EL-MOASSER

# Algebra and Probability

# نماذج امتحانات

## الصف 3 الإعدادى

الفصل الدراسى الثانى ٢٠٢١

# Model Exam 1

MATHEMATICS (Algebra and probability) — **30** Marks

## Answer the following questions :

**1 Choose the correct answer from those given :**

1. One of the solutions for the two equations : $x - y = 2$ , $x^2 + y^2 = 20$ in $\mathbb{R} \times \mathbb{R}$ is ..............

   (a) $(-4 , 2)$ (b) $(2 , -4)$ (c) $(3 , 1)$ (d) $(4 , 2)$

2. If $A \cap B = \varnothing$ , then $P(A - B) =$ ..............

   (a) $P(A)$ (b) $P(B)$ (c) $P(B - A)$ (d) $1$

3. If $x^2 + kx - 21 = (x - 3)(x + 7)$ , then $k =$ ..............

   (a) $-2$ (b) $4$ (c) $8$ (d) $20$

4. If $\frac{1}{x} + \frac{1}{y} + \frac{1}{xy} = \frac{k}{xy}$ , then $k =$ ..............

   (a) $2$ (b) $3$ (c) $x + y + 1$ (d) $x + y$

5. If $5^{x - 3} = 1$ , then $2x^2 =$ ..............

   (a) $36$ (b) $9$ (c) $18$ (d) $3$

6. If the width of the rectangle is 3 cm. , and its diagonal length is 5 cm. , then its length is .............. cm.

   (a) $2$ (b) $\frac{5}{3}$ (c) $4$ (d) $\frac{3}{5}$

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**2** [a] **By using the general formula , find in $\mathbb{R}$ the solution set of the equation :** $x(x - 2) = 1$

[b] If $n(x) = \frac{x^3 + x}{x^2 + 1} + \frac{x^2 + 2x + 4}{x^3 - 8}$ , find $n(x)$ in the simplest form , showing the domain.

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**3** [a] If the set of zeroes of the function $f : f(x) = \frac{x^2 - ax + 9}{bx + 4}$ is $\{3\}$ and its domain is $\mathbb{R} - \{2\}$

**, find the value of each of a and b**

[b] If $n(x) = \frac{x^3 - 8}{x^2 - 3x + 2} \div \frac{x^3 + 2x^2 + 4x}{2x^2 + x - 3}$ , find $n(x)$ in the simplest form , showing the domain.

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**4** [a] If $n_1(x) = \frac{x^2 + 5x + 6}{x^2 + x - 2}$ and $n_2(x) = \frac{x^2 - 2x - 15}{x^2 - 6x + 5}$ , is $n_1 = n_2$ ? and why ?

[b] If A and B are two events of the sample space of a random experiment , and

$P(A) = \frac{1}{4}$ , $P(B) = \frac{1}{2}$ and $P(A \cup B) = \frac{5}{8}$ , **find each of the following :**

1. $P(A \cap B)$ 2. $P(B - A)$ 3. $P(A \cup B)'$

**5** [a] **Find algebraically in $\mathbb{R} \times \mathbb{R}$ the solution set of the following two equations :**

$x - y = 3 \quad , \quad y^2 - x y = 21$

[b] **Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the following two equations algebraically or graphically :** $y = x + 4 \quad , \quad x + y = 4$

## Answer the following questions :

**1 Choose the correct answer from those given :**

1 In the experiment of tossing a piece of coin once , if A is the event of appearance of a head , B is the event of appearance of a tail , then P $(A \cup B) =$ ..............

(a) $\frac{1}{2}$ (b) 1 (c) zero (d) $\varnothing$

2 The number of solutions of the equation $x - y = 0$ in $\mathbb{R} \times \mathbb{R}$ is ..............

(a) 1 (b) 2 (c) 3 (d) infinite

3 The set of zeroes of $f : f(x) = \frac{-3}{x-2}$ is ..............

(a) $\mathbb{R} - \{2\}$ (b) $\mathbb{R} - \{3\}$ (c) $\{2\}$ (d) $\varnothing$

4 If the curve of the quadratic function $f$ passes through the points $(-1 , 0)$ , $(0 , -4)$ , $(4 , 0)$ , then the solution set of the equation $f(x) = 0$ in $\mathbb{R}$ is ..............

