2013 2014

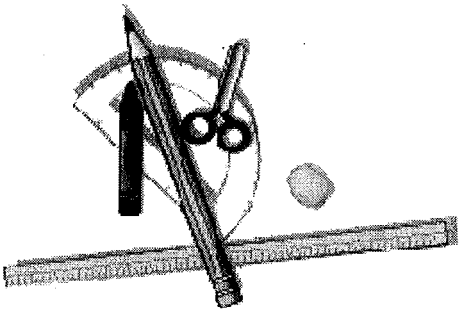
Mathematics Department

Algebra

Unit (1) : Numbers And Algebra

Second Term - Prep- stage

1 Prep.



1 Sheet No.

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اسم التلميذ : .....

خاص بالمجموعات الدراسية

Lesson (1) : Repeated Multiplication

We had known before in the set of integers that : $3^4 = 3 \times 3 \times 3 \times 3$ **where we found that the number 3 has repeated 4 times in the multiplication operation and we read it as «3 to the power 4»**

Also, in the set of rational $\mathbb{Q}$, we find that :

- $\left(\frac{2}{3}\right)^3 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{2 \times 2 \times 2}{3 \times 3 \times 3} = \frac{2^3}{3^3} = \frac{8}{27}$
- $\left(\frac{2}{3}\right)^4 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3} = \frac{2^4}{3^4} = \frac{16}{81}$

From the previous, we deduce that :

If $\frac{a}{b}$ is a rational number and n is a positive integer , then : $\left(\frac{a}{b}\right)^n = \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b}$ ... to n times

It is read as « $\frac{a}{b}$ to the power n » or «the n^{th} power of the number $\frac{a}{b}$ »

i.e. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

For example: • $\left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3} = \frac{8}{125}$ • $(0.7)^2 = \left(\frac{7}{10}\right)^2 = \frac{7^2}{10^2} = \frac{49}{100}$

Remark

If : $\frac{a}{b}$ is a rational number , then : $\left(\frac{a}{b}\right)^0 = 1$ where $a \neq 0$

For example: • $\left(\frac{1}{5}\right)^0 = 1$ • $\left(-\frac{3}{7}\right)^0 = 1$

Remark

If $\frac{a}{b}$ is a rational number , and m is a positive integer , then :

1 $\left(-\frac{a}{b}\right)^m = \left(\frac{a}{b}\right)^m$ when m is an even number.

For example: $\left(-\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

2 $\left(-\frac{a}{b}\right)^m = -\left(\frac{a}{b}\right)^m$ when m is an odd number.

For example: $\left(-\frac{1}{2}\right)^3 = -\left(\frac{1}{2}\right)^3 = -\frac{1}{8}$

Example (1) :

If $x = -\frac{1}{2}$, $y = \frac{1}{4}$ and $z = 4$, find the value of : $(x + y)^3 \times z^3$

Solution

$$(x + y)^3 \times z^3 = \left(-\frac{1}{2} + \frac{1}{4}\right)^3 \times 4^3 = \left(-\frac{2}{4} + \frac{1}{4}\right)^3 \times 4^3$$

$$= \left(-\frac{1}{4}\right)^3 \times 4^3 = -\frac{1^3}{4^3} \times 4^3 = -1$$

Lesson (2) : None Negative Integer Powers

If $\frac{a}{b}$ is a rational number , n and m are non-negative integers , then

$$\left(\frac{a}{b}\right)^n \times \left(\frac{a}{b}\right)^m = \left(\frac{a}{b}\right)^{n+m}$$

For example:

- $\left(\frac{2}{5}\right)^3 \times \left(\frac{2}{5}\right)^2 = \left(\frac{2}{5}\right)^{3+2} = \left(\frac{2}{5}\right)^5$
- $\left(-\frac{1}{2}\right)^4 \times \left(-\frac{1}{2}\right)^3 = \left(-\frac{1}{2}\right)^{4+3} = \left(-\frac{1}{2}\right)^7$

If $\frac{a}{b}$ is a rational number , where $\frac{a}{b} \neq 0$, n and m are non-negative numbers , $n \geq m$, then $\left(\frac{a}{b}\right)^n \div \left(\frac{a}{b}\right)^m = \left(\frac{a}{b}\right)^{n-m}$

For example:

- $\left(\frac{3}{8}\right)^5 \div \left(\frac{3}{8}\right)^2 = \left(\frac{3}{8}\right)^{5-2} = \left(\frac{3}{8}\right)^3$
- $\left(-\frac{2}{7}\right)^4 \div \left(-\frac{2}{7}\right)^2 = \left(-\frac{2}{7}\right)^{4-2} = \left(-\frac{2}{7}\right)^2$

Example (2) :

Calculate each of the following , then put the result in the simplest form :

1 $\left(\frac{4}{5}\right)^2 \times \left(\frac{4}{5}\right)^5 \div \left(\frac{4}{5}\right)^4$ **2** $\frac{2^5 \times 2^4}{2^6}$

Solution

1 $\left[\left(\frac{4}{5}\right)^2 \times \left(\frac{4}{5}\right)^5\right] \div \left(\frac{4}{5}\right)^4 = \left(\frac{4}{5}\right)^{2+5} \div \left(\frac{4}{5}\right)^4 = \left(\frac{4}{5}\right)^7 \div \left(\frac{4}{5}\right)^4 = \left(\frac{4}{5}\right)^{7-4} = \left(\frac{4}{5}\right)^3 = \frac{4^3}{5^3} = \frac{64}{125}$

2 $\frac{2^5 \times 2^4}{2^6} = \frac{2^{5+4}}{2^6} = \frac{2^9}{2^6} = 2^{9-6} = 2^3 = 8$

i.e. **If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers , n is a non-negative integer , then :**

$$\left(\frac{a}{b} \times \frac{c}{d}\right)^n = \left(\frac{a}{b}\right)^n \times \left(\frac{c}{d}\right)^n$$

i.e. **If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers , $\frac{c}{d} \neq 0$, n is a non-negative integer , then :**

$$\left(\frac{a}{b} \div \frac{c}{d}\right)^n = \left(\frac{a}{b}\right)^n \div \left(\frac{c}{d}\right)^n \quad \left(\text{where } \frac{c}{d} \neq 0\right)$$

