if $y = \log_a x$ then $x = a^y$

$\log_a(M\ N) = \log_a M + \log_a N$

$\log_a(a^x) = x$

$a^{-N} = \frac{1}{a^N}$

$\cos^2 x + \sin^2 x = 1$

$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$

$\log_a(M^p) = P\ \log_a M$

$x^{\log_b x} = b$

$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

$\sin(2\alpha) = 2\sin(\alpha)\ \cos(\alpha)$

$a^{\log_a x} = x$

$\log_a a = 1$

$\log_a x = \frac{\ln x}{\ln a}$

$a^{\left(\frac{M}{N}\right)} = \left(\sqrt{a}\right)^M$

$S_x = [x-(a+bi)]\ [x-(a-bi)] = x^2 - 2ax + (a^2 + b^2)$

$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$\tan\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$

$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$\cos(2\alpha) = 2\cos^2\alpha - 1$

$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$

$\cos(2\alpha) = 1 - 2\sin^2\alpha$

$\cos\alpha - \cos\beta = 2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$ax^2 + bx + c = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos\left(\frac{\pi}{2} + x\right) = -\sin x$

$\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$\cos(\pi \pm x) = -\cos x$

$\cos\left(\frac{3\pi}{2} + x\right) = \sin x$

$\cos\left(\frac{3\pi}{2} - x\right) = -\sin x$

$\tan\left(\frac{\pi}{2} + x\right) = -\cot x$

$\tan\left(\frac{\pi}{2} - x\right) = \cot x$

$\tan(\pi + x) = \tan x$

$\tan(\pi - x) = -\tan x$

$\tan\left(\frac{3\pi}{2} + x\right) = -\cot x$

$\tan\left(\frac{3\pi}{2} - x\right) = \cot x$

$\sin\left(\frac{\pi}{2} \pm x\right) = \cos x$

$\sin(\pi + x) = -\sin x$

$\sin(\pi - x) = \sin x$

$\sin\left(\frac{3\pi}{2} \pm x\right) = -\cos x$

- therefore
- ∵ since
- ¬ not
- and
- or

- $\prec$ precedes
- $\succ$ follows
- congruent to
- union
- $\cap$ intersection

- C (fancy) compliment
- implies
- $\Leftrightarrow$ double implication
- ~ negation (or ¬)

- subset
- superset
- proper subset
- propor superset

Q.E.D: Quod Erat Demonstandum
"that which was to be proved"

- for every
- element of
- there exists
- such that

- d derive
- $\int$ integrate
- $|_{x=a}$ evaluate with x = a
- proportional to

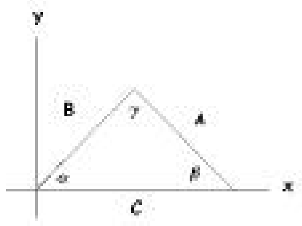
$Mid = \left(\frac{x_2 - x_1}{2}, \frac{y_2 - y_1}{2}\right)$

$Dist = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$A = \frac{1}{2}r^2\theta$, sector area



$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

$$\text{Frequency} = \frac{1}{\text{period}} = \frac{\omega}{2\pi}$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$
$$b^2 = a^2 + c^2 - 2ac\cos\beta$$
$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$\text{Phase Shift} = \frac{\beta}{\omega}$$

$$K = \frac{1}{2}bc\sin\alpha$$

$$\text{Critical Points} = \frac{\text{period}}{4}$$

$$y = A\ \sin(\omega x - \beta) + K,\quad \omega > 0$$

$$\text{Period} = \frac{2\pi}{\omega}$$

$$\text{Amplitude} = |A| = \frac{M - m}{2}$$

Unit Circle
(cos, sin)

$$0, 0^\circ = (1, 0)$$

$$\frac{\pi}{6}, 30^\circ = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

$$\frac{\pi}{4}, 45^\circ = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

$$\frac{\pi}{3}, 60^\circ = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

$$\frac{\pi}{2}, 90^\circ = (0, 1)$$

$$\frac{2\pi}{3}, 120^\circ = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$

$$\frac{3\pi}{4}, 135^\circ = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

$$\frac{5\pi}{6}, 150^\circ = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

$$\pi, 180^\circ = (-1, 0)$$

$$\frac{7\pi}{6}, 210^\circ = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

$$\frac{5\pi}{4}, 225^\circ = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$$

$$\frac{4\pi}{3}, 240^\circ = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

$$\frac{5\pi}{3}, 300^\circ = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

$$\frac{3\pi}{4}, 270^\circ = (0, -1)$$

$$\frac{11\pi}{6}, 330^\circ = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

$|ab| = |a|\,|b|$

$\sqrt{a^2} = |a|$

$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

$|a^n| = |a|^n$

$-|a| \le a \le |a|$

$|a| \le k$ iff $-k \le a \le k$

$|a+b| \le |a| + |b|$

$|a| \ge k$ iff $a \le -k$ of $a \ge k$

if $a < b$ & $b < c$, then $a < c$
if $a < b$ & $c < d$, then $a + c < b + d$
if $a < b$ then $a + k < b + k$
if $a < b$ & $k > 0$, then $ak < bk$
if $a < b$ & $k < 0$, then $ak > bk$

$\lim_{x \to o} \frac{1 - \cos x}{x} = 0$

$\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$

$\lim_{x \to o} \frac{\sin x}{x} = 1$

$\frac{d}{dx} x^n = nx^{n-1}$

$\frac{d}{dx} C = 0$

$\frac{d}{dx} \csc x = -\csc x \cot x$

$\frac{d}{dx} \sin x = \cos x$

$\frac{d}{dx} \tan x = \sec^2 x$

$\frac{d}{dx} \cot x = -\csc^2 x$

$f(t) = \frac{1}{2} g t^2 + v_0 t + s_0$
$v(t) = f'(t)$
$a(t) = v'(t) = f''(t)$

$\frac{d}{dx} |u| = u' \frac{u}{|u|}, u \ne 0$

$\int (k) dx = kx + C$

$\frac{d}{dx} \sec x = \sec x \tan x$

$\frac{d}{dx} \cos x = -\sin x$

$\frac{d}{dx}\left(f(x)\, g(x)\right) = f(x) g'(x) + g(x) f'(x)$

$f(x)$ increasing if $f'(x) > 0$
$f(x)$ decreasing if $f'(x) < 0$

$\Delta y \approx dy = f'(x)\, dx$

$\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{g(x) f'(x) - f(x) g'(x)}{\left(g(x)\right)^2}$

Profit = Revenue - Cost = Sold Price - Cost

Critical # when $f'(x) = 0$ or $f'(x)$ *DNE*

IPs if $f''(x) = 0$ and $f''(x)$ changes sign

$\int (x^n) dx = \frac{x^{n+1}}{n+1} + C$

MVT, $f'(c) = \frac{f(b) - f(a)}{b - a}$ on $[a, b]$

$\frac{d}{dx}\left(\int f(x) dx\right) = f(x)$

$f(x)$ concave down on (a, b)
if $f'(x)$ is DEC x in $[a, b]$, $f''(x) < 0$

TrapRule
$\int_a^b f(x) dx \approx \frac{b - a}{2n}\left(f(a) + f(b) + 2\left(f(x_1) + f(x_2) + ... + f(x_n)\right)\right)$

$f(x)$ concave up on (a, b)
if $f'(x)$ is INC x in $[a, b]$, $f''(x) > 0$

Euler's Method
Start @ x, y → $f'(x, y)$ → Δx → $\Delta y = \Delta x\ f'(x, y)$
Use Δy for change in next y

$\int \left(f'(x) \pm g'(x)\right) dx = f(x) \pm g(x)$

$\int (0) dx = C$

$\int \left(k\ f'(x)\right) dx = kf(x) + C$

$\sum_{i=1}^{n} c = cn$

$\int \left(\csc(x)\cot(x)\right) dx = C - \csc(x)$

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$\int \left(\cos(x)\right) dx = \sin(x) + C$
$\int \left(\sin(x)\right) dx = C - \cos(x)$

$\int \left(\sec^2(x)\right) dx = \tan(x) + C$
$\int \left(\csc^2(x)\right) dx = C - \cot(x)$

$\int \left(\sec(x)\tan(x)\right) dx = \sec(x) + C$

$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

$\lim_{x \to \infty} \sum_{i=1}^{n} f(c_i)\ \Delta x, x_{i-a} \le c_i \le x_i, \Delta x = \frac{b - a}{n}$

$\int_a^a f(x) dx = 0$

$\int_b^a f(x) dx = -\int_a^b f(x) dx$

$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$

$\int_a^b f(x) \pm g(x) dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$

$\int_a^b k\ f(x) dx = k \int_a^b f(x) dx$

$\int_a^b f(x) dx = F(b) - F(a), F'(x) = f(x)$

$F'(x) = \frac{d}{dx}\left(\int_a f(t)\right) = f(x)$

If f is even, $\int_{-a}^{a} f(x) dx = 2\int_0^a f(x) dx$

Average Value of a Function : $\frac{1}{b - a}\int_a^b f(x) dx$

If f is odd, $\int_{-a}^{a} f(x) dx = 0$

$\ln(x)=\int_1^x \frac{1}{t}\,dt,\ x>0$

$\ln(1)=0$

$\ln(a\cdot b)=\ln(a)+\ln(b)$

$e^{\pi i}+1=0$

$\ln(a)^n=n\cdot\ln(a)$

$\ln(e^x)=x$

$\ln\left(\frac{a}{b}\right)=\ln(a)-\ln(b)$

$\ln(e)=\int_1^e \frac{1}{t}\,dt=1$

$\int \frac{1}{u}\,du=\ln(u)+C$

$e^a\cdot e^b=e^{a+b}$

$\frac{e^a}{e^b}=e^{a-b}$

$\frac{d}{dx}\ln(u)=\frac{d}{dx}\ln|u|=\frac{du}{u}$

$\int \tan(x)dx=C-\ln|\cos(x)|=\ln|\sec(x)|+C$

$\frac{d}{dx}(e^e)=0$

$\int \cot(x)dx=\ln|\sin(x)|+C=C-\ln|\csc(x)|$

$\int \sec(x)dx=\ln|\sec(x)+\tan(x)|+C$

$e=\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

$f^{-1}(x)=g(x), g'(x)=\frac{1}{g'(g(x))}$

$\frac{d}{dx}(e^u)du=e^u\cdot du$

$\int \csc(x)dx=C-\ln|\csc(x)+\cot(x)|$

$f^{-1}(x)=g(x),(a,b)\epsilon f, g'(b)=\frac{1}{f'(a)}$

$\frac{d}{dx}(a^u)=\ln(a)\cdot a^u\cdot du$

$\log_a(x)=\frac{\ln(x)}{\ln(a)}$

$\frac{d}{dx}(\log_a(u))=\frac{du}{\ln(a)\cdot u}$

$\int(a^x)dx=\frac{a^x}{\ln(a)}+C$

$\frac{d}{dx}(u^n)=n\cdot u^{(n-1)}\cdot du$

$\frac{d}{dx}(x^x)=(\ln(x)+1)\cdot x^x$

Compounded n times a year

$A=P\left(1+\frac{r}{n}\right)^{nt}$

Compounded Continously

$A=P\cdot e^{rt}$

$y=\arcsin(x), iff \sin(y)=x$

$y=\arccos(x), iff \cos(y)=x$

$y=\arctan(x), iff \tan(y)=x$

$y=arc\cot(x), iff \cot(y)=x$

$y=arc\sec(x), iff \sec(y)=x$

$y=arc\csc(x), iff \csc(y)=x$

Newton's Law of Cooling

$\frac{dT}{dt}=k(T_o-T_s)$

$y=\arcsin(x), D:[-1,1], R:\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$y=\arccos(x), D:[-1,1], R:[0,\pi]$

$y=\arctan(x), D:(-\infty,\infty), R:\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

$y=arc\cot(x),\ D:(-\infty,\infty), R:(0,\pi)$

$y=arc\sec(x),\ D:(|x|\geq 1), R:(0,\pi)$

$y=arc\csc(x),\ D:(|x|\geq 1), R:\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

$\frac{d}{dx}(\arcsin(u))=\frac{1}{\sqrt{1-u^2}}\cdot du$

$\frac{d}{dx}(arc\cot(u))=\frac{-1}{1+u^2}\cdot du$

$\frac{d}{dx}(\arccos(u))=\frac{-1}{\sqrt{1-u^2}}\cdot du$

$\frac{d}{dx}(arc\csc(u))=\frac{-1}{|u|\sqrt{u^2-1}}\cdot du$

$\frac{d}{dx}(\arctan(u))=\frac{1}{1+u^2}\cdot du$

$\cosh(2x)=\cosh^2 x\;\sinh^2 x$

$\frac{d}{dx}(arc\sec(u))=\frac{1}{|u|\sqrt{u^2-1}}\cdot du$

$\int\frac{du}{a^2+u^2}=\frac{1}{a}\arctan\left(\frac{u}{a}\right)+C$

$\sinh(2x)=2\sinh(x)\cdot\cosh(x)$

$\sinh^2 x=\frac{\cosh(2x)-1}{2}$

$\sinh(x)=\frac{e^x-e^{-x}}{2}$

$\cosh(x)=\frac{e^x+e^{-x}}{2}$

$\int\frac{du}{|u|\sqrt{u^2-a^2}}=\frac{1}{a}arc\sec\left|\frac{u}{a}\right|+C$

$\int\frac{du}{\sqrt{a^2-u^2}}=\arcsin\left(\frac{u}{a}\right)+C$

$\cosh^2 x=\frac{\cosh(2x)+1}{2}$

$\sinh(x+y)=\sinh(x)\cdot\cosh(y)+\cosh(x)\cdot\sinh(y)$

$\sinh(x-y)=\sinh(x)\cdot\cosh(y)-\cosh(x)\cdot\sinh(y)$

$\cosh(x+y)=\cosh(x)\cdot\cosh(y)+\sinh(x)\cdot\sinh(y)$

$\cosh(x-y)=\cosh(x)\cdot\cosh(y)-\sinh(x)\cdot\sinh(y)$

$\cosh^2 x-\sinh^2 x=1$

$\tanh^2 x+\sec h^2 x=1$

$\coth^2 x-\csc h^2 x=1$

$\int\cosh(u)\cdot du=\sinh(u)+C$

$\int\sinh(u)\,du=\cosh(u)+C$

$\int\sec h^2 u\cdot du=\tanh(u)+C$

$\frac{d}{dx}\coth(u)=-\csc h^2 u\cdot du$

$\frac{d}{dx}\sec h(u)=-(\sec h(u)\cdot\tanh(u))\cdot du$

$\frac{d}{dx}\csc h(u)=-(\csc h(u)\cdot\coth(u))\cdot du$

$\frac{d}{dx}\sinh(u)=\cosh(u)\cdot du$

$\frac{d}{dx}\cosh(u)=\sinh(u)\cdot du$

$\frac{d}{dx}\tanh(u)=\sec h^2 u\cdot du$

$\int\csc h^2 u\cdot du=C-\coth(u)$

$\int\sec h(u)\cdot\tanh(u)\cdot du=C-\sec h(u)$

$\int\csc h(u)\cdot\coth(u)\cdot du=C-\csc h(u)$

$\operatorname{arcsin} h(x) = \ln\left(x + \sqrt{x^2+1}\right), D: (-\infty, \infty)$

