

Algebra

*Revision :-

1- Write down each of the sequences whose n^{th} term is given by the relation :-

a) $T_n = n^2 - 1$ (to five terms starting with first term)

b) $T_n = \frac{(-1)^n}{2n+1}$ (to an infinite number of terms starting with first term)

2- Write down the first five terms and the general terms and the general term of

the sequence (T_n) which is defined as follows :-

$$T_1 = 2 \text{ and } T_n + 1 = 2T_n \text{ where } n \geq 1$$

3- show which of the sequences (T_n) is in increasing , or otherwise .

a) $T_n = 2n + 3$

b) $T_n = \frac{1}{3n-1}$

c) $T_n = \frac{(-1)^n}{2n} + 4$

4- Expand each of the following series , then find the expansion sum .

a) $\sum_{r=1}^4 (r^2)$

b) $\sum_{r=1}^7 (2r - 1)$

c) $\sum_{r=1}^n \left(\frac{1}{r+1} - \frac{1}{r} \right)$

5- Find in two different methods $\sum_{r=1}^4 (3 - 2r + r^2)$

6- Which of the following is an arithmetic sequence ? why ?

a) (7,10,13,16,19)

b) (27,23,19,15,11,.....)

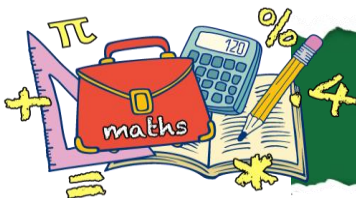
c) $\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6} \right)$

7- In the arithmetic sequence (13 , 16 , 19 , , 100)

a) Find the tenth term .

b) Find the number of the terms of the sequence .

8- Insert 5 arithmetic means between 6 and 48



Math

9- In the arithmetic series $5+8+11+\dots$ Find :

- a) The sum of its first twenty of the series .
- b) The sum of ten terms starting from the seventh term .
- c) The sum of the sequence terms starting from T_{10} up to T_{20}

10- Find the arithmetic sequence in which : $T_1 = 11$, $T_n = 87$ and $S_n = 980$

11- Show which of the following sequences (T_n) is geometric , then find the common ratio of each :

- a) $(T_n) = (2 \times 3^n)$
- b) $(T_n) = (4n^2)$
- c) The sequence (T_n) where : $T_1 = 12$, $T_n = \frac{1}{4} \times T_{n-1}$ (where $n > 1$)

12- In the geometric sequence (2 , 4 , 8 ,) , find :

- a) The fifth term
- b) the order of the term whose value is 512

13- (T_n) is a geometric sequence and all of its terms are positive .

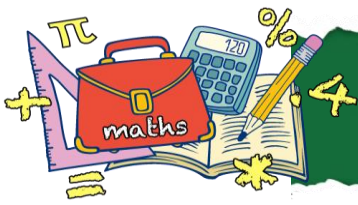
If $T_3 + T_4 = 6T_2$, $T_7 = 320$, Find This sequence .

14- Find the geometric means of the sequence : (4 , ... , ... , ... , ... , 2916)

15- Find the sum of the geometric sequence in which : $a = 3$, $r = 2$, $n = 8$.

16- If the sum of the first n terms of a geometric sequence is given by the rule :

$S_n = 128 - 2^{7-n}$, find the sequence and its seventh term .



Math

*Answer :-

1- a : (0 , 3 , 8 , 15 , 24)

b : ($\frac{-1}{3}$, $\frac{1}{5}$, $\frac{-1}{7}$, $\frac{1}{9}$,)

2- (2 , 4 , 8 , 16 , 32)

3- a) In Creasing b) de creasing c) neither in creasing nor de creasing

4- a) 30 b) 49 c) $\frac{-n}{n+1}$

5-A)(22) B) a) arithmetic b) arithmetic c)not arithe

7- a) 40 b) n = 30

8- 13 , 20 , 27 , 34 , 41

9- a) 670 b) 365 c) 517

10- (11 , 15 , 19 , , 87)