If $\frac{a}{b}$ is a rational number , n and m are non-negative integers , then

$$\left[\left(\frac{a}{b}\right)^n\right]^m = \left(\frac{a}{b}\right)^{n \times m}$$

Lesson (3) : Negative Integer Powers

Definition :

If a is a rational number , $a \neq 0$ and n is a positive integer , then $a^{-n} = \frac{1}{a^n}$

and $a^n = \frac{1}{a^{-n}}$

For example:

- $3^{-3} = \frac{1}{3^3} = \frac{1}{27}$
- $\frac{2}{5^{-2}} = 2 \times 5^2 = 2 \times 25 = 50$
- $0.1 = \frac{1}{10} = 10^{-1}$, $0.01 = \frac{1}{100} = \frac{1}{10^2} = 10^{-2}$, ... and so on.

Remark

1 **If a is a rational number , $a \neq 0$ and n is a positive integer , then :**

$a^n \times a^{-n} = a^n \times \frac{1}{a^n} = 1$ (the multiplicative neutral)

i.e. each of a^n and a^{-n} is the multiplicative inverse of the other.

2 **If $\frac{a}{b}$ is a rational number not equal to zero and n is a positive integer , then :**

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

For example: $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

Exercises on Lesson (1 , 2 & 3)

[A] Choose the correct answer from the gives ones :

1 Half the number 2^{20} = ...

(a) 2^{10} (b) 2^{21} (c) 2^{19} (d) 40

2 The third of 3^{20} = ...

(a) 3^{10} (b) 3^{19} (c) 3^{18} (d) 3^{17}

3 Third of 3^{12} = ...

(a) 3^{-1} (b) 3^{-11} (c) 3^{11} (d) 1

4 $\left(\frac{-1}{2}\right)^{0}$ = ...

(a) -1 (b) $\frac{-1}{2}$ (c) 1 (d) 0

5 $\left(\frac{2\,ab^{-2}}{3^{0}\,a^{-2}\,b}\right)^{0}$ = ...

(a) $\frac{a^3}{3\,b^3}$ (b) a^2 (c) 1 (d) $\frac{a^2}{b}$

6 The multiplicative inverse of $\left(\frac{2}{3}\right)^{0}$ is .....

(a) $\frac{3}{2}$ (b) $-\frac{2}{3}$ (c) 1 (d) 0

7 $\left(\frac{2}{3}\right)^{-2}$ = ...

(a) $\frac{4}{9}$ (b) $\frac{9}{4}$ (c) $\frac{-4}{9}$ (d) $-\frac{9}{4}$

8 5^{-1} = ...

(a) -1 (b) 5 (c) -5 (d) $\frac{1}{5}$

9 $\left(-\frac{1}{3}\right)^{-1}$ = ...

(a) $\frac{-1}{3}$ (b) -3 (c) 3 (d) $\frac{1}{3}$

10 $\left(\frac{2}{3}\right)^{-2}$ = ...

(a) $\frac{4}{9}$ (b) $\frac{9}{4}$ (c) $\frac{-4}{9}$ (d) $\frac{-9}{4}$

11 $\frac{1}{2}(2^{20}) =$ .....

(a) 2^5 (b) 2^{10} (c) 2^{19} (d) 2^{18}

12 $1\frac{9}{16} = (.....)^2$

(a) $1\frac{3}{4}$ (b) $\frac{4}{5}$ (c) $\frac{3}{4}$ (d) $1\frac{1}{4}$

13 $3^3 \times 3^{zero} =$ .....

(a) 9 (b) 27 (c) 1 (d) zero

14 $3^7 \times 3^{-5} =$ .....

(a) – 9 (b) 9 (c) 3 (d) – 3

15 $2^6 \div 2^3 =$ .....

(a) 3^9 (b) 2^2 (c) 3^2 (d) 8

16 $\left(\frac{-3}{7}\right)^7 \div \left(\frac{3}{7}\right)^5 =$ ..... (In the simplest form).

17 $\left(\frac{1}{2}\right)^5 \div \left(\frac{1}{2}\right)^3 =$ .....

(a) $\frac{1}{32}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) $\frac{1}{16}$

18 $\left(\frac{x}{y}\right)^n \div \left(\frac{x}{y}\right)^m =$ .....

(a) $\left(\frac{x}{y}\right)^{n+m}$ (b) $\left(\frac{x}{y}\right)^{n-m}$ (c) $\left(\frac{x}{y}\right)^{m-n}$ (d) $\left(\frac{x}{y}\right)^{\frac{n}{m}}$

19 If : $a = -3$, then $a^{-2} =$ .....

(a) $-\frac{1}{9}$ (b) $\frac{1}{9}$ (c) 9 (d) – 9

20 The multiplicative inverse of $\left(-\frac{3}{4}\right)^2$ is .....

(a) $\left(\frac{4}{3}\right)^2$ (b) $\left(-\frac{4}{3}\right)^2$ (c) $\left(\frac{3}{4}\right)^2$ (d) $-\left(\frac{3}{4}\right)^2$

21 $3x^0 =$ .....

[B] Complete the following :

1 The additive inverse of $\left(-\frac{2}{5}\right)^2$ is .....

2 The additive inverse of the number $\left(\frac{-1}{2}\right)^2$ is .....

3 $x = \frac{1}{2}$ and $y = \frac{3}{4}$, then the numerical value of : $x^2 + y$ = ...

4 $(2\,a^{-1})^{-1}$ = ... (In the simplest form).

5 $\left(\frac{-3}{7}\right)^7 \div \left(\frac{3}{7}\right)^5$ = ... (In the simplest form).

