

مراجعة ليلة الامتحان.. وبك لأهم المسائل المتوقعة في الامتحان

وصفة سحرية للتفوق.. أعدها خبراء في وضع الامتحانات وصناعة الأوائل



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[1] Choose the correct answer:

1) If $n(X)=3$ and $n(X \times Y)=15$, then $n(Y)=\dots\dots\dots$ (5, 12, 18 or 45)

2) If $(3, 5) \in \{3, 6\} \times \{x, 8\}$, then $x=\dots\dots\dots$ (3, 5, 6 or 8)

3) If $\{2\} \times \{x, y\} = \{(2, 4), (2, 3)\}$, then $x-y=\dots\dots\dots$ (1, -1, ± 1 or 0)

4) If $X-Y=\{7\}$, $Y-X=\{2, 4\}$ and $X \cap Y=\{6\}$, then $(X \times Y) \cap (Y \times X)=\dots\dots\dots$

($\{(6, 6)\}$, $\{(7, 2), (7, 4)\}$, $\{(7, 6)\}$ or $\{(2, 7), (4, 7)\}$)

5) If the point $(a-b, 5)$ lies on y-axis, then $\dots\dots\dots$

($a=b$, $a+b=0$, $a \neq b$ or $a-b=5$)

6) If the point $(5, b-7)$ is located on the x-axis then $b=\dots\dots\dots$ (2, 5, 7 or 12)

7) If the relation $R=\{(4, 3), (1, 3), (2, 5)\}$, then R represents a function where its range is $\dots\dots\dots$

($\{1, 2, 4\}$, $\{4, 1, 2, 3, 5\}$, $\{3, 5\}$ or \mathbb{N})

8) If the function $f: X \rightarrow Y$, then the range of the function $f \subset \dots\dots\dots$

($X \times Y$, $X, Y \times X$ or Y)

9) The function $f(x)=x(x-4x^2)$ is a polynomial of the $\dots\dots\dots$ degree.

(first, second, third or fourth)

10) If $f(x)=x^2 - \sqrt{3}x$, then $f(\sqrt{3})=\dots\dots\dots$ (0, 2, 4 or 6)

11) If $f(x)=kx+8$ and $f(2)=0$, then $k=\dots\dots\dots$ (-4, 4, 6 or 8)

12) If $f(x)=7$, then $f(-3)=\dots\dots\dots$ (-21, -7, 7 or 21)

13) The function f where $f(x)=3x$ is represented graphically by a straight line which passes through the point $\dots\dots\dots$

($(3, 3)$, $(3, 0)$, $(0, 0)$ or $(0, 3)$)

14) The function $f(x)=x^2-(x^2-2x)$ is a polynomial of the $\dots\dots\dots$ degree.

(first, second, third or fourth)

15) If $3a=5b$, then $\frac{3a}{b}=\dots\dots\dots$ ($\frac{3}{5}$, $\frac{3}{5}$, 3 or 5)

16) If $4x^2=9y^2$, then $\frac{x}{y}=\dots\dots\dots$ ($\frac{9}{4}$, $\frac{3}{2}$, $\pm \frac{2}{3}$ or $\pm \frac{3}{2}$)

17) If $\frac{a+2b}{a-b}=\frac{2}{3}$, then $\frac{b}{a}=\dots\dots\dots$ (-8, $-\frac{1}{8}$, $\frac{1}{8}$ or 8)

18) If $\frac{a}{b}=\frac{2}{3}$ and $\frac{a}{c}=\frac{4}{5}$, then $b:c=\dots\dots\dots$ (3:4, 5:6, 6:5 or 4:3)

19) The third proportion of the two numbers 9 and -12 is $\dots\dots\dots$

(-16, 8, 16 and 108)

20) The proportional mean between x and y is $\dots\dots\dots$ (\sqrt{xy} , $-\sqrt{xy}$, $\pm \sqrt{xy}$ or xy)

21) The proportional mean between $(x-2)$ and $(x+2)$ is $\dots\dots\dots$

($\sqrt{x+2}$, x^2-4 , $\pm \sqrt{x^2-4}$ or $\sqrt{x^2-4}$)

