



ANSWERS

Prep 3 - Second term 2021

Al Gebra

Al Basit in Mathematics

1 Choose the correct answer from those given

- | | | | |
|----------------------------------|--------------------------------|----------------------|----------------------------|
| 1 $\{(-5, 5)\}$ | 2 $\{(3, 1)\}$ | 3 The origin point | 4 $\{(3, 4)\}$ |
| 5 zero | 6 Parallel | 7 Coincident | 8 $a = 3$ |
| 9 $k \neq 4$ | 10 $k = 3$ | 11 Infinite numbers | 12 $k = 3$ |
| 13 9 and 4 | 14 $x + 8$ | 15 $x - 4$ | 16 $x + 10 y$ |
| 17 Φ | 18 zero | 19 $\{2, 3\}$ | 20 Φ |
| 21 Intersect X-axis in one point | 22 4 | 23 1 | 24 zero |
| 25 4 and 5 | 26 2 | 27 4 | 28 $\{(3, 3), (-3, -3)\}$ |
| 29 $(4, 2)$ | 30 $\{0\}$ | 31 \mathbb{R} | 32 $\{0, 1\}$ |
| 33 8 | 34 -50 | 35 5 | 36 \mathbb{R} |
| 37 $\mathbb{R} - \{4, -4\}$ | 38 \mathbb{R} | 39 Undefined | 40 $>$ |
| 41 $\{-3\}$ | 42 $\mathbb{R} - \{0, 1, -1\}$ | 43 3 | 44 7 |
| 45 $2x - 1$ | 46 1 | 47 $\frac{1-x}{x+3}$ | 48 $\mathbb{R} - \{7\}$ |
| 49 $\mathbb{R} - \{2, -5\}$ | 50 $\mathbb{R} - \{0\}$ | 51 x | 52 $\mathbb{R} - \{4, 7\}$ |
| 53 Undefined | 54 8 | | |

2 Find algebraically in $\mathbb{R} \times \mathbb{R}$ the solution set of each pair of the following equations

1 $2x - y = 3$ ① $x + 2y = 4$ ②

Multiply the two sides of equation ① by 2
We get: $4x - 2y = 6$ ③

adding ③ + ② $4x - 2y = 6$ ③
 $x + 2y = 4$ ②

$\therefore 5x = 10$ $\therefore x = 2$

By substituting in ② $\therefore 2 + 2y = 4$
 $\therefore 2y = 4 - 2 = 2$ $\therefore y = 1$
 $\therefore S.S = \{(2, 1)\}$

2 $3x + 4y = 24$ ① $x - 2y = -2$ ②

Multiply the two sides of equation ② by 2
We get: $2x - 4y = -4$ ③

adding ③ + ① $2x - 4y = -4$ ③
 $3x + 4y = 24$ ①

$\therefore 5x = 20$ $\therefore x = 4$

By substituting in ① $\therefore 3x + 4y = 24$
 $\therefore 4y + 12 = 24$ $\therefore 4y = 24 - 12 = 12$
 $\therefore y = 3$ $\therefore S.S = \{(4, 3)\}$

3 $3x + 2y = 11$ ① $2x + 3y = 14$ ②

Multiply the two sides of equation ① by 3
We get: $9x + 6y = 33$ ③

Multiply the two sides of equation ② by -2
We get: $-4x - 6y = -28$ ④

adding ③ + ④ $9x + 6y = 33$ ③
 $-4x - 6y = -28$ ④

$\therefore 5x = 5$ $\therefore x = 1$

By substituting in ① $\therefore 3 + 2y = 11$
 $\therefore 2y = 11 - 3 = 8$ $\therefore y = 4$
 $\therefore S.S = \{(1, 4)\}$

4 $\frac{x}{6} + \frac{y}{3} = \frac{1}{3}$ ① $\frac{x}{2} + \frac{2y}{3} = 1$ ②

Multiply the two sides of equation ① by 10
We get: $2x + 4y = 4$ ③

Multiply the two sides of equation ② by -6
We get: $-3x - 4y = -12$ ④

adding ③ + ④ $2x + 4y = 4$ ③
 $-3x - 4y = -12$ ④

$\therefore -x = -2$ $\therefore x = 2$

By substituting in ③ $\therefore 4 + 4y = 4$
 $\therefore 4y = 4 - 4 = 0$ $\therefore y = 0$
 $\therefore S.S = \{(2, 0)\}$