6 The additive inverse of $\left(\frac{-2}{3}\right)^2$ is .....

7 The multiplicative inverse of $\left(\frac{-2}{3}\right)^2$ .....

[C] Essay Problems :

[a] Simplify to it's simplest form :

① $\frac{(-3)^3 \times (-3)^5}{(-3)^6}$ ② $\left(\frac{-5}{4}\right)^2 \times \sqrt{\frac{25}{16}} \times \left(\frac{-4}{5}\right)^3$

[a] Calculate : $\left(\frac{1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{2}{5}\right)^0$

[a] Find the result in the simplest form : $\frac{4}{3} \times \sqrt{\frac{9}{25}} \times \left(-\frac{1}{2}\right)^2$

[a] Evaluate : $\left(\frac{5}{4}\right)^2 \times \sqrt{\frac{16}{25}} \times \left(\frac{-4}{5}\right)^0$

[a] Find the value of : $\left(\frac{5}{8}\right)^0 \times \sqrt{6\frac{1}{4}} \times \left(-\frac{2}{5}\right)^2$

[a] Find the result of : $\left(\frac{-2}{3}\right)^2 \times \sqrt{\frac{9}{4}} \times \left(\frac{-1}{4}\right)^{\text{zero}}$

[b] Find each of the following :

① $\left(-\frac{2}{3}\right)^2 \times \frac{9}{4} \times \left(-\frac{1}{4}\right)^0$ ② $\left(-\frac{2}{3}\right)^6 \div \left(-\frac{2}{3}\right)^7$

[a] Find in the simplest form :

① $\left(\frac{-2}{3}\right)^2 \times \sqrt{\frac{9}{4}} \times \left(\frac{1}{4}\right)^0$ ② $\left(\frac{-2}{3}\right)^6 \div \left(\frac{-2}{3}\right)^4$

[a] Calculate : $\frac{7^{-2} \times 7^5}{7^3}$

[a] Find the result of : $\frac{3^3 \times 3^{-4}}{3^{-2} \times 3^5}$

[b] If : $a = \frac{1}{2}$, $b = 2$ and $c = \frac{3}{4}$, then find the numerical value of : $a^2 b^3 + b^2 c$

[b] Simplify : $\frac{5^6 \times 5^2 \times 5^3}{5^4 \times 5^5}$

[a] Simplify each of the following :

① $\left(\frac{-2}{3}\right)^2 \times \sqrt{\frac{9}{4}} \times \left(\frac{2}{7}\right)^0$ ② $\frac{(2)^5 \times (-2)^4}{(2)^9}$

[a] Find the result of : $\frac{(-2)^6 \times (-2)^4}{(-2)^8}$

[a] Find : ① $\left(\frac{-5}{8}\right)^0 \times \sqrt{6\frac{1}{4}} \times \left(\frac{-2}{5}\right)^2$ ② $\frac{3^4 \times 3^3}{3^6}$

[a] Simplify : $\frac{x^7 \times x^5}{x^8}$, then find the numerical value of the result at $x = -2$

[a] Simplify : $\frac{y^3 \times y^{-4}}{y^{-2} \times y^5}$, then find its numerical value at $y = -1$

[b] If : $x = \left(-\frac{1}{2}\right)$, $y = \left(\frac{4}{3}\right)$, $z = \left(\frac{1}{5}\right)$, then find the numerical value of $x^3 y^2 z$

[a] If : $x = -\frac{3}{2}$, $y = \frac{1}{2}$ and $z = -\frac{4}{3}$, **find the numerical value of :** $x^2 - yz^2$

[a] If : $x = \frac{-2}{3}$, $y = \frac{1}{2}$ and $z = \frac{-4}{3}$ **Find the value of :** $x^2 - y^2 z$

[b] If : $x = \frac{1}{2}$, $y = \frac{4}{3}$ and $z = \frac{3}{2}$ **Find the numerical value of :** $8 x^3 y z^2$

[b] If : $\frac{a}{b} \in \mathbb{Q}$, $\frac{a^2}{b^2} = 0.16$, **Find the value of :** $\left(\frac{a}{b}\right)^3$

[a] If : $x = \frac{-1}{2}$, $y = \frac{3}{4}$ and $z = \frac{-3}{2}$, then **find :** $x^3 \div y^2 z^2$

[b] Answer the following :

If : $a = \frac{-1}{2}$, $b = 2$, $c = \frac{3}{4}$, then find the numerical value of : $a^2 b^3 + b^2 c$

[a] If : $a = -\frac{2}{3}$, $b = \frac{3}{4}$ and $c = \frac{1}{2}$ **Find the value of :** $(ab)^2 - c^2$

Lesson (4) : Scientific Notation Of The Rational Numbers

The standard scientific notation of a number :

The number is written in the standard form as :

$$a \times 10^n \text{ where } 1 \leq |a| < 10 \text{ and } n \in \mathbb{Z}$$

For example:

Each of the following two numbers is written in its standard form :

4.6×10^8

- 4.6 : a number included between 1 and 10
- 10^8 : a number expresses a power of 10

5.236×10^{-6}

- 5.236 : a number included between 1 and 10
- 10^{-6} : a number expresses a power of 10

Example 1 **Put each of the following two numbers in the standard form :**

1. 8 200 000 000
2. 0.000000135

Solution

1. $8,200,000,000,0 = 8.2 \times 10^9$
   - Moving the decimal point 9 places towards left
   - Using the power 9 of the number 10
2. $0,000,000,135 = 1.35 \times 10^{-7}$
   - Moving the decimal point 7 places towards right
   - Using the power -7 of the number 10

- Notice that the number 32.4×10^5 is not in the standard form because $32.4 > 10$ and to write it in the standard form , we move the decimal point one place towards left and multiply by 10

  i.e. $32,4 \times 10^5 = 3.24 \times 10^5 \times 10 = \boxed{3.24 \times 10^6}$ (the standard form)

- Notice that the number 0.032×10^{-4} is not in the standard form because $0.032 < 1$

and to write it in the standard form , we move the decimal point two places towards right and multiply by 10^{-2}

i.e. $0.032 \times 10^{-4} = 3.2 \times 10^{-4} \times 10^{-2} = \boxed{3.2 \times 10^{-6}}$ (the standard form)

- Notice that the standard form of the number 1 is 1×10^{0} , also the number 2 is 2×10^{0} , and so on ...