(a) $\{-1 , 0\}$ (b) $\{-4 , 0\}$ (c) $\{-1 , 4\}$ (d) $\{4 , -4\}$

5 If $2^{x+1} = 1$ , then $x \in$ ..............

(a) $\{0\}$ (b) $\{0 , 1\}$ (c) $\{-1\}$ (d) $\mathbb{R} - \{-1\}$

6 If $\sqrt{x^2} = 25$ , then $x =$ ..............

(a) 5 (b) $\pm 5$ (c) 25 (d) $\pm 25$

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**2** [a] If A , B are two events in a random experiment and P (A) = 0.6 , P (B) = 0.5 , P $(A \cap B) = 0.3$ , **find :** P $(A \cup B)$ , P $(\acute{B})$

**[b] Simplify to the simplest form , showing the domain :**

$$n(x) = \frac{x^3 - 1}{x^2 - 2x + 1} \times \frac{2x - 2}{x^2 + x + 1}$$

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**3** **[a] By using the general formula , find in $\mathbb{R}$ the solution set of the equation :**

$3x^2 - 6x = -1$ (approximating the result to the nearest two decimals)

[b] If the domain of the function n is $\mathbb{R} - \{3\}$ where $n(x) = \frac{x-1}{x^2 - ax + 9}$ , find the value of a

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**4** **[a] Find the solution set of the following two equations together in $\mathbb{R} \times \mathbb{R}$ :**

$$y - x = 2 \quad , \quad x^2 + xy - 4 = 0$$

**[b] Find n ($x$) in the simplest form , showing the domain of n :**

$$n(x) = \frac{x-3}{x^2 - 7x + 12} - \frac{x-3}{3-x}$$

**5** **[a]** Two acute angles in a right-angled triangle. The difference between their measures is 50°
Find the measure of each angle.

**[b]** If $n(x) = \dfrac{x^2 - 2x}{(x-2)(x^2+2)}$ , **find :**

1 $n^{-1}(x)$ in the simplest form , showing the domain of $n^{-1}$

2 The value of $x$ if $n^{-1}(x) = 3$

## Answer the following questions :

**1 Choose the correct answer from those given :**

1 If $n(X) = \frac{X^2 - 2X}{(X-2)(X^2+2)}$ , then the domain of $n^{-1}$ is ..............

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{0\}$ (d) $\mathbb{R} - \{0, 2\}$

2 If A and B are two mutually exclusive events from the sample space S of a random experiment , then P (A – B) = ..............

(a) P (B) (b) P (A) (c) P (Á) (d) P (B́)

3 In the equation : $aX^2 + bX + c = 0$ , if : $b^2 - 4ac > 0$ , then the equation has .............. roots in $\mathbb{R}$

(a) 1 (b) 2 (c) zero (d) $\infty$

4 The rule which describes the pattern $\left(\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, ...\right)$ where $n \in \mathbb{Z}_+$ is ..............

(a) $\frac{2}{n+1}$ (b) $n + \frac{1}{2}$ (c) $\frac{n}{n+1}$ (d) $\frac{2n-1}{n+1}$

5 If $2^7 \times 3^7 = 6^k$ , then k = ..............

(a) 14 (b) 7 (c) 6 (d) 5

6 If $3^X = 4$ , $4^y = 12$ , then $\frac{Xy}{X+1}$ = ..............

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

**2** [a] If A , B are two events from the sample space of a random experiment and

P (A) = 0.7 , P (B) = 0.5 and P (A ∩ B) = 0.3

, **find :** P (Á) , P (A – B) and P (A ∪ B)

[b] If the set of zeroes of the function $f$ where $f(X) = X^2 - 10X + a$ is $\{5\}$

, then find the value of a

**3** [a] **Find the S.S. in $\mathbb{R}^2$ of the two equations :** $X + y = 2$ , $\frac{1}{X} + \frac{1}{y} = 2$

[b] If $n_1(X) = \frac{X^2}{X^3 - X^2}$ , $n_2(X) = \frac{X^3 + X^2 + X}{X^4 - X}$ ,

**prove that :** $n_1 = n_2$

**4** [a] **Find n (X) in the simplest form and state the domain if :**

$$n(X) = \frac{X^2 - 3X}{2X^2 - X - 6} \div \frac{2X^2 - 3X}{4X^2 - 9}$$

**[b] Find graphically in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :**

$x + 2y = 8$ , $3x + y = 9$

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**5 [a] Using the general rule , find the solution set of the following equation in $\mathbb{R}$ :**