5 $x - y = 1$ ① $x^2 + y^2 = 25$ ②

From eq ① $x - y = 1$ We get $x = 1 + y$ ③

By substituting in ② $\therefore (1+y)^2 + y^2 = 25$

$$\therefore 1 + 2y + y^2 + y^2 = 25$$

$$\therefore 2y^2 + 2y + 1 - 25 = 0$$

$$\therefore 2y^2 + 2y - 24 = 0 \quad \text{divide both sides by 2}$$

$$\therefore y^2 + y - 12 = 0 \quad \therefore (y-3)(y+4) = 0$$

$$\therefore y = 3 \quad \text{or} \quad y = -4$$

By substituting in ③

At: $y = 3 \quad \therefore x = 1 + 3 = 4$

At: $y = -4 \quad \therefore x = 1 + (-4) = -3$

$\therefore \text{S.S} = \{(-3, -4), (4, 3)\}$

6 $x + y = 7$ ① $y^2 - x^2 = 7$ ②

From eq ① $x + y = 7$ We get $y = 7 - x$ ③

By substituting in ② $\therefore (7-x)^2 - x^2 = 7$

$$\therefore 49 - 14x + x^2 - x^2 = 7$$

$$\therefore -14x + 49 - 7 = 0$$

$$\therefore -14x + 42 = 0 \quad \therefore -14x = -42$$

$$\therefore x = 3$$

By substituting in ③

At: $x = 3 \quad \therefore y = 7 - 3 = 4$

$\therefore \text{S.S} = \{(3, 4)\}$

7 $y - x = 3$ ① $x^2 + y^2 - xy = 13$ ②

From eq ① $y - x = 3$ We get $y = 3 + x$ ③

By substituting in ②

$$\therefore x^2 + (3+x)^2 - x(3+x) = 13$$

$$\therefore x^2 + 9 + 6x + x^2 - 3x - x^2 - 13 = 0$$

$$\therefore x^2 + 3x - 4 = 0 \quad \therefore (x-1)(x+4) = 0$$

$$\therefore x = 1 \quad \text{or} \quad x = -4$$

By substituting in ③

At: $x = 1 \quad \therefore y = 3 + 1 = 4$

At: $x = -4 \quad \therefore y = 1 + (-4) = -3$

$\therefore \text{S.S} = \{(-4, -3), (1, 4)\}$

8 $x + y = 2$ ① $\frac{1}{x} + \frac{1}{y} = 2$ ②

Multiply the two sides of equation ② by xy

We get: $y + x = 2xy$ ③

From eq ① $y + x = 2$ We get $y = 2 - x$ ③

By substituting in ③

$$\therefore 2 - x + x = 2x(2 - x) \quad \therefore 2 = 4x - 2x^2$$

$$\therefore 2x^2 - 4x + 2 = 0 \quad \text{divide both sides by 2}$$

$$\therefore x^2 - 2x + 1 = 0 \quad \therefore (x-1)^2 = 0$$

$$\therefore x = 1$$

By substituting in ①

At: $x = 1 \quad \therefore y = 2 - 1 = 1$

3 Find in \mathbb{R} the solution set of each of the following equations using the general formula

1 $2x^2 - 4x + 1 = 0$

$a = 2, b = -4 \text{ and } c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16 - 4 \times 2 \times 1}}{2 \times 2}$$

$$= \frac{4 \pm \sqrt{8}}{4} \quad \therefore x = \frac{4 + \sqrt{8}}{4} = 1.707$$

or $x = \frac{4 - \sqrt{8}}{4} = 0.293$

$\therefore \text{S.S} = \{1.707, 0.293\}$

2 $x(x-1) = 4 \quad \therefore x^2 - x - 4 = 0$

$a = 1, b = -1 \text{ and } c = -4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1 - 4 \times 1 \times -4}}{2 \times 1}$$

$$= \frac{1 \pm \sqrt{17}}{2} \quad \therefore x = \frac{1 + \sqrt{17}}{2} = 2.562$$

or $x = \frac{1 - \sqrt{17}}{2} = -1.562$

$\therefore \text{S.S} = \{-1.562, 2.562\}$

3 $x - \frac{4}{x} = 4 \quad \text{Multiply both sides by } x$

$$\therefore x^2 - 4 = 4x \quad \therefore x^2 - 4x - 4 = 0$$

$a = 1, b = -4 \text{ and } c = -4$

Complete by yourself

4 $\frac{8}{x^2} - \frac{1}{x} = 1 \quad \text{Multiply both sides by } x^2$

$$\therefore 8 - x = x^2 \quad \therefore x^2 + x - 8 = 0$$

$a = 1, b = 1 \text{ and } c = -8$

Complete by yourself

5 $(x-3)^2 - 5x = 0 \quad \therefore x^2 - 6x + 9 - 5x = 0$

$a = 1, b = -11 \text{ and } c = 9$

Complete by yourself

4 in each of the following find $n(x)$ in the simplest form showing the domain of each of them

$$1 \quad n(x) = \frac{x^2 - 25}{x^2 - 3x - 10} = \frac{(x-5)(x+5)}{(x-5)(x+2)}$$