Example 2 **Write each of the following numbers in the standard form :**

1 45×10^{8}

2 706.4×10^{5}

3 0.248×10^{-7}

4 -0.0015×10^{-9}

Solution

1 $45 \times 10^{8} = 4.5 \times 10 \times 10^{8} = 4.5 \times 10^{9}$

2 $706.4 \times 10^{5} = 7.064 \times 10^{2} \times 10^{5} = 7.064 \times 10^{7}$

3 $0.248 \times 10^{-7} = 2.48 \times 10^{-1} \times 10^{-7} = 2.48 \times 10^{-8}$

4 $-0.0015 \times 10^{-9} = -1.5 \times 10^{-3} \times 10^{-9} = -1.5 \times 10^{-12}$

Example 3

Write the result of each of the following in the standard form :

1 $30\,000 \times 400\,000$

2 $140\,000 \times 0.005$

3 $0.000015 \div 30$

4 $(50\,000)^{3}$

5 $(0.0003)^{5}$

6 $(-0.001)^{6}$

Solution

1 $30\,000 \times 400\,000 = (3 \times 10^{4}) \times (4 \times 10^{5}) = (3 \times 4) \times (10^{4} \times 10^{5})$

$= 12 \times 10^{9} = 1.2 \times 10^{10}$

2 $140\,000 \times 0.005 = (1.4 \times 10^{5}) \times (5 \times 10^{-3}) = (1.4 \times 5) \times (10^{5} \times 10^{-3}) = 7 \times 10^{2}$

3 $0.000015 \div 30 = (1.5 \times 10^{-5}) \div 3 \times 10 = \frac{1.5}{3} \times \frac{10^{-5}}{10} = 0.5 \times 10^{-6} = 5 \times 10^{-7}$

4 $(50\,000)^{3} = (5 \times 10^{4})^{3} = 5^{3} \times 10^{12} = 125 \times 10^{12} = 1.25 \times 10^{14}$

5 $(0.0003)^{5} = (3 \times 10^{-4})^{5} = 3^{5} \times 10^{-20} = 243 \times 10^{-20} = 2.43 \times 10^{-18}$

6 $(-0.001)^{6} = (0.001)^{6} = (1 \times 10^{-3})^{6} = 1^{6} \times 10^{-18} = 10^{-18}$

Exercises on Lesson (4)

[A] Choose the correct answer from the gives ones :

(1) 0.00000027 = ... (In the standard form)

(a) 2.7×10^{-6} (b) 2.7×10^{6} (c) 2.7×10^{-7} (d) 2.7×10^{7}

(1) 0.00000052 = ...

(a) 5.2×10^{-6} (b) 5.2×10^{-7} (c) 5.2×10^{6} (d) 5.2×10^{2}

(2) 0.00000032 = ...

(a) 3.2×10^{-6} (b) 3.2×10^{6} (c) 3.2×10^{-7} (d) 3.2×10^{7}

(2) 2.37×10^{-4} = ...

(a) 0.00237 (b) 0.000237 (c) 23700 (d) 0.0000237

(1) If : $(0.0005)^2 = 25 \times 10^{n}$, then n = ...

(a) 4 (b) 8 (c) – 8 (d) – 6

[B] Complete the following :

(3) If : $0.00052 = 5.2 \times 10^{n}$, then n = ...

(4) If : $0.000024 = 2.4 \times 10^{n}$, then n = ...

(2) If : $0.000028 = 2.8 \times 10^{n}$, then n = ...

(2) If : $2700 = 2.7 \times 10^{n}$, then n = ...

(4) 0.000237 = ... $\times (10)^{.....}$ (in the standard form)

(2) If : $A = 0.000625$, then : $\sqrt{A} = 2.5 \times 10$ .....

(4) 0.00000027 = ... (In the standard form)

(5) The standard form of the number 7 millions is .....

[C] Essay Problems :

[a] Without using calculator find in form $a \times 10^{n}$, $n \in \mathbb{Z}$:

$(3.8 \times 10^{8}) \div (1.9 \times 10^{6})$

Lesson (5) : Order Of mathematical operations

First : Order of performing the mathematical operations in an expression has no parentheses

1. Evaluate the powers.
2. Perform multiplications and divisions in order from left to right.
3. Perform additions and subtractions in order from left to right.

Second : Order of performing the mathematical operations in an expression has parentheses

1. Perform the operations within parentheses (interior parentheses then exterior ones).
2. Evaluate the powers.
3. Perform multiplications and divisions in order from left to right.
4. Perform additions and subtractions in order from left to right.

Example 3

Calculate the value of each of the following :

1. $8 \times 2^2 - 7 \times (4 + 1)$
2. $2 + 3\left[5 + (4 - 1)^2\right]$
3. $3\left[(3^2 + 1) - (2^3 - 2)\right]$

Solution

1. $8 \times 2^2 - 7 \times (4 + 1) = 8 \times 2^2 - 7 \times 5$ (addition inside parentheses)
   $= 8 \times 4 - 7 \times 5$ (powers)
   $= 32 - 35$ (multiplication)
   $= -3$ (subtraction)

2. $2 + 3\left[5 + (4 - 1)^2\right] = 2 + 3\left[5 + 3^2\right]$ (subtraction inside interior parentheses)
   $= 2 + 3\left[5 + 9\right]$ (powers inside parentheses)
   $= 2 + 3 \times 14$ (addition inside parentheses)

$= 2 + 42$ (multiplication)

$= 44$ (addition)

3 $3\left[(3^2+1)-(2^3-2)\right] = 3\left[(9+1)-(8-2)\right]$ (powers)

$= 3\left[10-6\right]$ (the interior parentheses)

$= 3 \times 4$ (subtraction inside parentheses)

$= 12$ (multiplication)

Exercises on Lesson (5)

[A] Choose the correct answer from the gives ones :

③ $12 \div 3 + 3 \times 2 =$ .....

(a) 10 (b) 4 (c) 24 (d) 1

② $12\,(2^2) \div 24 + 3^2 =$ .....

(a) 4 (b) 5 (c) 11 (d) 13

② $2 \times 5 - 6 \div 2 =$ .....

(a) 2 (b) 8 (c) 7 (d) $\frac{1}{2}$

[B] Complete the following :

④ $2 \times 6 - 4 \div 2 =$ .....

