Unit Two: Linear Motion

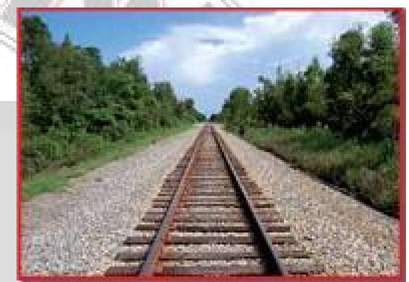
Chapter (1): Motion in a Straight Line

Objects around us can be sorted into stationary objects and moving objects. As we study the motion of different objects, it is necessary to describe and understand such motion. The vague ideas about motion convert travelling by ships, trains and planes into a mess. Schedules of departure and arrival of different transportations are mainly based on distances, times and speeds.

Accordingly, in this chapter we are going to investigate the concept of motion and the related physical quantities.

1- Motion:

It is the change in the position of the object with respect to a fixed point as the time passes.



If this motion is in one dimension along a straight path, it is then called **motion in a straight line.** This is considered the simplest type of motion

Motion diagrams: the motion of an object can be represented by a series of photos taken in equal intervals of time.

The pattern that represents the sequence of motion.

Types of Motion:

Translational motion: is the motion which is characterized by having a starting point and end point.

Examples are motion in a straight line, and projectile motion.

Periodic motion: is the motion that repeats itself over equal intervals of time.

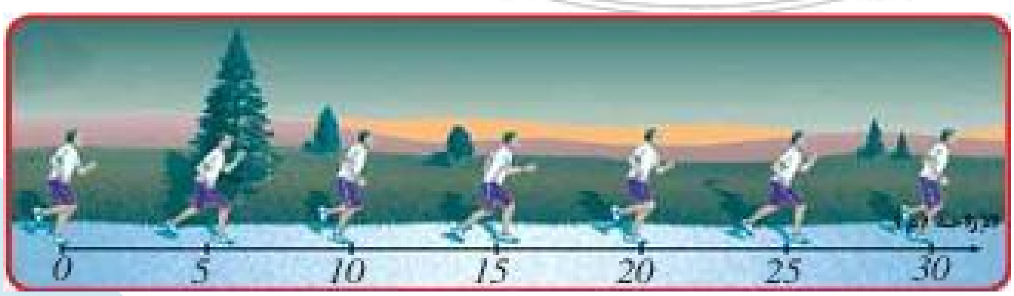
Examples are motion in a circle and vibrational motion.

2- Velocity:

Moving objects are described as being slow or fast. Scientifically this is not fairly accurate.

We can describe the object motion quantitatively through the concept of velocity.

To identify this concept, study the following motion diagram for athlete displacements every one second.



Velocity: is the displacement moved by the object in one second, or the rate of change of displacement. It is measured in (m/s) or (km/h).

- Through this diagram, the relation between displacement and time are recorded in the table below:

Time (s)	0	1	2	3	4	5	6
Displacement (m)	0	5	10	15	20	25	30

Using this table we can draw a conclusion that the athlete displacement is (5 m) every second. This quantity is known as velocity (v) that can be found by the slop of the relation:

$$\text{Velocity} = \frac{\text{Change of displacement}}{\text{Time of change}} \quad . \quad v = \frac{\Delta d}{\Delta t}$$

Applying this relation on the above mentioned example:

$$v = \frac{\Delta d}{\Delta t} = \frac{d_2 - d_1}{t_2 - t_1} = \frac{10 - 5}{2 - 1} = \frac{5}{1} = 5\ m/s$$

Graphical representation of the relationship between displacement and time:

-Displacement is represented on the ordinate (y-axis) and time on the abscissa (x-axis):

-Draw a vertical line starting from the point (1s) on time axis.

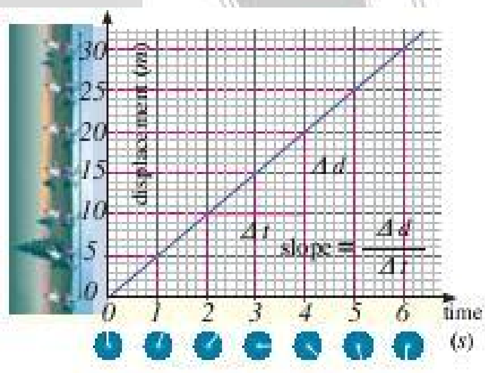
-Draw a horizontal line starting from the point (5 m) on displacement axis.

-Highlight the point of intersection of the two lines.

-Repeat with the other points of time and displacement.

-Draw a straight line passing through most of the highlighted points.

-Calculate velocity by getting the (slope) of the straight line.



Thinking Corner:

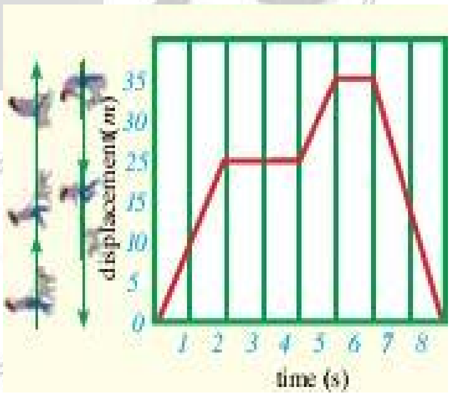
The opposite graph represents a walk of a girl from her house and returning back again. Study the graph and answer the questions:

When did the girl stop walking?

What is the maximum velocity of the girl motion?

Why is her velocity when moving back considered negative?

What is the difference between displacement and distance moved by the girl?



Types of velocity:

(A) Speed and Velocity:

Focusing on the speedometer of a car, its pointer swings right and left during car movement. The pointer reading specifies the value of the car speed (for example, 80 km/h) without defining the direction of the car motion. This value is known as (Speed).

Figure (7): Does the speedometer read speed or velocity?

However, just saying that a car moves at 80 km/h is an incomplete description since no hint is given about the direction of the car motion. Accordingly we need to define such direction to give a full description for the car motion. For instance, saying that the car moves at 80 km/h to east. in this case, we call this (Velocity).

Point of comparison	Speed	Velocity
Definition	The distance moved by the object per unit time.	The displacement of the object per unit time.
Its type	Scalar, defined by its magnitude only.	Vector, defined by its magnitude and direction.
Its sign	Always positive.	Positive in one direction and negative in the opposite direction.

It is worthy to mention that we are interested in velocity rather than speed when discussing the next texts, equations and problems since it offers a full description of motion.

(B) Uniform and Variable Velocity

As an athlete runs at uniform velocity, his displacements are equal in equal times. But if he moves at non uniform velocity, his displacements are unequal in equal times.