\therefore Domain = $\mathbb{R} - \{5, -2\}$

$$, n(x) = \frac{(x-5)(x+5)}{(x-5)(x+2)} = \frac{(x+5)}{(x+2)}$$

$$3 \quad n(x) = \frac{x}{x-4} + \frac{x+4}{x^2-16}$$

$$= \frac{x}{x-4} + \frac{x+4}{(x-4)(x+4)}$$

\therefore Domain = $\mathbb{R} - \{4, -4\}$

$$, n(x) = \frac{x}{x-4} + \frac{x+4}{(x-4)(x+4)} \\ = \frac{x}{x-4} + \frac{1}{x-4} = \frac{(x-1)}{(x-4)}$$

$$5 \quad n(x) = \frac{x^2 + 2x + 4}{x^3 - 8} - \frac{9 - x^2}{x^2 + x - 6} \\ = \frac{x^2 + 2x + 4}{(x-2)(x^2 + 2x + 4)} + \frac{x^2 - 9}{(2x-3)(x-5)} \\ = \frac{x^2 + 2x + 4}{(x-2)(x^2 + 2x + 4)} + \frac{(x-3)(x+3)}{(x+3)(x-2)}$$

\therefore Domain = $\mathbb{R} - \{2, -3\}$

$$, n(x) = \frac{1}{x-2} + \frac{x-3}{x-2} = \frac{x-2}{x-2} = 1$$

$$7 \quad n(x) = \frac{x^2 - 5x}{x^2 - 8x + 15} - \frac{x^2 + 3x + 9}{x^3 - 27} \\ = \frac{x(x-5)}{(x-3)(x-5)} - \frac{x^2 + 3x + 9}{(x-3)(x^2 + 3x + 9)}$$

\therefore Domain = $\mathbb{R} - \{3, 5\}$

$$, n(x) = \frac{x}{x-3} + \frac{1}{x-3} = \frac{x+1}{x-3}$$

$$\because 1 \in \text{Domain} \quad \therefore n(1) = \frac{1+1}{1-3} = -2$$

$\because 5 \notin \text{Domain} \quad \therefore n(5) \text{ undefined}$

$$2 \quad n(x) = \frac{x^3 - 4x}{x^3 - 5x^2 + 6x} = \frac{x(x-2)(x+2)}{x(x-3)(x-2)}$$

\therefore Domain = $\mathbb{R} - \{0, 3, 2\}$

$$, n(x) = \frac{x(x-2)(x+2)}{x(x-3)(x-2)} = \frac{(x+2)}{(x-3)}$$

$$4 \quad n(x) = \frac{x-6}{2x^2 - 15x + 18} + \frac{x-5}{15-13x+2x^2}$$

$$= \frac{x-6}{(2x-3)(x-6)} + \frac{x-5}{(2x-3)(x-5)}$$

\therefore Domain = $\mathbb{R} - \{6, 5, \frac{3}{2}\}$

$$, n(x) = \frac{x-6}{(2x-3)(x-6)} + \frac{x-5}{(2x-3)(x-5)} \\ = \frac{1}{2x-3} + \frac{1}{2x-3} = \frac{2}{2x-3}$$

$$6 \quad n(x) = \frac{x^2 - 3x + 2}{1-x^2} \div \frac{3x-15}{x^2 - 6x + 5}$$

$$= \frac{(x-1)(x-2)}{-(x^2-1)} \div \frac{3(x-5)}{(x-5)(x-1)}$$

$$= \frac{(x-1)(x-2)}{-(x-1)(x+1)} \times \frac{(x-5)(x-1)}{3(x-5)}$$

\therefore Domain = $\mathbb{R} - \{1, -1, 5\}$

$$, n(x) = \frac{x-2}{x+1} + \frac{x-1}{3} = \frac{(x-1)(x-2)}{3(x+1)}$$

Complete by yourself

5 Answer the following question

$$1 \quad \because \text{domain} = \mathbb{R} - \{3\} \quad \therefore x^2 + ax + 9 = 0 \quad \text{at } x = 3 \quad \text{substituting by 3 in the denominator} \\ \therefore 9 + 3a + 9 = 0 \quad \therefore 3a = -18 \quad \therefore a = -6$$

