - Y = 5$
Where X and Y and Z not equals zeros

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \frac{1}{11}\begin{pmatrix} 1 & -2 & 2 \\ 3 & 0 & 4 \\ 0 & -1 & 6 \end{pmatrix}\begin{pmatrix} 2 \\ 10 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

Q(15) prove that: $\begin{vmatrix} 1 & 1 & 1 \\ L & m & n \\ L^2 & m^2 & n^2 \end{vmatrix} = (L-m)(m-n)(n-L)$

$c_2 - c_1 \, , c_3 - c_1$

$$\begin{vmatrix} 1 & 0 & 0 \\ L & m-L & n-L \\ L^2 & m^2 - L^2 & n^2 - L^2 \end{vmatrix} = (m-L)(n-L)\begin{vmatrix} 1 & 0 & 0 \\ L & 1 & 1 \\ L^2 & m+L & n+L \end{vmatrix}$$

$c_3 - c_2$

$$\therefore (m-L)(n-L)\begin{vmatrix} 1 & 0 & 0 \\ L & 1 & 0 \\ L^2 & m+L & n-m \end{vmatrix} = (m-L)(n-L)(n-m)$$

Q(16) **A line passing through the origin with the direction cosines $\frac{2}{7}, \frac{-3}{7}, \frac{6}{7}$ intersect the plane 3X+5Y+2Z-6=0 at the point B find the length of OB**

$r = (0,0,0) + t(2,-3,6) = (2t,-3t,6t)$

$\therefore 3(2t) + 5(-3t) + 2(6t) - 6 = 0$

$6t - 15t + 12t - 6 = 0 \quad \therefore 3t = 6 \quad \therefore t = 2$

point of intersection(B) $= (4,-6,12)$

$OB = \sqrt{16 + 36 + 144} = \sqrt{196} = 14$

Q(17) **Find the equation of the line of intersection of the two planes X + 2Y – 2Z = 1 , 2 X + Y – 3Z = 5**

First method:

$$\begin{array}{l} -2X - 4Y + 4Z = -2 \\ \underline{\;2X + \;\; Y - 3Z = 5} \\ -3Y + Z = 3 \end{array}$$

put $Y = 0 \quad \therefore Z = 3 \quad \therefore X = 7 \quad \therefore P_1 = (7,0,3)$

Put $Z = 0 \quad \therefore Y = -1 \quad \therefore X = 3 \quad \therefore P_2 = (3,-1,0)$

$r = (7,0,3) - (3,-1,0) = (4,1,3)$

Equation : $(7,0,3) + t(4,1,3) \qquad \therefore \frac{X-7}{4} = \frac{Y}{1} = \frac{Z-3}{3}$

Q(18) If ${}^{n+1}P_{r+1} : {}^{n+1}C_{r+1} = 720$, ${}^{n}C_{r-2} + {}^{n}C_{r-3} = 56$ find the value of n , r

${}^{n+1}P_{r+1} \div \frac{{}^{n+1}P_{r+1}}{\lfloor r+1} = 720 \qquad \therefore \lfloor r+1 = 720 \quad \therefore r + 1 = 6 \quad \therefore r = 5$

${}^{n}C_{r-2} + {}^{n}C_{r-3} = {}^{n}C_{3} + {}^{n}C_{2} = {}^{n+1}C_{3} = 56 = {}^{8}C_{3} \quad \therefore \; n + 1 = 8 \quad \therefore n = 7$

Q(19) Find in the trigonometric and exponential forms the roots of the equation $Z^4 = 8(1 - \sqrt{3}\,i)$ then write the solution set

$X = 8$, $Y = -8\sqrt{3}$ $\therefore r = \sqrt{8^2 + (-8\sqrt{3})^2} = 16$

, $\tan\theta = \sqrt{3}$ in the 4^{th} quadrant $\therefore \theta = -\frac{\pi}{3}$

$$\therefore Z^4 = 16\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$$

$$\therefore Z = 2\left(\cos\frac{-\frac{\pi}{3} + 2\pi r}{4} + i\sin\frac{-\frac{\pi}{3} + 2\pi r}{4}\right)$$

When r=0 then $Z_1 = 2\left(\cos\frac{-\pi}{12} + i\sin\frac{-\pi}{12}\right) = 2e^{\frac{-\pi}{12}i}$

When r=1 then $Z_2 = 2\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right) = 2e^{\frac{5\pi}{12}i}$

When r=-1 then $Z_3 = 2\left(\cos\frac{-7\pi}{12} + i\sin\frac{-7\pi}{12}\right) = 2e^{\frac{-7\pi}{12}i}$

When r=2 then $Z_4 = 2\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right) = 2e^{\frac{11\pi}{12}i}$

S.S={ $2e^{\frac{-\pi}{12}i}, 2e^{\frac{5\pi}{12}i}, 2e^{\frac{-7\pi}{12}i}, 2e^{\frac{11\pi}{12}i}$ }

Q(20) In the expansion: $\left(X^2 - \frac{1}{X}\right)^9$ Find ❶general term

❷The term free of X

To get the General term:

$$T_{r+1} = {}^9C_r\left(-X^{-1}\right)^r \times \left(X^2\right)^{9-r} = {}^9C_r(-1)^r X^{-r} \times X^{18-2r} = {}^9C_r(-1)^r \times X^{18-3r}$$

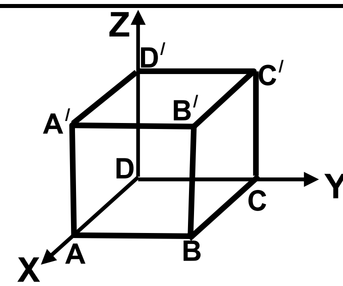
<u>To get the term free of X</u>

$18 - 3r = 0 \quad \therefore \quad 3r = 18 \quad \therefore r = 6 \qquad \therefore T_7 = \underline{{}^9C_6(-1)^6} \times X^{18-3\times6} = 84$

Answer the following questions 20 questions

From 1to 12 choose the correct answer

Q(1) In the opposite figure, A B C D A B C D is a cube of side length 2 units , then find $\overrightarrow{AB'} \bullet \overrightarrow{BD}$



① $\frac{1}{2}$ ② 1

③ $-\frac{1}{2}$ ④ -1

$AB' = (2,2,2) - (2,0,0) = (0,2,2)$ $BD = (0,0,0) - (2,2,0) = (-2,-2,0)$

$$\frac{(0,2,2)\bullet(-2,-2,0)}{\sqrt{0+4+4}\times\sqrt{4+4+4}} = \frac{-4}{8} = -\frac{1}{2}$$

Q(2) If Z is a complex number of unite modulus and argument θ then the argument of $\frac{1+Z}{1+\overline{Z}}$ is..............................

① $\frac{\pi}{2} - \theta$ ② $-\theta$

③ θ ④ $\theta - \pi$

$\frac{1+Z}{1+\overline{Z}} = \frac{1+Z}{1+\frac{1}{Z}} = Z \;\therefore$ argument is θ

Q(3) The amplitude of $\left(\frac{(\sqrt{3}+i)^{4n+1}}{(1-i\sqrt{3})^{4n}}\right) = \ldots.$

① $\frac{\pi}{3}$ ② $\frac{2\pi}{3}$

③ $\frac{\pi}{6}$ ④ $\frac{5\pi}{6}$

$\left(\frac{\sqrt{3}+i}{1-i\sqrt{3}}\right)\times(\sqrt{3}+i) = (i)^{4n}\times(\sqrt{3}+i) = \sqrt{3}+i$ $\therefore$ amplitude $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

Q(4) The co-efficient of X^3 in $(1 - X + X^2)^5$ is

① -30 ② -20

③ -10 ④ 30

$= (1 + X(X-1))^5 = {}^5C_0 + {}^5C_1X(X-1) + {}^5C_2X^2(X-1)^2 + {}^5C_3X^3(X-1)^3$

$= -2\,{}^5C_2 - {}^5C_3 = -20 - 10 = -30$

Q(5)
If θ is the measure of the angle included between
A = (2, 0, 2) , B = (0, 0, 4), then θ = ...

① 90° ② 60°

③ 45° ④ 30°

$$\cos\theta = \frac{(2,0,2)\bullet(0,0,4)}{\sqrt{2^2+0^2+2^2}\times\sqrt{0^2+0^2+4^2}} = \frac{8}{8\sqrt{2}} = \quad \therefore \theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

Q(6) If the point C(2,2,6) is the mid point of $\overline{AB}$ where $A(1,-4,0)$
Then the point B

① (1,10,11) ② (3,8,12)

③ (3,12,12) ④ (2,10,12)

B= $(2\times2-1, 2\times2+4, 2\times6-0) = (3,8,12)$

Q(7) If the vector A makes angles of measure α , β , θ with X , Y and Z axes then $\cos 2\alpha + \cos 2\beta + \cos 2\theta = ...$

① 1 ② 2

③ -1 ④ -2

$$2\cos^2\alpha - 1 + 2\cos^2\beta - 1 + 2\cos^2\theta - 1 = 2\left(\cos^2\alpha + 2\cos^2\beta + 2\cos^2\theta\right) - 3 =$$
$$= 2\times 1 - 3 = -1$$

Q(8) If $L_1 : \frac{X+2}{-1} = \frac{Y+3}{3} = \frac{Z+5}{2}$ is perpendicular to the line $L_2 : \frac{X}{2} = \frac{Y-5}{K} = \frac{Z-6}{m}$ then the value of 3K+2m=....

① -1 ② 2

③ 0 ④ 4

$(-1,3,2)\bullet(2,K,m) = 0 \quad \therefore 3K + 2m = 2$

Q(9) If A = (-1, 3, 4) , B (0, -2, 5) , then $\|\overrightarrow{AB}\|$ =................

① $3\sqrt{3}$ ② $4\sqrt{3}$

③ $3\sqrt{2}$ ④ $5\sqrt{3}$

$AB = B - A = (0,-2,5) - (-1,3,4)$ $\therefore \|\overrightarrow{AB}\| = \sqrt{(-1)^2 + (5)^2 + (-1)^2} = 3\sqrt{3}$

Q(10) How many triangles can be formed by joining the vertices of an octagon

① 10 ② 336

③ 8 ④ 56

$^8C_3 = 56$

Q(11) $\left(1 + 2\omega^5 + \frac{1}{\omega^2}\right)\left(1 + 2\omega + \frac{1}{\omega^4}\right) =$

① 0 ② -1

③ 1 ④ 2

$(1 + 2\omega^2 + \omega)(1 + 2\omega + \omega^2) = \omega^2 \times \omega = \omega^3 = 1$

Q(12) If $^nC_r = {}^nC_{r-1}$ and $^nP_r = {}^nP_{r+1}$ then the value of n is

① 3 ② 5

③ 4 ④ 2

$r + r - 1 = n \quad \therefore r = \frac{n+1}{2} \quad \because {}^nP_r = {}^nP_{r+1} \therefore \frac{\lfloor n}{\lfloor n-r} = \frac{\lfloor n}{\lfloor n-r-1} \quad \therefore n - r = 1$

$\therefore n - \frac{n+1}{2} = 1 \quad \therefore 2n - n - 1 = 2 \quad \therefore n - 1 = 2 \quad \therefore n = 3$

Q(13) If The two sphere $(X-3)^2+Y^2+(Z-3)^2=16$,
$(X+1)^2+(Y-4)^2+(Z-K)^2=25$ Touch each other find the value of K

$C_1=(3,0,3)\quad C_2=(-1,4,K)\quad r_1=\sqrt{16}=4\qquad r_1=\sqrt{25}=5$

$\therefore\sqrt{4^2+4^2+(K-3)^2}=5+4=9$

$\therefore\sqrt{32+(K-3)^2}=9$

$(K-3)^2+32=81\quad\therefore(K-3)^2=49$

$K-3=7\rightarrow K=10$ or $K-3=-7\ \therefore K=-4$

Q(14) Solve the following equations X + 3Y + 2Z = 13 , 2X - Y +Z = 3 , 3X + Y - Z = 2 using the multiplicative inverse of the matrix

$$\begin{pmatrix}X\\Y\\Z\end{pmatrix}=\frac{1}{25}\begin{pmatrix}0&5&5\\5&-7&3\\5&8&-7\end{pmatrix}\begin{pmatrix}13\\3\\2\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}\qquad\therefore\begin{pmatrix}X\\Y\\Z\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}$$

Q(15) prove that $\begin{vmatrix}a+x&y&z\\x&a+y&z\\x&y&a+z\end{vmatrix}=a^2(a+x+y+z)$

$$c_1+c_2+c_2\begin{vmatrix}a+x+y+z&y&z\\a+x+y+z&a+y&z\\a+x+y+z&y&a+z\end{vmatrix}$$

$$=(a+x+y+z)\begin{vmatrix}1&y&z\\1&a+y&z\\1&y&a+z\end{vmatrix}\ r_2-r_1\ ,r_3-r_1$$

$$(a+x+y+z)\begin{vmatrix}1&y&z\\0&a&0\\0&0&a\end{vmatrix}=a^2(a+x+y+z)$$

Q(16) Find the equation of the sphere has its center at the point (5,-2,3) and touch the plane 3X+2Y+Z=0