$\operatorname{arccos} h(x) = \ln\left(x + \sqrt{x^2-1}\right), D: (1, \infty)$

$\operatorname{arctan} h(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-X}\right), D: (-1, 1)$

$\operatorname{arc}\coth(x) = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right), D: (-\infty, -1) \quad (1, \infty)$

$\operatorname{arc}\operatorname{sec} h(x) = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right), D: (0, 1]$

$\operatorname{arc}\operatorname{csc} h(x) = \ln\left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{|x|}\right), D: (-\infty, 0) \quad (0, \infty)$

$\frac{d}{dx}\operatorname{arccos} h(u) = \frac{du}{\sqrt{u^2-1}}$

$\frac{d}{dx}\operatorname{arctan} h(u) = \frac{d}{dx}\operatorname{arc}\coth(u) = \frac{du}{1-u^2}$

$\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left(\frac{a+u}{a-u}\right) + C$

$\int u \ dv = uv - \int v \ du$

$\frac{d}{dx}\operatorname{arcsin} h(u) = \frac{du}{\sqrt{u^2+1}}$

$\int \frac{du}{u\sqrt{a^2 \pm u^2}} = C - \frac{1}{a}\ln\left(\frac{a+\sqrt{a^2 \pm u^2}}{|u|}\right)$

Disk : Line to axis

of revoluition, $\pi\int_a^b f(x)^2 \, dx$

$\frac{d}{dx}\operatorname{arc}\operatorname{sech}(u) = \frac{-du}{u\sqrt{1-u^2}}$

$\frac{d}{dx}\operatorname{arc}\operatorname{csc} h(u) = \frac{-du}{|u|\sqrt{1+u^2}}$

$\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln\left(u + \sqrt{u^2 \pm a^2}\right) + C$

Area Between Two Curves

$\int_a^b f(x) - g(x) dx$

Area of Revolved Surface :

$S = 2\pi\int_a^b r(x)\sqrt{1 + f'((x))^2} \, dx$

Washer : $\pi\int_a^b f(x)^2 - g(x)^2 \, dx, \ f(x) \geq g(x)$

Arc Length : $s = \int_a^b \sqrt{1 + f'(x)^2} \, dx$

$W = \int_a^b F(x) dx$

Shell : Line || to axis

of revolution, $2\pi\int_a^b r(x)h(x)dx$,

r = radius, h = height

Coulomb's Law for Charges :

$F = k\left(\frac{q_1 \ q_2}{d^2}\right)$

Gas Pressure: $F = \left(\frac{k}{v}\right)$

v = volume of gas

Hooke's Law : $F = k \ d$

Force needed to strech a spring d distance from its natural length

Law of Universal Gravitation :

$F = k\left(\frac{m_1 \ m_2}{d^2}\right)$ m_1 and m_2 are masses

Force of Gravity : $F = \left(\frac{C}{x^2}\right)$

x = distance from center of Earth

Weight = Volume Density

$\delta\int_a^b (\text{Cross Section Area} \ \ \text{Distance}) dy = \text{Work}$

Fluid Force F = Pressure Area

F = δ Depth Area

Work = Force Distance over which the force is applied

Fluid Pressure P = Weight Density h,

h = depth below surface

Force Exerted by a Fluid :

$F = w\int_a^b \left(h(y) \ \ l(y)\right) dy$

Center of Mass $(\bar{x}, \bar{y})$:

$m = \rho\int_a^b \left(f(x) - g(x)\right) dx$

$\bar{x} = \frac{M_y}{m}, \bar{y} = \frac{M_x}{m}$

Force = Mass Acceleration

Moment = m x

Theorem of Pappus :

$V = 2\pi \ r \ A$,

A = Area of Region R

Moments & Center of Mass of a Planar Lamina :

$M_x = \rho\int_a^b \left(\frac{f(x)+g(x)}{2}\right)\left(f(x) - g(x)\right) dx$

$M_y = \rho\int_a^b x \ \left(f(x) - g(x)\right) dx$

$\int \sin^{2k+1}(x)\cos^n(x) dx = \int \left(1 - \cos^2(x)\right)^k \cos^n(x)\sin(x) dx$

$\int \sin^m(x)\cos^{2x+1}(x) dx = \int \sin^m(x)\left(1 - \sin^2(x)\right)^k \cos(x) dx$

$$\int_0^{\frac{\pi}{2}} \cos^n(x)dx = \left(\frac{2}{3}\right)\left(\frac{4}{5}\right)\left(\frac{6}{7}\right)\cdots\left(\frac{n-1}{n}\right) \text{ n is odd}, n \geq 3$$

$$\int_0^{\frac{\pi}{2}} \cos^n(x)dx = \left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{5}{6}\right)\cdots\left(\frac{n-1}{n}\right)\left(\frac{\pi}{2}\right) \text{ n is even}, n \geq 2$$

$$\int \sec^{2k}(x)\tan^n(x)dx = \int\left(\sec^2(x)\right)^{k-1}\tan^n(x)\sec^2(x)dx$$

$$\int \sec^m(x)\tan^{2k+1}(x)dx = \int \sec^{m-1}\tan^{2k}(x)\sec(x)\tan(x)dx$$

$$\int \tan^n(x)dx = \int \tan^{n-2}(x)\tan^2(x)dx$$

For Integrals Invloving $\sqrt{a^2 + u^2}$, $u = a\tan(\theta), \sqrt{a^2 + u^2} = a\sec(\theta)$

For Integrals Invloving $\sqrt{u^2 - a^2}$, $u = a\sec(\theta), \sqrt{u^2 - a^2} = a\tan(\theta)$

For Integrals Invloving $\sqrt{a^2 - u^2}$, $u = a\sin(\theta), \sqrt{a^2 - u^2} = a\cos(\theta)$

$$\int_{-\infty}^{b} f(x)dx = \lim_{a\to-\infty}\int_a^b f(x)dx$$

$$\int_a^{\infty} f(x)dx = \lim_{b\to\infty}\int_a^b f(x)dx$$

$$\int \sqrt{a^2-u^2}\,du = \frac{1}{2}\left(a^2\arcsin\left(\frac{u}{a}\right) + u\sqrt{a^2-u^2}\right) + C, a > 0$$

$$\int \sqrt{u^2-a^2}\,du = \frac{1}{2}\left(u\sqrt{u^2-a^2} - a^2\ln\left|u + \sqrt{u^2-a^2}\right|\right) + C, u > a > 0$$

$$\int \sqrt{u^2+a^2}\,du = \frac{1}{2}\left(u\sqrt{u^2+a^2} + a^2\ln\left|u + \sqrt{u^2+a^2}\right|\right) + C, a > 0$$

Indeterminate form, L' Hôpital' s Rule

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$$

$$\int_{-\infty}^{\infty} f(x)dx = \lim_{a\to-\infty}\int_a^c f(x)dx + \lim_{b\to\infty}\int_c^b f(x)dx$$

$$\int_1^{\infty}\frac{1}{x^p}dx = \begin{cases}\frac{1}{p-1}, \text{if } p > 1\\ \text{Diverges, if p} \leq 1\end{cases}$$

Discontinuity at a on $(a, b]$

$$\int_a^b f(x)dx = \lim_{c\to a^+}\int_c^b f(x)dx$$

Discontinuity at b on $[a, b)$

$$\int_a^b f(x)dx = \lim_{c\to b^-}\int_a^c f(x)dx$$

Discontinuity at c in (a, b)

$$\int_a^b f(x)dx = \lim_{c\to b^-}\int_a^c f(x)dx + \lim_{c\to a^+}\int_c^b f(x)dx$$

$$\lim_{n\to\infty}(a_n \pm b_n) = L \pm K$$

$$\lim_{n\to\infty}(C\ a_n) = C\ L$$

$$\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = \frac{L}{K}, b_n \neq 0, k \neq 0$$

nth - term test $\sum_{n=1}^{\infty} a_n$, Diverges if $\lim_{n\to\infty} a_n \neq 0$

$$\lim_{n\to\infty}(a_n\ b_n) = L\ K$$

Telescoping, $\sum_{n=1}^{\infty}(b_n - b_{n+1})$, Converges if $\lim_{n\to\infty} b_n = L$

Sum is $S_n = b_1 - L$

Geometric $\sum_{n=0}^{\infty} ar^n$, Converges if $|r| < 1$, Diverges if $|r| \geq 1$

Sum is $S_n = \frac{a}{1-r}$, if it converges

Absolute Convergence, $\sum|a_n|$ Converges

Conditional Convergence, $\sum a_n$ Converges, but $\sum|a_n|$ Diverges

p -series, $\sum_{n=1}^{\infty}\frac{1}{n^p}$, Converges if $p > 1$, Diverges if $p \leq 1$

Alternating, $\sum_{n=1}^{\infty}(-1)^{n-1}a_n$, Converges if $0 < a_{n+1} \leq a_n, \lim_{n\to\infty} a_n = 0$

Remainder $|R_n| \leq a_{n+1}$

Definition of Taylor Series

$$\sum_{n=0}^{\infty}\frac{f^n(c)}{n!}(x-c)^n$$

if $\lim_{n\to\infty}|a_n| = 0$, then $\lim_{n\to\infty} a_n = 0$

Root, $\sum_{n=1}^{\infty} a_n$, Converges if $\lim_{n\to\infty}\sqrt[n]{|a_n|} < 1$, Diverges if $\lim_{n\to\infty}\sqrt[n]{|a_n|} > 1$

Fails if $\lim_{n\to\infty} = 1$

Power Series, $f(x) = \sum_{n=0}^{\infty} a_n(x-c)^n$

Radius is Δx about c : $c \pm \Delta x$

Integral (Constant, Positive, Decreasing),

$\sum_{n=1}^{\infty} a_n, a_n = f(n) \geq 0$, Converges if $\int_1^{\infty} f(n)\,dn$ Converges

Diverges if $\int_1^{\infty} f(n)\,dn$ Diverges, Remainder $0 < R_n < \int_n^{\infty} f(n)\,dn$

Power Series, $f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n$

$f'(x) = \sum_{n=1}^{\infty} n \; a_n (x-c)^{n-1}$

$\int f(x)dx = C + \sum_{n=0}^{\infty} \frac{a_n (x-c)^{n+1}}{n+1}$

Ratio, $\sum_{n=1}^{\infty} a_n$, Converges if $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$, Diverges if $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| > 1$

Fails if $\lim_{n\to\infty} = 1$

$e^x = 1 + x + \ldots + \frac{x^n}{n!} + \ldots$ Converges $(-\infty, \infty)$

Direct $(b_n > 0)$, $\sum_{n=1}^{\infty} a_n$,

Converges if $0 \leq a_n \leq b_n$, $\sum \mathrm{b_n}$ Converges,

Diverges if , $0 \leq b_n \leq a_n$, $\sum \mathrm{b_n}$ Diverges

Limit $(b_n > 0)$, $\sum_{n=1}^{\infty} a_n$,

Converges if $\lim_{n\to\infty} \frac{\mathrm{a_n}}{\mathrm{b_n}} = \mathrm{L} > 0$, $\sum \mathrm{b_n}$ Converges,

Diverges if , $\lim_{n\to\infty} \frac{\mathrm{a_n}}{\mathrm{b_n}} = \mathrm{L} > 0$, $\sum \mathrm{b_n}$ Diverges

$f(x) = P_n(x) + R_n, R_n(x) = (f(x) - P_n(x))$

$P_n(x) = f(c) + \frac{f'(c)(x-c)}{1!} + \frac{f'(c)(x-c)^2}{2!} + \frac{f^n(c)(x-c)^n}{n!} + R_n(x)$

$R_n(x) = \frac{f^{n+1}(z)(x-c)^{n+1}}{(x-1)!}$, z is between x and c

$\frac{dx}{d\theta} = \cos\theta \;\; f'(\theta) - f(\theta)\sin\theta$

$\frac{dy}{d\theta} = \cos\theta \;\; f(\theta) + f'(\theta)\sin\theta$

$\frac{dy}{dx} = \frac{\cos\theta \;\; f(\theta) + f'(\theta)\sin\theta}{\cos\theta \;\; f'(\theta) - f(\theta)\sin\theta}$