$2x^2 - 5x + 1 = 0$

**[b] Find n ($x$) in the simplest form , showing the domain of n where :**

$$n(x) = \frac{x^2 + 2x}{x^2 - 4} - \frac{2x - 6}{x^2 - 5x + 6}$$

# إجابات نماذج امتحانات

## الصف 3 الإعـــدادى

الفصل الدراسى الثانى ٢٠٢١

## Answers of model 1

**1**

| | | |
|---|---|---|
| 1 d | 2 a | 3 b |
| 4 c | 5 c | 6 c |

**2**

**[a]** $\because x(x-2)=1 \qquad \therefore x^2-2x-1=0$

$\therefore a=1 , b=-2 , c=-1$

$\therefore x=\dfrac{2\pm\sqrt{(-2)^2-4\times1\times(-1)}}{2\times1}$

$=\dfrac{2\pm\sqrt{8}}{2}=\dfrac{2\pm2\sqrt{2}}{2}=1\pm\sqrt{2}$

$\therefore x=1+\sqrt{2}$ or $x=1-\sqrt{2}$

$\therefore$ The S.S. $=\{1+\sqrt{2} , 1-\sqrt{2}\}$

**[b]** $\because n(x)=\dfrac{x(x^2+1)}{x^2+1}+\dfrac{x^2+2x+4}{(x-2)(x^2+2x+4)}$

$\therefore$ The domain of $n=\mathbb{R}-\{2\}$

$, n(x)=x+\dfrac{1}{x-2}=\dfrac{x(x-2)+1}{x-2}$

$=\dfrac{x^2-2x+1}{x-2}=\dfrac{(x-1)^2}{x-2}$

**3**

**[a]** $\because z(f)=\{3\} \qquad \therefore$ At $x=3$

$\therefore x^2-ax+9=0 \qquad \therefore 3^2-a\times3+9=0$

$\therefore 9-3a+9=0 \qquad \therefore -3a=-18 \qquad \therefore a=6$

$, \because$ The domain of $f=\mathbb{R}-\{2\}$

$\therefore$ At $x=2 \qquad \therefore bx+4=0$

$\therefore 2b+4=0 \qquad \therefore 2b=-4 \qquad \therefore b=-2$

**[b]** $\because n(x)=\dfrac{(x-2)(x^2+2x+4)}{(x-2)(x-1)}\div\dfrac{x(x^2+2x+4)}{(2x+3)(x-1)}$

$\therefore$ The domain of $n=\mathbb{R}-\{2 , 1 , 0 , -\frac{3}{2}\}$

$, n(x)=\dfrac{x^2+2x+4}{x-1}\times\dfrac{(2x+3)(x-1)}{x(x^2+2x+4)}=\dfrac{2x+3}{x}$

**4**

**[a]** $\because n_1=\dfrac{(x+3)(x+2)}{(x+2)(x-1)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_1=\mathbb{R}-\{-2 , 1\}\\ , n_1(x)=\dfrac{x+3}{x-1}\end{array}\right\}$ (1)

$, \because n_2(x)=\dfrac{(x-5)(x+3)}{(x-5)(x-1)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_2=\mathbb{R}-\{5 , 1\}\\ , n_2(x)=\dfrac{x+3}{x-1}\end{array}\right\}$ (2)

From (1) and (2) : $\therefore n_1\neq n_2$

because the domain of $n_1\neq$ the domain of $n_2$

**[b]** 1 $\because P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\therefore P(A\cap B)=P(A)+P(B)-P(A\cup B)$

$=\frac{1}{4}+\frac{1}{2}-\frac{5}{8}=\frac{1}{8}$

2 $P(B-A)=P(B)-P(A\cap B)=\frac{1}{2}-\frac{1}{8}=\frac{3}{8}$

3 $P(A\cup B)'=1-P(A\cup B)=1-\frac{5}{8}=\frac{3}{8}$

**5**

**[a]** $\because x-y=3 \qquad \therefore x=y+3 \qquad (1)$

$, y^2-xy=21 \qquad (2)$

substituting from (1) in (2) :