Radius $= \dfrac{|3\times 5+2\times -2+3|}{\sqrt{3^2+(2)^2+1^2}} = \sqrt{14}$

Equation $(X-5)^2+(Y+2)^2+(Z-3)^2 = 14$

Q(17) Find the equation of a plane which bisects perpendiculary the line joining the points (2,3,4) and (4,5,8)

Mid point $\left(\dfrac{2+4}{2}, \dfrac{3+5}{2}, \dfrac{4+8}{2}\right) = (3,4,6)$

$d = (4-2, 5-3, 8-4) = (2,2,4) = (1,1,2)$ is normal to the required plane

$n \bullet r = n \bullet A \quad \therefore (1,1,2)\bullet(X,Y,Z) = (1,1,2)\bullet(3,4,6)$

$X+Y+2Z = 3+4+12 = 19$

Q(18)In the expansion $(1+X)^5$ $8T_2$, $8T_3$, $7T_4$ in arithmatic sequence find X

$2(8T_3) = 7T_4 + 8T_2 \rightarrow$ dividing by $T_3 \quad \therefore 16 = 8\dfrac{T_2}{T_3} + 7\times\dfrac{T_4}{T_3}$

$16 = 8\times\dfrac{2}{5-2+1}\times\dfrac{1}{X} + 7\times\dfrac{5-3+1}{3}\times X$

$16 = \dfrac{4}{X} + 7X \therefore \rightarrow \times X \qquad \therefore 7X^2 - 16X + 4 = 0 \quad \therefore X = \dfrac{2}{7}$, or $X = 2$

(19) **Find in the trigonometric and exponential forms the roots of the equation $Z^4 = 8\left(1-\sqrt{3}\,i\right)$ then write the solution set**

$X = 8$, $Y = -8\sqrt{3}$ $\therefore r = \sqrt{8^2 + \left(-8\sqrt{3}\right)^2} = 16$

, $\tan\theta = \sqrt{3}$ in the 4^{th} quadrant $\therefore \theta = -\frac{\pi}{3}$

$$\therefore Z^4 = 16\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)$$

$$\therefore Z = 2\left(\cos\frac{-\frac{\pi}{3}+2\pi r}{4} + i\sin\frac{-\frac{\pi}{3}+2\pi r}{4}\right)$$

When r=0 then $Z_1 = 2\left(\cos\frac{-\pi}{12} + i\sin\frac{-\pi}{12}\right) = 2e^{\frac{-\pi}{12}i}$

When r=1 then $Z_2 = 2\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right) = 2e^{\frac{5\pi}{12}i}$

When r=-1 then $Z_3 = 2\left(\cos\frac{-7\pi}{12} + i\sin\frac{-7\pi}{12}\right) = 2e^{\frac{-7\pi}{12}i}$

When r=2 then $Z_4 = 2\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right) = 2e^{\frac{11\pi}{12}i}$

S.S={ $2e^{\frac{-\pi}{12}i}$, $2e^{\frac{5\pi}{12}i}$, $2e^{\frac{-7\pi}{12}i}$, $2e^{\frac{11\pi}{12}i}$ }

Q(20) If $^nC_r : {}^nC_{r+1} : {}^nC_{r+2} = 5:10:14$ find the values of n and r

$$\frac{^nC_{r+1}}{^nC_r} = \frac{n-(r+1)+1}{r+1} = \frac{10}{5} = 2 \quad \therefore \frac{n-r}{r+1} = 2 \quad \therefore n-3r-2=0 \quad \therefore n-3r=2$$

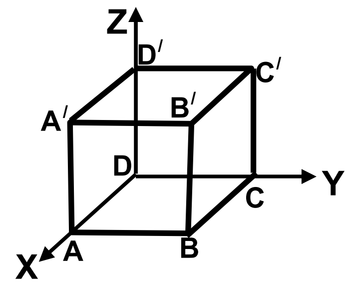
$$\frac{^nC_{r+2}}{^nC_{r+1}} = \frac{n-(r+2)+1}{r+2} = \frac{14}{10} = \quad \therefore \frac{n-r-1}{r+2} = \frac{7}{5} \qquad \therefore 5n-12r-19=0$$

$r = 3$, $n = 11$

Answer the following questions 20 questions

From 1to 12 choose the correct answer

Q(1) In the opposite figure , A B C D A\`B\`C\`D\` is a cuboid A (4, 0, 0) , C (0, 9, 0) , $D^{/}$ (0, 0, 7) , then find || $\overrightarrow{AC`}$ || =



① $\sqrt{114}$ ② 5

③ $\sqrt{146}$ ④ $2\sqrt{5}$

$$AC^{/} = C^{/} - A = (0,9,7) - (4,0,0) = (-4,9,7)$$
$$\therefore \left\| AC^{/} \right\| = \sqrt{16+81+49} = \sqrt{146}$$

Q(2) $\left[\dfrac{-1+\sqrt{-3}}{2}\right]^{3n} + \left[\dfrac{-1-\sqrt{-3}}{2}\right]^{3n} = \ldots\ldots$

① 0 ② 1

③ 2 ④ 3

$$(\omega)^{3n} + (\omega^2)^{3n} = 1 + 1 = 2$$

Q(3) If $Z = \left(-\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i\right)^n$ where n is a positive integer and Z=1 then the least value of n =

① 1 ② 3

③ 6 ④ 9

N=3

Q(4) If $X^2 + Y^2 + Z^2 + 6X - 4Y + 10Z - 8 = 0$ is equation of a sphere its centre Is M then M is ....

① (-3,2,-5) ② (4,-2,-5)

③ (-3,-2,-5) ④ (3,2,5)

(-3,2,-5)

Q(5)
If $\overrightarrow{A} = (1, -2, 1)$, $\overrightarrow{B} = (-2, 1, 2)$,then the component of $\overrightarrow{A}$ in the direction of B = ...

① $\frac{-2}{3}$ ② $\frac{4}{5}$

③ $\frac{1}{3}$ ④ $\frac{2}{3}$

$$\frac{A \bullet B}{\|B\|} = \frac{(1,-2,1)\bullet(-2,1,2)}{\sqrt{(-2)^2+1^2+2^2}} = -\frac{2}{3}$$

Q(6) If A=$(2\cos\theta, \log_5 X, \sin\theta)$ and A=$(\cos\theta, \log_3 27, 2\sin\theta)$ and $A \bullet B = 11$
Then X=

① 25 ② 500

③ 125 ④ 625

$$2\cos^2\theta + \log_5 X \times \log_3 27 + 2\sin^2\theta = 11$$

$$\therefore 2\cos^2\theta + 2\sin^2\theta + \log_5 X \times \log_3 27 = 11 \quad \therefore 2 + \log_5 X \times \log_3 27 = 11$$

$$\log_5 X \times \log_3 27 = 9 \quad \therefore \log_5 X = 3 \quad \therefore X = 5^3 = 125$$

Q(7) The rank of the matrix $A\begin{pmatrix} 0 & -2 & 3 \\ -2 & 4 & -6 \\ 3 & -6 & 9 \end{pmatrix}$ equals

① 3 ② 2

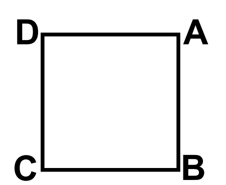
③ 1 ④ zero

Rank (2)

Q(8) ABCD is a square of side length =8cm then
$\overrightarrow{AB} \odot \overrightarrow{CD}$ =

① 0 ② -64

③ 64 ④ -8



Q(9) The vector A = 3 i + j -2 k makes an angle of measure with the +ve direction of x-axis.

① $\sin^{-1}\frac{3}{\sqrt{14}}$ ② $\sin^{-1}\frac{1}{\sqrt{14}}$

③ $\cos^{-1}\frac{3}{\sqrt{14}}$ ④ $\cos^{-1}\frac{1}{\sqrt{14}}$

$\|A\| = \sqrt{3^2 + 1^2 + (-2)^2} = \sqrt{14} \;\therefore \theta = \cos^{-1}\frac{3}{\sqrt{14}}$

Q(10) Find the number of diagonals of a decagon?

① 45 ② 25

③ 35 ④ 55

${}^{10}C_2 - 10 = 35$

Q(11) $\left(\frac{a - d\omega}{a\omega^2 - d} - \omega^2\right)^2 =$ ....

① 3i ② -3

③ $\pm\sqrt{3}i$ ④ 3

$\frac{a\omega^3 - d\omega}{a\omega^2 - d} - \omega^2 = \frac{\omega^2(a\omega^2 - d)}{(a\omega^2 - d)} - \omega^2 = (\omega^2 - \omega)^2 = (\pm\sqrt{3}i)^2 = -3$

Q(12) If the plane 2X-Y-Z+12=0 intersect the sphere $(X+3)^2 + (Y+2)^2 + (Z-1)^2 = 29$ then the area of the cross section equal ....units square

① 2π ② 8π

③ 4π ④ 25π

$\perp$ length from centre to the plane $\frac{|2\times -3 - 1\times -2 - 2\times 1|}{\sqrt{2^2 + 1 + 1}} = 2$

$r = \sqrt{29 - 4} = 5 \quad \therefore \text{area} = \pi r^2 = 25\pi$

Q(13)The force F has a magnitude 200N and acts with the octant shown Express F as a Cartesian vector

$\because (\cos 60^\circ)^2 + (\cos 45^\circ)^2 + (\cos\theta_Z)^2 = 1 \quad \therefore \frac{1}{4} + \frac{1}{2} + (\cos\theta_Z)^2 = 1$

$\therefore (\cos\theta_Z)^2 = \frac{1}{4} \quad \therefore \cos\theta_Z = \pm\frac{1}{2} \quad \therefore \theta_Z = 60^\circ$

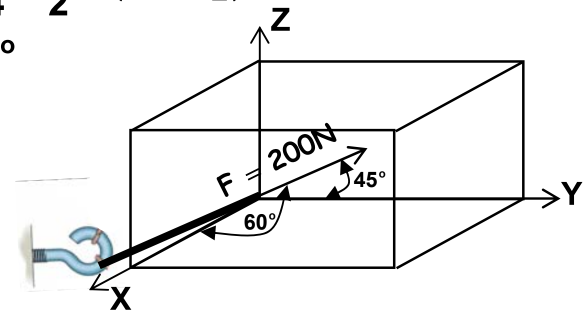
$F = \|F\|\cos\theta_X + \|F\|\cos\theta_Y + \|F\|\cos\theta_Z$

$= 200\cos 60^\circ\, i + 200\cos 45^\circ\, j + 200\cos 60^\circ\, K$

$= 100\, i + 100\sqrt{2}\, j + 100\, K$

To make sure that you are right

$\sqrt{100^2 + (100\sqrt{2})^2 + (100)^2} = 200$



Q(14) Find the equation of the straight line passing the point (3,-1,0) and intersect the line $r = (2,1,1) + t(1,2,-1)$ orthogonally

$BA = B - A = (1-t\ ,-2-2t\ ,-1+t)$

$AB \bullet d = 0$

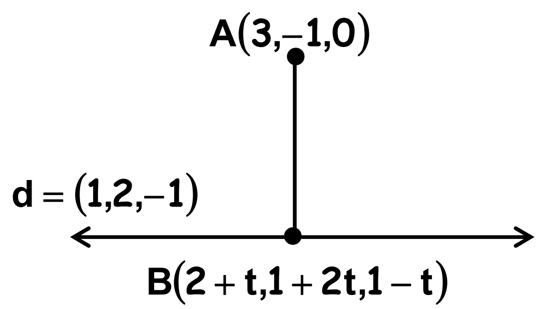
$(1-t\ ,-2-2t\ ,-1+t) \bullet (1,2,-1) = 0$

$1 - t - 4 - 4t + 1 - t = 0$

$t = -\frac{1}{3}$

$\therefore BA = \left(-\frac{4}{3}, \frac{4}{3}, \frac{4}{3}\right)$

$\therefore$ equation $r = (3,-1,0) + t\left(-\frac{4}{3}, \frac{4}{3}, \frac{4}{3}\right)$

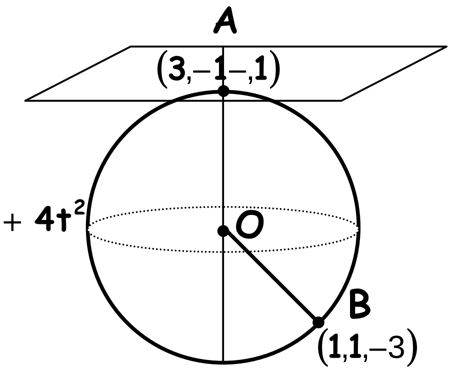


Q(15) Using the properties of the determinant prove that $\begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix} = (x+2)(x-1)^2$