⑤ $2^3 \div 4 \times 3 + (2-1) =$ .....

⑤ $2 \times 6 - 4 \div 2 =$ .....

⑤ $3 \times 7 - 15 \div 3 =$ .....

① $4 + 2 \times 3 =$ .....

[C] Essay Problems :

[b] Calculate the value of : $20 \div 5 + 8 - (4-1)$

[b] Simplify : $2^3 + \left[4 + (2^2 \div 2)\right]$

Lesson (6) : The Square Root

- **We know that :** $3^2 = 3 \times 3 = 9$ and it is read as the square of the number 3 is 9

And the square root is the inverse operation of finding the square of the number.

For example: To find the square root of the number 36 , we search for a number whose square is 36

- **We find that this number is :** 6 (because : $6^2 = 36$) or -6 [because : $(-6)^2 = 36$]

i.e. The number 36 has two square roots which are ± 6

- 6 is the positive square root of 36 and it is written as : $\sqrt{36} = 6$
- -6 is the negative square root of 36 and it is written as : $-\sqrt{36} = -6$

From the previous , we can deduce the following definition :

Definition :

The square root of the perfect square rational number "a" is the number whose square equals "a".

Generally :

*** The positive square root of the number a is symbolized by $\sqrt{a}$**

Notice that : $\sqrt{\text{Zero}} = \text{Zero}$

For example: The positive square root of 25 is $\sqrt{25} = 5$

*** The negative square root of the number a is symbolized by $-\sqrt{a}$**

For example: The negative square root of 16 is $-\sqrt{16} = -4$

*** The two square roots of the number a is symbolized by $\pm\sqrt{a}$, and each of them is the additive inverse of the other.**

For example: The two square roots of 49 are $\pm\sqrt{49} = \pm 7$

Remarks

1. It is meaningless to find $\sqrt{a}$ if a is a negative rational number because there is no rational number if it is multiplied by itself , the result will be negative.
2. $\sqrt{\left(\frac{a}{b}\right)^2} = \left|\frac{a}{b}\right|$

   For example: • $\sqrt{(-3)^2} = |-3| = 3$ • $\sqrt{\left(-\frac{4}{5}\right)^2} = \left|-\frac{4}{5}\right| = \frac{4}{5}$
3. $\sqrt{a^2 b^2} = \sqrt{(ab)^2} = |ab|$ **For example:** $\sqrt{a^4 b^6} = \sqrt{(a^2 b^3)^2} = |a^2 b^3|$
4. If : $x^2 = a$ "where a is a perfect square rational number" , then : $x = \pm\sqrt{a}$

Example (1) :

Simplify each of the following to the simplest form :

1 $-\frac{2}{7}\times\sqrt{\frac{49}{4}}\times\left(\frac{2}{7}\right)^2$ **2** $\left(-\frac{3}{2}\right)^2\times\sqrt{\frac{64}{9}}\times\left(\frac{5}{2}\right)^0$ **3** $\left(2\frac{7}{9}\right)^2\div\sqrt{\frac{25}{9}}$

Solution

1 $-\frac{2}{7}\times\sqrt{\frac{49}{4}}\times\left(\frac{2}{7}\right)^2=-\frac{2}{7}\times\frac{7}{2}\times\frac{4}{49}=-\frac{4}{49}$

2 $\left(-\frac{3}{2}\right)^2\times\sqrt{\frac{64}{9}}\times\left(\frac{5}{2}\right)^0=\frac{9}{4}\times\frac{8}{3}\times1=6$

3 $\left(2\frac{7}{9}\right)^2\div\sqrt{\frac{25}{9}}=\left(\frac{25}{9}\right)^2\div\frac{5}{3}=\left(\left(\frac{5}{3}\right)^2\right)^2\div\frac{5}{3}=\left(\frac{5}{3}\right)^4\div\frac{5}{3}=\left(\frac{5}{3}\right)^{4-1}=\left(\frac{5}{3}\right)^3=\frac{125}{27}$

Exercises on Lesson (6)

[A] Choose the correct answer from the gives ones :

(3) $\sqrt{\frac{25}{36}}=$ .....

(a) $\frac{\pm5}{6}$ (b) $\frac{-5}{6}$ (c) $\frac{5}{6}$ (d) otherwise

(3) $\sqrt{\left(\frac{-5}{6}\right)^2}=$ .....

(a) $\frac{-5}{6}$ (b) $\frac{5}{6}$ (c) $\pm\frac{5}{6}$ (d) $\frac{2}{6}$

(3) $\sqrt{\left(\frac{3}{4}\right)^2}=$ .....

(a) $\frac{3}{4}$ (b) $-\frac{6}{16}$ (c) $\pm\frac{3}{4}$ (d) $-\frac{3}{4}$

(1) $\sqrt{(-5)^2}=$ .....

(a) – 5 (b) 25 (c) 10 (d) 5

(1) $\sqrt{36+64}=6+$ .....

(a) 10 (b) 4 (c) 8 (d) 6

(2) If : $A=0.000625$, then : $\sqrt{A}=2.5\times10$ .....

(4) If the area of a square is 100 cm^2 , then its side length = ... cm.

(a) 10 (b) 12 (c) 13 (d) 25

[B] Complete the following :

(2) $\sqrt{(-9)^2}$ = ...

(5) $\sqrt{16+9}$ = ...

(3) $\sqrt{16+9} = 4 +$ .....

(2) $\sqrt{6^2+8^2} = 6 +$ .....

(2) $\sqrt{(91)^2 - 91 \times 2 + 1}$ = ...

(1) $\sqrt{\frac{81}{25}} \div \left(\frac{3}{5}\right)^2$ = ...

(1) The multiplicative inverse of $\sqrt{6\ \frac{1}{4}}$ is .....

(2) The multiplicative inverse of $\sqrt{\frac{16}{25}}$ is .....

(1) The multiplicative inverse of $\sqrt{\frac{10}{2.5}}$ is .....

(5) The multiplicative inverse of $\sqrt{\frac{25}{64}}$ is .....