If f has n derivatives at x = c, then the polynomial

$P_n(x) = f(c) + \frac{f'(c)(x-c)}{1!} + \frac{f'(c)(x-c)^2}{2!} + \frac{f^n(c)(x-c)^n}{n!} + \ldots$

This is the n - th Taylor polynomial for f at c

If $c = 0$, then the polynomial is called Maclaurin

Position Function for a Projectile

$r(t) = (v_0 \cos\theta) t i + \left(h + (v_0 \sin\theta) t - \frac{1}{2} g t^2\right) j$

g = gravitational constant

h = initial height

$\mathrm{v_0}$ = initial velocity

θ = angle of elevation

$\frac{1}{x} = 1 - (x-1) + \ldots + (-1)^n (x-1)^n + \ldots$ Converges $(0,2)$

$\frac{1}{1+x} = 1 - x + \ldots + (-1)^n x^n + \ldots$ Converges $(-1,1)$

$\arcsin x = x + \frac{x^3}{2\;3} + \ldots + \frac{(2n)!\,x^{2n+1}}{(2^n n!)^2 (2n+1)} + \ldots$ Converges $[-1,1]$

$\sin x = x - \frac{x^3}{3!} + \ldots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \ldots$ Converges $(-\infty, \infty)$

Polar

$x = r \;\; \cos\theta, y = r \;\; \sin\theta$

$\tan\theta = \frac{y}{x}, x^2 + y^2 = r^2$

$\ln x = (x-1) + \ldots + \frac{(-1)^{n+1}(x-1)^n}{n} + \ldots$ Converges $(0,2)$

$\cos x = 1 - \frac{x^2}{2!} + \ldots + \frac{(-1)^n x^{2n}}{(2n)!} + \ldots$ Converges $(-\infty, \infty)$

$\arctan x = x - \frac{x^3}{3!} + \ldots + \frac{(-1)^n x^{2n+1}}{2n+1} + \ldots$ Converges $[-1,1]$

Smooth Curve C, $x = f(t),\ y = g(t)$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} \quad \& \quad \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}},\ \frac{dx}{dt} \neq 0$$

$$\|\vec{v}\| = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2} = \text{Norm}$$

Horizontal Tangent Lines

$$\frac{\text{dy}}{\text{dx}} = 0, \frac{dy}{d\theta} = 0, \frac{dx}{d\theta} \neq 0$$

Vertical Tangent Lines

$$\frac{\text{dy}}{\text{dx}} = DNE, \frac{dx}{d\theta} = 0, \frac{dy}{d\theta} \neq 0$$

$$Velocity = r'(t), Acceleration = r''(t)$$

$$Speed = \|r'(t)\|$$

Limaçon : $r = \text{a} \pm \text{b} \cos\theta$

if a > b, limaçon; if a < b, limaçon w/ loop;

if a = b, cardioid

Rose Curve : $r = a \cos(n\theta), r = a \sin(n\theta)$

if n is odd, n petals; even, 2n petals

Circles and Lemniscates

$$r = a \cos(\theta), r = a \sin(\theta)$$

$$r^2 = a^2 \sin(2\theta), r^2 = a^2 \cos(2\theta)$$

Unit Vector in the Direction of v

$$\vec{u} = \frac{\vec{v}}{\|\vec{v}\|}$$

Angle between two Vector

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}$$

$$\int r(t)dt = \left(\int f(t)dt\right)i + \left(\int g(t)dt\right)j$$

$$r(t) = f(t)i + g(t)j$$

$$r_1(t) \pm r_2(t) = (f_1(t)i + g_1(t)j) \pm (f_2(t)i + g_2(t)j)$$

$$\lim_{t \to a}(r(t)) = \lim_{t \to a}(f(t))i + \lim_{t \to a}(g(t))j$$

Continuous at point $t = a$ if

$\lim_{t \to a}(r(t))$ exists and $\lim_{t \to a}(r(t)) = r(a)$

POLAR

$$Area = \frac{1}{2}\int_{\alpha}^{\beta}(f(\theta))^2 d\theta$$

$$Arc = \int_{\alpha}^{\beta}\sqrt{(f(\theta))^2 + (f'(\theta))^2}\, d\theta$$

$$SurfaceX = 2\pi\int_{\alpha}^{\beta} f(\theta)\sin\theta\sqrt{(f(\theta))^2 + (f'(\theta))^2}\, d\theta$$

$$SurfaceY = 2\pi\int_{\alpha}^{\beta} f(\theta)\cos\theta\sqrt{(f(\theta))^2 + (f'(\theta))^2}\, d\theta$$

$$Arc = \int_{a}^{b}\sqrt{(f'(t))^2 + (g'(t))^2}\, dt$$

$$SurfaceX = 2\pi\int_{a}^{b} g(t)\sqrt{(f'(t))^2 + (g'(t))^2}\, dt$$

$$SurfaceY = 2\pi\int_{a}^{b} f(t)\sqrt{(f'(t))^2 + (g'(t))^2}\, dt$$

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2$$

$$r'(t) = f'(t)i + g'(t)j$$

Square Matrix
Rows = # Cols

Diagonal Matrix
Square with all entries not on the main diagonal being zero

Upper Triangular
Lower left triangle from main diagonal is all zeros

Lower Triangular
Upper right triangle from main diagonal is all zeros

Matrix Elements
Rows X Columns

Transpose A: A^t or A'
Switch rows and columns positions with eachother

Row Vector
A 1 × n Matrix

Column Vector
A m × 1 Matrix

Scalar
A real number

Identity Matrix : I
Diagonal matrix with ones on main diagonal

Dot Product
Sum of elements in two or more matrices multiplied by each corresponding element

$I \cdot A = A = A \cdot I$

Length (Norm)
$\|v\| = \sqrt{v \bullet v} = \sqrt{v_1^2 + v_2^2 + ... + v_n^2}$

Angle Between Two Vectors
$\cos(\theta) = \dfrac{x \bullet y}{\|x\| \cdot \|y\|}$

Matrix Multiplication
Multiply rows of A and columns of B to get an entry of the product C
i^{th} row of A · j^{th} column of B = C_{ij}
$C_{ij} = \sum_{k=1}^{N} A_{ik} \cdot B_{kj}$

How to solve a system
1) Add a multiple of one equation to another equation
2) Multiply an equation by a non - zero scalar (constant)
3) Switch the orger of the equations

Gaussian Elimination
$\left(A \mid \underset{\sim}{b}\right) \to \left(\Omega \mid \underset{\sim}{c}\right)$
Using Elementary Row Ops Into Row Echelon Form

Back Substitution
Solving from row echelon form to row reduced echelon form

Pivot
A one on the main diagonal used as a reference point for solving a system of equations and for determining the number of equations in the system

Free Variables
Columns without pivots

Basic Variables
Columns with pivots

Matrices define linear functions
$\underset{\sim}{f}\left(\underset{\sim}{x} + \underset{\sim}{y}\right) = \underset{\sim}{f}\left(\underset{\sim}{x}\right) + \underset{\sim}{f}\left(\underset{\sim}{y}\right)$
$\underset{\sim}{f}\left(c \cdot \underset{\sim}{x}\right) = c \cdot \underset{\sim}{f}\left(\underset{\sim}{x}\right)$

$A^{-1} \cdot A = I$

Superposition Principle of Homogenous Systems
If $\underset{\sim}{x}$ and $\underset{\sim}{y}$ are solutions, so is $\underset{\sim}{x} + \underset{\sim}{y}$

Every Linear Function
$\underset{\sim}{f} : \Re^N \to \Re^M$
$\underset{\sim}{x} \to \underset{\sim}{f}\left(\underset{\sim}{x}\right)$

$f^{-1}\left(\underset{\sim}{y}\right) = A^{-1} \cdot \underset{\sim}{y}$

Rank =# of Pivots

$\exists A^{-1} \Leftrightarrow \det A \neq 0$

Every Linear Equations is given by a Matrix Multiplication
$\underset{\sim}{f}\left(\underset{\sim}{x}\right) = A\left(\underset{\sim}{x}\right)$

Composition = Matrix Multiplication

Gauss - Jordan Method for A^{-1}
$(A \mid I) \to \text{Row Ops} \to (I \mid A^{-1})$

A is invertible (A^{-1} *exists*) iff A has N pivots ($rank\,A = N$)

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$A^{-1} = \dfrac{1}{a \cdot d - b \cdot c} \cdot \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Determinant of A
$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$
$\det A = a \cdot d - b \cdot c$

$\det \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & j \end{pmatrix} = aej + bfg + cdh - ceg - afh - bdj$

Equilibrium
$Stable \to f'(x^*) < 0$
$Unstable \to f'(x^*) > 0$
$Need\ More\ Info \to f'(x^*) = 0$

Solve Initial Value Problem
1) Solve the differential equation
2) Plug in initial conditions

Autonomous Differential Equations
$\dfrac{dx}{dt} = f(x)$

Autonomous Ordinary Differential Equations
$\dfrac{dx}{dt} = g(x)$

Equilibrium Solutions
$x(t) = \text{constant}$
$\dfrac{dx}{dt} = 0$

Equilibrium Point
If $\lim_{t \to \infty} x(t) = x^*$ then x^* is an equilibrium point

$$\frac{dx}{dt} = F(t,x,y)$$
$$\frac{dy}{dt} = G(t,x,y)$$
$(x(t), y(t))$ parametrize the curve

Uncoupled Linear Systems
$$\frac{dx}{dt} = a \cdot x,\ x(t_0) = x_0$$
$$\frac{dy}{dt} = b \cdot y,\ y(t_0) = y_0$$

Sink: $a < 0,\ b < 0$
Source: $a > 0,\ b > 0$
Saddle: $a < 0,\ b > 0$

$$\frac{d\underset{\sim}{x}}{dt} = A \cdot \underset{\sim}{x},\ \underset{\sim}{x}(t_0) = \underset{\sim}{x}_0$$
$$\underset{\sim}{x}(t) = e^{\lambda \cdot t} \cdot \underset{\sim}{v}$$

Crucial Equation
$A \cdot \underset{\sim}{v} = \lambda \cdot \underset{\sim}{v}$

$$\begin{pmatrix} x'(t) \\ y'(t) \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$
λ is an eigenvalue of A
$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ is an eigenvector
$$\begin{pmatrix} x \\ y \end{pmatrix} = e^{\lambda \cdot t} \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

λ is a scalar called an eigenvalue of the matrix A and $\underset{\sim}{v}$ is the corresponding eigenvector and is not equal to 0

Characteristic Equation
λ is an eigenvalue of A iff $\det(A - \lambda \cdot I) = 0$

If $\underset{\sim}{0} \notin$ V then V is not a subspace

Eigenspace
All multiples of an eigenvector

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \lambda \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

Characteristic Polynomial
$$\det\begin{pmatrix} a-\lambda & b \\ c & d-\lambda \end{pmatrix} = 0$$
$$(a-\lambda)(d-\lambda) - b \cdot c = 0$$

Subspace w (Linearity)
1) If $\underset{\sim}{x}$ and $\underset{\sim}{y}$ are two vectors in w, then $\underset{\sim}{x} + \underset{\sim}{y}$ is a vector in w
2) If $\underset{\sim}{x}$ is a vector in w, then $c \cdot \underset{\sim}{x}$ is a vector in w for any scalar c

A set of vectors $v_1 \ldots v_k$ is linearly dependent if there are scalars $c_1 \ldots c_k = \underset{\sim}{0}$

Vectors span a plane (all linear combinations)

Let A be an $m \times n$ matrix, the null space (kernel) of A is the set of solutions to the homogenous system of linear equations

$$e^{i \cdot \pi} + 1 = 0$$

A set of vectors $v_1 \ldots v_k$ is a basis for a subspace $v^{C}\mathfrak{R}^{n}$ if they span v and are linearly independent

Dimension of v
The number of vectors in any basis of v

Homogenous Linear Systems of First Order Differential Equations
$$\frac{d\underset{\sim}{x}}{dt} = A \cdot \underset{\sim}{x},\ \underset{\sim}{x}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

$$\frac{1}{i} = -i$$

Solving Systems
1) Compute eigenvalues & eigenvectors
2) Solve Equation (Real of Imaginary)
3) Find General Solutions
4) Solve the Initial Conditions

If A is real and $\lambda = a + i \cdot b$ is an eigenvalue, then so is its complex conjugate $\lambda = a - i \cdot b$

$$\frac{a - i \cdot b}{a^2 + b^2} = \frac{1}{a + i \cdot b}$$

Euler's Formula
$$e^{i \cdot x} = \cos(x) + i \cdot \sin(x)$$

$$e^{(a + i \cdot b) \cdot t} = e^{a \cdot t} \cdot \cos(b \cdot t) + e^{i \cdot b \cdot t} \cdot \sin(b \cdot t)$$

Independent if $\det A \neq 0$

Classification of planar hyperbolic equilibria for given eigenvalues
Real and of opposite sign : Saddle
Complex with negative real part : Spiral Sink
Complex with positive real part : Spiral Source
Real, unequal, and negative : Nodal Sink
Real, unequal, and positive : Nodal Source
Real, equal, negative, only one : Improper Nodal Sink
Real, equal, positive, only one : Improper Nodal Source
Real, equal, negative, two : Focus Sink
Real, equal, positive, two : Focus Source

Spring Equation
$$F_{ext}(t) = m \cdot \frac{d^2x}{dt^2} + \mu \cdot \frac{dx}{dt} + k \cdot x$$
m = mass, μ = friction, k = spring constant