$C_1 + C_2 + C_3 \quad \therefore \begin{vmatrix} X+2 & 1 & 1 \\ X+2 & X & 1 \\ X+2 & 1 & X \end{vmatrix} = (X+2)\begin{vmatrix} 1 & 1 & 1 \\ 1 & X & 1 \\ 1 & 1 & X \end{vmatrix} \quad r_2 - r_1, r_3 - r_1$

$(X+2)\begin{vmatrix} 1 & 1 & 1 \\ 0 & X-1 & 0 \\ 0 & 0 & X-1 \end{vmatrix} = (X+2)(X-1)^2$

Q(16) Sphere is tangent to the plane $X - 2y - 2Z = 7$ in the point (3,-1,-1) and goes through the point (1,1,-3) find its equation



$O = (3,-1,-1) + t(1,-2,-2)$
$= (3 + t, -1 - 2t, -1 - 2t)$
$OB = OA$
$(2 + t)^2 + (-2 - 2t)^2 + (2 - 2t)^2 = t^2 + 4t^2 + 4t^2$
$4 + t^2 + 4t + 4 + 4t^2 + 8t + 4 + 4t^2 - 8t = t^2 + 4t^2 + 4t^2$
$4t + 12 = 0 \quad \therefore t = -3$
$\therefore O = (0,5,5) \quad \therefore r = \sqrt{9 + 6^2 + 6^2} = 9$
$\therefore X^2 + Y^2 + Z^2 - 10Y - 10Z - 31 = 0$

Q(17) Solve the following equations 2X + Y – 2Z = 10 , X + 2Y + 2Z = 1, 5X + 4Y + 3Z = 6 using the multiplicative inverse of the matrix

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \frac{1}{15}\begin{pmatrix} -2 & -11 & 6 \\ 7 & 16 & -6 \\ -6 & -3 & 3 \end{pmatrix}\begin{pmatrix} 13 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{10}{3} \\ -3 \end{pmatrix}$$

Q(18) let the two middle terms in the expansion $\left(X - \frac{1}{2X^2}\right)^7$ are a ,b find the value of $aX^2 + 4bX^5$

$$a = T_4 = {}^7C_3\left(-\frac{1}{2}X^{-2}\right)^3 (X)^4 = -{}^7C_3 \times \frac{1}{8}X^{-2}$$

$$b = T_5 = {}^7C_4\left(-\frac{1}{2}X^{-2}\right)^4 (X)^3 = {}^7C_4 \times \frac{1}{16}X^{-5}$$

$$aX^2 + 4bX^5 = -{}^7C_3 \times \frac{1}{8}X^{-2} \times X^2 + 4 \times {}^7C_4 \times \frac{1}{16}X^{-5} \times X^5 = \frac{35}{8}$$

Q(19) Put the number : $Z=\dfrac{\sqrt{3}+i}{\sqrt{3}-i}$, $i^2=-1$ in the exponential form hence find the cubic roots of the number Z

$$Z=\frac{\sqrt{3}+i}{\sqrt{3}-i}\times\frac{\sqrt{3}+i}{\sqrt{3}+i}=\frac{3+\sqrt{3}i+\sqrt{3}i+i^2}{3-i^2}=\frac{2+2\sqrt{3}i}{4}=\frac{1}{2}+\frac{\sqrt{3}}{2}i$$

$$X=\frac{1}{2}\ ,\ Y=\frac{\sqrt{3}}{2}\ ,\ r=\sqrt{\frac{1}{4}+\frac{3}{4}}=1\ ,\ \tan\theta=\frac{Y}{X}=\sqrt{3}\quad \therefore \theta=60^\circ\ \therefore Z=\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}$$

$$\sqrt[3]{Z}=\left(\cos\frac{60^\circ+2K\pi}{3}+i\sin\frac{60^\circ+2K\pi}{3}\right)$$

$$\text{When } K=0\ \therefore Z_1=\left(\cos\frac{60^\circ}{3}+i\sin\frac{60^\circ}{3}\right)=e^{\frac{\pi}{9}i}$$

$$\text{When } K=1\ \therefore Z_2=\left(\cos\frac{60^\circ+2\times1\times180}{3}+i\sin\frac{60^\circ+2\times1\times180}{3}\right)=e^{\frac{7\pi}{9}i}$$

when $K=-1$

Q(20) In the expansion $\left(2X+\dfrac{3}{X^2}\right)^n$ the ninth and the tenth terms are equal and the ratio between the sixth term and the seventh term is 8:15 find the value of n and prove that there is no term free of X in this expansion

$$\frac{T_{10}}{T_9}=\frac{n-9+1}{9}\times\frac{\frac{3}{X^2}}{2X}=1\qquad \therefore \frac{n-8}{9}\times\frac{3}{2X^3}=1\rightarrow(1)$$

$$\frac{T_7}{T_6}=\frac{n-6+1}{6}\times\frac{\frac{3}{X^2}}{2X}=\frac{15}{8}\qquad \therefore \frac{n-5}{6}\times\frac{3}{2X^3}=\frac{15}{8}\rightarrow(2)$$

dividing 1 , 2 $\quad\therefore\ \dfrac{n-8}{9}\times\dfrac{6}{n-5}=\dfrac{8}{15}\ \therefore\rightarrow\ \dfrac{n-8}{n-5}=\dfrac{4}{5}$

$5n-40=4n-20 \qquad\qquad \therefore n=20$

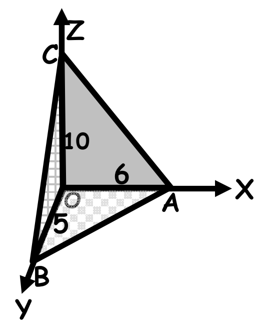
$$T_{r+1}={}^{20}c_r\left(3X^{-2}\right)^r(2X)^{20-r}={}^{20}c_r(3)^r(2)^{20-r}X^{20-3r}$$

$20-3r=0 \qquad$ when $r=\dfrac{20}{3}\notin Z^+\quad \therefore$ no rterm free of X

Answer the following questions 20 questions

From 1to 12 choose the correct answer

Q(1)If the plane $10X + 12Y + 6Z = 60$ intersected with the axes X,Y and Z at the points A ,B and C respectively then the volume of the solid ABCO where O is the origin point equals …..cube unit



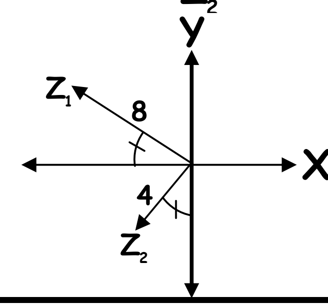
① 20 ② 30

③ 50 ④ other wises

Volume of pyramid $= \frac{1}{3}$ base area $\times$ height

$= \frac{1}{3}\left(\frac{1}{2} \times 5 \times 6\right) \times 10 = 50$

Q(2) In the opposite figure: Z_1, Z_2 are two complex numbers then $\frac{Z_1}{Z_2} =$



① 2 ② 2i

③ -2 ④ -2i

$$\frac{8(\cos 180^\circ - \theta + i\sin 180^\circ - \theta)}{4(\cos -90^\circ - \theta + i\sin -90^\circ - \theta)} = 2(\cos 270^\circ + i\sin 270^\circ) = -2i$$

Q(3) If Z_1, Z_2 are two complex numbers the amplitude of $(Z_1Z_2) = \frac{5\pi}{18}$

And the amplitude of $\left(\frac{Z_1}{Z_2}\right) = \frac{\pi}{9}$ then the amplitude of $Z_1 =$

① $\frac{7\pi}{36}$ ② $\frac{5\pi}{36}$

③ $\frac{\pi}{3}$ ④ $\frac{\pi}{4}$

$\theta_1 + \theta_2 = \frac{5\pi}{18}$ and $\theta_1 - \theta_2 = \frac{\pi}{9}$ $\therefore \theta_1 = 35^\circ = \frac{7\pi}{36}$

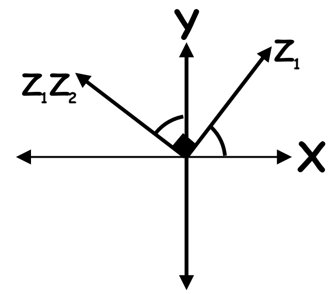
Q(4) If the number of terms in the expansion $(X + Y)^{2n-1}$ equals 12 terms then n=…..

① 5 ② 6

③ 7 ④ 8

$2n - 1 + 1 = 12 \quad \therefore 2n = 12 \quad \therefore n = 6$

Q(5) In the opposite figure: If Z_1 and Z_2 and Z_1Z_2 are complex numbers then $Z_2 =$



① $-2i$ ② $-i$

③ i ④ $2i$

$$Z_2 = \frac{Z_1Z_2}{Z_1} = \frac{re^{\left(\frac{1}{2}\pi+\theta\right)i}}{re^{\theta i}} = e^{\frac{1}{2}\pi i} = i$$

Q(6) The radius length of the sphere $X^2 + Y^2 + Z^2 - 2X - 6Y + 10Z - 1 = 0$ equals .... length unit

① 3 ② 4

③ 5 ④ 6

Center $=(1,3,-5)$ radius $= \sqrt{1^2 + 3^2 + (-5)^2 + 1} = 6$

Q(7) If A=(2, -1,3) and B=(-2,2,-9) then the length of $\overline{AB}$ equals

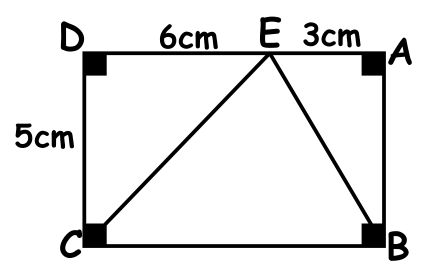
① 12 ② 13

③ 14 ④ 15

$$AB = \sqrt{(2+2)^2 + (-1-2)^2 + (3+9)^2} = 13$$

Q(8) In the opposite figure ABCD is a rectangle

$E \in \overline{AD}$ then $\overrightarrow{EB} \bullet \overrightarrow{EC} =$



① 7 ② 8

③ 9 ④ 10

$C = (0,0)$, $B = (9,0)$ $E = (6,5)$

$EB = B - E = (3,-5)$ $EC = C - E = (-6,-5)$

$EB \bullet EC = (3,-5) \bullet (-6,-5) = -18 + 25 = 7$

Q(9)

If $\overrightarrow{A}=(2,3,-4)$ and $\overrightarrow{B}=(4,2,m)$ and $\overrightarrow{A} \perp \overrightarrow{B}$ then m=…

① 1
② 2
③ 3
④ $\frac{7}{2}$

$A \bullet B = 0 \quad \therefore 2 \times 4 + 3 \times 2 - 4m = 0 \quad \therefore 8 + 6 - 4m = 0$

$\therefore 14 - 4m = 0 \qquad \therefore m = \frac{7}{2}$

Q(10) Number of ways that a team of six members can be formed from eight girls and six boys such that the team must include exaltedly 3 boys

① 2110
② 1120
③ 1008
④ 810

${}^{8}C_{3} \times {}^{6}C_{3} = 1120$

Q(11) $\sqrt{5+12i} =$ ………….