[C] Essay Problems :

[a] Calculate : $\left(\frac{1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{2}{5}\right)^0$

[a] Find the result in the simplest form : $\frac{4}{3} \times \sqrt{\frac{9}{25}} \times \left(-\frac{1}{2}\right)^2$

[a] Evaluate : $\left(\frac{5}{4}\right)^2 \times \sqrt{\frac{16}{25}} \times \left(\frac{-4}{5}\right)^0$

[b] Simplify : $\sqrt{3 \times 7 - 15 \div 3}$

[a] Find the result of : $\left(\frac{-2}{3}\right)^2 \times \sqrt{\frac{9}{4}} \times \left(\frac{-1}{4}\right)^{\text{zero}}$

[a] Find the value of : $\left(\frac{5}{8}\right)^0 \times \sqrt{6\frac{1}{4}} \times \left(-\frac{2}{5}\right)^2$

Lesson (7) : Solving Equations

Prelude

The equation is a mathematical statement which contains one variable as x (or more as x and y) and contains equality relation « = »

as : $2x = 6$, $x + 3 = 5$, $2x - y = 3$ and $x^2 = 25$

The degree of the equation is determined by the heighest degree of the terms forming the equation.

For example:

- $5x + 2 = 7$ is an equation of the first degree in one unknown x
- $x^2 + x - 3 = 0$ is an equation of the second degree in one unknown x
- $2x + 3y = 5$ is an equation of the first degree in two unknowns x and y

The substituting set : is the set that contains the probable values of the unknown.

The solution set (the S.S.) : is the set whose elements satisfy the equality of the equation and it is a subset of the substitution set.

- We can add any rational number to both sides of the equation.

  For example: If $x - 1 = 5$, then $x - 1 \underline{+1} = 5 \underline{+1}$

  i.e. $x = 6$

- We can subtract any rational number from both sides of the equation.

  For example: If $x + 3 = 2$, then $x + 3 \underline{-3} = 2 \underline{-3}$

  i.e. $x = -1$

- We can multiply both sides of the equation by the same rational number.

  For example: If $\frac{1}{5}x = 2$, then $\frac{1}{5}x \underline{\times 5} = 2 \underline{\times 5}$

  i.e. $x = 10$

- We can divide both sides of the equation by the same rational number not equal to zero.

  For example: If $7x = 14$, then $\frac{7x}{7} = \frac{14}{7}$

  i.e. $x = 2$

Then by applying any of the previous properties in any equation , then we will get an equivalent equation to the origin equation that has the same solution.

Generally :

If a , b and c are three rational numbers , then these numbers have the following properties :

1. If $a = b$, then $a + c = b + c$
2. If $a + c = b + c$, then $a = b$
3. If $a = b$, then $a \times c = b \times c$
4. If $a \times c = b \times c$, $c \neq 0$, then $a = b$

Example 3

Find the S.S. of each of the following equations :

1. $2(x + 3) = 4$, where $x \in \mathbb{Z}$
2. $5(x + 2) - 1 = 19$, where $x \in \mathbb{Q}$

Solution

1. $\because 2(x + 3) = 4$

Dividing both sides by 2

$\therefore \frac{2(x+3)}{2} = \frac{4}{2}$

$\therefore x + 3 = 2$

Adding (-3) to both sides

$\therefore x + 3 - 3 = 2 - 3$

$\therefore x = -1$

$\therefore$ The S.S. $= \{-1\}$

2. $\because 5(x + 2) - 1 = 19$

Using the distribution property

$\therefore 5x + 10 - 1 = 19$

$\therefore 5x + 9 = 19$

Notice that :

$5(x + 2) - 1 = 19$, $5x + 9 = 19$ and $5x = 10$ are equivalent equations.

Adding (-9) to both sides

$\therefore 5x + 9 - 9 = 19 - 9$

$\therefore 5x = 10$

Dividing both sides by 5

$\therefore \frac{5x}{5} = \frac{10}{5}$

$\therefore x = 2$

$\therefore$ The S.S. $= \{2\}$

Remarks for solving life problems :

- If a number $= x$, then its twice $= 2x$ and its three times $= 3x$, ...
- If a number $= x$ and another number exceeds it by 5 , then the other number $= x + 5$

- If a number = x and another number decreases than it by 5 , then the other number = $x - 5$
- If the age of a man now = x years , then :
  * His age after 3 years = $(x + 3)$ years.
  * His age 3 years ago = $(x - 3)$ years.
- Three consecutive integers are : x , $x + 1$ and $x + 2$
- Three consecutive natural (even or odd) numbers are x , $x + 2$ and $x + 4$
- The perimeter of a rectangle = 2 (length + width)
- The perimeter of a square = side length × 4
- The perimeter of the triangle = the sum of its sides lengths
- The area of the triangle = $\frac{1}{2}$ the base length × the height.
- The sum of measures of the interior angles of the triangle = 180°

Three natural consecutive odd numbers whose sum is 27, find these numbers.

Solution

Let the smallest odd number = x

$\because$ Each odd number exceeds the odd number just before it by 2

$\therefore$ The next odd number = $x + 2$ and the third odd number = $x + 4$

$\because$ The sum of the numbers = 27

$\therefore x + (x + 2) + (x + 4) = 27$ $\quad\quad$ $\therefore 3x + 6 = 27$

$\therefore 3x = 27 - 6$ $\quad\quad$ $\therefore 3x = 21$

$\therefore x = \frac{21}{3}$ $\quad\quad$ $\therefore x = 7$

i.e. The numbers are 7 , 9 and 11

To check the solution : the numbers 7 , 9 and 11 are natural consecutive odd numbers , $7 + 9 + 11 = 27$ $\quad\quad$ $\therefore$ The solution is true.

A rectangle with length equals twice its width and its perimeter = 18 cm. Find the dimensions of the rectangle.

Solution

Let the width of the rectangle = x cm. $\quad\quad$ $\therefore$ Its length = $2x$ cm.

$\because$ The perimeter of the rectangle = 2 (length + width)

$\therefore 18 = 2(2x + x)$ $\quad\quad$ $\therefore 18 = 2 \times 3x$

$\therefore 18 = 6x$ $\quad\quad$ $\therefore x = 3$

i.e. The width of the rectangle = 3 cm. and its length = 6 cm.