22) If y varies inversely with x and $x=\sqrt{3}$ where $y=\frac{2}{\sqrt{3}}$, then the constant proportional $=\dots\dots\dots$ ($\frac{1}{2}$, $\frac{2}{3}$, 2 or 6)

23) If $y=3x-6$, then $y \propto \dots\dots\dots$ (x , $3x$, $x-2$ or $3x+6$)

24) If $y^2+4x^2=4xy$, then $\dots\dots\dots$ ($y \propto x$, $y \propto x^2$, $y \propto \frac{1}{x}$ or $y \propto \frac{1}{x^2}$)

25) If $\frac{x}{2}=\frac{y}{3}=\frac{4x-2y}{z}$, then $z=\dots\dots\dots$ (-2 , $-\frac{1}{2}$, $\frac{1}{2}$ or 2)

26) Selecting a sample of layers of a statistical society is called $\dots\dots\dots$ sample. (random, layer, deliberate or bunch)

27) The mean of the values 7, 3, 6, 9 and 5 equals $\dots\dots\dots$ (3, 4, 6 or 12)

28) The positive square root of the average of squares of deviations of values from their mean is called the $\dots\dots\dots$

(range, arithmetic mean, median or standard deviation)

29) If all individuals are equal in values, then $\dots\dots\dots$

($x-\bar{x}>0$, $x-\bar{x}<0$, $\sigma=0$ or $\bar{x}=0$)

30) The range of the set of values 5, 14, 4, 37, 15, 16, and 7 is $\dots\dots\dots$ (22, 30, 32 or 33)

Answers

1) 5 2) 5 3) ± 1 4) $\{(6, 6)\}$

5) $a=b$ 6) 7 7) $\{3, 5\}$ 8) Y

9) third 10) 0 11) -4 12) 7

13) (0, 0) 14) first 15) 5 16) $\pm \frac{3}{2}$

17) $-\frac{1}{8}$ 18) 6:5 19) 16 20) $\pm \sqrt{xy}$

21) $\pm \sqrt{x^2-4}$ 22) 2 23) $x-2$

24) $y \propto x$ 25) 2 26) layer 27) 6

28) standard deviation 29) $\sigma=0$ 30) 33

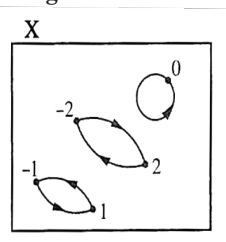
[2] If $X=\{-2, -1, 0, 1, 2\}$ and R is a relation on X where "a R b" means "a is the additive inverse of the number b" for every $a \in X$ and $b \in X$.

State R, then represent it by arrow diagram and Cartesian diagram.

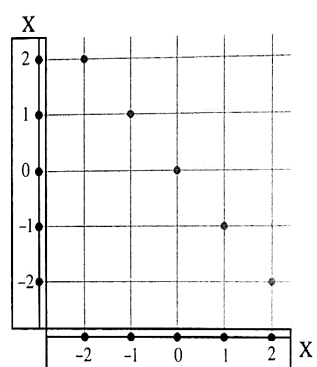
Answer

$R=\{(-2, 2), (-1, 1), (0, 0), (1, -1), (2, -2)\}$

The arrow diagram:



The Cartesian diagram:



[3] If $X=\{0, 1, 2, 3\}$, $Y=\{2, 3, 4, 5, 6\}$ and R is a relation from X to Y where "a R b" means "a + b = 5" for each $a \in X$ and $b \in Y$.

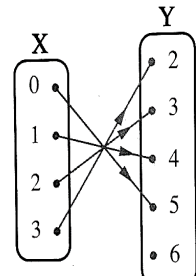
Write the relation R and represent it by an arrow diagram.

Mention giving reasons if R is a function from X to Y or not? And if it is a function, find its range.

Answer

$R=\{(0, 5), (1, 4), (2, 3), (3, 2)\}$

R represents a function from X to Y because each element of X connects with only one element of Y.