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$
$$\lambda = \frac{tr(A) \pm \sqrt{tr(A)^2 - 4 \cdot \det(A)}}{2}$$

$$tr\begin{pmatrix} a & b \\ c & d \end{pmatrix} = a + d$$

Trace is the sum of the diagonal elements of a matrix

Spring Equation
$F_{ext}(t) = 0$, homogenous
$F_{ext}(t) \neq 0$, inhomogenous

Second Order Scalar ODEs

$$a \cdot \frac{d^2x}{dt^2} + b \cdot \frac{dx}{dt} + c \cdot x = d$$

General Solution to Inhomogenous

$$x(t) = x_{particular}(t) + c_1 \cdot x_{ind\#1}(t) + c_2 \cdot x_{ind\#2}(t)$$

Spring (or RCL Circuit)

$\mu^2 > 4 \cdot m \cdot k$, Overdamped

$\mu^2 = 4 \cdot m \cdot k$, Critically Damped

$\mu^2 < 4 \cdot m \cdot k$, Underdamped

Spring (or RCL Circuit)

$\mu^2 > 4 \cdot m \cdot k,\ x_1 = e^{\lambda_1 \cdot t},\ x_2 = e^{\lambda_2 \cdot t}$

$\mu^2 = 4 \cdot m \cdot k,\ x_1 = e^{\lambda_1 \cdot t},\ x_2 = t \cdot e^{\lambda_2 \cdot t}$

$\mu^2 < 4 \cdot m \cdot k,\ x_1 = e^{\alpha \cdot t}\cos(\beta \cdot t),\ x_2 = e^{\alpha \cdot t}\sin(\beta \cdot t)$

If you are unable to solve the particular solution, try increasing the power of t

For Spring, $B = \sqrt{k/m}$

If $\omega \approx B$, Beats Occurs

If $\omega = B$, Resonance Occurs

Solving 2nd Order ODEs

1) Solve Homogenous
2) Find Particular Solution
3) Find General Solution
4) Solve Initial Conditions
5) Combine for Solution

Quasi-Periodic

Close to periodic

Laplace Derivatives

$$\mathcal{L}(f'(t)) = s \cdot F(s) - f(0)$$

$$\mathcal{L}(f''(t)) = s^2 \cdot F(s) - s \cdot f(0) - f'(0)$$

Laplace Transforms

$$F(s) = \int_0^\infty e^{-s \cdot t} \cdot f(t)\,dt$$

Dirac Delta Function

$$\delta_C(t) = \begin{cases} \infty & \text{for } t = c \\ 0 & \text{for } t < c - \delta \\ 1/2\delta & \text{for } c - \delta < t < c + \delta \\ 0 & \text{for } t > c + \delta \end{cases}$$

Step Function

$$H_C(t) = \begin{cases} 0 & \text{for } 0 \le t < c \\ 1 & \text{for } t \ge c \end{cases}$$

Laplace Transforms

$$f(t) = 1,\ F(s) = \frac{1}{s}$$

$$f(t) = t^n,\ F(s) = \frac{n!}{s^{n+1}}$$

$$f(t) = e^{a \cdot t},\ F(s) = \frac{1}{s - a}$$

$$f(t) = \cos(\tau \cdot t),\ F(s) = \frac{s}{s^2 + \tau^2}$$

$$f(t) = \sin(\tau \cdot t),\ F(s) = \frac{\tau}{s^2 + \tau^2}$$

$$f(t) = H_C(t),\ F(s) = \frac{e^{-c \cdot s}}{s}$$

$$f(t) = \delta_C(t),\ F(s) = e^{-c \cdot s}$$

Jacobian Matrix

$$F(\underset{\sim}{x}) = F(\underset{\sim 0}{x}) + dF_{\underset{\sim 0}{x}} \cdot (\underset{\sim}{x} - \underset{\sim 0}{x})$$

$$dF_{\underset{\sim 0}{x}} = \begin{pmatrix} \frac{\partial f}{\partial x}(x_0, y_0) & \frac{\partial f}{\partial y}(x_0, y_0) \\ \frac{\partial g}{\partial x}(x_0, y_0) & \frac{\partial g}{\partial y}(x_0, y_0) \end{pmatrix}$$

The fixed point $\underset{\sim}{z} = \underset{\sim}{0}$ is called hyperbolic if no eigenvalue of the jacobian has 0 as a real part (no 0 or purely imaginary)

If you are near a hyperbolic fixed point, the phase portrait of the nonlinear system is essentially the same as its linearization

$$\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$$

$$\frac{dz}{dx} = \lim_{\Delta x\to 0}\frac{\Delta z}{\Delta x}$$

$$\frac{dz}{dy} = \lim_{\Delta y\to 0}\frac{\Delta z}{\Delta y}$$

$z = f(x,b) \quad y = b$ (x-curve)

$z = f(a,y) \quad x = a$ (y-curve)

$$f_x(x,y) = \lim_{h\to 0}\frac{f(x+h,y) - f(x,y)}{h}$$

$$\frac{\partial z}{\partial x} = \frac{\partial f}{\partial x} = f_x(x,y) = \frac{\partial}{\partial x} f(x,y) = D_x[f(x,y)] = D_1[f(x,y)]$$

$$f_y(x,y) = \lim_{k\to 0}\frac{f(x,y+k) - f(x,y)}{k}$$

$$\frac{\partial z}{\partial y} = \frac{\partial f}{\partial y} = f_y(x,y) = \frac{\partial}{\partial y} f(x,y) = D_y[f(x,y)] = D_2[f(x,y)]$$

Plane Tangent to the surface $z = f(x,y)$ at the point $P = (a,b,f(a,b))$ has the equation:

$$z - f(a,b) = f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

Normal Vector to Tangent Plane

$$\mathbf{n} = f_x(x_0,y_0)\mathbf{i} + f_y(x_0,y_0)\mathbf{j} - \mathbf{k} = \left\langle \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1 \right\rangle$$

$$(f_x)_x = f_{xx} = \frac{\partial f_x}{\partial x} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \left(\frac{\partial^2 f}{\partial x^2}\right)$$

$$(f_y)_y = f_{yy} = \frac{\partial f_y}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \left(\frac{\partial^2 f}{\partial y^2}\right)$$

$$(f_x)_y = f_{xy} = \frac{\partial f_x}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \left(\frac{\partial^2 f}{\partial x \partial y}\right)$$

$$(f_y)_x = f_{yx} = \frac{\partial f_y}{\partial x} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \left(\frac{\partial^2 f}{\partial y \partial x}\right)$$

$$f_{xy}(a,b) = f_{yx}(a,b)$$

Conditions for Extrema for $f(x,y)$

$$f_x(a,b) = 0 = f_y(a,b)$$

$$df = f_x(x,y)\Delta x + f_y(x,y)\Delta y$$

$f(x+\Delta x, y+\Delta y) = f(x,y) + \Delta f$ (exact)

$f(x+\Delta x, y+\Delta y) \approx f(x,y) + df$ (appoximation)

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

$$f(a+\Delta x, b+\Delta y) \approx f(a,b) + f_x(a,b)\Delta x + f_y(a,b)\Delta y$$

$$dz = \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

$$dw = \frac{\partial w}{\partial x}dx + \frac{\partial w}{\partial y}dy + \frac{\partial w}{\partial z}dz$$

$$x = f(x,y,z) = f(g(u,v), h(u,v), k(u,v))$$

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial u}$$

$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial v}$$

w is a funtion of $x_1, x_2, \ldots, x_m$ and each is a funtion of the variables $t_1, t_2, \ldots, t_n$

$$\frac{\partial w}{\partial t_i} = \frac{\partial w}{\partial x_1}\frac{\partial x_1}{\partial t_i} + \frac{\partial w}{\partial x_2}\frac{\partial x_2}{\partial t_i} + \ldots + \frac{\partial w}{\partial x_m}\frac{\partial x_m}{\partial t_i}$$

for each i, $1 \le i \le n$

$$D_u f(P) = \quad f(P) \quad \mathbf{u}, \quad \mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|}$$

$$D_1 f(r(t)) = \quad f(r(t)) \quad \mathbf{r}'(t)$$

$$\frac{\partial x}{\partial r} = \cos\theta, \frac{\partial y}{\partial r} = \sin\theta, \frac{\partial x}{\partial \theta} = -r\sin\theta, \frac{\partial y}{\partial \theta} = r\cos\theta$$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r} = \frac{\partial w}{\partial x}\cos\theta + \frac{\partial w}{\partial y}\sin\theta$$

$$\frac{\partial w}{\partial \theta} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial \theta} = -r\frac{\partial w}{\partial x}\sin\theta + r\frac{\partial w}{\partial y}\cos\theta$$

$$\frac{\partial^2 w}{\partial r^2} = \frac{\partial^2 w}{\partial x^2}\cos^2\theta + 2\frac{\partial^2 w}{\partial x \partial y}\cos\theta\sin\theta + \frac{\partial^2 w}{\partial y^2}\sin^2\theta$$

$$f(x,y,z) = f_x(x,y,z)\mathbf{i} + f_y(x,y,z)\mathbf{j} + f_z(x,y,z)\mathbf{k}$$

$$f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial y} \right\rangle = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$$

$$\Delta w \approx \quad f(P) \quad \mathbf{v}, \mathbf{v} = \overrightarrow{PQ} = \langle \Delta x, \Delta y, \Delta z \rangle$$

Tangent Plane to a Surfave $F(x,y,z)$ at $P = (a,b,c)$

$$F_x(x,y,z)(x-a) + F_y(x,y,z)(y-b) + F_z(x,y,z)(z-c) = 0$$

$$Volume = V = \iint_R f(x,y)\,dA$$

$$\iint_R f(x,y)\,dA = \int_a^b \left(\int_c^d f(x,y)\,dy\right)dx = \int_c^d \left(\int_a^b f(x,y)\,dx\right)dy$$

$$A = f_{xx}(a,b), \quad B = f_{xy}(a,b), \quad C = f_{yy}(a,b)$$

$$\Delta = D = AC - B^2 = f_{xx}(a,b)\, f_{yy}(a,b) - [f_{xy}(a,b)]^2$$

If $\Delta > 0$ and $A > 0$, then f has a local minimum at (a,b)

If $\Delta > 0$ and $A < 0$, then f has a local maximum at (a,b)

If $\Delta < 0$, then f has a saddle point at (a,b)

If $\Delta = 0$, no information is known for f at point (a,b)

LaGrange Multipliers

Constraint: $g(x,y)=0$

Check Critical Points of $F(x,y,\lambda) = f(x,y) - \lambda g(x,y)$

$$\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = \lambda\left(\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}\right)$$

$$\left(\frac{\partial f}{\partial x} - \lambda\frac{\partial g}{\partial x}\right) = 0,\ \left(\frac{\partial f}{\partial y} - \lambda\frac{\partial g}{\partial y}\right) = 0$$

$x^2+y^2+z^2=a^2$ Sphere w/ Radius a

$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ Ellipsoid

$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ Hyperboloid, 1 Sheet, z-axis

$\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ Hyperboloid, 1 Sheet, y-axis

$-\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ Hyperboloid, 1 Sheet, x-axis

$\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ Hyperboloid, 2 Sheets, yz-plane

$-\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ Hyperboloid, 2 Sheets, xz-plane

$-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ Hyperboloid, 2 Sheets, xy-pla

Vertically Simple: $\iint_R f(x,y)\,dA$

$$V = \int_a^b \int_{y_1(x)}^{y_2(x)} f(x,y)\,dy\,dx$$

Horizontally Simple: $\iint_R f(x,y)\,dA$

$$V = \int_c^d \int_{x_1(y)}^{x_2(y)} f(x,y)\,dx\,dy$$

$$A(y) = \int_a^b f(x,y)\,dx$$

$$A(x) = \int_c^d f(x,y)\,dy$$

Polar: $a \le r \le b,\ \alpha \le \theta \le \beta$

$$A = \frac{1}{2}(a+b)(a-b)(\beta-\alpha) = \bar{r}\Delta r\Delta\theta$$

$$V = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$

Volume Between Two Surfaces

$$V = \iint_R (z_{top} - z_{bottom})\,dA$$

$$mass = m = \iint_R \delta(x,y)\,dA$$

Centroid: $(\bar{x},\bar{y})$

$$\bar{x} = \frac{1}{m}\iint_R x\delta(x,y)\,dA$$

$$\bar{y} = \frac{1}{m}\iint_R y\delta(x,y)\,dA$$

Polar Moments of Inertia

$$I_x = \iint_R y^2\delta(x,y)\,dA$$

$$I_y = \iint_R x^2\delta(x,y)\,dA$$

Kinetic Energy due to Rotation

$$KE_{rot} = \iint_R \frac{1}{2}\omega^2 r^2\delta\,dA = \frac{1}{2}I_0\omega^2$$

ω is angular speed

Radius of Gyration

$$\hat{r} = \sqrt{\frac{I}{m}}$$

I is moment of inertia, m is mass around axis

$$\hat{x} = \sqrt{\frac{I_x}{m}} \quad \hat{y} = \sqrt{\frac{I_y}{m}}$$

$$I_0 = m\hat{r}^2 \quad KE = \frac{1}{2}m(\hat{r}\omega)^2$$

Sphererical $x=\rho\sin\phi\cos\theta, y=\rho\sin\phi\sin\theta, z=\rho\cos\phi$

$$V = \iiint_U f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

Cylindrical $x=r\cos\theta,\ y=r\sin\theta,\ z=z$

$$V = \iiint_U f(r\cos\theta, r\sin\theta, z)\,r\,dz\,dr\,d\theta$$

$$mass = m = \iiint_T \delta(x,y,z)\,dV$$

$$Volume = V = \iiint_T dV$$

Centroid: $(\bar{x},\bar{y},\bar{z})$

$$\bar{x} = \frac{1}{m}\iiint_T x\delta(x,y,z)\,dA$$

$$\bar{y} = \frac{1}{m}\iiint_T y\delta(x,y,z)\,dA$$

$$\bar{z} = \frac{1}{m}\iiint_T z\delta(x,y,z)\,dA$$

Parametric

$$\mathbf{r}_u = \frac{\partial \mathbf{r}}{\partial \mathbf{u}} = \langle x_u, y_u, z_u\rangle = \frac{\partial x}{\partial u}\mathbf{i} + \frac{\partial y}{\partial u}\mathbf{j} + \frac{\partial z}{\partial u}\mathbf{k}$$

$$\mathbf{r}_v = \frac{\partial \mathbf{r}}{\partial v} = \langle x_v, y_v, z_v\rangle = \frac{\partial x}{\partial v}\mathbf{i} + \frac{\partial y}{\partial v}\mathbf{j} + \frac{\partial z}{\partial v}\mathbf{k}$$