$\therefore y^2-(y+3)y=21 \qquad \therefore y^2-y^2-3y=21$

$\therefore -3y=21 \qquad \therefore y=-7$

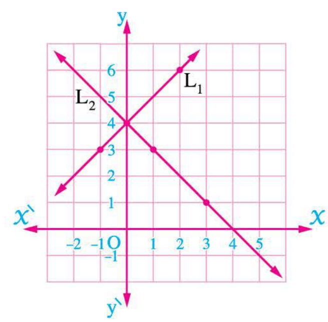
substituting in (1) : $\therefore x=-4$

$\therefore$ The S.S. $=\{(-4 , -7)\}$

**[b]** $y=x+4 \qquad , \qquad x=4-y$

| $x$ | $-1$ | 0 | 2 |
|---|---|---|---|
| $y$ | 3 | 4 | 6 |

| $x$ | 3 | 1 | 0 |
|---|---|---|---|
| $y$ | 1 | 3 | 4 |




From the graph : $\therefore$ The S.S. $=\{(0 , 4)\}$

## Answers of model 2

**1**

| | | |
|---|---|---|
| 1 b | 2 d | 3 d |
| 4 c | 5 c | 6 d |

**2**

**[a]** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.6 + 0.5 - 0.3 = 0.8$

$\because P(\grave{B}) = 1 - P(B)$ $\quad \therefore P(\grave{B}) = 1 - 0.5 = 0.5$

**[b]** $\because n(x) = \dfrac{(x-1)(x^2+x+1)}{(x-1)^2} \times \dfrac{2(x-1)}{x^2+x+1}$

$\therefore$ The domain of $n = \mathbb{R} - \{1\}$ , $n(x) = 2$

**3**

**[a]** $\because 3x^2 - 6x + 1 = 0$

$\therefore a = 3$ , $b = -6$ , $c = 1$

$\therefore x = \dfrac{6 \pm \sqrt{(-6)^2 - 4 \times 3 \times 1}}{2 \times 3} = \dfrac{6 \pm \sqrt{24}}{6}$

$= \dfrac{6 \pm 2\sqrt{6}}{6} = \dfrac{3 \pm \sqrt{6}}{3}$

$\therefore x \approx 1.82$ or $x \approx 0.18$

The S.S. $= \{1.82 , 0.18\}$

**[b]** $\because$ The domain of $n = \mathbb{R} - \{3\}$

$\therefore$ At $x = 3$ $\quad \therefore x^2 - a x + 9 = 0$

$\therefore 9 - 3a + 9 = 0$ $\quad \therefore -3a = -18$ $\quad \therefore a = 6$

**4**

**[a]** $\because y - x = 2$ $\quad \therefore y = x + 2$ $\quad$ (1)

$, x^2 + xy - 4 = 0$ $\quad$ (2)

Substituting from (1) in (2) :

$\therefore x^2 + x(x+2) - 4 = 0$

$\therefore x^2 + x^2 + 2x - 4 = 0$

$\therefore 2x^2 + 2x - 4 = 0$ (Dividing by 2)

$\therefore x^2 + x - 2 = 0$

$(x - 1)(x + 2) = 0$

$\therefore x = 1$ or $x = -2$

Substituting in (1) : $\therefore y = 3$ or $y = 0$

$\therefore$ The S.S. $= \{(1 , 3) , (-2 , 0)\}$

**[b]** $\because n(x) = \dfrac{x-3}{(x-4)(x-3)} + \dfrac{x-3}{x-3}$

$\therefore$ The domain of $n = \mathbb{R} - \{4 , 3\}$

$, n(x) = \dfrac{1}{x-4} + 1 = \dfrac{1 + x - 4}{x-4} = \dfrac{x-3}{x-4}$

**5**

**[a]** Let the measure of the first angle be $x°$

, the measure of the second angle be $y°$

$\therefore x + y = 90°$ $\quad$ (1)

$, x - y = 50°$ $\quad$ (2)

Adding (1) and (2) : $\therefore 2x = 140°$ $\quad \therefore x = 70°$

Substituting in (1) : $\therefore y = 20°$

$\therefore$ The measures of the two angles are 70° , 20°

**[b]** $\boxed{1}$ $\because n(x) = \dfrac{x(x-2)}{(x-2)(x^2+2)}$

$\therefore n^{-1}(x) = \dfrac{(x-2)(x^2+2)}{x(x-2)}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{0 , 2\}$

$, n^{-1}(x) = \dfrac{x^2+2}{x}$

$\boxed{2}$ $\because n^{-1}(x) = 3$ $\quad \therefore \dfrac{x^2+2}{x} = 3$

$\therefore x^2 - 3x + 2 = 0$ $\quad \therefore (x-2)(x-1) = 0$

$\therefore x = 2$ (refused) or $x = 1$

## Answers of model 3

**1**

| | | |
|---|---|---|
| 1 d | 2 b | 3 b |
| 4 c | 5 b | 6 b |

**2**

**[a]** $P(\grave{A}) = 1 - P(A) = 1 - 0.7 = 0.3$

$P(A - B) = P(A) - P(A \cap B)$

$= 0.7 - 0.3 = 0.4$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.7 + 0.5 - 0.3 = 0.9$