Uniform velocity: the object velocity when it is displaced through equal displacements in equal times. Both the velocity magnitude and direction are constant (when the object moves in a straight line).

Non-uniform velocity: the object velocity when it is displaced through unequal displacements in equal times. Its velocity may change in magnitude or direction.

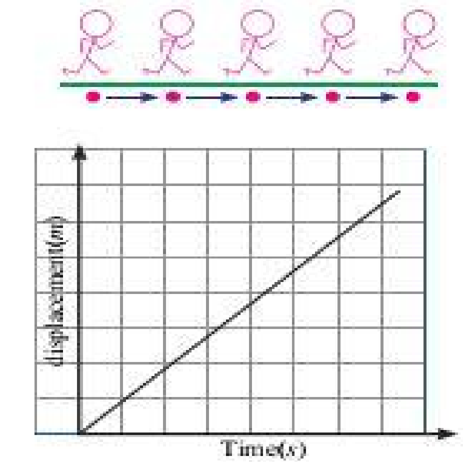


Figure (8): motion at uniform velocity

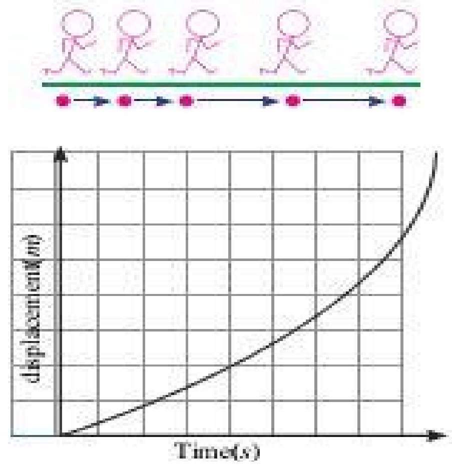
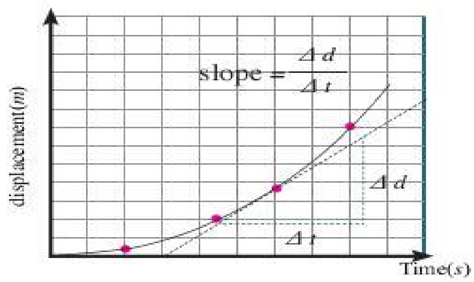


Figure (9): motion at variable velocity

(C) Instantaneous and Average Velocity



Instantaneous velocity $(v) = \frac{\text{Change of displacement } (\Delta d)}{\text{Time of change } (\Delta t)}$

The motion of a car on roads is certainly variable. The car speeds up or slows down responding to the traffic conditions. Consequently, we are going now to distinguish between the Instantaneous Velocity and Average Velocity of the object (car).

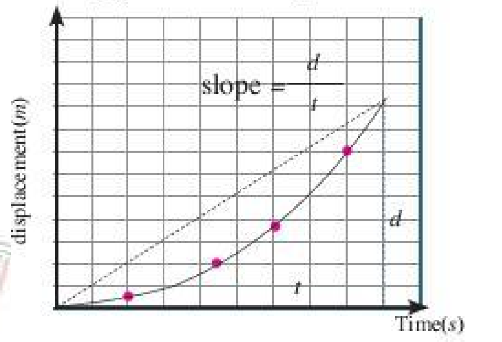
Instantaneous Velocity (V): It is the velocity of the object at a given instant.

For example, the reading of the speedometer pointer at a given instant.

It can be determined graphically by the slope of the tangent drawn to the velocity curve at that instant.

Average Velocity ($\overline{v}$): It is given by dividing the total displacement of the object from the starting point to the end point by the total time of motion.

It can be determined graphically by the slope of the straight line joining the starting point to the end point.



- Average velocity $(v) = \frac{\text{Total displacement } (d)}{\text{Total Time } (t)}$

Correcting misconceptions

One of the common misconceptions is the confusion between average velocity which is a vector quantity and average speed which is a scalar quantity:

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total Time}} \qquad \text{Average speed} = \frac{\text{Total distance}}{\text{Total Time}}$$

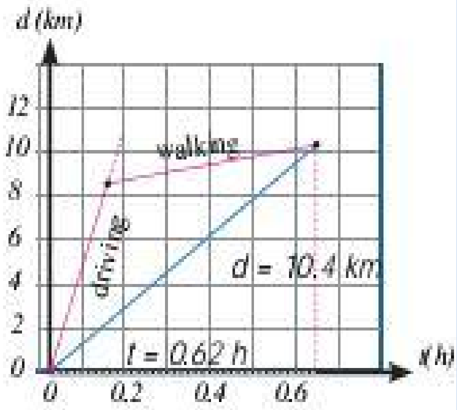
Time management:

Set a target for each task you are going to accomplish. Examine your targets. Are they realistic or not? Decide what you want to achieve and why.

Design your daily or weekly schedule to arrange your activities and define deadlines. Keep a memo or reminder to record activities, dates and duties.

Solved Examples:

1- A person drove a car in a straight line to cover (8.4 km) in (0.12 h). Because the fuel had run out, he walked through (2 km) along the same straight line to reach the nearest gas station after (0.5 h). Calculate the average velocity of this journey.



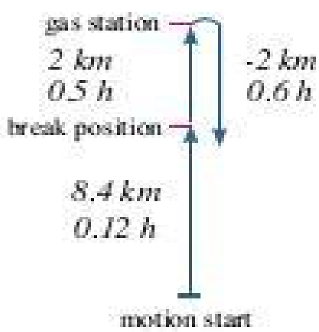
Solution:

$$\text{Average velocity} = \frac{\text{Total displacement } (d)}{\text{Total Time } (t)}$$

$$\overline{v} = \frac{d}{t} = \frac{8.4 + 2}{0.12 + 0.5} = \frac{10.4}{0.62} = 16.8\ km/h$$

We can get the same result by finding the slope of the straight line joining the starting point to the end point.

2- If the person in the previous example returned back to his car in 0.6 h, find the average velocity during the whole story.



Solution:

The total displacement = (8.4 km)

$$\overline{v} = \frac{d}{t} = \frac{8.4}{0.12 + 0.5 + 0.6} = \frac{8.4}{1.22} = 6.88\ km/h$$

3-Acceleration

We have discussed the concept of the variable velocity (magnitude, direction or both).

Motion in which velocity changes with time is called the accelerated motion and the quantity that expresses the change of velocity per unit time is called acceleration (a).

To investigate the concept of acceleration, study the following motion diagram that illustrates the readings of the speedometer of a car moving from rest and speeds up in a straight line.