11- a) Geometric b) not geometric c) Geometric

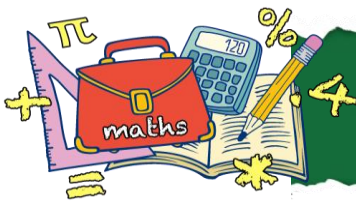
12- (32) , n=9

13- (5 , 10 , 20 ,)

14- 12 , 36 , 108 , 324 , 972

15- (765)

16- (64 , 32 , 16 ,) , $T_7 = 1$



*Calculus and trig :-

*Revision

1- If $f(x) = 3x^2 + x - 2$

and x varies from 2 to $2 + h$, find the function of variation V , then calculate the change in f when :

- a) $H = 0.3$ b) $h = -0.1$ c) 4.17 d) -1.27

2- Find the rate of change function in f when $x = x_1$ for each of the following then find this rate at the given values of x .

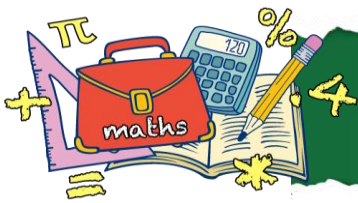
- a) $f(x) = 3x^2 + 2$ when $x = 2$ b) $f(x) = \frac{2}{x-1}$ when $x = 3$
c) 12 d) $-\frac{1}{2}$

3- Find the average rate of change function in f where

$f(x) = \sqrt{x+3}$ when $x = x_1$, then find the rate of change in f when :

- a) $X = 6$ b) $x = -2$ c) $\frac{1}{6}$ d) $\frac{1}{2}$

4- A piece of stone has fallen in calm water, a circular wave has been formed and expanded regularly so that the wave has kept its circular shape. Find the rate of change in its surface area with respect to its radius length when the radius length is 3.5 ($\pi = \frac{22}{7}$).



Math

5- Find the derivative function of the function f where

$f(x) = x^2 - x + 1$ using the definition of the derivative , then find the slope of the tangent at the point $(-2, 7)$

$F(x) = 2x - 1$, the shope of the taugent = -5

6- Prove that $f(x) = \frac{x-1}{x+1}$ is differentiable when $x = 2$

7- Show that the function f where $f(x) = \begin{cases} x^2 & \text{when } x \leq 2 \\ x + 2 & \text{when } x < 2 \end{cases}$

is not – differentiable when $x = 2$

Solution

8- Discuss the differentiability of the function f at $x = 3$ where $f(x) =$

$$\begin{cases} 2x - 1 & \text{when } x < 3 \\ 7 - x & \text{when } x \geq 3 \end{cases} \text{ not differentiable}$$

Solution

9- Find $\frac{dy}{dx}$ in each of the following :

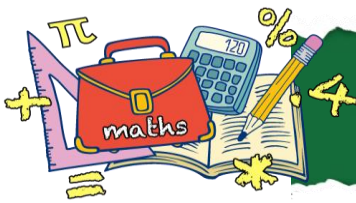
a) $Y = 2x^6 + x^{-9}$

b) $Y = \frac{\sqrt{x} - 2x}{\sqrt{x}}$

c) $12x^5 - 9x^{-10}$

d) $-x^{-\frac{1}{2}}$

10- Find $\frac{dy}{dx}$ if $Y = (x^2 + 1)(x^3 + 3)$, then find $\frac{dy}{dx}$ when $x = -1$



Math

11- Find $\frac{dy}{dx}$ If $Y = \frac{x^2-1}{x^3+1}$

$$\frac{dy}{dx} = \frac{-x^4 + 3x^2 + 2x}{(x^3+1)^2}$$

12- If $Y = (x^2 - 3x + 1)^5$, find $\frac{dy}{dx}$

$$= 5(x^2 - 3x + 1)^4(2x - 3)$$

13- From a point on the ground surface , a man observed the top of a tower of an angle of elevation of 25° , then he walked straight ahead for 57 m at the horizontal level toward the tower base to find that the angle of elevation of the tower top is $52^\circ 30'$. Find the height of the tower to the nearest meter .

