Test 1

1-a) A die is designed such that it carries the numbers from 0 to 5, if the die is tossed twice and the number on the face up is observed in each time. Find the probability that the sum of the scores in ≥ 8. Find also the probability that the one of the two appeared number is 4 and their sum is greater than 7.

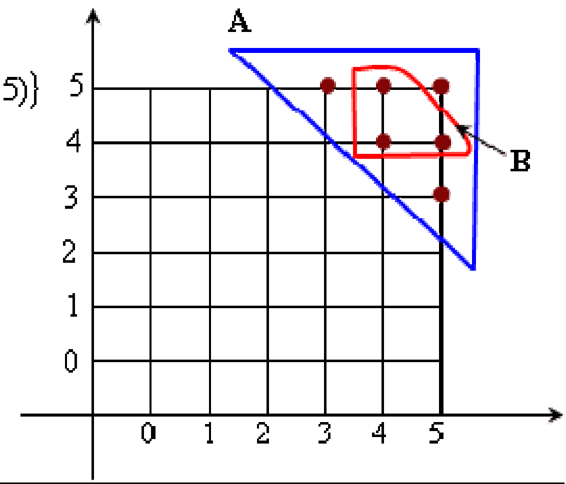
1-a) Number of the outcomes = 6 × 6 = 36

$$\because\ A = \{(3,5), (4,4), (4,5), (5,3), (5,4), (5,5)\}$$

$$\therefore\ P(A) = \frac{6}{36}$$

$$B = \{(4,4), (4,5), (5,4)\}$$

$$P(B) = \frac{3}{36}$$



1-b) Let x be a normal random variable with mean 100 and the variance 16 find:

1) the value of a if : P(x > a) = 0.5636

2) P (95 < x < 105)

1-b)

$$P(x > a) = 0.5636$$

$$\therefore\ P\left(\frac{x-\mu}{\sigma} > \frac{a-100}{4}\right) = 0.5636$$

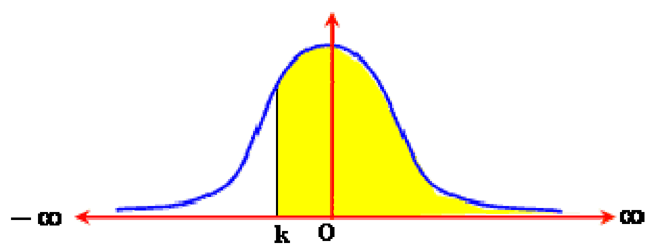
$$P(z > k) = 0.5636$$

$$0.5 + P(0 < Z < k) = 0.56636$$

$$P(0 < Z < k) = 0.0636$$

$$\therefore\ k = 0.16$$

$$\therefore\ \frac{a-100}{4} = 0.16 \qquad a = 100.64$$



$$2)\ P(95 < x < 105) = P\left(\frac{95-100}{4} < \frac{x-\mu}{\sigma} < \frac{105-100}{4}\right) = P(-1.25 < Z < 1.25)$$

$$= 2P(0 < Z < 1.25) = 2(0.3944) = 0.7888$$

2-a) If A and B are two events in S such that

$P(A) = \frac{5}{8}$, $P(B') = \frac{2}{3}$ and $P(A \cup B') = \frac{7}{8}$ find $P(A \cap B')$, $P(A' \cap B')$ and $P(A \cap B)$

2-a)

$$P(B) = 1 - P(B') = 1 - \frac{2}{3} = \frac{1}{3} ,$$

$\because \quad P(A \cup B') = P(A) + P(B') - P(A \cap B')$

$\therefore \quad \frac{7}{8} = \frac{5}{8} + \frac{2}{3} - P(A \cap B') \qquad \therefore \; P(A \cap B') = \frac{5}{12}$

$\because \quad P(A' \cap B') = 1 - P(A \cup B) = 1 - \left(P(A) + P(B) - P(A \cap B)\right)$

$$= 1 - \left(P(A) + P(B) - P(A) + P(A \cap B')\right)$$

$$= 1 - P(B) - P(A \cap B')$$

$$= 1 - \frac{1}{3} - \frac{5}{12} = \frac{1}{4}$$

$\because \quad P(A \cap B') = \frac{5}{12} \qquad \therefore \; P(A) - P(A \cap B) = \frac{5}{12}$