Moments of Inertia

$$I_x = \iiint_T (y^2+z^2)\delta(x,y,z)\,dA$$

$$I_y = \iiint_T (x^2+z^2)\delta(x,y,z)\,dA$$

$$I_z = \iiint_T (x^2+y^2)\delta(x,y,z)\,dA$$

Surface Area

$$A = a(S) = \iint_R \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}\,dx\,dy$$

Cylindrical Surface Area

$$A = a(S) = \iint_R \sqrt{r^2 + \left(r\frac{\partial f}{\partial r}\right)^2 + \left(\frac{\partial f}{\partial \theta}\right)^2}\,dr\,d\theta$$

Change of Variable

$$\iint_R F(x,y)\,dx\,dy$$

$x = f(u,v) \quad y = g(u,v)$

$u = h(x,y) \quad v = k(x,y)$

$$\text{Jacobian}: J_T(u,v) = \begin{vmatrix} f_u(u,v) & f_v(u,v) \\ g_u(u,v) & g_v(u,v) \end{vmatrix} = \frac{\partial(x,y)}{\partial(u,v)}$$

$$\iint_R F(x,y)\,dx\,dy = \iint_S F\big(f(u,v),g(u,v)\big)\left|J_T(u,v)\right|du\,dv$$

$$\iint_R F(x,y)\,dx\,dy = \iint_S G(u,v)\left|\frac{\partial(x,y)}{\partial(u,v)}\right|du\,dv$$

$x = f(u,v,w) \quad y = g(u,v,w) \quad z = h(u,v,w)$

$$\text{Jacobian}: J_T(u,v) = \frac{\partial(x,y,z)}{\partial(u,v,w)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix}$$

$$\iiint_T F(x,y,z)\,dx\,dy\,dz = \iiint_S G(u,v,w)\left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right|du\,dv\,dw$$

$$\mathbf{F}(x,y,z) = P(x,y,z)\mathbf{i} + Q(x,y,z)\mathbf{j} + R(x,y,z)\mathbf{k}$$

Force Field

$$\mathbf{F}(x,y,z) = -\frac{k\mathbf{r}}{r^3}$$

$k = G \quad M$

$$(af+bg) = a \quad f + b \quad g$$

$$(fg) = f \quad g + g \quad f$$

Velocity Vector

$$\mathbf{v}(x,y) = \omega(-y\mathbf{i} + x\mathbf{j})$$

$$\left|\mathbf{F}(x,y)\right| = \left|x\mathbf{i} + y\mathbf{j}\right| = \sqrt{x^2+y^2} = r$$

$$\int_C f(x,y,z)\,ds = \lim_{\Delta t \to 1} f\big(x(t_i^*),y(t_i^*),z(t_i^*)\big)\Delta s_i$$

$$div\,\mathbf{F} = \quad \mathbf{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$$\times(a\mathbf{F} + b\mathbf{G}) = a(\quad \times\mathbf{F}) + b(\quad \times\mathbf{G}),$$

$$\times(f\mathbf{G}) = (\ f)(\quad \times\mathbf{G}) + (\ f) \times \mathbf{G}$$

$$mass = \int_C \delta(x,y,z)\,ds$$

$$\bar{x} = \frac{1}{m}\int_C x\delta(x,y,z)\,ds$$

$$\bar{y} = \frac{1}{m}\int_C y\delta(x,y,z)\,ds$$

$$\bar{z} = \frac{1}{m}\int_C z\delta(x,y,z)\,ds$$

Centroid : $(\bar{x},\bar{y},\bar{z})$

$$curl\,\mathbf{F} = \quad \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

$$curl\,\mathbf{F} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\mathbf{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\mathbf{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\mathbf{k}$$

$$(a\mathbf{F} + b\mathbf{G}) = a \quad \mathbf{F} + b \quad \mathbf{G},$$

$$(f\mathbf{G}) = (\ f)(\quad \mathbf{G}) + (\ f) \quad \mathbf{G}$$

$$f(x,y,z) = \int_C \mathbf{F} \quad \mathbf{T}\,ds = \int_{(x_0,y_0,z)_0}^{(x,y,z)} \mathbf{F} \quad \mathbf{T}\,ds$$

$$\int_C f(x,y,z)\,ds = \int_a^b f\big(x(t),y(t),z(t)\big)\sqrt{\big(x'(t)\big)^2 + \big(y'(t)\big)^2 + \big(z'(t)\big)^2}\,dt$$

$$\int_C \mathbf{F} \quad \mathbf{T}\,ds = \int_A^B \mathbf{F} \quad \mathbf{T}\,ds$$

$$\int_C \mathbf{F} \quad \mathbf{T}\,ds = \int_C \mathbf{F}\,d\mathbf{r} = P\,dx + Q\,dy + R\,dz$$

$$\int_C P(x,y,z)\,dx + Q(x,y,z)\,dy + R(x,y,z)\,dz =$$

$$\int_a^b P\big(x(t),y(t),z(t)\big) \quad x'(t)\,dt +$$

$$\int_a^b Q\big(x(t),y(t),z(t)\big) \quad y'(t)\,dt +$$

$$\int_a^b R\big(x(t),y(t),z(t)\big) \quad z'(t)\,dt$$

$$\int_C f(x,y,z)\,dx = \int_a^b f\big(x(t),y(t),z(t)\big) \quad x'(t)\,dt$$

$$\int_C f(x,y,z)\,dy = \int_a^b f\big(x(t),y(t),z(t)\big) \quad y'(t)\,dt$$

$$\int_C f(x,y,z)\,dz = \int_a^b f\big(x(t),y(t),z(t)\big) \quad z'(t)\,dt$$

Flux of a Vector Field across C

$$\oint_C \mathbf{F} \quad \mathrm{n}\,ds$$

$$\mathbf{n} = \frac{dy}{ds}\mathbf{i} - \frac{dx}{ds}\mathbf{j}$$

Work

$$W = \int_a^b \mathbf{F} \quad \mathbf{T}\,ds$$

$$W = \int_a^b \mathbf{F}\,d\mathbf{r}$$

$$W = \int_a^b P\,dx + Q\,dy + R\,dz$$

$$\int_{-C} f(x,y,z)\,ds = \int_C f(x,y,z)\,ds$$

$$\int_{-C} P\,dx + Q\,dy + R\,dz = -\int_C P\,dx + Q\,dy + R\,dz$$

$$\int_C \quad f \quad d\mathbf{r} = \int_C f\big(r(b)\big) - f\big(r(a)\big) = f(B) - f(A)$$

$$W = \int_C \mathbf{F} \quad \mathbf{T}\,ds = k\int_C \mathbf{w} \quad \mathbf{T}\,ds$$

$\mathbf{w}$ = velocity vector

Moment of Inertia

$$I = \int_C w^2\delta(x,y,z)\,ds$$

$w = w(x,y,z) =$ distance from (x,y,z) to axis

$$W = \int_A^B \mathbf{F} \quad d\mathbf{r} = V(A) - V(B)$$

The Line Integral $\int_C \mathbf{F}\cdot\mathbf{T}\,ds$ is independent of path in the region D iff $\mathbf{F} = \nabla f$ for some funtion f defined on D

The vector field **F** defined on region D is conservative provided that there exists a scalar funtion f defined on D such that

$\mathbf{F} = \nabla f$

at each point of D, f is a potential funtion for **F**

$$\int_A^B \mathbf{F}\cdot\mathbf{T}\,ds = \int_A^B \nabla f \cdot d\mathbf{r} = f(B) - f(A)$$

Continuous funtions $P(x,y)$ and $Q(x,y)$ have continuous 1st order partials in R = $\{(x,y) \mid a < x < b,\ c < y < d\}$ Then the vector field $\mathbf{F} = P\mathbf{i} + Q\mathbf{j}$ is conservative in R and has a potential funtions $f(x,y)$ on R irr at each point of R

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

Newton's First Law gives:

$$\mathbf{F}(\mathbf{r}(t)) = m\mathbf{r}''(t) = m\mathbf{v}'(t)$$

$$d\mathbf{r} = \mathbf{r}'(t)\,dt = \mathbf{v}(t)$$

$$\int_A^B \mathbf{F}\cdot d\mathbf{r} = \frac{1}{2}m(v_B)^2 - \frac{1}{2}m(v_A)^2$$

$$\oint_C P\,dx = -\iint_R \frac{\partial P}{\partial y}\,dA$$

$$\oint_C Q\,dy = +\iint_R \frac{\partial Q}{\partial x}\,dA$$

$$div\,\mathbf{F} = \nabla\cdot\mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}$$

$$\frac{1}{2}m(v_A)^2 + V(A) = \frac{1}{2}m(v_B)^2 + V(B)$$

$$A = \frac{1}{2}\oint_C -y\,dx + x\,dy = -\oint_C y\,dx = \oint_C x\,dy$$

$$\oint_C \mathbf{F}\cdot\mathbf{n}\,ds = \iint_R \nabla\cdot\mathbf{F}\,dA$$

Green's Theorem

$$\oint_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dA$$

$$\oint_C \mathbf{F}\cdot\mathbf{n}\,ds = \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right)dA$$

$$\{\nabla\cdot\mathbf{F}\}(x_0,y_0) = \lim_{r\to 0}\frac{1}{\pi r^2}\oint_{C_r}\mathbf{F}\cdot\mathbf{n}\,ds$$

$$\mathbf{N} = \frac{\partial \mathbf{r}}{\partial \mathbf{u}} \times \frac{\partial \mathbf{r}}{\partial \mathbf{v}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & \frac{\partial z}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} & \frac{\partial z}{\partial v} \end{vmatrix}$$

$$\iint_S f(x,y,z)\,dS = \iint_D f(\mathbf{r}(u_i,v_i))\,|\mathbf{N}(u_i,v_i)|\,du\,dv$$

$$= \iint_D f(\mathbf{r}(u_i,v_i))\left|\frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v}\right| du\,dv$$

$$dS = |\mathbf{N}(u_i,v_i)|\,du\,dv = \left|\frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v}\right| du\,dv$$

$z = h(x,y)$ in xy-plane

$$dS = \sqrt{1 + \left(\frac{\partial h}{\partial x}\right)^2 + \left(\frac{\partial h}{\partial y}\right)^2}\,dx\,dy$$

Flux ϕ across S in the direction of **n**

$$= \iint_S \mathbf{F}\cdot\mathbf{n}\,dS$$

$$A \approx \sum_{i=1}^{n} |\mathbf{N}(u_i,v_i)|\,\Delta u\,\Delta v$$

$$m \approx \sum_{i=1}^{n} f(\mathbf{r}(u_i,v_i))\,|\mathbf{N}(u_i,v_i)|\,\Delta u\,\Delta v$$

$$\mathbf{N} = \frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v} = \frac{\partial(y,z)}{\partial(u,v)}\mathbf{i} + \frac{\partial(z,x)}{\partial(u,v)}\mathbf{j} + \frac{\partial(x,y)}{\partial(u,v)}\mathbf{k}$$

$$\iint_S f(x,y,z)\,dS$$

$$= \iint_S f(x(u,v),y(u,v),z(u,v))\sqrt{\left(\frac{\partial(y,z)}{\partial(u,v)}\right)^2 + \left(\frac{\partial(z,x)}{\partial(u,v)}\right)^2 + \left(\frac{\partial(x,y)}{\partial(u,v)}\right)^2}\,du\,dv$$

$$\sum_{i=1}^{n} R(\mathbf{r}(u_i,v_i))\cos\gamma\,|\mathbf{N}(u_i,v_i)|\,\Delta u\,\Delta v$$

$$\approx \iint_D R(\mathbf{r}(u_i,v_i))\,|\mathbf{N}(u_i,v_i)|\,du\,dv$$

$$\mathbf{n} = \frac{\mathbf{N}}{|\mathbf{N}|} = (\cos\alpha)\mathbf{i} + (\cos\beta)\mathbf{j} + (\cos\gamma)\mathbf{k}$$

$$\iint_S f(x,y,z)\,dS = \iint_S f(x,y,h(x,y))\sqrt{1 + \left(\frac{\partial h}{\partial x}\right)^2 + \left(\frac{\partial h}{\partial y}\right)^2}\,dx\,dy$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{k}\frac{\partial u}{\partial t}$$

k = Thermal Diffusivity

Divergence Theorem

$$\iint_S \mathbf{F}\cdot\mathbf{n}\,dS = \iint_T \nabla\cdot\mathbf{F}\,dV$$

$$\{div\,\mathbf{F}\}(P) = \lim_{r\to 0}\frac{1}{V_r}\iint_{S_r}\mathbf{F}\cdot\mathbf{n}\,dS$$