① $\pm(2+3i)$
② $\pm(3+2i)$
③ $\pm(2-3i)$
④ $\pm(3-2i)$

$\sqrt{5+12i} = X + iY \quad \therefore 5 + 12i = X^2 - Y^2 + 2XYi \quad \therefore 2XYi = 12 \therefore XY = 6$

$\therefore Y = \frac{6}{X} \qquad \therefore X^2 - Y^2 = 5 \qquad \therefore X^2 - \left(\frac{6}{X}\right)^2 = 5 \quad \therefore X^2 - \frac{36}{X^2} = 5$

$X^4 - 5X^2 - 36 = 0 \quad \therefore X = \pm 3 \quad , Y = \pm 2$

Q(12) If the side length of a triangle are $\frac{1}{2}\lfloor n$, $\lfloor n-2$ and $\lfloor 2-n$ cm

Then the numerical value of the area of the triangle =….cm^2

① $\sqrt{3}$
② $\frac{\sqrt{3}}{4}$
③ $\frac{\sqrt{3}}{2}$
④ $\frac{2\sqrt{3}}{3}$

$n-2 \geq 0 \quad ,2-n \geq 0 \quad \therefore n = 2$ or 3

if $n = 2$ sides are 1, 1, 1 area $= \frac{1}{2} \times 1 \times 1 \times \sin 60^\circ = \frac{\sqrt{3}}{4}$

Q(13) Find measure of the angle included between the straight line $L : \frac{X-3}{\sqrt{2}} = \frac{Y-1}{1} = \frac{-Z-2}{1}$ and the plane $\sqrt{2}X - Y - Z + 5 = 0$

$$\cos\theta = \frac{(\sqrt{2},1,1)\bullet(\sqrt{2},-1,-1)}{\sqrt{(\sqrt{2})^2+1^2+1^2}\times\sqrt{(\sqrt{2})^2+1^2+1^2}} = \frac{2-1-1}{} = 0$$

$\therefore \theta = 90^\circ$

Q(14) Find the standard form and the general form of the equation of the plane passing through point (3 , -5 , 2) and the vector n = (2 , 1, 1) is normal to the plane.

$$\boxed{n \bullet r = n \bullet A}$$

$$(2,1,1)\bullet[(X,Y,Z)-(3,-5,2)] = 0 \quad \therefore (2,1,1)\bullet(X-3,Y+5,Z-2) = 0$$

$$2(X-3)+1(Y+5)+1(Z-2) = 0 \quad \therefore 2X - 6 + Y + 5 + Z - 2 = 0$$

$$2X + Y + Z - 3 = 0$$

Q(15) Without expanding the determinant prove: $\begin{vmatrix} 1 & 4 & 1 \\ 0 & -1 & 2 \\ 4 & 18 & 0 \end{vmatrix} = 0$

$r_3 + 2r_2 \quad \therefore \begin{vmatrix} 1 & 4 & 1 \\ 0 & -1 & 2 \\ 4 & 16 & 4 \end{vmatrix}$ take 4 common factor from r_3

$$\therefore 4\begin{vmatrix} 1 & 4 & 1 \\ 0 & -1 & 2 \\ 4 & 16 & 4 \end{vmatrix} = 4\begin{vmatrix} 1 & 4 & 1 \\ 0 & -1 & 2 \\ 1 & 4 & 1 \end{vmatrix}$$

Q(16)

Find the volume of the parallelepiped which three of its adjacent sides are represented by the vectors
$-12i-3k$, $3j-K$ and $2i+j-15K$

$$V=\begin{vmatrix}-12 & 0 & -3\\ 0 & 3 & -1\\ 2 & 1 & -15\end{vmatrix}=546$$

Q(17)Find the vector form of the equation of the straight line passing through point (3, -1, 0) and the vector (-2, 4 , 3) is a direction vector for it.

$$\boxed{\vec{r} = \text{point} + t\ (\text{direction vector})}$$

❶The vector equation:

$$\vec{r}=(3,-1,0)+t(-2,4,3)$$

❷Parametric equations:

$$(X,Y,Z)=(3,-1,0)+t(-2,4,3)$$

$$\therefore X=3-2t \quad , \quad Y=-1+4t \quad , \quad Z=0+3t$$

❸The Cartesian equation:

$$\therefore t=\frac{X-3}{-2} \quad , \quad t=\frac{Y+1}{4} \quad , \quad t=\frac{Z}{3} \qquad \therefore \frac{X-3}{-2}=\frac{Y+1}{4}=\frac{Z}{3}$$

Q(18) Solve the following equations
X-Y+Z=2 , 2X+3Y-Z=5 , 3X-5Y+2Z=-1 using the multiplicative inverse of the matrix

$$\begin{pmatrix}X\\Y\\Z\end{pmatrix}=\frac{1}{11}\begin{pmatrix}-1 & 3 & 2\\ 7 & 1 & -3\\ 19 & -2 & -5\end{pmatrix}\begin{pmatrix}2\\5\\-1\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}$$

Q(19) If Z is a complex number , the amplitude of $(Z + i) = \frac{\pi}{4}$

And the amplitude of $(Z - 3) = \frac{3\pi}{4}$ find Z in the algebraic form

Let $Z = X + iY \quad \therefore (X + iY + i) = X + i(Y + 1)$

$\therefore \tan\theta = \frac{Y + 1}{X} = \tan 45^\circ = 1 \quad \therefore X = Y + 1$

$\therefore (X + iY - 3) = (X - 3) + iY$

$\therefore \frac{Y}{X - 3} = \tan 135^\circ = -1 \quad \therefore Y = -X + 3$

$\therefore Z = 2 + i$

Q(20) If the coefficients of the 4^{th} , 5^{th} and 6^{th} term respectively in the expansion of $(2X + Y)^n$ form an arithmetic sequence find the value of n

$2\underline{\text{coeff.}}T_5 = \underline{\text{coeff.}}T_4 + \underline{\text{coeff.}}T_6 \quad \therefore \div T_5$

$$\frac{\underline{\underline{\text{coeff.}}}T_4}{\underline{\text{coeff.}}T_5} + \frac{\underline{\underline{\text{coeff.}}}T_6}{\underline{\text{coeff.}}T_5} = 2$$

$$\frac{4}{n - 4 + 1} \times 2 + \frac{n - 5 + 1}{5} \times \frac{1}{2} = 2$$

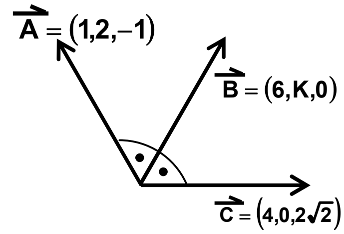
$$\frac{8}{n - 3} + \frac{n - 4}{10} = 2$$

$\therefore n = 19 \quad \text{or } n = 8$

Answer the following questions | 20 questions

From 1to 12 choose the correct answer

Q(1) In the opposite figure, the value of k = ...



① 9 ② 3

③ 6 ④ 4

$(6,K,0)\bullet(4,0,2\sqrt{2}) = (6,K,0)\bullet(1,2,-1)$

$24 = 6 + 2K \quad \therefore 2K = 18 \quad \therefore K = 9$

Q(2) $\left(1-\frac{1}{\omega}\right)\left(1-\frac{1}{\omega^2}\right)\left(1-\frac{1}{\omega^4}\right)\left(1-\frac{1}{\omega^8}\right)\ldots\ldots\ldots \times$ up to 10 factors

① 243 ② 0

③ 200 ④ 201

$= \left[(1-\omega^2)(1-\omega)\right]^5 = (1-\omega-\omega^2+\omega^3)^5 = (2+1)^5 = 243$

243

Q(3) The rank of the matrix A= $\begin{pmatrix} -1 & 0 & 0 \\ -1 & -1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ equals

① 1 ② 0

③ 2 ④ 3

$\because |A| \neq 0$ rank is 3

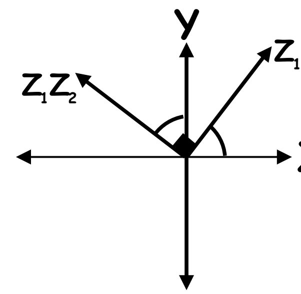
Q(4) $1 - 6X + \frac{6\times5}{2\times1}X^2 - \frac{6\times5\times4}{3\times2\times1}X^3 + \ldots\ldots\ldots + X^6 = 64$ then X=.....

① -1 ② 2

③ 3 ④ {-1,3}

$(1-X)^6 = 64 \quad \therefore 1-X = \pm 2 \quad \therefore X = \{-1,3\}$

Q(5) In the opposite figure: If Z_1 and Z_2 and Z_1Z_2 are complex numbers then $Z_2 =$



① -2i ② -i

③ i ④ 2i

$$Z_2 = \frac{Z_1Z_2}{Z_1} = \frac{re^{\left(\frac{1}{2}\pi+\theta\right)i}}{re^{\theta i}} = e^{\frac{1}{2}\pi i}$$

Q(6) If A form an equal angles θ with the three axis and $\|A\| = 9$ then $\theta = \ldots\ldots..$

① $\cos^{-1}\frac{1}{\sqrt{2}}$ ② $\cos^{-1}\frac{1}{9}$

③ $\cos^{-1}\frac{1}{\sqrt{3}}$ ④ $\cos^{-1}\frac{1}{3}$

$$3\cos^2\theta = 1 \therefore \cos\theta = \frac{1}{\sqrt{3}}$$

Q(7) If $(a+b)\bullet(a-b) = 63$ and $|a| = 8|b|$ then $|a| =$

① 8 ② 64

③ 16 ④ 4

$$a^2 - b^2 = 63 \quad \therefore a^2 - \frac{1}{64}a^2 = 63 \quad \therefore a^2 = 64 \quad \therefore |a| = 8$$

Q(8) The direction cosine of the line
$X = 3 - 2Y$, $Z = 2Y - 1$ is ……..

① $\frac{-2}{3}, \frac{1}{3}, \frac{2}{3}$ ② $\frac{-2}{3}, \frac{-1}{3}, \frac{2}{3}$

③ $\frac{2}{3}, \frac{1}{3}, \frac{2}{3}$ ④ $\frac{-2}{3}, -\frac{1}{3}, \frac{2}{3}$

$$\frac{X-3}{-2} = \frac{Y}{1} = \frac{Z+1}{2}$$

Q(9) If A(7,-1,,8) and B(11,2,-4) then the length of $\overline{AB} =$

① 10 ② 11

③ 12 ④ 13

$AB\sqrt{(11-7)^2+(2+1)^2+(-4-8)^2} = 13$

Q(10) From a class of 9 boys and 6 girls it's required to select a 4-person team of the same gender.

① 3384 ② 143

③ 145 ④ 141

${}^{9}C_4 + {}^{6}C_4 = 126 + 15 = 141$

Q(11) If $(1+\omega^2)^n = (1+\omega)^n$ then the least value of the positive integer n equals

① 2 ② 3

③ 6 ④ 5

$(-\omega)^6 = (-\omega^2)^6 \quad \therefore n = 6$

Q(12) If $\frac{a^2+b^2}{a+bi} = 2+3i$ then $a \times b =$….. where a and b $\in R$

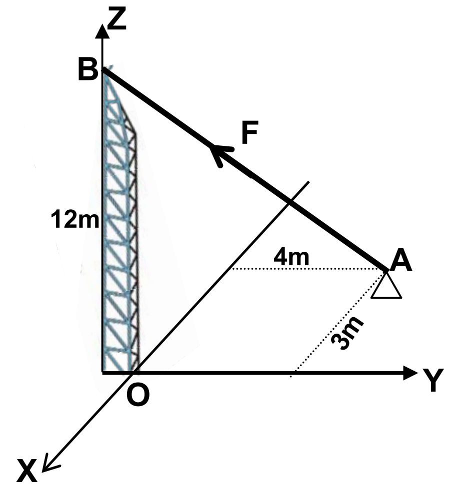
① -6 ② 5

③ -5 ④ 6

$\frac{a^2+b^2}{a+bi} \times \frac{a-bi}{a-bi} = \frac{(a^2+b^2)(a-bi)}{(a^2+b^2)} = 2+3i \quad \therefore a - bi = 2+3i \therefore a \times b = -6$

Q(13)A Tower guy wire is anchored by mean of a ring at A .The tension in the wire is 2600N determine the components

F_X , F_Y , F_Z



$A \to (-3,4,0)$, $B \to (0,0,12)$

$AB = B - A = (0,0,12) - (-3,4,0) = (3,-4,12)$

$$F = \frac{2600}{\sqrt{3^2+4^2+12^2}}(3,-4,12) = \frac{2600}{13}(3i - 4j + 12K)$$

$$F = 600\,i - 800\,j + 2400\,K$$

Q(14) Solve the following system of linear equations using the inverse matrix where 4X+Y=0 , X+2Z=15 , Y-7Z=0

$$|A| = \begin{vmatrix} 4 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & -7 \end{vmatrix} = -1 \qquad \text{The cofactor matrix} \begin{pmatrix} -2 & 7 & 1 \\ 7 & -28 & -4 \\ 2 & -8 & -1 \end{pmatrix}$$

$$\therefore \text{Adj }(A) = \begin{pmatrix} -2 & 7 & 2 \\ 7 & -28 & -8 \\ 1 & -4 & -1 \end{pmatrix} \qquad \therefore A^{-1} = \frac{1}{|A|}\text{ Adj }(A) = \begin{pmatrix} 2 & -7 & -2 \\ -7 & 28 & 8 \\ --1 & 4 & 1 \end{pmatrix}$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 2 & -7 & -2 \\ -7 & 28 & 8 \\ --1 & 4 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 15 \\ 0 \end{pmatrix} = \begin{pmatrix} -105 \\ 480 \\ 60 \end{pmatrix}$$

Q(15)Prove that : $\begin{vmatrix} X & a & a \\ a & X & a \\ a & a & X \end{vmatrix} = (X+2a)(X-a)^2$

$C_1 + C_2 + C_3$

$$\begin{vmatrix} X+2a & a & a \\ X+2a & X & a \\ X+2a & a & X \end{vmatrix} = (X+2a)\begin{vmatrix} 1 & a & a \\ 1 & X & a \\ 1 & a & X \end{vmatrix} = (X+2a)\begin{vmatrix} 1 & a & a \\ 0 & X-a & 0 \\ 0 & 0 & X-a \end{vmatrix}$$

$$= (X+2a)(X-a)^2$$

Q(16) Prove that: $r_1 = j + t_1(i + 2j - K)$, $r_2 = (i + j + K) + t_2(-2i - 2j)$ Intersect at a point , then find their intersection point