(41 m.)

14- From the top of a hill , a man observed the measures of the two angles of depression of the top and the base of a tower to give 22° and 30° respectively . If the height of the tower is 50 meters , find the height of the hill to the nearest meter known that the two bases of the tower and hill are in the same horizontal level .

(167 m)

15- Find :

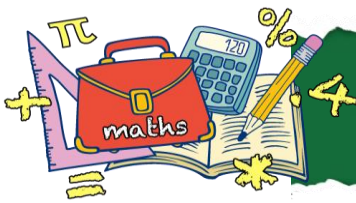
a) $\sin 75^\circ$

b) $\cos 15^\circ$

a) $a = \frac{r^2+r6}{4}$

b) $\frac{r^2+r6}{4}$

16- If $\sin A = \frac{3}{5}$ where $90^\circ < A < 180^\circ$, $\cos B = \frac{-5}{13}$



Math

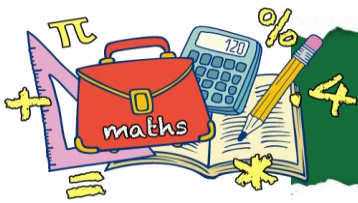
Where $180^\circ < B < 270^\circ$

Find $\cos (A-B)$, $\sin (A+B)$

$$\cos(A-B) = \frac{-16}{65} \quad , \quad \sin (A+B) = \frac{33}{65}$$

17- Without using the calculator , prove that :

$$\tan 50^\circ = \frac{1+\tan 5^\circ}{1-\tan 5^\circ}$$



Math

Applied mathematics

1- If \vec{r} is the position vector of a particle moving in a straight line from the point

O , r is given as a function in time by the relation $\vec{r} = (t^2 + 3t - 2) \vec{I}$ where I is a fixed unit vector . Find the displacement vector after 4 seconds.

2- A particle moves such that its position vector r is given as a function of time in terms of the two fundamental unit vectors I and j by the relation

$$r = (6t - 3) \vec{i} + (8t + 1) \vec{j}$$

Find the displacement vector at $t = 3$ sec .

3- A cyclist covered 40 km . on a straight road with a velocity of 20 km ./hr , then he returned back and covered 15 km. in the opposite direction with a velocity of 15 km./hr .

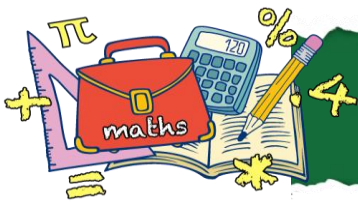
Find the average velocity during his while trip .

" $8\frac{1}{3}$ km./hr. in the direction of the first motion "

4- A traveller moved on a straight road , he covered 800 meters with velocity 9km./hr . and then he covered the same distance in the same direction in the same direction with velocity 4.5 km./hr.

Find the magnitude of the average speed of the traveller during the whole trip.

($\frac{5}{3}$ m./sec.)



Math

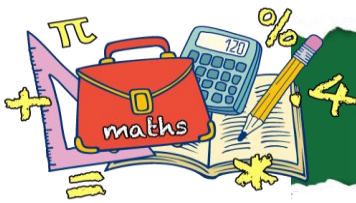
5- A body moved on a straight line a distance 100m. with velocity 5 m./sec., then it moved with velocity 8 m/sec. in the same direction for 10 seconds during the whole trip. Find the avelocity **(6m./sec)**

6- A train of length 150 metres intered a straight tunnel of length s metres . It took the entire crossing of the tunnel in a time of 15 seconds . Find the length of the tunnel if the train moves with uniform velocity equals 90 km./hr. **(225 metres)**

7- A moving radar car to monitor the velocity on the desert road moves with velocity 40 km./hr. this car observes the movement of a truck coming in the opposite direction . It seems like it is moving with velocity 120 km./hr What is the actual velocity for the truck ? **(80 km./hr.)**

8- The distance between two cities A and B is 120 km. A car moved from the city A towards the city B with a velocity of 88 km./hr the same moment , anither car moved from the city B towards the city A with a uniform velocity of 72 km./hr.