Acceleration: the change of the object velocity per unit time, or the rate of change of velocity. It is measured in (m/s^2) or (km/h^2).

Do you know?

You can convert the speedometer reading from km/h into m/s by the relation:

$$\therefore 1\ km/h = \frac{1\ km}{h} = \frac{1000\ m}{60 \times 60\ s} = \frac{5}{18}\ m/s$$

Recording the data of the given diagram that include velocity (m/s)and time (s), we obtain the table below:

Time (s)	0	1	2	3	4
Velocity (m/s)	0	5	10	15	20

It is obvious that the car speeds up at a constant rate where its velocity increases by (5m/s)، every second. This value expresses the acceleration of motion that can be found by the slop of relation:

$$\text{Acceleration} = \frac{\text{Change of velocity}}{\text{Time of change}} = \frac{\text{Final velocity} - \text{initial velocity}}{\text{Final time} - \text{initial time}}$$

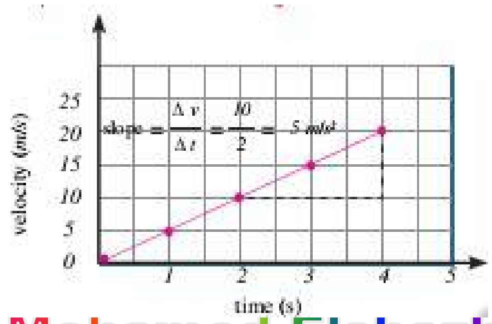
$$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

Applying this relation on the above mentioned example:

$$a = \frac{\Delta v}{\Delta t} = \frac{10 - 5}{2 - 1} = 5\ m/s^2$$

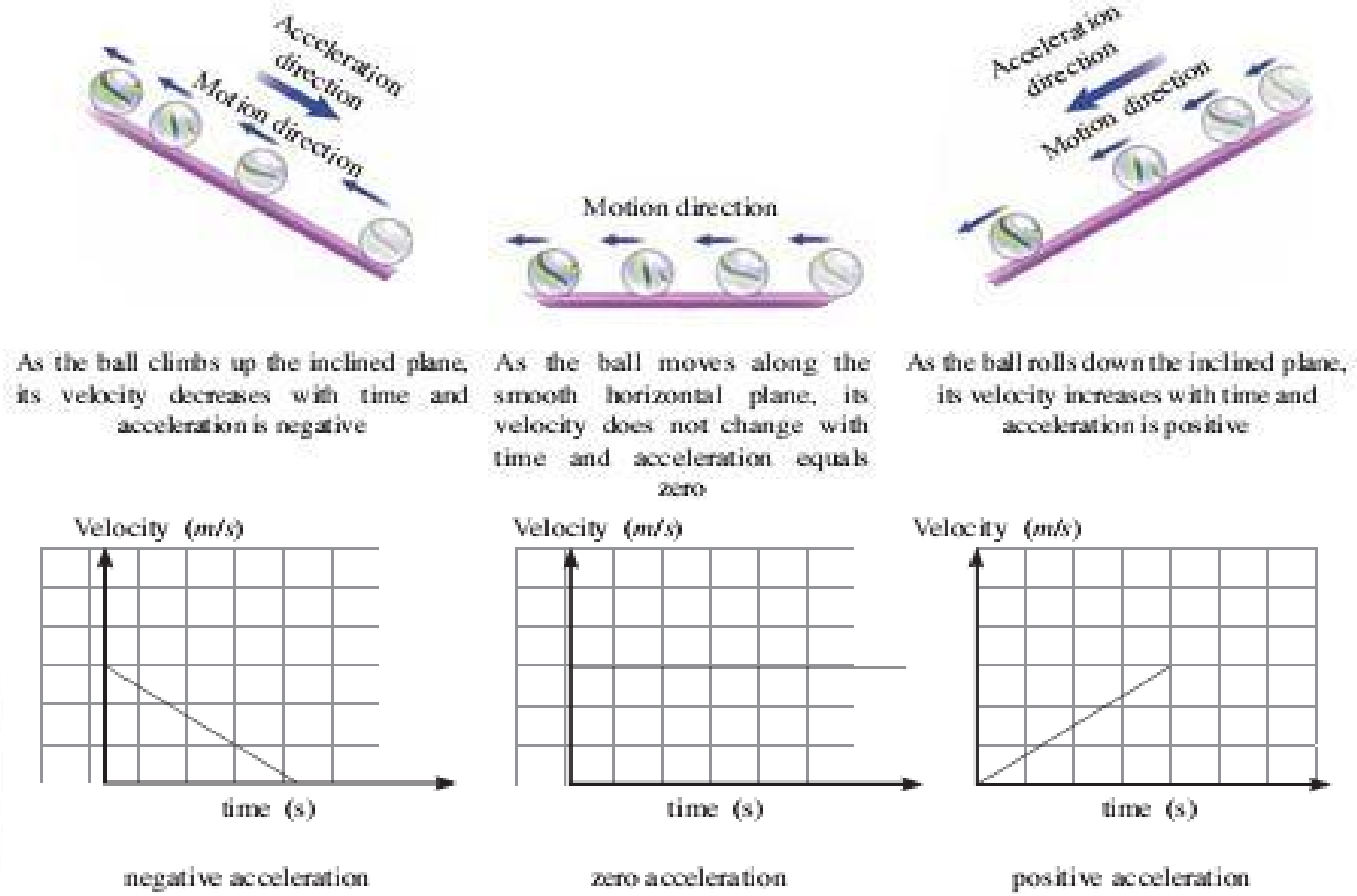
Graphical representation of the relationship between velocity and time:

The graph (velocity – time) expresses the motion of the car in the previous motion diagram. Notice that a straight line is obtained indicating that the velocity of the car increases uniformly and acceleration can be found by the slope of the straight line.



Types of Acceleration:

Objects may move at positive acceleration (increasing velocity), negative acceleration or deceleration (decreasing velocity) or zero acceleration (uniform velocity). These types can be identified by studying the following motion diagram that shows the motion of a small ball along frictionless planes of different inclination.



Life Applications:

✦ Three tools in the car can control the magnitude or the direction of the car velocity. These are the accelerator to pump more fuel, the brakes to slow down and the steering wheel to change direction.

Chapter (2): Motion with Uniform Acceleration

You have studied in the previous chapter that acceleration is the change in velocity per unit time. This acceleration may be uniform (constant) or varying.