$r_1 = (0,1,0) + t_1(1,2,-1) \rightarrow (1)$, $r_2 = (1,1,1) + t_2(-2,-2,0) \rightarrow (2)$

$d_1 \rightarrow (1,2,-1)$, $d_2 \rightarrow (-2,-2,0)$ $\because \frac{a_1}{a_2} = \frac{1}{-2} \neq \frac{b_1}{b_2} = \frac{2}{-2}$

$\therefore$ The two straight lines are not parallel to get the point of intersection

$r_1 = r_2 \quad \therefore \quad (0 + t_1 \;,\; 1 + 2t_1 \;,\; 0 - t_1) = (1 - 2t_2 \;,\; 1 - 2t_2 \;,\; 1)$

$\therefore \; t_2 = 1$ substitute in (2) $\therefore \text{point} = r_2 = (1,1,1) + 1(-2,-2,0) = (-1,-1,1)$

Q(17) Find the volume of the parallelepiped which three of its adjacent sides are represented by the vectors

$-12\hat{i} - 3\hat{k}$, $3\hat{j} - \hat{K}$ and $2\hat{i} + \hat{j} - 15\hat{K}$

$$V = \begin{vmatrix} -12 & 0 & -3 \\ 0 & 3 & -1 \\ 2 & 1 & -15 \end{vmatrix} = 546$$

Q(18) If the coefficients of the 4^{th} , 5^{th} and 6^{th} term respectively in the expansion of $(2X + Y)^n$ form an arithmetic sequence find the value of n

$2\,\text{coeff.}T_5 = \text{coeff.}T_4 + \text{coeff.}T_6 \quad \therefore \div T_5$

$$\frac{\text{coeff.}T_4}{\text{coeff.}T_5} + \frac{\text{coeff.}T_6}{\text{coeff.}T_5} = 2$$

$$\frac{4}{n-4+1} \times 2 + \frac{n-5+1}{5} \times \frac{1}{2} = 2$$

$$\frac{8}{n-3} + \frac{n-4}{10} = 2$$

$\therefore n = 19$ or $n = 8$

Q(19) Put the number $Z_1 = -1 + \sqrt{3}\,i$,if $Z_1 Z_2 = 8e^{\frac{11}{3}\pi i}$ in the exponential form then find the square root of Z_2 in the trigonometric form

$$r = \sqrt{1^2 + \left(\sqrt{3}\right)^2} = 2 \quad ,\tan\theta = \frac{\sqrt{3}}{1} \therefore \to \theta = \frac{2\pi}{3} \quad (-,+)\,2^{nd} \therefore \theta = 120^o = \frac{2\pi}{3}$$

$$\therefore Z_1 = 2e^{\frac{2\pi}{3}i} \qquad Z_1 = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

$$Z_2 = \frac{8\left(\cos\frac{11}{3}\pi + i\sin\frac{11}{3}\right)}{2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)} = 4(\cos 3\pi + i\sin 3\pi)$$

$$= 4(\cos\pi + i\sin\pi) = \sqrt{Z} = 2\left[\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right], 2\left[\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right]$$

Q(20) In the expansion of $\left(aX + \frac{1}{bX}\right)^{10}$ in descending powers of X if the term free of X is equal to the coefficient of the 7^{th} term prove that $6ab = 5$

$$T_{r+1} = {}^{10}C_r\left(\frac{1}{b}X^{-1}\right)^r (aX)^{10-r} = {}^{10}C_r(b)^{-r}(a)^{10-r}X^{10-2r} \quad 10 - 2r = 0 \quad \therefore r = 5 \quad \therefore T_6 = {}^{10}C_5(b)^{-5}(a)^5$$

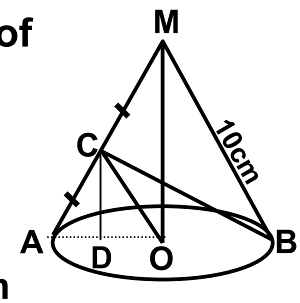
$$T_7 = {}^{10}C_6(b)^{-6}(a)^4X^{-5} \qquad \therefore {}^{10}C_6(b)^{-6}(a)^4 = {}^{10}C_5(b)^{-5}(a)^5 \therefore \frac{{}^{10}C_6(b)^{-6}(a)^4}{{}^{10}C_5(b)^{-5}(a)^5} = 1$$

$$\therefore \to \frac{10-6+1}{6} \times \frac{1}{ab} = 1 \qquad \therefore \frac{5}{6} \times \frac{1}{ab} = 1 \quad \therefore \quad 6ab = 5$$

Answer the following questions 20 questions

From 1to 12 choose the correct answer

Q(1) In the opposite figure, a right circular cone, the perimeter of its base =12π cm C is the midpoint of AM then $\overrightarrow{OC} \bullet \overrightarrow{OA} =$



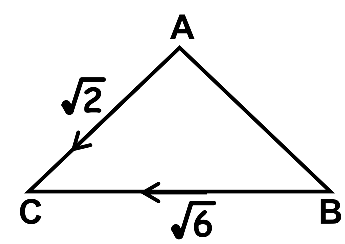
① 9 ② 18

③ 36 ④ 54

$2\pi r = 12\pi \quad \therefore r = 6 \quad OA = 6 \quad \therefore MO = 8cm \;, CO = 5 \quad \therefore DO = 3cm$

$CD = 4cm \quad \therefore \overrightarrow{OC} \bullet \overrightarrow{OA} = \|\overrightarrow{OC}\| \times \|\overrightarrow{OA}\| \cos(\angle COA) = 5 \times 6 \times \frac{3}{5} = 18$

Q(2) In the given figure $\|\overrightarrow{BC}\| = \sqrt{6}$ and $\|\overrightarrow{AC}\| = \sqrt{2}$ $\overrightarrow{BA} = (-1,0,1)$ then $\overrightarrow{BA} \bullet \overrightarrow{BC} =$….



① 1 ② 2

③ 4 ④ 3

$\because \|\overrightarrow{BA}\| = \sqrt{1+0+1} = \sqrt{2} \quad \therefore m(\angle A) = m(\angle B)$

$\cos(\angle B) = \cos(\angle C) = \frac{2+6-2}{2\sqrt{2} \times \sqrt{6}} = \frac{\sqrt{3}}{2} \quad \therefore \overrightarrow{BA} \bullet \overrightarrow{BC} = \sqrt{2} \times \sqrt{6} \times \frac{\sqrt{3}}{2} = 3$

Q(3) The coefficient of the middle term in the expansion $\left(3X - \frac{1}{6}\right)^{10}$ equals

① $-\frac{63}{8}$ ② $-\frac{67}{8}$

③ $\frac{63}{8}$ ④ $\frac{67}{8}$

$coeff.T_6 = {}^{10}C_5 \left(-\frac{1}{6}\right) \times 3^5 = -\frac{63}{8}$

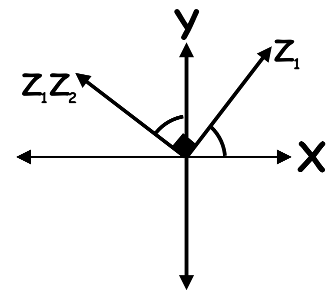
Q(4) The distance between the two planes Y=4 and Y=-2 is

① 3units ② 6units

③ 2units ④ 8units

$4 + 2 = 6$

Q(5) In the opposite figure: If Z_1 and Z_2 and Z_1Z_2 are complex numbers then $Z_2 =$



① $-2i$ ② $-i$

③ i ④ $2i$

$$Z_2 = \frac{Z_1Z_2}{Z_1} = \frac{re^{\left(\frac{1}{2}\pi+\theta\right)i}}{re^{\theta i}} = e^{\frac{1}{2}\pi i}$$

Q(6) If $(X-2)^2+(Y+4)^2+(Z-2)^2=1$, $(X+4)^2+(Y-4)^2+(Z-2)^2=4$ Are the equations of two spheres then the distance between their centers

① 10 ② $\sqrt{10}$

③ 20 ④ $2\sqrt{5}$

$$\sqrt{(2+4)^2+(-4-4)^2+(2-2)^2}=10$$

Q(7) $\left(\frac{3+5\omega}{5+3\omega^2}+\frac{5+3\omega^2}{3+5\omega}\right)^8 =$

① 81 ② 27

③ 9 ④ 3

81

Q(8) The equation of the line of intersection of the planes $X+2Y-3Z=6$ and $2X-Y+Z=7$

① $\frac{X-4}{1}=\frac{Y-1}{7}=\frac{Z}{5}$ ② $\frac{X+4}{1}=\frac{Y-1}{7}=\frac{Z}{5}$

③ $\frac{X+1}{1}=\frac{Y+1}{7}=\frac{Z}{5}$ ④ $\frac{X-1}{-1}=\frac{Y-1}{-7}=\frac{Z}{5}$

$$\frac{X-4}{1}=\frac{Y-1}{7}=\frac{Z}{5}$$

Q(9) If $\overrightarrow{A} = 2i + 3j + mk$, $\overrightarrow{B} = -6i - 4j + 4k$ and $\overrightarrow{A} \perp \overrightarrow{B}$, then m = ……………

① 8 ② 4

③ 6 ④ -4

$(2\times -6+3)\times(-4+m\times 4)=0 \quad \therefore 4m=24 \quad \therefore m=6$

Q(10) Number of solution(natural) such that a + b + c =7

① 24 ② 35

③ 36 ④ 210

${}^{n+r-1}C_r = {}^{3+7-1}C_7 = 36$ (n) number of variables ,(r) their sum

Similar example

Number of ways to distribute 4 identical balls among 3 boxes

${}^{n+r-1}C_r = {}^{3+4-1}C_3$ (n) number of boxes ,(r) number of balls

Q(11) $\sum_{r=1}^{6} 1+\omega^r =$

① 0 ② 6

③ $1+\omega$ ④ 1

$1+\omega+\omega^2+\omega^3+\omega^4+\omega^5+\omega^6=1$

Q(12)

${}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots\ldots\ldots\ldots\ldots + {}^nC_n =$

① 1 ② 2

③ 2^n ④ 4^n

$\left({}^nC_0 + {}^nC_n\right)^n = 2^n$

Q(13) Find the volume of the parallelepiped in which three adjacent sides are represented by the vectors
A = (2, 1, 3), B = (-1 , 3 , 2) C = (1 , 1 , -2)

$$\text{The volume} = |A \bullet B \times C| = \begin{vmatrix} 2 & 1 & 3 \\ -1 & 3 & 2 \\ 1 & 1 & -2 \end{vmatrix} = |-28| = 28$$

Q(14) prove that $\begin{vmatrix} X+y+2 & X & Y \\ 1 & 2X+Y+1 & Y \\ 1 & X & X+2Y+1 \end{vmatrix}$

$= 2(X+Y+1)^3$

$C_1 + C_2 + C_3$

$$\begin{vmatrix} 2X+2Y+2 & X & Y \\ 2X+2Y+2 & 2X+Y+1 & Y \\ 2X+2Y+2 & X & X+2Y+1 \end{vmatrix} = (2X+2Y+2)\begin{vmatrix} 1 & X & Y \\ 1 & 2X+Y+1 & Y \\ 1 & X & X+2Y+1 \end{vmatrix}$$

$r_2 - r_1 , r_3 - r_1$

$$= 2(X+Y+1)\begin{vmatrix} 1 & X & Y \\ 0 & X+Y+1 & 0 \\ 0 & 0 & X+Y+1 \end{vmatrix} = 2(X+Y+1)^2$$

Q(15)

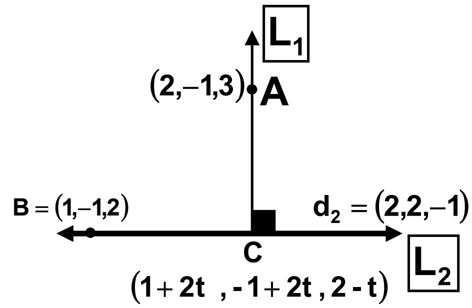
If $\begin{vmatrix} X & Y & Z \\ l & m & n \\ K & f & g \end{vmatrix} = 2$ find the value o $\begin{vmatrix} 2X & 2Y & 2Z \\ 5l+X & 5m+Y & 5n+Z \\ 7K-3l & 7F-3m & 7g-3n \end{vmatrix}$

$$\begin{vmatrix} 2X & 2Y & 2Z \\ 5l+X & 5m+Y & 5n+Z \\ 7K-3l & 7F-3m & 7g-3n \end{vmatrix} = 2\begin{vmatrix} X & Y & Z \\ 5l+X & 5m+Y & 5n+Z \\ 7K-3l & 7F-3m & 7g-3n \end{vmatrix} r_2 - r_1 \rightarrow$$

$$2\begin{vmatrix} X & Y & Z \\ 5l & 5m & 5n \\ 7K-3l & 7F-3m & 7g-3n \end{vmatrix} = 2(5)\begin{vmatrix} X & Y & Z \\ l & m & n \\ 7K-3l & 7F-3m & 7g-3n \end{vmatrix} \rightarrow r_3 + 3r_2$$

$$= 10\begin{vmatrix} X & Y & Z \\ l & m & n \\ 7K & 7F & 7g \end{vmatrix} = 70\begin{vmatrix} X & Y & Z \\ l & m & n \\ K & F & g \end{vmatrix} = 70 \times 2 = 140$$

Q(16) Find the equation of the straight line passing through the point (2 , -1 , 3) and intersects the straight line $r_1 = (1 , -1 , 2) + t (2 , 2 , -1)$ orthogonally.