$$2 \quad \because \text{domain} = \mathbb{R} - \{2, 3\}$$

$$\therefore x^2 + ax + b = 0 \quad \text{at } x = 2 \text{ and } 3$$

$$\text{substituting by 2 in the denominator} \quad \therefore 4 + 2a + b = 0 \quad \therefore 2a + b = -4 \quad ①$$

$$\text{substituting by 3 in the denominator} \quad \therefore 9 + 3a + b = 0 \quad \therefore 3a + b = -9 \quad ②$$

$$\text{Multiply the two sides of equation } ① \text{ by } -1 \quad \text{We get: } -2a - b = 4 \quad ③$$

$$\text{adding } ③ + ② \quad \text{We get: } a = -5 \quad \text{By substituting in } ① \quad \therefore b = 6$$

$$3 \quad \because n_1(x) = \frac{x^2 - x}{x^3 - 2x^2} = \frac{x(x-1)}{x^2(x-2)} = \frac{(x-1)}{x(x-2)} \quad \text{and its domain} = \mathbb{R} - \{0, 2\} \quad ①$$



$$, n_1(x) = \frac{x^2 - 3x + 2}{x^3 - 4x^2 + 4x} = \frac{(x-1)(x-2)}{x(x-2)(x-2)} = \frac{(x-1)}{x(x-2)} \text{ and its domain} = \mathbb{R} - \{0, 2\} \quad (2)$$

From (1) and (2) $\therefore n_1 = n_2$

4 $\because z(f) = \{3, 5\}$

$$\therefore f(3) = 0 \quad \therefore 9a + 3b + 15 = 0 \quad \therefore 9a + 3b = -15 \quad (1)$$

$$, f(5) = 0 \quad \therefore 25a + 5b + 15 = 0 \quad \therefore 25a + 5b = -15 \quad (2)$$

Multiply the two sides of equation (1) by -5 We get: $-45a - 15b = 75$ (3)

Multiply the two sides of equation (2) by 3 We get: $75a + 15b = -45$ (4)

$$\text{adding (3) + (4) We get: } 30a = 30 \quad \therefore a = 1$$

$$\text{By substituting in (1) } \therefore b = -8$$

5 \because let length = x and width = y

$$\text{A length of a rectangle is 3 cm. more than its width means: } x - y = 3 \quad (1)$$

$$\text{area is } 28 \text{ cm}^2 \quad \text{means: } xy = 28 \quad (2)$$

solve the two equations together by yourself $x = 7$ and $y = 3$

6 \because let the lengths of the two sides of the right-angle are x and y

$$\text{the length of the hypotenuse} = 13 \text{ cm.}$$

$$\text{perimeter} = 30 \text{ cm.} \Rightarrow x + y + 13 = 30$$

$$\Rightarrow x^2 + y^2 = 169 \quad (1)$$

$$\Rightarrow x + y = 17 \quad (2)$$

solve the two equations together by yourself $x = 12$ and $y = 5$

7 try yourself

8 \because let the digit of ones is x and the digit of tens is y then: the number is $x + 10y$

$$\text{the sum of its digits is } 11 \Rightarrow x + y = 11 \quad (1)$$

if the two digits reversed ($y + 10x$), then the resulted number is 27 more than the original number

$$(y + 10x) - (x + 10y) = 27 \Rightarrow 9x - 9y = 27 \text{ divide both sides by 9} \Rightarrow x - y = 3 \quad (2)$$

solve the two equations together by yourself $x = 7$ and $y = 4$

9 \because let the measures of the two angles are x and y

$$\text{Two acute angles in a right-angled triangle} \Rightarrow x + y = 90 \quad (1)$$

$$\text{the difference between their measures} = 50^\circ \Rightarrow x - y = 50 \quad (2)$$

solve the two equations together by yourself $x = 70$ and $y = 20$

10 $\because (3, -1)$ is the solution set of the equation: $a x + b y = 5 \quad \therefore 3a - b = 5 \quad (1)$

$$\because (3, -1)$$
 is the solution set of the equation: $3a x + b y = 17 \quad \therefore 9a - b = 17 \quad (2)$

$$\text{Multiply the two sides of equation (1) by -1 We get: } -3a + b = -5 \quad (3)$$

$$\text{adding (3) + (2) We get: } 6a = 12 \quad \therefore a = 2$$

$$\text{By substituting in (1) } \therefore b = 1$$

Al Basit in mathematics A New Starting