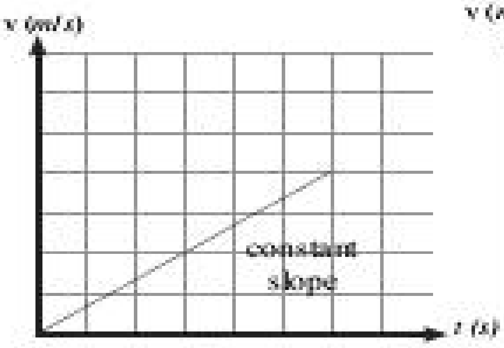


Figure (11): motion at uniform acceleration

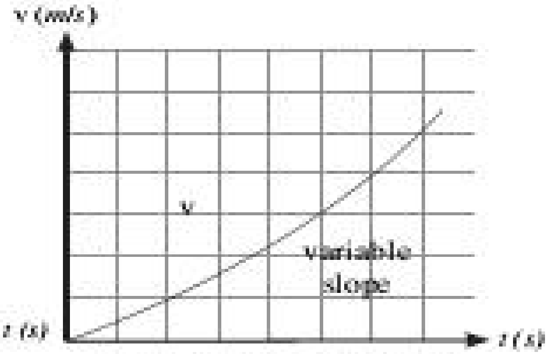


Figure (12): motion at non uniform acceleration

Motion of an object at uniform acceleration has a special importance since it represents the motion of a number of objects in our experience. Examples may include those objects falling near the Earth's surface and projectiles.

Figure (13): falling of water from the top of a waterfall is at uniform acceleration

Figure (14): skating in air is at uniform acceleration

Assuming that an object moved in a straight line at uniform acceleration (a)، and started motion from rest at initial velocity (v_i) It reached a final velocity (v_f) after an interval (t) during which it was displaced through a displacement (d)، We can describe such motion using certain equations called equations of motion as follows:

1- Equation of (Velocity- Time):

You know that acceleration (a) is given by the relation:

$$a = \frac{v_f - v_i}{t}$$

The change of velocity (v_f - v_i) can be obtained by multiplying both sides by (t)

$$v_f - v_i = at$$

$$\boxed{v_f = v_i + at} \qquad (1)$$

Thinking Corner:

Compare the acceleration of motion of the fastest animal on Earth and that of one of the fastest cars using the previous equation.

Figure (15): Cheetah can change its speed from zero to 110 km/s in 3 seconds

Figure (16): Bugatti Veyron car changes its speed from zero to 100 km/s in 2.4 seconds

2- Equation of (Displacement- time):

The average velocity of a moving object can be given by the relation :(v)

$$\bar{v} = \frac{d}{t}$$

Since the object moves at uniform acceleration, the average velocity can be given by the relation:

$$\bar{v} = \frac{v_f + v_i}{2}$$

From the two forms

$$\frac{d}{t} = \frac{v_f + v_i}{2}$$

Substituting (v_f) from the first equation of motion:

$$\therefore \frac{d}{t} = \frac{(v_i + at) + v_i}{2} = \frac{2v_i + at}{2} = v_i + \frac{1}{2}at$$

Multiplying both sides by (t), we obtain:

$$\therefore \boxed{d = v_i t + \frac{1}{2}at^2} \qquad (2)$$

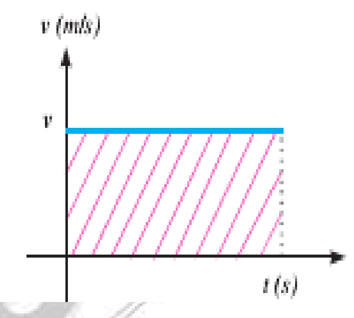
Pay Attention:

❋When the object moves in a straight line in one direction, its displacement (d)equals the distance it covers (s).

❋When the object moves in a straight line but changes direction, such as the object that is projected vertically upwards then falls back to surface, its displacement (d) does not equal the distance it covers (s).

Deriving the second equation of motion graphically:

Since displacement = velocity x time, this corresponds the numeral value of the product length x width in the (velocity – time) graph, or in other words the area below the curve.



Accordingly, we can deduce the second equation of motion and find the displacement of the object by getting the area below the curve in (velocity – time) graph i.e. the area of both the rectangle and the triangle.

The area of rectangle $= v_i\, t$

The area of the triangle $= \frac{1}{2}(v_f - v_i)\, t$

Since the change in velocity ($v_f - v_i$) equals (at)

The area of the triangle becomes: $\frac{1}{2}\, at^2$

The object displacement (d) = sum of the areas of the rectangle and the triangle

$$d = v_i t + \frac{1}{2}at^2$$

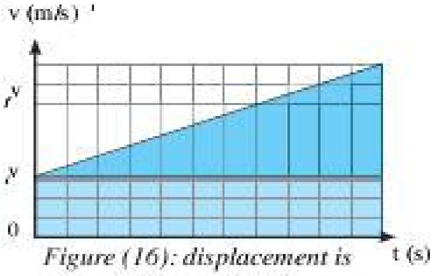


Figure (16): displacement is numerically equal to the area below the curve.

3- Equation of (Displacement- Velocity):

Sometimes time of motion is unknown. Because of this we need to deduce an equation independent on time as follows:

Displacement of object can be determined by the relation: $d = \overline{V}\, t$

Substituting $(\overline{V})$ and (t) using the following formulae:

$$\overline{v} = \frac{v_f + v_i}{2} \qquad t = \frac{v_f - v_i}{a}$$

Thus, displacement can be found as follows:

$$d = \overline{v}\, t = \frac{v_f + v_i}{2} \times \frac{v_f - v_i}{a} = \frac{v_f^2 - v_i^2}{2a}$$

The third equation of motion can be obtained:

$$\therefore 2ad = v_f^2 - v_i^2 \qquad (3)$$

Three equations of motion are obtained right now that can be applied to the motion of an object at uniform acceleration. Except for time, all the included quantities are vectors. So,

First of all a positive direction should be agreed at. For instance, if the direction to right is considered positive, displacement, velocity and acceleration are positive if their directions are to right and negative if their directions are to left. The table below summarizes some cases based on equations of motion:

General formula	Motion starts from rest $v_i = 0$	Stopping at the end of motion $v_f = 0$	Motion at uniform velocity $a = 0$
$v_f = v_i + at$	$v_f = at$	$v_i = -at$	$v_f = v_i$
$d = v_i t + \frac{1}{2}at^2$	$d = \frac{1}{2}at^2$	$d = -\frac{1}{2}at^2$	$d = v_i t$
$2ad = v_f^2 - v_i^2$	$2ad = v_f^2$	$2ad = -v_i^2$	$0 = v_f^2 - v_i^2$

Overcoming Learning Difficulties:

You may find difficulty in converting the word problems into a mathematical form. The following guidelines may help:

❋ Its speed increases means: acceleration is positive (if velocity is positive)

❋ Its speed decreases means: acceleration is negative (if velocity is positive)

❋ When? Means find the time. (t)

❋ Where? Means find the displacement. (d)

Time management:

- Estimate the time interval expected to finish a particular activity.