To check the solution :

$\because$ The length of the rectangle = 6 cm. equals twice its width 3 cm.

, The perimeter of the rectangle = $2(6 + 3) = 2 \times 9 = 18$ cm.

$\therefore$ The solution is true.

Exercises on Lesson (7)

[A] Choose the correct answer from the gives ones :

(5) The solution set of the equation : $-2x + 1 = -3$ in $\mathbb{N}$ is .....

(a) $\{1\}$ (b) $\{2\}$ (c) $\{3\}$ (d) $\{4\}$

(5) The solution set of : $-2x + 1 = -3$ in $\mathbb{Z}$ is .....

(a) $\{2\}$ (b) $\{-2\}$ (c) $\{-4\}$ (d) $\varnothing$

(5) The solution set of the equation : $2x + 1 = -3$ in $\mathbb{N}$ is .....

(a) $\{1\}$ (b) $\{2\}$ (c) $\{4\}$ (d) $\varnothing$

(4) The solution set of the equation : $3x - 13 = 26$ in $\mathbb{Q}$ is .....

(a) $\{36\}$ (b) $\{13\}$ (c) $\{3\}$ (d) $\{31\}$

(5) The S.S. of $x + 1 = 9$ in $\mathbb{N}$ is .....

(a) $\{9\}$ (b) $\{10\}$ (c) $\{8\}$ (d) $\varnothing$

(3) If : $5x = 20$, then $x + 3 =$ .....

(a) 16 (b) 12 (c) 17 (d) 7

(5) If : $5x = 20$, then $x + 5 =$ .....

(a) 10 (b) 15 (c) 9 (d) 20

(3) If : $3x = 5$, then the value of $12x =$ .....

(a) 4 (b) 36 (c) 20 (d) 60

(3) If the age of Ahmed now is x years then this age 10 years ago is ..... years.

(a) $10x$ (b) $x + 10$ (c) $10 - x$ (d) $x - 10$

(3) If the age of Mohamed now is x years then his age after five years is ..... years.

(a) $5x$ (b) $x+5$ (c) $x-5$ (d) x^5

(5) Ali's age 2 years ago was x , then his age now is ..... years.

(a) $x+2$ (b) $x-2$ (c) $2-x$ (d) $2x$

(5) If x is an odd natural number then the next odd number directly is .....

(a) $x+1$ (b) $x+2$ (c) $2x+1$ (d) $2x$

[B] Complete the following :

(3) If : $x \in \mathbb{N}$, then the solution set of the equation : $x + 7 = 1$ is .....

(3) If : $x + 9 = 11$, then the value of $7x =$ .....

(5) If : $3x = 6$, then $6x =$ .....

(4) If : $3x = 6$, then $5x =$ .....

(3) If the age of Ahmed now is x years , then his age after three years is ..... years.

(4) If the age of Mona now is x years , then her age 5 years ago is ..... years.

(1) If the age of a man now is x years , then his age 3 years ago is .....

(5) If : $\frac{x}{5} = \frac{8}{5}$, then $\frac{1}{4}x =$ .....

[C] Essay Problems :

[a] Solve the equation : $5x + 8 = 13 - 2x$, where $x \in \mathbb{Q}$

[b] Find the solution set of the equation : $3x - 1 = 5$ in $\mathbb{Q}$

[b] Find in $\mathbb{Q}$ the S.S. of the equation : $3x + 7 = 13$

[a] Solve the equation : $3x - 3 = 9$, where $x \in \mathbb{Q}$

[a] Solve the equation : $3x + 5 = 26$, where $x \in \mathbb{Z}$

[a] Two natural numbers , one of them is twice the other and their sum is 45
Find the two numbers.

[b] Three consecutive natural numbers their sum is 33 Find these numbers.

[a] Three consecutive odd natural numbers are with sum 117 **Find this numbers.**

[a] The age of a man is three times the age of his son , if the sum of their ages is 60 years. **Find the age of each of the man and his son.**

[a] Two consecutive even numbers , their sum is 150 Find them

3 Two natural numbers , one of them is twice the other and their sum is 108
Find the two numbers.

[b] The sum of three consecutive numbers is 42 **Find them.**

[a] The length of a rectangle exceeds its width by 4 meters and its perimeter is 68 meters
Find the dimensions of the rectangle.

[b] Three consecutive even numbers are of sum 78 **Find these numbers.**

[b] The sum of three consecutive numbers is 33 Find the three numbers.

Lesson (8) : Solving Inequalities in Q

Assuming that a , b , c are three rational numbers , then :

1. If $a < b$, then $a + c < b + c$
2. If $a < b$, then $a - c < b - c$
3. If $a < b$, c is a positive number , then $ac < bc$
4. If $a < b$, c is a positive number , then $\frac{a}{c} < \frac{b}{c}$
5. If $a < b$, c is a negative number , then $ac > bc$
6. If $a < b$, c is a negative number , then $\frac{a}{c} > \frac{b}{c}$

Remark

If a and b are two rational numbers not equal to zero and $a > b$, then : $\frac{1}{a} < \frac{1}{b}$

Example 1

Find the solution set of the inequality :

$x + 2 < 5$, **where :** 1 $x \in \mathbb{Z}$ 2 $x \in \mathbb{N}$

then represent the solution set on the number line in each case.