Find when and where do the two cars meet . **($\frac{3}{4}$ hour , 66 km. from A)**



Math

9- A police car (A) moves on a straight road with velocity of magnitude 25 km./hr. It observes another car (B) moving on the same road with velocity of magnitude 75 km./hr

Find the relative velocity of the car (B) with respect to the car (A) if :-

(1) The two cars move in the same direction .

(2) The car (B) moves in the opposite direction of the car (A) .

(50km./hr. in direction of motion of motion , 100 km./hr in the opposite direction of the police car)

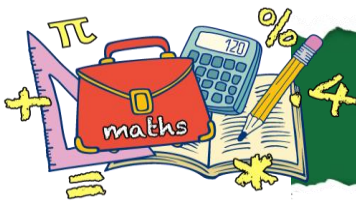
10- A particle started its motion in constant direction with velocity 20 cm./sec. and a uniform acceleration of magnitude 2 cm./sec^2 in the opposite direction of the initial velocity vector . Find :

(1) Its velocity at the end of 10 seconds from starting motion.

(2) Time elapsed from starting motion till its velocity becomes 3.6 km./hr . in the opposite direction of starting motion's direction . (zero , 60 seconds)

11- If the velocity of a moving car (A) in a straight line changes from 24km./hr to 36km./hr. within 5 seconds , the velocity of another car (B) moving in the same straight line changes from 12 km./hr. to 30 km./hr. within the same time interval , which of them move with more acceleration explain your answer.

(the car B)



Math

12- A particle moves in a straight line with a uniform acceleration so its velocity increased from 15m./sec . into 25 m./sec. after covering 125 meter.

Find the time takes for that . **($6\frac{1}{4}$ seconds)**

13- A body started its motion from rest in a horizontal straight line with uniform acceleration of magnitude 4 cm./sec². For 30 seconds , then it moves with the velocity it gained for another 40 seconds. Find the magnitude of its average speed.

($94\frac{2}{7}$ cm./sec.)

14-A small ball is projected with velocity 20 m./sec. horizontally . It moved in a straight line with retardation where the uniform acceleration = $\frac{1}{2}$ m./sec² .

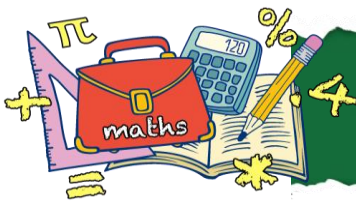
Determine the position of the ball and its velocity after 2 seconds from starting motion .

(39 m. , 19 m./sec.)

15- Karim Practices on riding the bicycle . His father pushes him to gain a constant acceleration of magnitude $\frac{1}{2}$ m./sec². For 6 seconds and after that Karim rides the bicycle alone with the velocity gained for another 6 seconds before he falls on the ground . Find the distance that Karim will cover .

(27 metres)

16-Find the coordinates of the centre , also find the radius length for each of the following circles :-



Math

$$(1) x^2 + y^2 - 8 = 0$$

$$(2) (x + 3)^2 + (y - 5)^2 = 49$$

$$(3) (x + 4)^2 + y^2 = 9$$

$$(4) x^2 + (y + 7)^2 = 24$$

$$(5) x^2 + y^2 - 4x + 6y - 12 = 0$$

$$(6) x^2 + y^2 + 4y = 8$$

$$1) (0,0) \quad r = 2\sqrt{2} \text{ cm}$$

$$2) (-3,5) \quad r = 7 \text{ cm}$$

$$3) (-4,0) \quad r = 3 \text{ cm.}$$

$$4) (0,-7) \quad r = 2\sqrt{6} \text{ cm}$$

$$5) (2,-3) , \quad r = 5 \text{ cm.}$$

$$6) (0,-2) , r = 2\sqrt{3} \text{ cm.}$$