Source $\{div\,\mathbf{F}\}(P) > 0$

Sink $\{div\,\mathbf{F}\}(P) < 0$

$$\oint_C \mathbf{F}\cdot\mathbf{T}\,dS = \iint_R (curl\,\mathbf{F})\cdot\mathbf{k}\,dA$$

$$\mathbf{F}\cdot\mathbf{n} = P\cos\alpha + Q\cos\beta + R\cos\gamma$$

$$\cos\alpha = \mathbf{n}\cdot\mathbf{i} = \frac{\mathbf{N}\cdot\mathbf{i}}{|\mathbf{N}|} = \frac{1}{|\mathbf{N}|}\frac{\partial(y,z)}{\partial(u,v)}$$
$$\cos\beta = \mathbf{n}\cdot\mathbf{j} = \frac{\mathbf{N}\cdot\mathbf{i}}{|\mathbf{N}|} = \frac{1}{|\mathbf{N}|}\frac{\partial(z,x)}{\partial(u,v)}$$
$$\cos\gamma = \mathbf{n}\cdot\mathbf{k} = \frac{\mathbf{N}\cdot\mathbf{i}}{|\mathbf{N}|} = \frac{1}{|\mathbf{N}|}\frac{\partial(x,y)}{\partial(u,v)}$$

$$\iint_S P(x,y,z)\,dydz = \iint_S P(x,y,z)\cos\alpha\, dS = \iint_D P(\mathbf{r}(u,v))\frac{\partial(y,z)}{\partial(u,v)}\,dudv$$
$$\iint_S Q(x,y,z)\,dydx = \iint_S Q(x,y,z)\cos\beta\, dS = \iint_D Q(\mathbf{r}(u,v))\frac{\partial(z,x)}{\partial(u,v)}\,dudv$$
$$\iint_S R(x,y,z)\,dxdy = \iint_S R(x,y,z)\cos\gamma\, dS = \iint_D R(\mathbf{r}(u,v))\frac{\partial(x,y)}{\partial(u,v)}\,dudv$$

$$\iint_S Pdydz + Qdzdx + Rdxdy$$
$$= \iint_S (P\cos\alpha + Q\cos\beta + R\cos\gamma)\,dS$$
$$= \iint_D \left(P\frac{\partial(y,z)}{\partial(u,v)} + Q\frac{\partial(z,x)}{\partial(u,v)} + R\frac{\partial(x,y)}{\partial(u,v)}\right)dudv$$

Gauss' Law for flux

$$\phi = \iint_S \mathbf{F}\cdot\mathbf{n}\,dS = -4\pi GM$$

Gauss' Law for electric fields

$$\phi = \iint_S \mathbf{E}\cdot\mathbf{n}\,dS = \frac{Q}{\varepsilon_0}$$

Heat -Flow Vector

$\mathbf{q} = -K\nabla u$

K =Heat Conductivity

$$\iint_S \mathbf{q}\cdot\mathbf{n}\,dS = -\iint_S K\nabla u\cdot\mathbf{n}\,dS$$

$$\iint_S \mathbf{F}\cdot\mathbf{n}\,dS = \iint_S Pdydz + Qdzdx + Rdxdy$$
$$= \iint_D \left(-P\frac{\partial z}{\partial x} - Q\frac{\partial z}{\partial y} + R\frac{\partial z}{\partial z}\right)dxdy$$

$$\iint_S (P\cos\alpha + Q\cos\beta + R\cos\gamma)\,dS$$
$$= \iint_S (Pdydz + Qdzdx + Rdxdy)$$
$$= \iiint_T \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}\right)dV$$

$$\iint_S Pdydz = \iiint_T \frac{\partial P}{\partial x}\,dV$$
$$\iint_S Qdzdx = \iiint_T \frac{\partial Q}{\partial y}\,dV$$
$$\iint_S Rdxdy = \iiint_T \frac{\partial R}{\partial z}\,dV$$

S_3 is SA between S_1 and S_2

$$\iint_{S_3} Rdxdy = \iint_{S_3} R\cos\gamma\, dS = 0$$
$$\iint_{S_2} Rdxdy = \iint_{S_2} R(x,y,z_2(x,y))\,dxdy$$
$$\iint_{S_1} Rdxdy = -\iint_D R(x,y,z_1(x,y))\,dxdy$$

$$\iiint_T \nabla\cdot\mathbf{F}\,dV = \iint_S \mathbf{F}\cdot\mathbf{n}\,\mathrm{d}S$$
$$= \iint_{S_2} \mathbf{F}\cdot\mathbf{n}_2\,\mathrm{d}S - \iint_{S1} \mathbf{F}\cdot\mathbf{n}_1\,\mathrm{d}S$$

$$\iint_S \mathbf{F}\cdot\mathbf{n}\,dS = \iint_{S_a} \mathbf{F}\cdot\mathbf{n}\,dS$$
$$= \iint_{S_a} -\frac{GM\mathbf{r}}{|\mathbf{r}|^3}\cdot\frac{\mathbf{r}}{|\mathbf{r}|}\,dS$$
$$= -\frac{GM\mathbf{r}}{a^2}\iint_{S_a} 1\,dS$$
$$= -4\pi GM$$

Stokes' Theorem

$$\oint_C \mathbf{F}\cdot\mathbf{T}\,dS = \iint_S (curl\,\mathbf{F})\cdot\mathbf{n}\,dA$$

$$\oint_C Pdx = -\iint_D \left(\frac{\partial P}{\partial y}\frac{\partial y}{\partial y} + \frac{\partial P}{\partial z}\frac{\partial z}{\partial y}\right)dxdy$$

$$\oint_C Pdx + Qdy + Rdz = \iint_S \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)dydz + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)dzdx + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dxdy$$

$$\iint_S \left(\frac{\partial P}{\partial z}\,dzdy - \frac{\partial P}{\partial z}\,dxdy\right)$$
$$= \iint_D \left(-\frac{\partial P}{\partial z}\frac{\partial z}{\partial y} - \frac{\partial P}{\partial y}\frac{\partial y}{\partial y}\right)dxdy$$

$$\{(curl\,\mathbf{F})\cdot\mathbf{n}\}(P) = \lim_{r\to 0}\frac{1}{\pi r^2}\oint_{C_r}\mathbf{F}\cdot\mathbf{T}\,ds$$

$$\{(curl\,\mathbf{F})\cdot\mathbf{n}\}(P^*) \approx \frac{\Gamma(C_r)}{\pi r^2}$$

$$\Gamma(C) = \oint_C \mathbf{F}\cdot\mathbf{T}\,ds$$

$$\phi(x,y,z) = \int_{C_1}\mathbf{F}\cdot\mathbf{T}\,ds$$

$$\iiint_T div\,\mathbf{v}\,dV = \iint_S \mathbf{v}\cdot\mathbf{n}\,dS = 0$$
$$div(\nabla\phi) = \frac{\partial^2\phi}{dx^2} + \frac{\partial^2\phi}{dy^2} + \frac{\partial^2\phi}{dz^2} = 0$$
$$\frac{\partial^2 u}{dx^2} + \frac{\partial^2 u}{dy^2} + \frac{\partial^2 u}{dz^2} = 0$$

A vector field is irrotational iff

It is conservative and

$\mathbf{F} = \nabla\phi$ for some scalar ϕ

Electric Charge
Charges with the same electrical sign repel each other, and charges with opposite electrical sign attract

$$F = \frac{q_1 \; q_2}{4\pi \; \varepsilon_0 \; r^2}$$

$$e^- \approx 1.6 \times 10^{-19} C$$

$$q = n \; e, \quad n = \pm 1, \pm 2, \ldots$$

$$k = \frac{1}{4\pi \; \varepsilon_0} \approx 9 \times 10^9 \frac{N \; m^2}{C}$$

Point Charge
$$\mathbf{E} = k \frac{q}{r^2}$$

A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at its center

A shell of uniform charge exerts no electrostatic force on a charged particle that is located inside the shell.

Electric Fields
$$\mathbf{E} = \frac{\mathbf{F}}{q_0}$$

Electric field lines extend away from positive charge and toward negative charge

Torque
$$\mathbf{t} = \mathbf{p} \times \mathbf{E}$$

Electric Dipole
$\mathbf{p} =$ Dipole Moment
$$\mathbf{E} = k \frac{2\mathbf{p}}{z^3} \approx \frac{3(\mathbf{p} \cdot \hat{r}) - \mathbf{p}}{4\pi \; \varepsilon_0 \; z^3}$$

Charged Ring
$$\mathbf{E} = \frac{q \; z}{4\pi \; \varepsilon_0 \left(z^2 + R^2\right)^{3/2}}$$

Charged Disk
$$\mathbf{E} = \frac{\sigma}{2\varepsilon_0} \left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

Conducting Surface
$$\mathbf{E} = \frac{\sigma}{\varepsilon_0}$$

Line Of Charge
$$\mathbf{E} = \frac{\lambda}{2\pi \; \varepsilon_0 \; r}$$

Sheet Of Charge
$$\mathbf{E} = \frac{\sigma}{2\varepsilon_0}$$

Potential Energy Of A Dipole
$$U = -\mathbf{p} \cdot \mathbf{E}$$

Gauss' Law : $\varepsilon_0 \; \Phi = Q_{enclosed}$
$$Flux = \Phi = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{enclosed}}{\varepsilon_0}$$

Electric Potential
$$\Delta U = q \; \Delta V$$
$$\Delta V = V_f - V_i = -\frac{W}{q}$$
$$V_i = -\frac{W_\infty}{q}$$

Electric Potential
$$V_f - V_i = -\int_i^f \mathbf{E} \cdot d\mathbf{s} = V$$

Electric Diplole
$$V = k \frac{p \; \cos(\theta)}{r^2}$$

Spherical Shell Field At r ≥ R
$$\mathbf{E} = \frac{q}{4\pi \; \varepsilon_0 \; r^2}$$

Spherical Shell Field At r < R
$$\mathbf{E} = 0$$

Electric Potential
$$V = k \frac{q}{r} = k \sum_{i=1}^{n} \frac{q_i}{r_i}$$

$$V = k \int \frac{dq}{r}$$

$$E_S = -\frac{\partial V}{\partial s}$$

$$\mathbf{E} = -\nabla V$$

$$U = W = k \frac{q_1 \; q_2}{r}$$

Isolated Sphere
$$C = 4\pi \; \varepsilon_0 \; R$$

Capacitance
$$C = \frac{Q}{V}$$

Parallel Plate Capacitor
$$C = \frac{\varepsilon_0 \; A}{d}$$

Cylindrical Capacitor
$$C = 2\pi \; \varepsilon_0 \frac{L}{\ln(b/a)}$$

Spherical Capacitor
$$C = 4\pi \; \varepsilon_0 \frac{a \; b}{b - a}$$

Parallel
$$C_{eff} = \sum_{i=1}^{n} C_i$$

Series
$$\frac{1}{C_{eff}} = \sum_{i=1}^{n} \frac{1}{C_i}$$

Current
$$i = \frac{dq}{dt}$$

Potential Energy
$$U = \frac{Q^2}{2C} = \frac{1}{2} C \; V^2$$

Energy Density
$$u = \frac{Q^2}{2C} = \frac{1}{2} \varepsilon_0 \; E^2$$

Dielectric
$$C = \kappa \; C_{air}$$
$$\varepsilon_0 \oint \kappa \; \mathbf{E} \cdot d\mathbf{A} = Q_{enclosed}$$

Resistance
$$R = \frac{V}{i}, \text{ Ohm's Law}$$

$$\mathbf{E} = \rho \; \mathbf{J}$$

$$\mathbf{J} = n \; e \; \mathbf{v}_{drift}$$

Current Density
$$i = \int \mathbf{J} \cdot d\mathbf{A}$$

Current Density
$$J = \frac{i}{A}, \text{ Constan}$$

$$\rho - \rho_0 = \rho_0 \; \alpha (T - T_0)$$

Resistance is a property of an object
Resistivity is a property of a material

$$R = \rho \frac{L}{A}$$

Conductivity
$$\sigma = \frac{1}{\rho}$$

Resistivity
$$\rho = \frac{E}{J}$$

Power, Rate Of Transfer Of Electrical Energy
$$P = i \; V$$

Resistive Dissapation
$$P = i^2 \; R = \frac{V^2}{R}$$

Resistivity Of A Conductor (Such As Metal)
$$\rho = \frac{m}{e^2 \; n \; \tau}$$

EMF
$$\mathcal{E} = \frac{dW}{dq}$$

$$i = \frac{\mathcal{E}}{R}$$

Parallel
$$\frac{1}{R_{eff}} = \sum_{i=1}^{n} \frac{1}{R_i}$$

EMF Power
$$P_{emf} = i \; \mathcal{E}$$

Series
$$R_{eff} = \sum_{i=1}^{n} R_i$$

Kirchoff's Loop Rule
The sum of the changes in potential in a loops of a circuit must be zero

Kirchoff's Junction Rule
The sum of the currents entering any junction must equal the sum of the currents leaving that junction

$$i(t) = \frac{dq}{dt}$$

Discharging A Capacitor
$$q(t) = q_0 \; e^{-t/R\,C}$$
$$i(t) = -\frac{q_0}{R \; C} \; e^{-t/R\,C}$$

Charging A Capacitor
$$q(t) = C \; \mathcal{E}\left(1 - e^{-t/R\,C}\right)$$
$$i(t) = \frac{\mathcal{E}}{R} \; e^{-t/R\,C}$$

Charging A Capacitor
$$V_C = \mathcal{E}\left(1 - e^{-t/R\,C}\right)$$

Magnetic Fields, B
$\mathbf{F}_B = q\ \mathbf{v} \times \mathbf{B}$

Opposite magnetic poles attract
Like magnetic poles repel

Circular Charged Path
$q\ v\ B = \frac{m\ v^2}{r}$

Mass Spectrometer
$v = \sqrt{\frac{2\ q\ V}{m}}$
$m = \frac{B^2\ q\ x^2}{8V}$

Right Hand Rule, Positive Charge
Thumb Up, Pointer Pointing
and Middle Finger Perpendicular
Thumb = Magnetic Force Direction
Pointer = Velocity Direction
Middle = Magnetic Field Direction