**[b]** $\because z(f) = \{5\}$ $\quad \therefore$ At $x = 5$

$\therefore x^2 - 10x + a = 0$ $\quad \therefore (5)^2 - 10 \times 5 + a = 0$

$\therefore 25 - 50 + a = 0$ $\quad \therefore a = 25$

**3**

**[a]** $\because x + y = 2$ $\quad$ (1)

$, \dfrac{1}{x} + \dfrac{1}{y} = 2$ $\quad \therefore x + y = 2xy$ $\quad$ (2)

Substituting in (1) from (2) : $\therefore 2 = 2\,x\,y$

$\therefore x\,y = 1$ $\qquad \therefore x = \frac{1}{y}$

Substituting in (1) : $\therefore \frac{1}{y} + y = 2$

Multiplying by y : $\therefore 1 + y^2 = 2\,y$

$\therefore y^2 - 2\,y + 1 = 0$ $\qquad \therefore (y-1)^2 = 0$

$\therefore y = 1$

Substituting in (1) : $\therefore x = 1$

$\therefore$ The S.S. $= \{(1 \,,\, 1)\}$

**[b]** $\because n_1(x) = \frac{x^2}{x^2(x-1)}$

$\left.\begin{array}{l} \therefore \text{The domain of } n_1 = \mathbb{R} - \{0 \,,\, 1\} \\ \therefore n_1(x) = \frac{1}{x-1} \end{array}\right\}$ (1)

$, \because n_2(x) = \frac{x(x^2+x+1)}{x(x^3-1)}$

$= \frac{x(x^2+x+1)}{x(x-1)(x^2+x+1)}$

$\left.\begin{array}{l} \therefore \text{The domain of } n_2 = \mathbb{R} - \{0 \,,\, 1\} \\ , n_2(x) = \frac{1}{x-1} \end{array}\right\}$ (2)

From (1) and (2) : $\therefore n_1 = n_2$

## 4

**[a]** $\because n(x) = \frac{x(x-3)}{(2x+3)(x-2)} \div \frac{x(2x-3)}{(2x-3)(2x+3)}$

$\therefore$ The domain of $n = \mathbb{R} - \left\{-\frac{3}{2} \,,\, 2 \,,\, 0 \,,\, \frac{3}{2}\right\}$

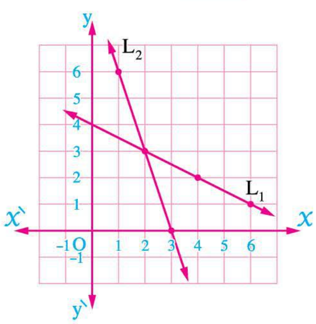
$, n(x) = \frac{x(x-3)}{(2x+3)(x-2)} \times \frac{(2x-3)(2x+3)}{x(2x-3)}$

$= \frac{x-3}{x-2}$

**[b]** $x = 8 - 2\,y$ $\qquad , y = 9 - 3\,x$

| $x$ | 6 | 4 | 2 |
|---|---|---|---|
| y | 1 | 2 | 3 |

| $x$ | 1 | 2 | 3 |
|---|---|---|---|
| y | 6 | 3 | 0 |




From the graph : $\therefore$ The S.S. $= \{(2 \,,\, 3)\}$

## 5

**[a]** $\because 2\,x^2 - 5\,x + 1 = 0$

$\therefore a = 2 \,,\, b = -5 \,,\, c = 1$

$\therefore x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 2 \times 1}}{2 \times 2} = \frac{5 \pm \sqrt{17}}{4}$

$\therefore$ The S.S. $= \left\{\frac{5+\sqrt{17}}{4} \,,\, \frac{5-\sqrt{17}}{4}\right\}$

**[b]** $\because n(x) = \frac{x(x+2)}{(x+2)(x-2)} - \frac{2(x-3)}{(x-2)(x-3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-2 \,,\, 2 \,,\, 3\}$

$, n(x) = \frac{x}{x-2} - \frac{2}{x-2} = \frac{x-2}{x-2} = 1$