- Make balance between studying, doing assignments, and homework at one hand, social events, hobbies and fun at the other hand. Evaluate the importance of various duties and tasks and arrange priorities.

Solved Examples:

1- An aeroplane lands on the runway at velocity 162 km/h and decelerates uniformly at ($0.5 m/S^2$)Find the time it takes till stops.

Solution:

$$v_i = 162 \times \frac{5}{18} = 45 \text{ m/s} \qquad v_f = 0$$

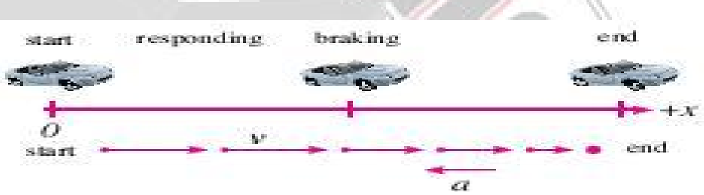
$$a = -0.5 \text{ m/s}^2 \qquad v_f = v_i + a t$$

$$0 = 45 + (-0.5)t \qquad -45 = (-0.5)t$$

$$t = 90 \text{ s}$$

2- Mohamed drove a car at uniform velocity (30 m/s). Suddenly, he saw a child crossing the street and he applied the brakes to decelerate the car uniformly at ($9 m/s^2$). If Mohamed's reaction time to use the brakes is (0.5 S), find the displacement of the car till it stopped.

Solution:



Displacement of the car during reaction time (uniform velocity):

$$d_1 = v \cdot t = (30) \times (0.5) = 15m$$

Displacement of the car when applying the brakes (uniform acceleration):

Refering to the table in page: (38)

$$2\, a d_2 = -v_i^2$$

Since, v_f (of reaction time) = v_i (when braking)

$$\therefore 2\, a d_2 = -v_f^2 \text{ (of reaction)}$$

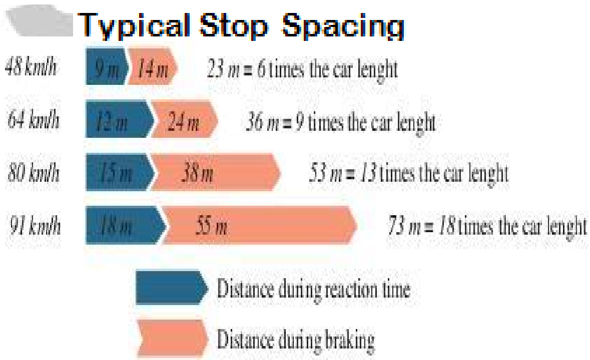
$$\therefore d_2 = \frac{-v_f^2}{2a} = \frac{-(30)^2}{2 \times -9} = 50m$$

The total displacement

$$d = d_1 + d_2 = 15 + 50 = 65\ m$$

Note that the total displacement of the car is the same as the total distance covered by the car if the car keeps moving in a straight line.

Safety Skills:



✦ To save souls, avoid the dangers of skipping prescribed speeds, and traffic instructions should be followed. One of these rules is leaving an appropriate distance between vehicles in order to allow the driver to stop safely in case of emergency. Obviously, more spacing is required as the speed of cars get higher, or the road is wet or has oil stains. Also, trucks should leave larger spacing than small cars do.

Applications of motion at uniform acceleration:

Free fall:

Figure (18): would two objects reach the ground at the same instant in a vacuum?

If we drop a book and a sheet of paper at the same instant from the same height, which of them reach the ground first? But, when the sheet of paper is placed adjacent to the book topside and allowed to fall, what would you observe? Explain your observation.

When an object falls to ground, its motion is affected by the air resistance due to collisions between the object and air molecules. The impact of this resistance is greater on the velocity of falling light objects than that of heavier objects.

Note that no air resistance affected the sheet of paper when it was placed adjacent to the topside of the book during falling.

To simplify this issue, we are going to study the fall of objects under the effect of their weights, only neglecting the effect of air resistance. This motion is called free fall. It is worthy to mention that at the absence of air resistance, all objects fall to the ground at the same acceleration.

Distinguished Scientists:

Figure (19): Galileo's Experiment of free fall

Galileo proved that falling objects of different masses reach the ground at the same time, when air resistance is neglected.

By dropping two objects of different masses down Tower of Pisa. This experiment put an end for Aristotle thoughts that implied that heavier objects would reach the ground first.

Free Fall Acceleration (g):

It is the uniform acceleration of objects that fall freely.

This acceleration equals (9.8 m/s^2) and means that the object velocity when falling freely increases by (9.8 m/s) every second.

Figure (20) Does this person fall at acceleration 9.8 m/s^2?

This acceleration (g) varies from one position to another depending on its distance from the Earth's centre. For simplicity, it can be considered (10 m/s^2)

Solved Examples:

Study the table then answer the questions below:

Time (s)	Displacement (m)	Velocity (m/s)
0	0	0
0.5	1.25	5
1	5	10
1.5	11.25	15
2	20	20

1. Use equations of motion to calculate displacement and velocity after 3 s.

2. What do you conclude from the spacing increase between the object positions with time?

3. using the data recorded, plot the graphical relationships (displacement – time) and (velocity – time)

2- A box fell from a helicopter that was staying still at 78.4 m high above sea level. Find the velocity by which the box hit water giving that acceleration due to gravity 9.8 m/s^2 neglecting the air resistance. Also,

Find the time it took till splash.

Solution:

$v_i = 0$, $g = 9.8$ m/s^2 , $d = 78.4$ m

$2\,g\,d = v_f^2 - v_i^2 \qquad 2 \times 9.8 \times 78.4 = v_f^2$

$v_f = 39.2$ m/s

$t = \dfrac{v_f - v_i}{d} = \dfrac{v_f}{d} = \dfrac{39.2}{9.8} \qquad t = 4$ s

3- A stone fell from the roof of a building. If the stone passed by a person standing in a balcony 5m high above the ground 4 s later (consider g = 10 m/s^2), find:

A- The building height.
B- The stone velocity when passed by the person.