$\therefore \qquad P(A \cap B) = \frac{5}{8} - \frac{5}{12} = \frac{5}{24}$

2-b) Let x be a continuous random variable with density function:

$$f(x) = \begin{cases} \frac{x+2}{24} & , \; 2 \le x \le 6 \\ 0 & , \; \text{otherwise} \end{cases}$$

i) Show that the area under the curve of the density function bounded by the x-axis, between x = 2 , x = 6 is equal to 1

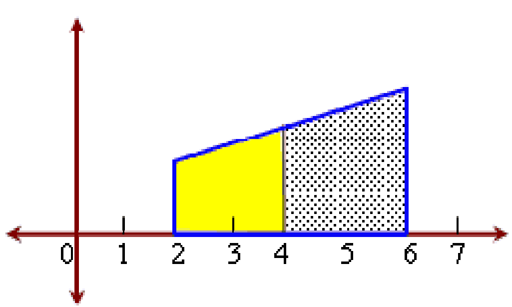
ii) find $P(1 \le x \le 4)$

2-b)

$$P(2 < x < 6) = \left(\frac{f(2) + f(6)}{2}\right) \times 4$$

$$= \left(\frac{4}{24} + \frac{8}{24}\right) \times 2 = 1$$

$\therefore$ f(x) is a probability density function



$P(1 \le x \le 4) = P(2 \le x \le 4)$

$$= \left(\frac{f(2) + f(4)}{2}\right) \times 2 = \frac{4}{24} + \frac{6}{24} = \frac{10}{24}$$

3-a) Let x be a discrete random variable its range is {0, 1, 2, 3, 4} if

$P(x = 1) = P(x = 3) = \frac{1}{4}$, $P(x = 0) = P(x = 4) = \frac{1}{6}$

find : 1) P(x =2)

2) find the coefficient of variation.

x	0	1	2	3	4
f(x)	$\frac{1}{6}$	$\frac{1}{4}$	b	$\frac{1}{4}$	$\frac{1}{6}$

$\because \frac{1}{6}+\frac{1}{4}+b+\frac{1}{4}+\frac{1}{6} = 1$

$\therefore b = \frac{1}{6}$

$\therefore$ the mean $\mu = 2$

x	f(x)	x.f(x)	x^2.f(x)
0	$\frac{1}{6}$	0	0
1	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
2	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{4}{6}$
3	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{9}{4}$
4	$\frac{1}{6}$	$\frac{4}{6}$	$\frac{16}{6}$
		2	$\frac{35}{6}$

The variance $\sigma^2 = \frac{35}{6} - (2)^2 = \frac{11}{6}$

The standard deviation $\sigma = \sqrt{\frac{11}{6}} = 1.35$

The coefficient of variation $= \frac{\sigma}{\mu} \times 100 = 67.7\,\%$

3-b) from the following data:

x	pass	exc	pass	pass	weak	v-good	good	weak
y	weak	v-good	good	good	pass	good	exc	pass

calculate spearman's rank correlation coefficient between x , y and determine its type.

3-b)

Ranks of x	Ranks of y	D	D^2
5	3	-3	9
1	2	-1	1
5	4	1	1
5	4	1	1
7.5	6.5	1	1
2	4	-2	4
3	1	2	4
7.5	6.5	1	1
			22
			$\sum D^2$

$$\because r_s = 1 - \frac{6\sum D^2}{n(n^2-1)} \quad , n = 8$$

$$r_s = 1 - \frac{6 \times 22}{8(63)} = 0.74$$

it is a direct correlation.

4-a) In studying the relation between the demanded quantity "y" of certain commodity and its price "x" LE, the following data are obtained :

$\sum x = 50$, $\sum y = 70$, $\sum xy = 374$, $\sum x^2 = 406$, $\sum y^3 = 536$, $n - 10$ find:

i) the linear correlation coefficient between x , y

ii) Estimate the demanded quantity when the price is 7 LE

4-i) $$r = \frac{n\sum xy - (\sum x)(\sum y)}{\sqrt{n\sum x^2 - (\sum x)^2}\sqrt{n\sum y^2 - (\sum y)^2}}$$

$$r = \frac{10(374) - (60)(70)}{\sqrt{10(406) - (60)^2} - \sqrt{10(536) - (70)^2}} = -1$$

it is a perfect inverse correlation.

4-ii) $y = ax + b$

$$a = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2} = \frac{10(374) - (60)(70)}{10(406) - (60)^2} = -1$$

$$b = \frac{\sum y - a\sum x}{n} = \frac{70 - (-1)(60)}{10} = 13$$

$\therefore$ $y = -x + 13$ when $x = 7$ $\therefore$ $y = -7 + 13 = 6$