$d_1 = AC = C - A = (2t-1 \; , 2t \; , -1-t)$

$\because L_1 \perp L_2 \;\; \therefore d_1 \bullet d_2 = 0$

$(2t-1 \; , 2t \; , -1-t) \bullet (2,2,-1) = 0$

$4t - 2 + 4t + 1 + t = 0 \quad \therefore 9t - 1 = 0 \quad \therefore \; t = \frac{1}{9}$

$\therefore d_1 = \left(-\frac{7}{9}, \frac{2}{9}, \frac{-10}{9} \right) = (-7, 2, -10)$

Equitation of the line is $r = (2,-1,3) + t(-7,2,-10)$

Q(17) Find all the different forms of the equation of the straight line $\frac{3X+1}{2} = \frac{Y-1}{2} = \frac{5-Z}{3}$

Let $\frac{3X+1}{2} = \frac{Y-1}{2} = \frac{5-Z}{3} = t \;\; \therefore \begin{cases} \frac{3X+1}{2} = t \;\; \therefore X = \frac{2t-1}{3} = -\frac{1}{3} + \frac{2}{3}t \\ \frac{Y-1}{2} = t \;\; \therefore Y = \frac{2t+1}{1} = 1 + 2t \\ \frac{5-Z}{3} = t \;\; \therefore Z = \frac{3t-5}{-1} = 5 - 3t \end{cases}$

$\therefore r = \left(-\frac{1}{3}, 1, 5 \right) + t\left(\frac{2}{3}, 2, -3 \right)$

Q(18) If ${}^{n+1}C_r : {}^{n+1}C_{r-1} = 3 : 5$, $\underline{|n} = 720$ calculate the value of ${}^{n+1}P_{r-2}$

$\underline{|n} = 720 = \underline{|6} \therefore n = 6$, $\frac{{}^7C_r}{{}^7C_{r-1}} = \frac{3}{5} \therefore \rightarrow \frac{7-r+1}{r} = \frac{3}{5}$

$3r = 40 - 5r \therefore r = 5$ $\qquad \therefore {}^{n+1}P_{r-2} = {}^7P_3 = 210$

Q(19)

If $Z=\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$, find each of the two numbers $Z_1=Z-1$, $,Z_2=Z+1$

then prove that $\frac{Z_1}{Z_2}$ is a pure imaginary number

$$Z=\frac{1}{2}+\frac{\sqrt{3}}{2}i \qquad Z_1=-\frac{1}{2}+\frac{\sqrt{3}}{2}i \qquad |Z_1|=\sqrt{\frac{1}{4}+\frac{3}{4}}=1$$

$$,\tan\theta=\frac{\sqrt{3}}{-1}(-,+)\ 2^{nd}\rightarrow\theta=120^\circ$$

$$Z_1=\cos 120^\circ+i\sin 120^\circ$$

$$Z_2=\frac{3}{2}+\frac{\sqrt{3}}{2}i \qquad |Z_2|=\sqrt{\frac{9}{4}+\frac{3}{4}}=\sqrt{3}\ ,\tan\theta=\frac{1}{\sqrt{3}}\rightarrow\theta=30^\circ$$

$$Z_2=\sqrt{3}\left(\cos 30^\circ+i\sin 30^\circ\right)$$

$$\frac{Z_1}{Z_2}=\frac{1}{\sqrt{3}}\left[\cos\left(120^\circ-30^\circ\right)+i\sin\left(120^\circ-30^\circ\right)\right]$$

$$=\frac{1}{\sqrt{3}}\left[\cos 90^\circ+i\sin 90^\circ\right]=\frac{1}{\sqrt{3}}i$$

Q(20) If 35 , 21 , 7 are the coefficient of three consecutive terms in the expansion $(1+X)^n$ find the value of n and the order of these terms

<u>Let the terms are T_r , T_{r+1} , T_{r+2}</u>

$$\frac{\text{coof}T_{r+1}}{\text{coof}T_r}=\frac{n-r+1}{r}\times\frac{1}{1}=\frac{21}{35}=\frac{3}{5}\rightarrow 5n-8r+5=0$$

$$\frac{\text{coof}T_{r+2}}{\text{coof}T_{r+1}}=\frac{n-(r+1)+1}{r+1}\times\frac{1}{1}=\frac{7}{21}=\frac{1}{3}\rightarrow 3n-4r-1=0$$

$$\therefore n=7,\ r=5$$

Answer the following questions 20 questions

From 1to 12 choose the correct answer

Q(1) The two lines $\frac{X-1}{2}=\frac{Y-2}{3}=\frac{Z-3}{4}$ and $\frac{X-4}{5}=\frac{Y-1}{2}=Z$

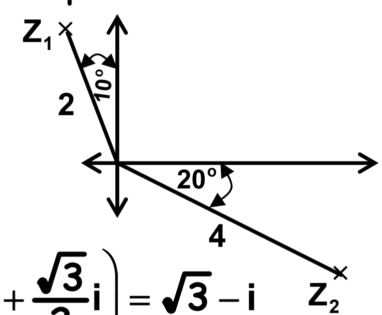
Are intersecting in the point ………

① (1,1,1) ② (-1,-1,1)

③ (-1,1,-1) ④ (-1,-1,-1)

(-1,-1,-1)

Q(2) If Z_1 , Z_2 , are two complex numbers represented On argend's plane as in the opposite figure Find the form of X+iY the number $\frac{Z_2}{7}$



① $1+\sqrt{3}i$ ② $\sqrt{3}+i$

③ $1-\sqrt{3}i$ ④ $\sqrt{3}-i$

$$\frac{4(\cos 100^\circ + i\sin 100^\circ)}{2(\cos -20^\circ + i\sin -20^\circ)} = 2(\cos 120^\circ + i\sin 120^\circ) = 2\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right) = \sqrt{3}-i$$

Q(3) $|-11+60i| = ...$

① 60 ② 61

③ 3721 ④ 49

$$\sqrt{(-11)^2+(60)^2} = 61$$

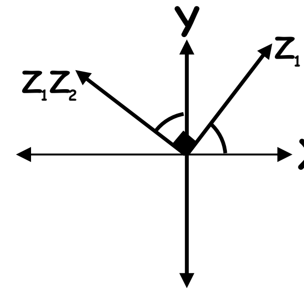
Q(4) The term free of x in the expansion of $\left(2X+\frac{1}{2X}\right)^8$

① 35 ② 140

③ 70 ④ 56

70

Q(5) In the opposite figure: If Z_1 and Z_2 and Z_1Z_2 are complex numbers then $Z_2 =$



① $-2i$ ② $-i$

③ i ④ $2i$

$$Z_2 = \frac{Z_1Z_2}{Z_1} = \frac{re^{\left(\frac{1}{2}\pi+\theta\right)i}}{re^{\theta i}} = e^{\frac{1}{2}\pi i}$$

Q(6) The radius length of the sphere $(X-2)^2+(Y+4)^2+(Z-5)^2=64$ is …

① 64 ② 4

③ 8 ④ 16

$$\sqrt{2^2+4^2+5^2-64} = 4$$

Q(7) If $A=(-4,-2,3)$, $B=(1,2,3)$ and the length of $\overline{AB}=\sqrt{77}$ then one value of K is

① 2 ② 4

③ 6 ④ 9

$$\sqrt{(1+4)^2+(2+2)^2+(K-3)^2} = \sqrt{77}$$
$$\therefore 25+16+(K-3)^2 = 77 \quad \therefore (K-3)^2 = 36 \quad \therefore K = 9 \text{ or } K = -3$$

Q(8) In the expansion of $(3X-2Y)^{13}$ if the ratio between the two consecutive middle terms equals $\frac{-2}{3}$ then Y:X=……

① 9:4 ② 4:9

③ 3:2 ④ 2:3

$$\frac{T_7}{T_8} = \frac{-2}{3} = \frac{7}{13-7+1} \times \frac{3X}{-2Y} = \frac{-2}{3} \qquad \therefore \frac{Y}{X} = 9:4$$

Q(9)

If $\overrightarrow{A} = (1, -1, 2)$, $\overrightarrow{B} = (3, -2, 0)$, $\overrightarrow{C} = (0, 2, 4)$, then $\overrightarrow{A} \bullet \overrightarrow{B} \times \overrightarrow{C} =$ ....

① 16 ② 24

③ 8 ④ 20

$$\begin{vmatrix} 1 & -1 & 2 \\ 3 & -2 & 0 \\ 0 & 2 & 4 \end{vmatrix} = 16$$

Q(10) How many4 different digits even number Can be form using the digits 1,2,3,4,5,6,7

① ②

③ ④

6X5X4X3=360

Q(11) The S.S of the equation $X^3 = 27$ in set of complex number

① $\{3\}$ ② $\{ 3, 9\omega, 3\omega^2 \}$

③ $\{ 3, 3\omega, 3\omega^2 \}$ ④ $\{ 3+\omega, 9\omega, 3\omega^2 \}$

Q(12) If ${}^{30}C_r = {}^{30}C_{r+10}$, ${}^{n}P_7 = 90 \times {}^{n-2}P_5$ then $\underline{|n-r} =$

① zero ② 1

③ 10 ④ $\underline{|20}$

$r + r + 10 = 30 \quad \therefore r = 10$

$$\frac{\underline{|n}}{\underline{|n-7}} = 90\frac{\underline{|n-2}}{\underline{|n-7}} \quad \therefore n = 10 \quad \therefore \underline{|n-7} = \underline{|0} = 1$$

Q(13) Find the equation of the line passing through the point (1,2,3) Perpendicular to the plane $2X - 3Y + Z + 1 = 0$

$$r = (1,2,3) + t(2,-3,1)$$

Q(14) If $\begin{pmatrix} 3a & b & c \\ b & 3c & a \\ c & a & 3b \end{pmatrix}$, $a,b,c \in R$ and $abc=1$ and $AA^t = 64I$

$|A| > 0$ Find the value of $a^3 + b^3 + c^3$

$\because A^t = A \quad \therefore A^2 = 64I \quad \therefore |A| = 8$

$$\begin{vmatrix} 3a & b & c \\ b & 3c & a \\ c & a & 3b \end{vmatrix} = 8$$

$$3(a^3 + b^3 + c^3) = 21 \quad \therefore a^3 + b^3 + c^3 = 7$$

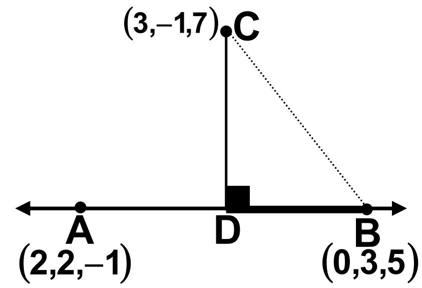
Q(15) Prove that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = a^2 + b^2 + c^2 + 1$

$$\underbrace{\begin{vmatrix} a^2 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix}}_{\substack{\text{a common from } r_1 \\ \text{a common from } c_1}} + \begin{vmatrix} 1 & ab & ac \\ 0 & b^2+1 & bc \\ 0 & bc & c^2+1 \end{vmatrix} = a^2\begin{vmatrix} a & b & c \\ b & b^2+1 & bc \\ c & bc & c^2+1 \end{vmatrix} + \begin{vmatrix} 1 & ab & ac \\ 0 & b^2 & bc \\ 0 & bc & c^2+1 \end{vmatrix} + \begin{vmatrix} 1 & 0 & ac \\ 0 & 1 & bc \\ 0 & 0 & c^2+1 \end{vmatrix}$$

$$= a^2\begin{vmatrix} 1 & b & c \\ b & b^2+1 & bc \\ c & bc & c^2+1 \end{vmatrix} + \underbrace{\begin{vmatrix} 1 & ab & ac \\ 0 & b^2 & bc \\ 0 & bc & c^2+1 \end{vmatrix}}_{\substack{\text{b common from } c_2 \\ \text{b common from } r_2}} + \begin{vmatrix} 1 & 0 & ac \\ 0 & 1 & bc \\ 0 & 0 & c^2+1 \end{vmatrix}$$

$$= a^2\underbrace{\begin{vmatrix} 1 & b & c \\ b & b^2+1 & bc \\ c & bc & c^2+1 \end{vmatrix}}_{\substack{r_2 - br_1 \\ r_3 - cr_1}} + b^2\begin{vmatrix} 1 & a & ac \\ 0 & 1 & c \\ 0 & c & c^2+1 \end{vmatrix} + c^2 + 1 = a^2\begin{vmatrix} 1 & b & c \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} + b^2\begin{vmatrix} 1 & a & ac \\ 0 & 1 & c \\ 0 & c & c^2+1 \end{vmatrix} + c^2 + 1$$