Solution:

$d = v_i\,t + \dfrac{1}{2}\,g\,t^2$

$d = 0 + (\dfrac{1}{2} \times 10 \times 16) = 80$ m

A The building height: $h = 80 + 5 = 85$ m

B The stone velocity when passed by the person:

$v_f = v_i + g\,t$

$v_f = 0 + (10 \times 4) = 40$ m/s

3- An apple has fallen from a tree. Find its velocity when it reached the ground if it took 1 second to the ground. Then, find the average velocity of the apple during falling and the height from which it fell.

Solution:

Given data: $v_i = 0 \quad g = 10\ m/s^2 \quad t = 1\ s$

Velocity of reaching the ground $v_f = v_i + gt = gt$

$v_f = 10 \times 1 = 10\ m/s$

The average velocity of the apple during falling $\overline{v} = \dfrac{v_f + v_i}{2}$

$$\bar{v} = \frac{10+0}{2} = 5\ m/s$$

The height from which the apple fell:

$$d = v_i t + \frac{1}{2} g t^2 = \frac{1}{2} g t^2$$

$$\therefore d = (\frac{1}{2})(10)(1)^2 = 5\ m$$

Mini – lab

Determination of the acceleration due to gravity:

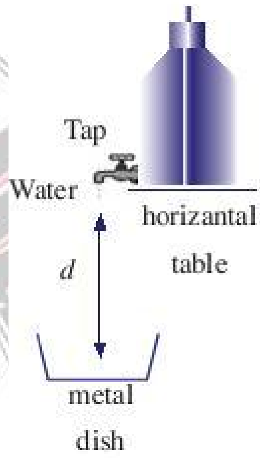
- The idea of the experiment is based on determining both of (d) And (t) to find (g) using the second equation of motion.

- Adjust the tap to allow a water drop to hit the plate base just as the next drop starts to fall from the tap.

- Use a stopwatch to record the time taken by 50 successive drops to fall. Divide the total time by the number of falling drops to find the time taken by one drop (t).

- Determine the acceleration (g) = $\frac{2d}{t^2}$

- Communicate with colleagues on the book site to compare the results each of you got.



Solved Example:

In an experiment to determine the acceleration due to gravity using falling water drops, the distance between the tap and the plate base is (1m). If the time taken by (100) drops is (45 s), find the acceleration due to gravity.

Solution:

Given data: $d = 1m$, $v_i = 0$, $t = ?$, $a = ?$

Time taken by one drop to fall $(t) = \frac{\text{Total time}}{\text{Number of drops}} = \frac{45}{100} = 0.45\ s$

Substituting in the second equation of motion:

$$d = \frac{1}{2} g t^2$$

$$g = \frac{2d}{t^2} = \frac{2 \times 1}{0.45 \times 0.45} = 9.88\ m/s^2$$

Projectiles:

(a) Vertical projectiles:

- When an object is projected vertically upwards, it leaves the hand at initial velocity (V_i).

The object moves at uniform acceleration ($-10\ m/s^2$). The negative sign indicates that the direction of acceleration is opposite to that of velocity.

- Velocity decreases gradually as the object gets higher till its velocity reaches zero at maximum height.

- Direction of velocity changes when the object returns back to the ground under the effect of the Earth's gravity that makes the object accelerate.

- Velocity of the object when projected up = - its velocity at the same point on falling.

The negative sign indicates that the two velocities are in opposite directions.

- Time of rise = time of fall.

Solved Example:

Time, displacement, and velocity of an object projected vertically upwards at initial velocity (20 m/s) are recorded in the table below:

Time (s)	0	0.5	1	1.5	2	2.5	3	3.5	4
Displacement (m)	0	8.75	15	18.75	20	18.75	15	8.75	0
Velocity (m/s)	20	15	10	5	0	-5	-10	-15	-20

This motion can be represented by the following diagrams:

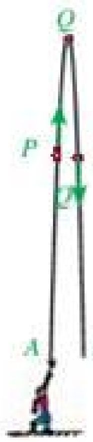


Figure (21): projectile path

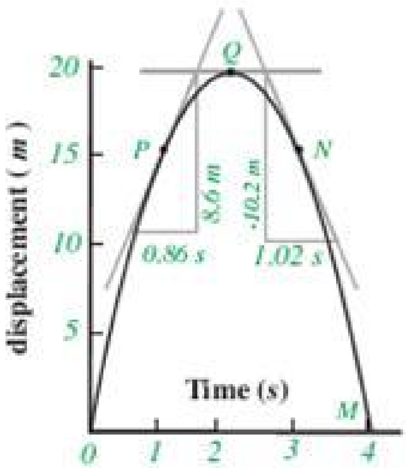


Figure (22): change of displacement with time

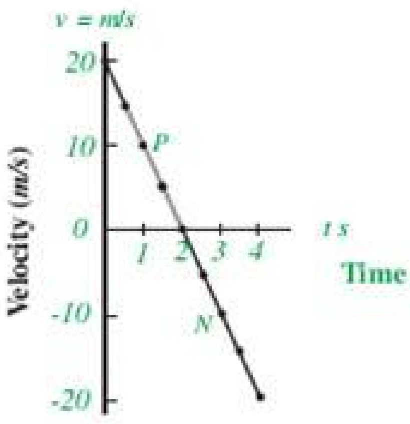


Figure (23): change of velocity with time

1- Determine the object velocity at the points P, Q and N in the (displacement – time) and (velocity – time) graphs.

2- What is the slope of the line in (velocity – time) graph? What does it represent?

Why has it got a negative sign?

Solution:

1- Finding velocity by the slope of tangent at points Q, P and N in (displacement-time) graph.

$$v_Q = 0 \qquad v_P = \frac{8.6}{0.86} = 10 \text{ m/s} \qquad v_N = \frac{-10.2}{1.02} = -10 \text{ m/s}$$

These values are the same as those obtained in (velocity - time) graph.

2- Acceleration (a) is the slope of line in (velocity - time) graph:

$$a = \frac{\Delta v}{\Delta t} = \frac{-20}{2} = -10 \text{ m/s}^2$$

The negative sign indicates that the object velocity is decreasing as it goes further from the ground.

(b) Projectiles when projected at an angle (Motion in two dimensions):

You have studied the motion of objects at uniform acceleration in a straight line either in a horizontal, inclined or vertical plane. At the moment, we are going to study objects motion when projected at an angle to the horizontal under the effect of gravity

Studying the projectile motion such as that of a ball or a cannon shell shows that it takes a curved path (figure 26). It starts motion at initial velocity (V_i) at angle (θ) to the horizontal.