Hall Effect
$V = E\ d$
$n = \frac{B\ i}{V\ l\ e}$

Resonance Condition
$f = f_{osc}$
$f = \frac{\omega}{2\pi} = \frac{1}{T} = \frac{q\ B}{2\pi\ m}$

Force On A Current
$d\mathbf{F}_B = i\ d\mathbf{L} \times \mathbf{B}$

Parallel currents attract and AntiParallel currents repel

Magnetic Dipole Moment
$\mu = N\ i\ A$, Moment
$\mathbf{t} = \mu \times \mathbf{B}$

Magnetic Potential Energy
$U(\theta) = -\mu\ \mathbf{B}$

Biot-Savart Law
Like "Leo Bazaar"
$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{i\ d\mathbf{s} \times \mathbf{r}}{r^3}$

$\mu_0 = 4\pi \times 10^{-7} \frac{T\ m}{A}$

Long Straight Wire
$\mathbf{B} = \frac{\mu_0\ i}{2\pi\ r}$

Torid
$\mathbf{B} = \frac{\mu_0\ i\ N_{turns}}{2\pi\ r}$

Right Hand Rule
Grasp the element with thumb pointing in the direction of the current, fingers curl in the direction of the magnetic field

Center Of Circular Arc
$\mathbf{B} = \frac{\mu_0\ i\ \phi}{4\pi\ R}$

Ideal Solenoid
$n =$ turns per unit length
$\mathbf{B} = \mu_0\ i\ n$

Force Between Two Parallel Wires
$F = \frac{\mu_0\ i_1\ i_2}{2\pi\ d} L$

Ampére's Law
$\oint \mathbf{B}\ d\mathbf{s} = \mu_0\ i_{enclosed}$

Magnetic Flux
$\Phi_B = \int \mathbf{B}\ d\mathbf{A}$

Current Carrying Coil
$\mathbf{B}(z) = \frac{\mu_0\ \mu}{2\pi\ z^3}$

A changing magnetic field produces an electric field

Faraday's Law
$\mathcal{E} = -\frac{d\Phi_B}{dt}$

Coil Of N Turns
$\mathcal{E} = -N \frac{d\Phi_B}{dt}$

Faraday's Law
$\oint \mathbf{E}\ d\mathbf{s} = \frac{d\Phi_B}{dt} = \mathcal{E}$

Current Loop + Magnetic Field → Torque
Torque + Magnetic Field → Current? YES

Inductance
$L = \frac{N\ \Phi_B}{i}$

Solenoid
$L = \mu_0\ n^2\ A\ l$

Self Induced emf
$\mathcal{E}_L = -L \frac{di}{dt}$

Rise Of Current (Inductor)
$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right),\ \tau_L = \frac{L}{R}$

Decay Of Current (Inductor)
$i = \frac{\mathcal{E}}{R} e^{-t/\tau_L} = i_0\ e^{-t/\tau_L},\ \tau_L = \frac{L}{R}$

Magnetic Energy
$U_B = \frac{1}{2} L\ i^2$

Magnetic Energy Density
$u_B = \frac{B^2}{2\mu_0}$

Mutual Inductance
$\mathcal{E}_2 = -M \frac{di_1}{dt} \Rightarrow \mathcal{E}_1 = -M \frac{di_2}{dt}$

Gauss' Law, Magnetic Fields
$\Phi_B = \oint \mathbf{B}\ d\mathbf{A} = 0$

Maxwell's Equations

Gauss' Law For Electricity
$\oint \mathbf{E}\ d\mathbf{A} = \frac{q_{enc}}{e_0}$

Gauss' Law For Magnetism
$\oint \mathbf{B}\ d\mathbf{A} = 0$

Faraday's Law
$\oint \mathbf{E}\ d\mathbf{s} = -\frac{d\Phi_B}{dt}$

Ampére-Maxwell Law
$\oint \mathbf{B}\ d\mathbf{s} = \mu_0\ \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0\ i_{enc}$

Spin Magnetic Dipole Moment
$\mu_s = -\frac{e}{m}\ \mathbf{S}$

Bohr Magneton
$\mu_B = -\frac{e\ h}{4\pi\ m} \approx 9.27 \times 10^{-24} \frac{J}{T}$

Speed Of Light
$c = \frac{1}{\sqrt{\mu_0\ \varepsilon_0}}$

Potential Energy
$U = -\mu_s\ \mathbf{B}_{ext} = -\mu_{s,z}\ B$

Potential Energy
$U = -\mu_{orb}\ \mathbf{B}_{ext} = -\mu_{orb,z}\ B_{ext}$

Orbital Magnetic Dipole Moment
$\mu_{orb} = -\frac{e}{2m} \mathbf{L}_{orb}$

Inside A Circular Capacitor
$\mathbf{B} = \frac{\mu_0\ i_d}{2\pi\ R^2} r$

Outside A Circular Capacitor
$\mathbf{B} = \frac{\mu_0\ i_d}{2\pi\ r}$

Displacement Current
$i_d = \varepsilon_0 \frac{d\Phi_E}{dt}$
$i = i_d$, Capacitor

Potential
$U_E = \frac{q^2}{2C}$

Potential
$U_B = \frac{L\ i}{2}$

LC Circuit
$\omega_{LC} = \frac{1}{\sqrt{L\ C}}$

LC Circuit

$$L\frac{d^2q}{dt^2}+\frac{q}{C}=0$$
$$q(t)=Q\ \cos(\omega\ \ t+\phi)$$
$$i(t)=-\omega\ \ Q\ \ \sin(\omega\ \ t+\phi)$$

$$U_E=\frac{Q^2}{2C}\ \cos^2(\omega\ \ t+\phi)$$
$$U_B=\frac{Q^2}{2C}\ \sin^2(\omega\ \ t+\phi)$$

RLC Circuit

$$L\frac{di}{dt}+i\ \ R+\frac{Q}{C}=0$$
$$q(t)=A\ \ e^{-t/2\tau_{LR}}\ \cos(\omega_{LC}\ \ t-\phi)$$

$$\omega=\sqrt{\omega_{LC}{}^2-\left(\frac{1}{2\tau_{LR}}\right)^2}$$

$$\tau_{LR}=\frac{L}{R}$$

$$U=\frac{Q^2}{2C}e^{-t/\tau_{LR}}$$

RLC Circuit w/ AC

$$L\frac{di}{dt}+i\ \ R+\frac{Q}{C}=\mathcal{E}_B\ \sin(\omega_d\ \ t)$$
$$i(t)=i_B\ \sin(\omega_d\ \ t-\phi)$$
$$i_B=\mathcal{E}_B/Z$$

Capacitive Reactance

$$X_C=\frac{1}{\omega_d\ \ C}$$

Impedance

$$Z=\sqrt{(X_L-X_C)^2+R^2}$$

Phase Constant

$$\tan(\phi)=\frac{X_L-X_C}{R}$$

Current Amplitude

$$i=\frac{\mathcal{E}_B}{Z}$$

Inductive Reactance

$$X_L=\omega_d\ \ L$$

Resonance

$$\omega_d=\omega=\frac{1}{\sqrt{L\ \ C}}$$

rms current

$$i_{rms}=\frac{i}{\sqrt{2}}$$

Average Power

$$P_{av}=i_{rms}{}^2\ \ R$$

Not "rms Power"

rms potential

$$V_{rms}=\frac{V}{\sqrt{2}},\mathcal{E}_{rms}=\frac{\mathcal{E}_B}{\sqrt{2}}$$

Average Power

$$P_{av}=\mathcal{E}_{rms}\ \ i_{rms}\ \ \cos(\phi)$$

Resistance Load at Generator

$$R_{eq}=\left(\frac{N_{1st}}{N_{2nd}}\right)^2\ R$$

Transformer Voltage

$$V_{2nd}=V_{1st}\frac{N_{2nd}}{N_{1st}}$$

Transformer Current

$$i_{2nd}=i_{1st}\frac{N_{1st}}{N_{2nd}}$$

Average Power relates to the heating effect RMS Power, while it can be calculated is worthless

Maxwell's Equations (Integral)

$$\oint\mathbf{E}\ \ d\mathbf{A}=\frac{q_{enc}}{\varepsilon_0}$$
$$\oint\mathbf{B}\ \ d\mathbf{A}=0$$
$$\oint\mathbf{E}\ \ d\mathbf{s}=-\frac{d}{dt}\int\mathbf{B}\ \ d\mathbf{A}$$
$$\oint\mathbf{B}\ \ d\mathbf{s}=\mu_0\ \ i_{enc}+\frac{1}{c^2}\frac{d}{dt}\int\mathbf{E}\ \ d\mathbf{A}$$

Maxwell's Equations (Derviative)

$$\cdot\mathbf{E}=\frac{\rho}{\varepsilon_0}$$
$$\cdot\mathbf{B}=0$$
$$\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}$$
$$\times\mathbf{B}=\mu_0\ \ \mathbf{J}+\frac{1}{c^2}\ \ \frac{\partial\mathbf{E}}{dt}$$

P and Q is P Q
P or Q is P Q
not P is ¬P
if P then Q is P Q
P if and only if Q is P ⇔ Q

¬Q ¬P is contrapositive of P Q
and these are equivalent
Q P is converse of P Q
and these may not be equivalent

N = *Set of positive integers*
Z = *Set of all integers*
Q = *Set of rational numbers*
R = *Set of real numbers*

The set A is finite if A = or for some n **N**,
A has exactly n members.
The set A is infinite if A is not finite.

Mathematical Induction:
1) *n* **N**, Prove *Base Case that* $P(1)$ *is true*
2) *Assume* $P(n)$ *and n* **N**, *show* $P(n)$ $P(n+1)$

Existential Quantifier:
x: for all/every x
Universal Quantifier:
x: there exists x
: element of

Well Ordering Property of **N**:
Every nonempty subset of **N** has a smallest memeber

Principle of Mathematical Induction
Let $P(n)$ *be a mathematical statement, if*
a) $P(1)$ *is true, and*
b) for every n **N**
$P(n)$ $P(n+1)$ *is true,*
then $P(n)$ *is true for every n* **N**

Strong Induction
1) $P(1)$ *is true, and*
2) *for each n* **N**, *if* $[$*each* $P(1),\ldots,P(n)$ *is true*$]$,
then $P(n+1)$ *is true, then* $P(n)$ *is true n* **N**

P is a necessary condition for Q: Q P
P is a sufficient condition for Q: P Q

$\neg(P \;\; Q)$ *is* $\neg P \;\; \neg Q$
$\neg(P \;\; Q)$ *is* $\neg P \;\; \neg Q$
$\neg(P \;\; Q)$ *is* $P \;\; \neg Q$

The least upper bound, r, of A is the supremum of and is denoted sup A, that is r = sup A if:
a) r is an upper bound of A, and
b) r ≤ r' for every upper bound r' of A
The greatest lower bound, r, of A is the infimum of and is denoted inf A, that is r = inf A if:
a) r is a lower bound of A, and
b) r ≥ r' for every upper bound r' of A
sup A exists if A has a least upper bound and
inf A exists if A has a greatest lower bound

Archimedean Property of **N**:
N is not bounded above

Completeness Axiom: Every nonempty subset of **R** which is bounded above has a least upper bound

For every positive real number x, there is some positive integer n such that $0 < \frac{1}{n} < x$

For all real numbers x and y such that $x < y$, *there is a rational number r such that* $x < r < y$

Bounds: Suppose A is a set of real numbers
1) *r* **R** *is an upper bound of A if no member of A is bigger than r:* $x \;\; A[x \le r]$
2) *r* **R** *is a lower bound of A if no member of A is smaller than r:* $x \;\; A[r \le x]$
3) *A is bounded above if r* **R** *which is an upper bound of A*
A is bounded below if r **R** *which is a lower bound of A*
A is bounded if if is bounded above and below

For all real numbers x and y such that $x < y$, *there is an irrational number* i_r *such that* $x < i_r < y$

There is no rational number r such that $r^2 = 2$
There is a real number x such that $x^2 = 2$

Let n **N**. *For every real number* $y > 0$, *there is a real number* $x > 0$ *such that* $x^n = y$

Completeness Axiom Corollary: Every nonempty subset of **R** which is bounded below has a greatest lower bound

A sequence is a function whose domain is a set of the form $\{n \quad Z : n \geq k\}$, *where* $k \quad Z$

If s is a sequence, s_n *is the value of the sequence at argument n.*

Given $\{a_n\}$ *and* $\{b_n\}$ *with domain D:*

$\{a_n + b_n\}$ *is* $s_n = a_n + b_n \quad n \quad D$

$\{a_n - b_n\}$ *is* $s_n = a_n - b_n \quad n \quad D$

$\{a_n \quad b_n\}$ *is* $s_n = a_n \quad b_n \quad n \quad D$

$\left\{\frac{a_n}{b_n}\right\}$ *is* $s_n = \frac{a_n}{b_n} \quad n \quad D, b_n \neq 0$

$\{c \quad a_n\}$ *is* $s_n = c \quad a_n \quad n \quad D$

If a sequence is convergent, then it is bounded

Definition of a Limit:
Let L be a real number.
$\lim_{n\to\infty} s_n = L$ *iff*
$\varepsilon > 0 \quad n_0 \quad n \geq n_0 \left[|s_n - L| < \varepsilon\right]$

$\lim_{n\to\infty} s_n = L$ *is*
$\lim s_n = L$ *is*
$s_n \to L$

The sequence s converges to L if $\lim_{n\to\infty} s_n = L$.
The sequence s is convergent if there is some L such that s converges to L.
The sequence s is divergent if is is not convergent

If $\lim s_n = L_0$ *and* $s_n = L_1$, *then* $L_0 = L_1$

The sequence $\{s_n\}$ is bounded if there is some r **R** such that $|s_n| \leq r$ for all n in the domain of $\{s_n\}$.