Q(16) Find the perpendicular distance from point (3 , -1 , 7) to the straight line passing through the two points (2 , 2 , -1) and (0 , 3 , 5)



First method :

$BA = A - B = (2,2,-1) - (0,3,5) = (2,-1,-6)$

$BC = C - B = (3,-1,7) - (0,3,5) = (3,-4,2)$

Projection of CB on AB

$$BD = \frac{BC \bullet BA}{\|BA\|} = \frac{|(3,-4,2) \bullet (2,-1,-6)|}{\sqrt{4+1+36}} = \frac{2}{\sqrt{41}}$$

$$\|BC\| = \sqrt{9+16+4} = \sqrt{29}$$

$$CD = \sqrt{(BC)^2 - (BD)^2} = \sqrt{29 - \frac{4}{41}} = 5.4$$

Q(17) A force $\overrightarrow{F} = \hat{i} - 2\hat{j} + 3\hat{K}$ Newton act on a body it move it from the point A(-3,1,0) to the point B(2,0,-2) find the work done by this force such that displacement is measured by meter

$$\overrightarrow{AB} = \overrightarrow{B} - \overrightarrow{A} = (2,0,-2) - (-3,1,0) = (5,-1,-2)$$

$$W = \overrightarrow{F} \bullet \overrightarrow{S} = (1,-2,3) \bullet (5,-1,-2) = 1 \times 5 + (-2) \times (-1) + 3 \times -2 = 1 \text{ N.m}$$

Q(18) If ${}^{n+2}P_r : {}^{n+2}C_r = 2 : 1$, ${}^{n}C_{r+1} : {}^{n}C_{r-1}$ 5 : 3

find the value of ${}^{2n}C_{n-r}$

$$\frac{{}^{n+2}P_r}{{}^{n+2}C_r} = \frac{{}^{n+2}P_r}{} \div \frac{{}^{n+2}P_r}{\lfloor r} = 2 \quad \therefore \lfloor r = 2 \quad \therefore r = 2$$

$$\frac{{}^{n}C_{r+1}}{{}^{n}C_{r-1}} = \frac{5}{3} \quad \therefore \frac{n-(r+1)+1}{r+1} = \frac{5}{3}$$

$$\therefore \frac{n-2}{3} = \frac{5}{3} \quad \therefore n - 2 = 5 \quad \therefore n = 7$$

Q(19) If $Z_1 = \frac{6+4i}{1+i}$ and $Z_2 = \frac{26}{5-i}$ if $Z = 4(Z_1 - Z_2)$ Find the cubic roots of Z in the exponential form

$$Z = -8i = 8(\cos -90^\circ + i\sin -90^\circ) = 8e^{-\frac{\pi}{2}i}$$

$$\sqrt[3]{Z} = 2e^{\frac{\frac{-\pi}{2}+2K\pi}{3}i}$$

At K=0 $\sqrt[3]{Z} = 2e^{\frac{-\pi}{6}i}$

At K=1 $\sqrt[3]{Z} = 2e^{\frac{\pi}{2}i}$

At K=-1 $\sqrt[3]{Z} = 2e^{\frac{-5\pi}{6}i}$

Q(20) In the expansion of $(X+Y)^n$ in descending power of X if T_2, T_3, T_4 are respectively 240 , 720 , 1080 evaluate the value of each of X , Y , n

Answer :

$\frac{T_3}{T_2} = \frac{n-2+1}{2} \times \frac{Y}{X} = \frac{720}{240} = 3 \rightarrow (1)$, $\frac{T_4}{T_3} = \frac{n-3+1}{3} \times \frac{Y}{X} = \frac{1080}{720} = \frac{3}{2} \rightarrow (2)$

dividing 1,2 $\therefore \frac{n-1}{2} \times \frac{3}{n-2} = 2 \quad \therefore \frac{3n-3}{2n-4} = 2 \quad \therefore 4n-8 = 3n-3 \quad \therefore n = 5$

$\therefore$ from 1 $\therefore \frac{Y}{X} = \frac{3}{2} \quad \therefore Y = \frac{3X}{2}$

$T_2 = {}^5C_1 \times \left(\frac{3X}{2}\right)^1 X^4 = 240 \qquad \therefore X^5 = 32 \quad \therefore X = 2 \quad \therefore Y = 3$

Answer the following questions 20 questions

From 1to 12 choose the correct answer

Q(1) If The Y axis cut the circle which center (3,-4,12) and its radius 13cm at the points A and B then AB=…..

① 6 ② 8

③ 26 ④ 24

$(X-3)^2+(Y+4)^2+(Z-12)^2=13^2 \quad \therefore (0-3)^2+(Y+4)^2+(0-12)^2=13^2$

$\therefore (Y+4)^2=16 \quad \therefore Y+4=\pm 4 \quad \therefore Y=0$ or $Y=-8$

$A=(0,0,0)$ and $B=(0,-8,0) \quad \therefore AB=\sqrt{0+8^2+0}=8$

Q(2) The sum of the coefficient of the expansion of $(1+X)^5$ equals

① 0 ② 30

③ 32 ④ 36

Put X=1 $(1+X)^5=32$

Q(3) If $A=\begin{pmatrix} 1 & -2 & 3 \\ m & 0 & 1 \\ 3 & 2 & -1 \end{pmatrix}$ and If rank(A) =2 then m=……..

① -2 ② zero

③ 2 ④ 6

$\begin{vmatrix} 1 & -2 & 3 \\ m & 0 & 1 \\ 3 & 2 & -1 \end{vmatrix}=0 \quad \therefore 4m=8 \ \therefore m=2$

Q(4) The equation of the plane passing the points (2,3,5) , (-1,3,1) And (4,3,-2)

① $X+Y-Z=0$ ② X=-1

③ Y=3 ④ Z=-2

$Y=3$

Q(5) The equation of the plane passing through the points (1,-2,5) and vector (2,1,3) is perpendicular to it is ……

① 2X+Y+3Z=1 ② 2X+Y+3Z=15

③ X-2Y+5Z=15 ④ X+Y+Z=4

$$\mathbf{n}\bullet\mathbf{r}=\mathbf{n}\bullet A \quad \therefore (2,1,3)\bullet(X,Y,Z)=(2,1,3)\bullet(1,-2,5)$$
$$\therefore 2X+Y+3Z=15$$

Q(6) $e^{\theta i}+e^{-\theta i}=$……..

① $e^{2\theta i}$ ② $2\cos\theta$

③ $2\sin\theta$ ④ $e^{-2\theta i}$

$$\cos\theta+i\sin\theta+\cos-\theta+i\sin-\theta=\cos\theta+i\sin\theta+\cos\theta-i\sin\theta=2\cos\theta$$

Q(7) If $\begin{vmatrix}1&1&1\\1&2&3\\a&b&c\end{vmatrix}=5$ then $\begin{vmatrix}1&1&1\\1&2&3\\a+5&b+5&c+5\end{vmatrix}=$

① zero ② 1

③ 5 ④ 10

$$\begin{vmatrix}1&1&1\\1&2&3\\a&b&c\end{vmatrix}+\begin{vmatrix}1&1&1\\1&2&3\\5&5&5\end{vmatrix}=5+0=5$$

Q(8) The principle amplitude of the number $Z=1-i$ is …..

① $\frac{\pi}{4}$ ② $\frac{-\pi}{4}$

③ $-\frac{7\pi}{4}$ ④ $\frac{7\pi}{4}$

The number in 4th quadrant $\therefore\theta=-\tan^{-1}1=-\frac{\pi}{4}$

Q(9) The equation of the plane intercepting from coordinates axes(X,Y,Z) parts a,b,c respectively such that ab=2 , ac=3 ,bc=6

① $2X + 3Y + Z - 6 = 0$ ② $2X + 3Y + 6Z - 6 = 0$

③ $2X + 3Y + 6Z = 0$ ④ $6X + 3Y - 2Z - 6 = 0$

$a^2b^2c^2 = 36 \quad \therefore abc = 6 \quad \therefore \frac{abc}{ab} = c = 3 \; , \; \frac{abc}{ac} = b = 2 \; , \; \frac{abc}{bc} = a = 1$

$\frac{X}{1} + \frac{Y}{2} + \frac{Z}{3} = 1 \quad \therefore 6X + 3Y + 2Z - 6 = 0$

Q(10) If X={A,B,C,D,E,F} then The number of triangles whose vertices $\in X$ equals

① 15 ② 20

③ 17 ④ 21

${}^6C_3 = 20$

Q(11) $(2 + 7\omega + 2\omega^2)(2 + 7\omega^2 + 2\omega^4) = \ldots\ldots$

① 25ω ② 25

③ 125 ④ 20

$(-2\omega + 7\omega)(-2\omega^2 + 7\omega^2) = 5\omega \times 5\omega^2 = 25\omega^3 = 25$

Q(12) If the two straight lines L_1: X=2t-1 , Y=t+1 , Z=t-1 L_2:X=at-1 ,Y=2t+1, Z=bt-2 are parallel then a+b=….

① 4 ② -2

③ 6 ④ 2

$\frac{a}{2} = \frac{1}{2} = \frac{1}{b} \qquad \therefore a = 4 \; , b = 2 \qquad a + b = 6$

Q(13)Find the equation of the plane passing through the point (2,1,4) and perpendicular to each of the two planes 7X+Y+2Z=6 , 3X+5Y-6Z=8

$\because$ required plane $\perp$ to the planes

$\therefore (7,1,2)$, $(3,5,-6)$ are direction vectors lies in the required plane

$$\therefore \begin{vmatrix} \hat{i} & \hat{j} & \hat{K} \\ 7 & 1 & 2 \\ 3 & 5 & -6 \end{vmatrix} = -16\hat{i} + 48\hat{j} + 32\hat{K} \qquad \div 16 \qquad \therefore \text{required (n) is } (1,-3,-2)$$

$$r \bullet (1,-3,-2) = (1,-3,-2) \bullet (2,1,4)$$

Q(14) Put the number $Z = \dfrac{8}{1+\sqrt{3}i}$ in the trigonometric form

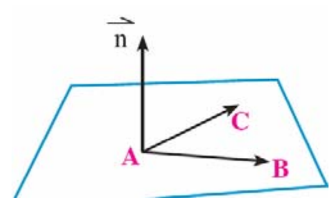
And hence find its two square roots in the exponential form

$$Z = \frac{8}{1+\sqrt{3}i} \times \frac{1-\sqrt{3}i}{1-\sqrt{3}i} = 2 - 2\sqrt{3}i \qquad \therefore r = 4 \qquad 4^{th} \text{ quadrant}$$

$$\theta = -\tan^{-1}\sqrt{3} = -\frac{\pi}{3}$$

$$\sqrt{Z} = 2e^{\frac{-\frac{\pi}{3}+2K\pi}{2}} \qquad \text{put } K=0 \ , \ K=1$$

Q(15) Find the equation of the plane passing through points (3 , -1, 0) , (2 , 1 , 4) and (0 , 3 , 3).



$$n = AB \times AC = \begin{vmatrix} \hat{i} & \hat{j} & \hat{K} \\ -1 & 2 & 4 \\ -3 & 4 & 3 \end{vmatrix} = -10\hat{i} - 9\hat{j} + 2\hat{K}$$

$$n \bullet r = n \bullet A$$

$$(-10,-9,2) \bullet r = (-10,-9,2) \bullet (3,-1,0) = -21$$

$$-10X - 9Y + 2Z + 21 = 0$$

Q(16)

Prove that: $\begin{vmatrix} c & b & a \\ a & c & b \\ a & b & c \end{vmatrix} = (b-c)(a-c)(a+b+c)$

$$(a+b+c)\begin{vmatrix} 1 & b & a \\ 1 & c & b \\ 1 & b & c \end{vmatrix} = (a+b+c)\begin{vmatrix} 1 & b & a \\ 0 & b-c & b-a \\ 0 & 0 & c-a \end{vmatrix}$$

$$= (b-c)(a-c)(a+b+c)$$

Q(17) Find the equation of the line of intersection of the two planes X + 2Y – 2Z = 1 , 2 X + Y – 3Z = 5

$$\begin{array}{l} -2X-4Y+4Z=-2 \\ \underline{\;\;2X+\;\;Y-3Z=5} \\ -3Y+Z=3 \end{array}$$

put $Y=0$ $\therefore Z=3$ $\therefore X=7$ $\therefore P_1=(7,0,3)$

Put $Z=0$ $\therefore Y=-1$ $\therefore X=3$ $\therefore P_2=(3,-1,0)$

$r=(7,0,3)-(3,-1,0)=(4,1,3)$

Equation : $(7,0,3)+t(4,1,3)$ $\therefore \frac{X-7}{4}=\frac{Y}{1}=\frac{Z-3}{3}$