We can resolve velocity into two dimensions; horizontal (x) and vertical (y) as shown:

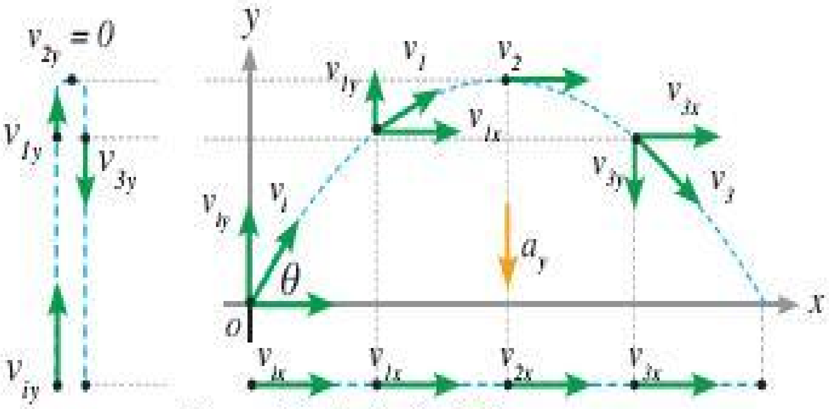


Figure (26): Projectile Motion

The horizontal dimension (x): the ball velocity is uniform (V_{ix}) neglecting any friction.

This velocity can be found by the relation:

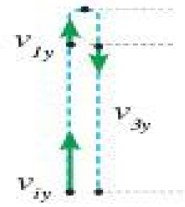
$$v_{ix} = v_i \cos\theta$$

Substituting in the three equations of motion by the value of (V_{ix}), considering (a_x=0):

The vertical dimension (y): the ball moves at the acceleration due to gravity. Consequently, velocity varies. We can find the initial velocity in the vertical dimension (V_{iy}) by the relation:

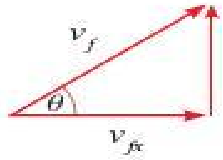
$$V_{iy} = V_i \sin\theta$$

Substituting in the three equations of motion by the value of (v_{iy}) considering $(a_y = g = -10\ m/s^2)$



The velocity of the projectile at any instant is given by Pythagoras' relation:

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2}$$



Finding the time of reaching maximum height (t):

Substituting in the first equation of motion by $(v_{fy} = 0)$, we obtain:

$$0 = v_{iy} + gt$$

i.e. :

$$\boxed{t = \frac{-v_{iy}}{g}}$$

Time taken till returning back to the plane of projection (flight time):

$$\boxed{T = 2t = \frac{-2v_{iy}}{g}}$$

Finding the maximum height reached by the projectile (h):

Substituting in the third equation of motion by $(v_{fy} = 0)$, we obtain:

$$2g\,h = -v^2_{iy}$$

i.e

$$\boxed{h = \frac{-v^2_{iy}}{2\,g}}$$

Finding the range (the horizontal distance reached by the projectile) (R):

Note that: T = time of the horizontal range = flight time

Substituting $(a_x = o)$ and $(d = R)$ in the second equation of motion, we find:

$$\boxed{R = v_{ix}T = 2v_{ix}\,t}$$

Solved Example

A motorcycle is launched at 15 m/sin a direction at an angle 30°to the horizontal.

A. What is maximum height reached by the motorcycle?

B. Find the time of its flight.

C. What is the horizontal range reached by the motorcycle?

Solution:

First, find the value of (v_{ix}) and (v_{iy}):

$$v_{ix} = v_i \cos 30 = 15 \times 0.866 = 13\ m/s$$

$$v_{iy} = v_i \sin 30 = 15 \times 0.5 = 7.5\ m/s$$

Finding maximum height (h):

$$h = \frac{-v^2_{iy}}{2\,g} = \frac{-(7.5)^2}{2\times(-10)} = 2.8\ m$$

Finding the time of flight (T):

$$T = 2t = \frac{-2 \times v_{iy}}{g} = \frac{-2\times 7.5}{(-10)} = 1.5\ s$$

Finding the horizontal range (R):

$$R = v_{ix}\,T = 13 \times 1.5 = 21.5\ m$$

Chapter (3): Force and Motion

Previously we have described motion by studying the concepts of velocity and acceleration without getting into the reasons beyond.

In this chapter we are going to discuss the existence of acceleration due to the impact of a force. This leads us to Newton's laws of motion that are considered as basic laws in physics.

Force:

Force is a common term used in everyday life. For instance, the force exerted by your muscles helps to pull things, the car engine forces the car to move and the force of brakes act to stop the moving car.

Force: is an external influence that affects the object to change its state or direction of motion.

Force is measured by the spring balance in Newton (N).

Distinguished Scientists:

Although a lot of ancient philosophers tried to explore and explain motion and its causes, no reliable theory had been achieved before the end of the seventeenth century. In this aspect, we appreciate the contributions of two great scientists; Galileo and Newton.

Newton's first law:

May you have returned home one day after a long absence and looked around and wondered:

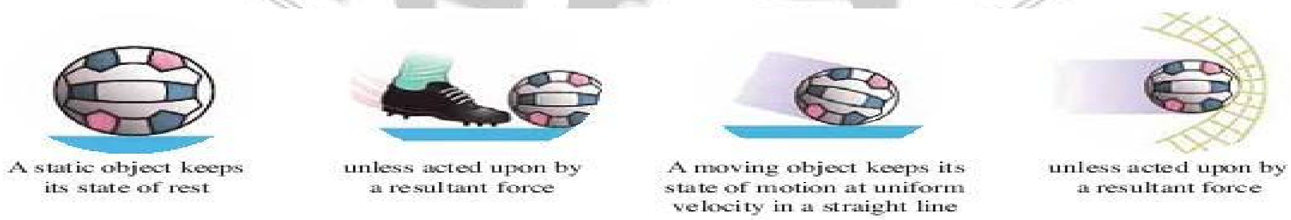
"Alright, everything is in place, isn't it?" Have you ever thought that you have just stated one of the most important laws in nature!

Moreover, it is known that a rolling object on the floor would move for a certain distance, slowing down, then stops. Ancient people thought that the normal nature of an object was being static; meaning that every motion devolves to rest. But, experiments show that the rolling object experiences forces of friction that slow it down till stop. If these forces do not exist, the object would keep moving and would not stop. This principle is known as

Newton's First Law of motion: "A static object keeps its state of rest, and a moving object keeps its state of motion at uniform velocity in a straight line unless acted upon by a resultant force." The mathematical formula that

Expresses the law: $\sum F = 0$

The term ΣF is the resultant force that may equal zero when the forces acting on an object may cancel the effect of each other.



Applying Newton's First Law, we can draw a conclusion that when the resultant force = 0, acceleration = 0, and no change happens in object velocity either being static or dynamic.