Suppose $\lim s_n = L_1$, *and* $\lim t_n = L_2$, *and* $c \quad R$.

a) $\lim(s_n + t_n) = L_1 + L_2$

b) $\lim(c \quad s_n) = c \quad L_1$

c) $\lim(s_n \quad t_n) = L_1 \quad L_2$

d) $\lim\left(\frac{s_n}{t_n}\right) = \frac{L_1}{L_2}$, $L_2 \neq 0$ *and* $n[t_n \neq 0]$

Suppose that for all sufficiently large *n*,
$a_n \leq b_n \leq c_n$
If $\lim a_n = L$ and $\lim c_n = L$ then $\lim b_n = L$

If $\lim|s_n| = 0$, *then* $\lim s_n = 0$

Suppose: $\lim a_n = L$, *and f is a function which is continuous at L, and for each n,* a_n *is in the domain of f. Then* $\lim f(a_n) = f(L)$

f is continuous at x if for any $\varepsilon > 0$ *there exists some* $\delta > 0$ *so that if* $|z - x| < \delta$, *then* $|f(z) - f(x)| < \varepsilon$

The set of reals **R** *is the completion of the set of integers* **Q**. *Or* $\mathbf{R} = \{\sup A \mid A \quad \mathbf{Q}\}$

Let $s_n = f(n)$. *If* $\lim_{x\to\infty} f(x) = L$, *then* $\lim s_n = L$

Let $s_n = f\left(\frac{1}{n}\right)$. *If* $\lim_{x\to 0^+} f(x) = L$, *then* $\lim s_n = L$

Let $\{s_n\}$ *be a sequence.* $\lim s_n = \infty$ *if:*
$M \quad n_0 \quad n \geq n_0 [s_n \geq M]$
$\lim s_n = -\infty$ *if:*
$M \quad n_0 \quad n \geq n_0 [s_n \leq M]$

The sequence $\{s_n\}$ *is:*
increasing if for all n, $s_n \leq s_{n+1}$
strictly increasing if for all n, $s_n < s_{n+1}$
decreasing if for all n, $s_n \geq s_{n+1}$
strictly decreasing if for all n, $s_n > s_{n+1}$
monotonic if any of the above are true

Suppose $\{s_n\}$ is monotonic. Then $\{s_n\}$ converges iff it is bounded.

Recursively Defined Sequence is defined in terms of s_n *for each n.*

The sequence $\{s_n\}$ *is a Cauchy sequence if for every* $\varepsilon > 0$, *there is some* n_0 *such that* $|s_m - s_n| < \varepsilon$ *for all m,* $n \leq n_0$

f is continuous and $x \geq a$
If $\int_a^\infty |f(x)|dx$ *converges*
then $\int_a^\infty f(x)dx$ *converges*

Cauchy sequences are bounded and converge

$$\int_a^\infty f(x)dx = \lim s_n = \lim_{n\to\infty}\int_a^n f(x)dx$$

Geometric Series: a and r are real
$\sum_{i=0}^{\infty} a \quad r^i$, *with r being the ratio*

If $\lim a_i \neq 0$, then $\sum_{i=1}^{\infty} a_i$ is divergent

Comparison Test

$0 \le f(x) \le g(x), x \ge a$

i) If $\int_a^\infty f(x)dx$ converges, then $\int_a^\infty g(x)dx$ converges

and $\int_a^\infty f(x)dx \le \int_a^\infty g(x)dx$

ii) If $\int_a^\infty f(x)dx$ diverges, then $\int_a^\infty g(x)dx$ diverges

i) Suppose $\sum_{i=1}^\infty a_i$ is convergent, for $c \in \mathbf{R}$,

$$\sum_{i=1}^\infty c\, a_i = c\left(\sum_{i=1}^\infty a_i\right)$$

ii) If $\sum_{i=1}^\infty a_i$ is convergent, and $\sum_{i=1}^\infty b_i$ is convergent

then $\sum_{i=1}^\infty (a_i + b_i)$ is convergent and:

$$\sum_{i=1}^\infty (a_i + b_i) = \sum_{i=1}^\infty a_i + \sum_{i=1}^\infty b_i$$

Integral Test

f is continuous and decreasing on the interval $[1, \infty)$

and $f(x) \ge 0$ for $x \ge 1$

$\sum_{n=1}^\infty f(n)$ converges iff $\int_1^\infty f(n)$ converges.

$\sum_{n=1}^\infty a_n$ is absolutely convergent if

$\sum_{n=1}^\infty |a_n|$ is convergent

So $\sum_{n=1}^\infty a_n$ is convergent

$\int_1^\infty \frac{1}{x^p}dx$ converges if $p > 1$

and diverges if $p \le 1$

$\sum_{n=1}^\infty \frac{1}{x^p}$ converges if $p > 1$

and diverges if $p \le 1$

Comparison Test $0 \le a_n \le b_n$

$\{a_n\}$ and $\{b_n\}$ are sequences

If $\sum_{n=1}^\infty b_n$ converges then $\sum_{n=1}^\infty a_n$ converges

If $\sum_{n=1}^\infty a_n$ diverges then $\sum_{n=1}^\infty b_n$ diverges

Given a sequence $\{A_n\}$ we form $\{S_n\}$ by:

$$S_n = \sum_{i=1}^n a_i$$

i) If $\{S_n\}$ converges, we say the infinite series:

$\sum_{i=1}^n a_i$ converges to $\sum_{i=1}^n a_i = \lim S_n$, the value of the series

ii) If $\{S_n\}$ diverges, we say the infinite series:

$\sum_{i=1}^n a_i$ diverges also.

iii) If $\lim S_n = \infty$ we say: $\sum_{i=1}^n a_i = \infty$ and

the series diverges to infinity.

iv) If $\lim S_n = \infty$ we say: $\sum_{i=1}^n a_i = -\infty$ and

the series diverges to minus infinity.

i) If $|r| < 1$ then the geometric series

$\sum_{i=0}^\infty a\, r^i$ converges to $\frac{a}{1-r}$

ii) If $|r| \ge 1$ and $a \ne 0$ then

$\sum_{i=0}^\infty a\, r^i$ diverges

Ratio Test For all n, $0 < a_n$

If $\lim \frac{a_{n+1}}{a_n} = L < 1$ then $\sum_{n=1}^\infty a_n$ converges

If $\lim \frac{a_{n+1}}{a_n} = L > 1$ or $\to \infty$ then $\sum_{n=1}^\infty a_n$ diverges

Root Test For all n, $0 < a_n$

If $\lim (a_n)^{\frac{1}{n}} = L < 1$ then $\sum_{n=1}^\infty a_n$ converges

If $\lim (a_n)^{\frac{1}{n}} = L > 1$ or $\to \infty$ then $\sum_{n=1}^\infty a_n$ diverges

$n-th$ term test for divergence

If $\lim a_n \ne 0$, then $\sum_{n=1}^\infty a_n$ is divergent

Limit - Comparison Test

For all n, $0 < a_n$ and $0 < b_n$

If $\lim \frac{a_n}{b_n} = L > 0$, L

$\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} b_n$ converges

If $\lim \frac{a_n}{b_n} = 0$

$\sum_{n=1}^{\infty} b_n$ converges then $\sum_{n=1}^{\infty} a_n$ converges

If $\lim \frac{a_n}{b_n} \to \infty$

$\sum_{n=1}^{\infty} b_n$ diverges then $\sum_{n=1}^{\infty} a_n$ diverges

Alternating - Series Test

For all n, $a_n > 0$

$\{a_n\}$ is a strictly decreasing sequence

$a_n \to 0$

Then $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ is convergent

The interval of convergence of the power series $\sum_{n=0}^{\infty} a_n x^n$ is the set

$$\left\{ x : \sum_{n=0}^{\infty} a_n x^n \text{ is convergent at } x \right\}$$

The power series $\sum_{n=0}^{\infty} a_n x^n$ converges at the real number x_1 if $\sum_{n=0}^{\infty} a_n x_1^n$ converges and diverges at x_1 if $\sum_{n=0}^{\infty} a_n x_1^n$ diverges

Differentiation and Integration of power series can be done term by term

If the power series $\sum_{n=0}^{\infty} a_n x^n$ is:

convergent only at $x = 0$, then the radius of convergence is 0

Absolutely convergent at all x, then the radius of convergence is ∞

Converges absolutely at all x such that $|x| < r$ and diverges at all x such that $|x| > r$, then the radius of convergence is r

For a given power series $\sum_{n=0}^{\infty} a_n x^n$, one of the following is true:

The power series converges only at $x = 0$

The power series absolutely converges at all x

There is a number $r > 0$ such that the power series converges absolutely at all x such that $|x| < r$ and diverges at all x such that $|x| > r$

Abel's Theorem

Suppose $f(x) = \sum_{n=0}^{\infty} a_n x^n$ for $|x| < 1$

If $\sum_{n=0}^{\infty} a_n$ converges, then

$$\lim_{x \to -1} f(x) = \sum_{n=0}^{\infty} a_n$$

Suppose that $r > 0$, and

$f(x) = \sum_{n=0}^{\infty} a_n x^n$ for $|x| < r$

Then for all $n \geq 0$, $a_n = \frac{f^{(n)}(0)}{n!}$

If $f(x) = a_0 + a_1 x + \ldots + a_n x^n + \ldots$ for $|x| < r$

$$f(x) = f(0) + f'(0)x + f^{(n)}(0)\frac{x^n}{n!} + \ldots$$

Taylor's Theorem

Assume that f has continous derivatives of order n +1 on $[0, x]$, then

$$R_n(x) = \frac{1}{n!}\int_0^x f^{(n+1)}(t)(x-t)^n \, dt$$

Math Reference SAT IIC James Lamberg

Repeated Percent Increase

$$Final\ Amount = Original\ (1+rate)^{\#\ changes}$$

Repeated Percent Decrease

$$Final\ Amount = Original\ (1-rate)^{\#changes}$$

Percent Change

$$\frac{\text{Change}}{\text{Original}} = \frac{x}{100}$$

$$\log_b(xy) = \log_b(x) + \log_b(y)$$

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$

$$\log_b(x^n) = n\log_b(x)$$

30-60-90 Triangle

$1 : \sqrt{3} : 2$

45-45-90 Triangle

$1 : 1 : \sqrt{2}$

distance = rate time

$\ln n = x$

$\log_e n = x$

$e^x = n$

$(x+y)^2 = x^2 + 2xy + y^2$

$(x-y)^2 = x^2 - 2xy + y^2$

$(x+y)(x-y) = x^2 - y^2$

$y = ax^2 + bx + c$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$AverageSpeed = \frac{TotalDistance}{TotalTime}$$

Area of Square

$$A = s^2 \text{ or } A = \frac{d^2}{2}$$

$TriangleInternalAngles = 180°$

Freds Theorem

2 II lines intersected make only 2 unique angles

3rd triangle side between sum and difference of other two

$a^2 + b^2 = c^2$

Triagle Area

$$A = \frac{1}{2}bh$$

Equilateral Triagle Area

$$A = \frac{s^2\sqrt{3}}{4}$$

$Domain: x - values$

$Range: y - values$

$Roots: f(x) = 0$

$\log_b n = x$

$b^x = n$

Sum Internal Polygon w/ n Sides

$SumAngles = (n-2)180°$

Rect Solid Surface Area

$SA = 2lw + 2wh + 2lh$

Long Diagonal

$a^2 + b^2 + c^2 = d^2$

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

Parabola $y = a(x-h)^2 + k$

Circle $r^2 = (x-h)^2 + (y-k)^2$

Ellipse $1 = \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2}$

Hyperbola $1 = \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2}$

$y = mx + b$

$y - y_1 = m(x - x_1)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$midpt = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$

Area of Trapezoid

$$A = \left(\frac{b_1 + b_2}{2}\right)h$$

$$\frac{degrees}{360} = \frac{radians}{2\pi}$$

Work Done=rate of work time

Arithmetic Series

$a_n = a_1 + (n-1)d$

Arithmetic Sum

$$Sum = n\left(\frac{a_1 + a_n}{2}\right)$$

$\sin = \frac{opp}{hyp}$ $\cos = \frac{adj}{hyp}$ $\tan = \frac{opp}{adj} = \frac{\sin}{\cos}$

$\csc = \frac{1}{\sin}$ $\sec = \frac{1}{\cos}$ $\cot = \frac{1}{\cos}$

$\sin^2 x + \cos^2 x = 1$

$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

Polar

$x = r\cos\theta$ $y = r\sin\theta$

$r^2 = x^2 + y^2$

$\tan\theta = \frac{x}{y}$

x-axis symmetry

$f(x)$ and $-f(x)$ x

Infinite Geometric

$$Sum = \frac{a_1}{1-r}, -1 < r < 1$$

$Even: f(x) = f(-x), y - axis$

$Odd: -f(x) = f(-x), origin$

Contrapositive

$A \to B$ $\sim B \to \sim A$

Probabilty of Multiple Events

$P(x_n) = P(x_1)\ P(x_2)\ P(x_3)\ P(x_4)...$

Group Problem

$Total = Group_1 + Group_2 + Neither - Both$

Geometric Series

$a_n = a_1 r^{(n-1)}$

Geometric Sum

$$Sum = \frac{a_1(1-r^n)}{1-r}$$

$$\text{Standard Deviation} = \sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}}$$

Find the mean of the set

Find difference between each value and mean

Square differences

Average results

Square root the average

Mean: Average of set elements

Median: Middle Value

Mode: Most Often

Range: Highest-Lowest

Cube $V = s^3$ $SA = 6s^2$

$LongDiagonal = s\sqrt{3}$

Cylinder $V = \pi r^2 h$

$SA = 2\pi r^2 + 2\pi rh$

$$\text{Probability}(x) = \frac{\text{Number of outcomes that are x}}{\text{Total possible outcomes}}$$