Q(18) Using matrix solve : 3X+Y-2Z=-3 , 2X+7Y+3Z=9 , 4X-3Y-Z=7

$$|A| = \begin{vmatrix} 3 & 1 & -2 \\ 2 & 7 & 3 \\ 4 & -3 & -1 \end{vmatrix} = 88$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \frac{1}{88}\begin{pmatrix} 2 & 7 & 17 \\ 14 & 5 & -13 \\ -34 & 13 & 19 \end{pmatrix}\begin{pmatrix} -3 \\ 9 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$$

Q(19) **In the expansion** of $\left(2X^2+\frac{1}{2}\right)^7$ **Find the value of X which makes the third term equals the sixth term**

$T_3 = T_6 \quad \therefore \ {}^7C_2\left(\frac{1}{2}\right)^2\left(2X^2\right)^5 = {}^7C_5\ \left(\frac{1}{2}\right)^5\left(2X^2\right)^2$

$168X^{10} = \frac{21}{8}X^4 \quad \therefore \quad \frac{X^{10}}{X^4} = \frac{21}{8} \div 168 = \frac{1}{64} \quad \therefore \ X^6 = \frac{1}{64} \quad \therefore \ X = \frac{1}{2}$

Q(20) **Find the coordinates of the point on intersection of the line $r = (2,-1,2) + t(3,4,5)$ with the plane $r \bullet (1,-1,1) = 5$**

The line $r = (2+3t \quad ,-1+4t \ , \ 2+5t \ )$

Plane $(X,Y,Z)\bullet(1,-1,1) = 5 \quad \therefore X - Y + Z = 5$

$\therefore 2+3t+1-4t+2+5t = 5 \quad \therefore \quad 4t+5 = 5 \quad \therefore t = 0$

Point of intersection is $(2,-1,2)$

Answer the following questions 20 questions

From 1to 12 choose the correct answer

Q(1) number of ways to select a team formed from 6 members to chosen from 8 girls and five boys such that the team contains two boys only

① ${}^{12}C_6$ ② ${}^{8}C_4 \times {}^{5}C_2$

③ ${}^{8}C_4 \times {}^{8}C_2$ ④ ${}^{8}C_4 + {}^{8}C_2$

${}^{8}C_4 \times {}^{5}C_2$

Q(2) The amplitude of $(Z_1Z_2) = \frac{\pi}{6}$ and The amplitude of $(Z_1Z_3) = \frac{2\pi}{9}$ and The amplitude of $(Z_2Z_3) = \frac{5\pi}{18}$ then amplitude$(Z_1Z_2Z_3)$

① $\frac{\pi}{2}$ ② $\frac{\pi}{3}$

③ $\frac{\pi}{5}$ ④ $\frac{\pi}{6}$

$\theta_1 + \theta_2 = 30^\circ$, $\theta_1 + \theta_3 = 40^\circ$, $\theta_2 + \theta_3 = 50^\circ$

$\therefore 2(\theta_1 + \theta_2 + \theta_3) = 120^\circ \quad \therefore \theta_1 + \theta_2 + \theta_3 = 60^\circ = \frac{\pi}{3}$

Q(3) If the plane $20X + 15Y + 12Z = 60$ intersected with the axes X,Y and Z at the points A ,B and C respectively then the volume of the solid ABCO where O is the origin point equals …..cube unit

① 30 ② 90

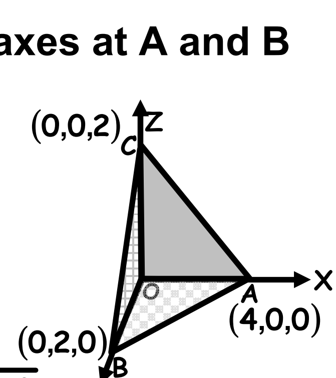
③ 30 ④ 10

$\frac{1}{3} \times \left(\frac{1}{2} \times 3 \times 4\right) \times 5 = 10$

Q(4) If the plane $\frac{X}{4} + \frac{Y}{2} + \frac{Z}{2} = 1$ cut the coordinates axes at A and B and C then the area of Δ Abc equal

① 12 ② 10

③ 6 ④ 4



$AC = C - A = (-4,0,2)$, $AB = B - A = (-4,2,0)$

$A = \frac{1}{2} AC \times AB = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{K} \\ -4 & 0 & 2 \\ -4 & 2 & 0 \end{vmatrix} = -4\hat{i} + 8\hat{j} - 8\hat{K} \quad \therefore \text{Area} = \frac{1}{2}\sqrt{4^2 + 8^2 + 8^2} = 6$

Q(5) If the two planes X-3Y+mZ=5 and 3X+KY+6Z=10 are parallel then Km=……

① 6 ② -20

③ 9 ④ -18

$\frac{1}{3}=\frac{-3}{K}=\frac{m}{6} \quad \therefore K=-9 \quad , m=2 \quad \therefore mK=-18$

Q(6) If $\begin{pmatrix} X-1 & 4 \\ 2 & X+1 \end{pmatrix}$ is singular matrix then X=

① -3 ② ± 3

③ 3 ④ 9

$X^2-1=8 \quad \therefore X^2=9 \quad \therefore X=\pm 3$

Q(7) If $(X-2)\times {}^nC_3 = {}^nP_3$ then X=…….

① 5 ② 8

③ 6 ④ 3n

$(X-2)=6 \quad \therefore \quad X=8$

Q(8) A sphere of center (2,-1,-2) and radius 3units placed on the plane 2X+6Y-3Z+K=0 then K=…….

① 13 ② 14

③ 16 ④ 17

$\frac{2(2)+6(-1)-3(-2)+K}{\sqrt{4+36+9}}=3 \quad \therefore \frac{4-6+6+K}{7}=3 \quad \therefore 4+K=21 \quad \therefore K=21$

Q(9) $\omega^{98}+\frac{1}{\omega^{98}}=\ldots\ldots$

① ω ② -1

③ 0 ④ 1

$\omega^{98}+\frac{1}{\omega^{98}}=\omega^{2}+\frac{1}{\omega^{2}}=\omega^{2}+\omega=-1$

Q(10) the direction vector of the line $L:\frac{X-2}{3}=\frac{Y+2}{2}$ and Z=4

① (3,2,4) ② (4,2,3)

③ (3,2,0) ④ (2,3,4)

(3,2,0)

Q(11) Measure of the angle between the two planes :X+Y-1=0 and Y+Z-1=0 equals ………..°

① 30 ② 45

③ 60 ④ 75

Angle between the normal $\cos\theta=\frac{(1,1,0)\bullet(0,1,1)}{\sqrt{1^2+1^2+0}\sqrt{0^2+1^2+1^2}}=\frac{1}{2}\therefore\theta=60^\circ$

$\therefore$angle between planes $90^\circ-60^\circ=30^\circ$

Q(12) The equation of the sphere its center M(1,-2,-5) and touch the XY plane …..

① $(X-1)^2+(Y+2)^2+(Z+5)^2=1$ ② $(X-1)^2+(Y+2)^2+(Z+5)^2=4$

③ $(X-1)^2+(Y+2)^2+(Z+5)^2=25$ ④ $(X-1)^2+(Y+2)^2+(Z-5)^2=25$

Q(13) Find the Cartesian equation of the plane
$(X,Y,Z)=(2,3,5)+t_1(-1,3,4)+t_2(6,1,-2)$ where t_1 and t_2 are parameters

$(-1,3,4)$, $(6,1,-2)$ are the direction vectors of two lines in the plane
To get the normal to this plane

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{K} \\ -1 & 3 & 4 \\ 6 & 1 & -2 \end{vmatrix} = -10\hat{i}+22\hat{j}-19\hat{K}$$

$$\therefore (X,Y,Z)\bullet(-10,22,-19)=(2,3,5)\bullet(-10,22,-19)$$

$$10X-22Y+19Z=49$$

Q(14) Find the measure of the angle between the two straight lines :
$r_1=(2,-1,3)+t_1(-2,0,2)$ and $r_2 :\ X=1\,,,\frac{Y-4}{3}=\frac{Z+5}{-3}$

$d_1=(-2,0,2)$, $d_2=(0,3,-3)$

$$\cos\theta=\frac{d_1\bullet d_2}{\|d_1\|\times\|d_2\|}=\frac{(-2,,0,2)\bullet(0,3,-3)}{\sqrt{(-2)^2+0^2+2^2}\sqrt{0^2+3^2+(-3)^2}}=\frac{1}{2}$$

$$\therefore \theta=\cos^{-1}\frac{1}{2}=60^\circ$$

Q(15) If 35 , 21 , 7 are the coefficient of three consecutive terms in the expansion $(1+X)^n$ find the value of n and the order of these terms

Let the terms are T_r , T_{r+1} , T_{r+2}

$$\frac{coofT_{r+1}}{coofT_r}=\frac{n-r+1}{r}\times\frac{1}{1}=\frac{21}{35}=\frac{3}{5}\rightarrow 5n-8r+5=0$$

$$\frac{coofT_{r+2}}{coofT_{r+1}}=\frac{n-(r+1)+1}{r+1}\times\frac{1}{1}=\frac{7}{21}=\frac{1}{3}\rightarrow 3n-4r-1=0$$

$$\therefore n=7,\ r=5$$

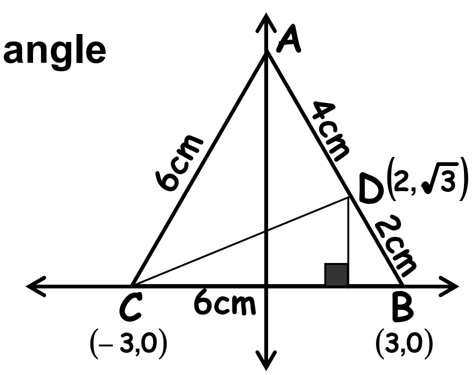
Q(16) Solve the following system of linear equations using the inverse matrix where X+Y-Z=1 , 2X-Y+2Z=3 , 3X+2Y-4Z=1

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 0 & 2 & 1 \\ 14 & -1 & -4 \\ 7 & 1 & -3 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Q(17)

<u>In the opposite figure</u>: ABC is an equilateral triangle

AD=4cm , DB=2cm find $CD \bullet CB$



$$CD = D - C = (2,\sqrt{3}) - (-3,0) = (5,\sqrt{3})$$
$$CB = B - C = (3,0) - (-3,0) = (6,0)$$
$$(5,\sqrt{3}) \bullet (6,0) = 30$$

Q(18) A sphere of centre (1,2,1) touches the plane X+Y+Z=1 find the equation of the sphere

Radius of the sphere is the perpendicular length from the centre
To the plane

$$= \frac{|1+2+1-1|}{\sqrt{1+1+1}} = \frac{3}{\sqrt{3}}$$

Equation of the sphere

$$(X-1)^2 + (Y-2)^2 + (Z-1)^2 = 3$$

Q(19) without expanding the determinant prove that

$$\begin{vmatrix} a+b+2 & a & b \\ 1 & 2a+b+1 & b \\ 1 & a & a+2b+1 \end{vmatrix} = 2(a+b+1)^3$$

$c_1 + c_2 + c_3$

$$\begin{vmatrix} 2a+2b+2 & a & b \\ 2a+2b+2 & 2a+b+1 & b \\ 2a+2b+21 & a & a+2b+1 \end{vmatrix} = (2a+2b+2)\begin{vmatrix} 1 & a & b \\ 1 & 2a+b+1 & b \\ 1 & a & a+2b+1 \end{vmatrix}$$

$$= 2(a+b+1)\begin{vmatrix} 1 & a & b \\ 0 & a+b+1 & 0 \\ 0 & 0 & a+b+1 \end{vmatrix} = 2(a+b+1)^3$$

Q(20) If $Z = \sin\frac{\pi}{9} + i\cos\frac{\pi}{9}$ find the cubic roots of $(\overline{Z})^9$

$Z = \cos 70^\circ + i\sin 70^\circ$

$\overline{Z} = \cos 70^\circ - i\sin 70^\circ \quad \therefore \overline{Z} = \cos -70^\circ + i\sin -70^\circ$

$(\overline{Z})^7 = \cos -70^\circ \times 9 + i\sin -70^\circ \times 9 = \cos -630^\circ + i\sin -630^\circ$

$= \cos 90^\circ + i\sin 90^\circ$

$\sqrt[3]{} = \cos\frac{90^\circ + 2\pi r}{3} + i\sin\frac{90^\circ + 2\pi r}{3}$

When r=0 $Z_1 = \cos 30^\circ + i\sin 30^\circ$ $e^{\frac{\pi}{6}i}$

When r=1 $Z_2 = \cos 150^\circ + i\sin 150^\circ$ $e^{\frac{5\pi}{6}i}$

When r=-1 $Z_2 = \cos -90^\circ + i\sin -90^\circ$ $e^{\frac{-\pi}{2}i}$