Solution

$\because x + 2 < 5$ Subtracting 2 from the two sides

$\therefore x + 2 - 2 < 5 - 2$ *i.e.* $x < 3$

1 When $x \in \mathbb{Z}$

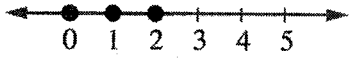
The solution set is all the integers which are less than 3

i.e. The S.S. $= \{2 , 1 , 0 , -1 , ...\}$

2 When $x \in \mathbb{N}$

The solution set is all the natural numbers which are less than 3

i.e. The S.S. $= \{2 , 1 , 0\}$



We notice from the previous example that :

The solution set of the inequality depends on the substitution set, we find that :
The solution set in $\mathbb{N}$ differs from the solution set in $\mathbb{Z}$

Example 2

Find the solution set of the inequality $2x - 5 > 5$, where :

1 $x \in \mathbb{Q}$ **2** $x \in \mathbb{Z}$

Solution

$\because 2x - 5 > 5$ Adding 5 to both sides

$\therefore 2x - 5 + 5 > 5 + 5$ $\therefore 2x > 10$

Multiplying both sides by $\frac{1}{2}$

$\therefore \frac{1}{2} \times 2x > \frac{1}{2} \times 10$ ***i.e.*** $x > 5$

1 When $x \in \mathbb{Q}$

The S.S. is all the rational numbers which are greater than 5 , then we write it by characterized property method because it is difficult to list all its members.

i.e. The S.S. $= \{x : x \in \mathbb{Q} , x > 5\}$

2 When $x \in \mathbb{Z}$

The solution set is all the integers which are greater than 5

i.e. The S.S. $= \{6 , 7 , 8 , \cdots\}$

Example 3

Find in $\mathbb{Q}$ the solution set of each of the two following inequalities :

1 $4 - 2x \leq 2$ **2** $7(x - 1) > 9x - 6$

Solution

1 $\because 4 - 2x \leq 2$ Adding -4 to both sides

$\therefore -4 + 4 - 2x \leq -4 + 2$ $\therefore -2x \leq -2$

Dividing both sides by (-2)

$\therefore \frac{-2x}{-2} \geq \frac{-2}{-2}$ **Notice that:** The change of inequality sign.

$\therefore x \geq 1$ ***i.e.*** The S.S. $= \{x : x \in \mathbb{Q} , x \geq 1\}$

2 $\because\ 7(x-1) > 9x-6$ $\quad\therefore\ 7x-7 > 9x-6$

Subtracting $(9x)$ from both sides

$\therefore\ 7x-9x-7 > 9x-9x-6$ $\quad\therefore\ -2x-7 > -6$

Adding 7 to both sides

$\therefore\ -2x-7+7 > -6+7$ $\quad\therefore\ -2x > 1$

Dividing both sides by (-2)

$\therefore\ \frac{-2x}{-2} < \frac{1}{-2}$

$\therefore\ x < -\frac{1}{2}$

Notice that: The change of inequality sign.

i.e. The S.S. $= \{x : x \in \mathbb{Q}\ ,\ x < -\frac{1}{2}\}$

Example 4

Find in $\mathbb{Z}$ the solution set of the inequality $-11 \leq 3x-5 < 4$, then represent it on the number line.

Solution

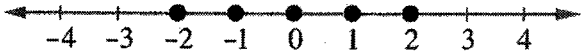
$\because\ -11 \leq 3x-5 < 4$ $\quad$ Adding 5 to the three sides

$\therefore\ -11+5 \leq 3x-5+5 < 4+5$ $\quad\therefore\ -6 \leq 3x < 9$

Dividing all sides by 3

$\therefore\ \frac{-6}{3} \leq \frac{3x}{3} < \frac{9}{3}$ $\quad\therefore\ -2 \leq x < 3$

i.e. The S.S. $= \{-2\ ,-1\ ,0\ ,1\ ,2\}$



Exercises on Lesson (8)

[A] Choose the correct answer from the gives ones :

(5) If : $x + 3 > 5$, then x may be equal .....

(a) 5 (b) 1 (c) -5 (d) -2

(3) If : $z > y$ and $y > x$, then $z >$ .....

(1) If : $X > Y$, then $-X$ ..... $-Y$

(a) = (b) ≥ (c) > (d) <

(4) If : $-x < 3$, then x ..... -3

(a) < (b) = (c) > (d) ≤

(2) If : $-2x < 4$, then .....

(a) $x < -2$ (b) $x < -6$ (c) $x < 2$ (d) $x > -2$

[B] Complete the following :

(2) If : $a - 3 < 0$, then ..... > .....

(5) If : $x < y$ and z is negative then $x z$ ..... yz

(3) If : $-x < 5$, then x ..... (-5)

(3) The solution set of : $1 > x \geq -2$ in $\mathbb{N}$ is .....

(3) If : $z > y$ and $y > x$, then $z >$ .....

[C] Essay Problems :

[a] Find in $\mathbb{Z}$ the S.S. of the inequality : $x - 1 < 3$, then represent the solution on number line.

[a] Find the S.S. of each of the following :

(1) $8x + 4 = 12$ where $x \in \mathbb{Z}$ (2) $2x + 3 \leq 7$ where $x \in \mathbb{N}$

[b] Find the solution set of the following ($x \in \mathbb{Q}$) :

(1) $2x - 5 = 1$ (2) $4x + 9 \leq 1$

[b] Solve the inequality : $5x - 7 \leq 3$, $x \in \mathbb{Q}$

[a] Solve the inequality : $4x + 5 \geq -3$ and represent the solution set on the number line where $x \in \mathbb{N}$

4 Find in $\mathbb{Q}$ the S.S of :

(1) $4x - 1 \geq 7$ (2) $2(x + 5) = 16$

[a] Solve the inequality : $4x - 7 \geq 1$, where $x \in \mathbb{Q}$

[a] Find the solution set of each of the following in $\mathbb{Z}$:

(1) $2x + 14 = 12$ (2) $3x + 1 \leq 7$

[b] Find the solution set of the inequality , where $x \in \mathbb{Z}$: $4x + 5 \geq 1$

4 **Find the solution set of each of the following where $x \in \mathbb{Q}$:**

(1) $2x + 1 = 13$ (2) $3x - 1 \leq 2x + 4$

[a] Solve each of the following inequalities and represent the solution set on the number line where $x \in \mathbb{Z}$:

(1) $8x - 3x - 1 \leq 29$ (2) $x + 4 > 1$

[b] Solve the inequality in $\mathbb{Q}$: $3(x + 2) < -x + 4$

[a] Find the solution set of the inequality : $3x - 2 > x + 4$, where $x \in \mathbb{Q}$

[b] Find S.S. of : $3x - 2 \leq 3 - 2x$ in $\mathbb{Q}$

[b] Solve the following inequality in $\mathbb{Q}$: $1 - (4x - 1) > 2(x - 3)$