Also, a resultant force is needed to move a static object or to stop a moving one. No need for a resultant force to move objects at uniform velocity in a straight line.

Newton's First Law is related to the concept of inertia. Accordingly, it is called "Law of Inertia"

Inertia: the tendency of an object to keep either its state of rest or state of motion at its original velocity uniformly in a straight line. This means that objects resist changing its static or dynamic state.

Exercise

Based on the concept of inertia explain the following daily observations:

Fall of crayon into the bottle when the ring is removed rapidly

Motorcycle rider flies off the motorcycle when it hits an obstacle.

Seat belt should be fastened on driving.

Technological Applications

When being away from the Earth's gravity, a space rocket does not need to consume fuel to keep moving because inertia keeps it moving at uniform velocity in a straight line.

Newton's second law:

We have learnt that when no resultant force affects the object, it does not move at acceleration. Consequently, when a resultant force acts on the object ($\Sigma F \neq 0$), it moves at acceleration. Accordingly, its velocity changes and it acquires acceleration ($a \neq 0$).

Through his second law, Newton defined the factors affecting such acceleration; where it is directly proportional to the resultant force on the object and inversely proportional to the object mass.

Less force, less acceleration. greater force, greater acceleration.

Figure (30): increase of acceleration with force.

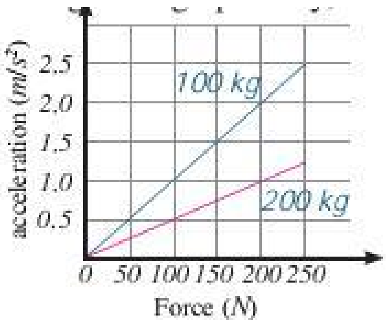
Less mass, more acceleration greater mass, less acceleration.

Figure (31): decrease of acceleration with mass

Newton's Second Law of motion: "when a resultant force affects an object, the object acquires an acceleration which is directly proportional to the resultant force and inversely proportional to the object mass." The mathematical formula that.

Expresses the law: $a = \Sigma F/m$ or $\Sigma F = ma$

Representing the relationship between acceleration and the acting force graphically, we notice that the acceleration of motion increases as the force increases. Also, the less mass object (say, 100 kg) moves at a greater acceleration than a heavier one (200 kg) does, when affected by the same force.



Through Newton's Second Law, we can state a definition for the unit of force, "The Newton"

The Newton: is the force that when acts on an object of mass 1 kg accelerates it at 1 m/s^2.

i.e. 1 Newton = 1 kg. m/s^2

Critical Thinking Skills

A force of 1N acts on a wooden cube and accelerates it at certain acceleration (a).

When this force acts on another cube and accelerates it at acceleration (3a). What do you conclude about the masses of the two cubes?

Mass and Weight:

Through Newton's Second Law, we can infer that moving or stopping a heavy body as a plane is much difficult than a lighter body as a bicycle. In other words, the plane resists the change in its Kinematic state more than that done by the bicycle. Therefore, mass of an object is defined by its resistance to change its Kinematic state.

Also we can infer that acquiring acceleration implies the existence of force acting on the object.

In case of moving at free fall acceleration, the object is under the effect of the gravitational force of Earth.

Therefore, weight of an object is defined as the force of gravity acting on the body. Its direction is towards the Earth's centre and determined by the relation: W = mg

Newton's third law:

Figure (34): what happens when air in a inflated balloon is allowed to rush out?

Figure (35): what happens when kicking an opposite wall while sitting on a desk chair?

Figure (36): what happens to the rifle when the bullet goes out?

Newton formulated an explanation for the above mentioned situations through his third law of motion that studies the nature of forces acting on objects. He noticed that forces act in pairs of equal magnitude and opposite directions.

Figure (37): the two spring balances read the same

Newton's Third Law of motion: "when an object acts on another object by a force, the second object reacts with an equal force on the first object in a direction opposite to that of action."

I.e. For every action there is a reaction equal in magnitude and opposite in direction.

The mathematical formula that expresses the law: $F_1 = -F_2$

Third law of motion implies that:

- No single force exists in the universe. Action and reaction are paired; originate and vanish together.

- Action and reaction are of the same type; if the action is a gravitational force, reaction is a gravitational force, as well.

- It is not a must that action and reaction are at equilibrium since they may act on different bodies.

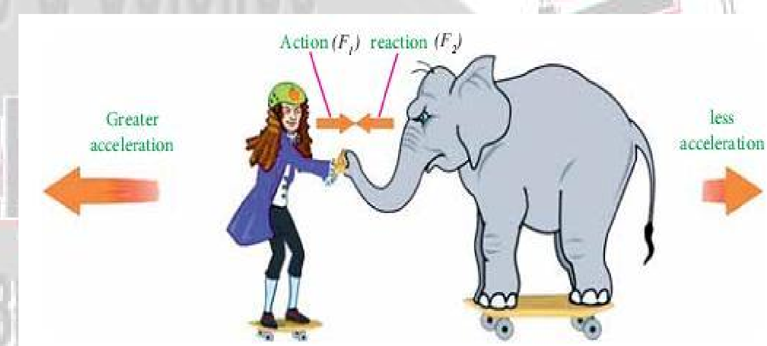
Practical Applications

✦ launching a rocket is based on Newton's third law of motion. A huge amount of burning gases rush down the rocket to generate a reaction pushing the rocket upwards.

Time management:

Take care and manage your examination time properly. Finishing a test in a hurry does not grantee any extra marks. Answer carefully and neatly. Revise answers repeatedly to avoid mistakes due to hastiness.

Solved Example:



1. What is the relation between the force acting on the elephant and that on the man?

2. Why are not action on the elephant and reaction on the man at equilibrium?

3. If the elephant's mass is 6times heavier than the man's mass, calculate the acceleration by which the elephant moves giving that the man moves at an acceleration $2 m/s^2$.Why is the elephant acceleration negative?

Solution:

1. The force acting on the elephant = the force acting on the man.F1= -F2

2. For two forces to be at equilibrium, they must be equal, opposite, having one line of action and act on the same body. All these conditions except the last one may be applied on action and reaction; since the action acts on the elephant and the reaction is on the man.

3 Finding acceleration of elephant's motion:

$$F_1 = -F_2$$

$$m_1 a_1 = -m_2 a_2$$

$$\frac{-a_1}{a_2} = \frac{m_2}{m_1}$$

$$\text{since} \quad m_2 = 6m_1$$

$$\frac{-a_1}{2} = 6$$

$$a_1 = -12\ m/s^2$$

The negative sign indicates that the elephant motion is opposite to the man motion.