The range of the function = $\{5, 4, 3, 2\}$

[4] If $f: f(x)=x^2-2x+5$, prove that: $f(2\sqrt{2}+1)=2f(1-\sqrt{2})$

Answer

$\therefore f(2\sqrt{2}+1)=(2\sqrt{2}+1)^2-2(2\sqrt{2}+1)+5$

$=8+4\sqrt{2}-4\sqrt{2}-2+5=12$ (1)

$f(1-\sqrt{2})=(1-\sqrt{2})^2-2(1-\sqrt{2})+5$

$=1+2-2\sqrt{2}-2+2\sqrt{2}+5=6$ (2)

From (1) and (2): $\therefore f(2\sqrt{2}+1)=2f(1-\sqrt{2})$

[5] Graph the function $f: f(x)=x^2-2x-3$ taking $x \in [-2, 4]$ from the graph, find:

- The point of the vertex of the curve.
- The equation of the line of symmetry.
- The maximum or minimum value of the function.

Answer

$f(x)=x^2-2x-3$

x	-2	-1	0	1	2	3	4
f(x)	5	0	-3	-4	-3	0	5

From the graph, we deduce that:

- The point of the vertex of the curve is (1, -4)
- The equation of the line of symmetry is $x=1$
- The maximum value of the function = -4

Answer

[6] Find the number that if we add to the two terms of the ratio 17 : 22 the result will be 6 : 7.

$\therefore \frac{17+x}{22+x}=\frac{6}{7} \therefore 7(17+x)=6(22+x)$

$\therefore 119+7x=132+6x$

$\therefore 7x-6x=132-119 \therefore x=13$

\therefore The required number = 13

[7] If $4x-3y:2x+y=4:7$, find in the simplest form the ratio $x:y$.

Answer

$\therefore \frac{4x-3y}{2x+y}=\frac{4}{7} \therefore 7(4x-3y)=4(2x+y)$

$\therefore 28x-21y=8x+4y$

$\therefore 28x-8x=21y+4y$

$\therefore 25x=25y \therefore \frac{x}{y}=\frac{25}{20} \therefore \frac{x}{y}=\frac{5}{4}$

[8] If a, b, c and d are proportional quantities, prove that:

(1) $\frac{2a+3c}{7a-5c}=\frac{2b+3d}{7b-5d}$

(2) $\frac{a+c}{b+d}=\frac{a^2+c^2}{ab+cd}$

Answer

(1) Let $\frac{a}{b}=\frac{c}{d}=m \therefore \textcircled{a}=bm, \textcircled{c}=dm$

L.H.S. = $\frac{2bm+3dm}{7bm-5dm}=\frac{m(2b+3d)}{m(7b-5d)}$

$=\frac{2b+3d}{7b-5d}=\text{R.H.S.}$

(2) Let $\frac{a}{b}=\frac{c}{d}=m \therefore \textcircled{a}=bm, \textcircled{c}=dm$

$\therefore \frac{a+c}{b+d}=\frac{bm+dm}{b+d}=\frac{m(b+d)}{(b+d)}=m$ (1)

$\therefore \frac{a^2+c^2}{ab+cd}=\frac{(bm)^2+(dm)^2}{bm \times b+dm \times d}=\frac{b^2m^2+d^2m^2}{b^2m+d^2m}$

$=\frac{m^2(b^2+d^2)}{m(b^2+d^2)}=m$ (2)

From (1) and (2): $\frac{a+c}{b+d}=\frac{a^2+c^2}{ab+cd}$

[9] If $\frac{a}{4}=\frac{b}{5}=\frac{c}{3}$, Prove that:

$\frac{a-b+c}{a+b-c}=\frac{1}{3}$

Answer

Multiplying the two terms of the 2nd ratio by (-1), then add the antecedents and the consequents of the three ratios:

$\therefore \frac{a-b+c}{4-5+3}=\frac{a-b+c}{2}=\text{one of the given ratios.}$ (1)

Multiplying the two terms of the 3rd ratio by (-1), then add the antecedents and the consequents of the three ratios:

$\therefore \frac{a+b-c}{4+5-3}=\frac{a+b-c}{6}=\text{one of the given ratios.}$ (2)

From (1) and (2): $\frac{a-b+c}{2}=\frac{a+b-c}{6}$

$\therefore \frac{a-b+c}{a+b-c}=\frac{2}{6}=\frac{1}{3}$

[10] If b is a middle proportion between a and c, prove that:

(1) $\frac{a-b}{a}=\frac{a-c}{a+b}$

(2) $ab-c^2=(b-c)(a+b+c)$

Answer

$\therefore b$ is the middle proportion between a and c

$\therefore a, b$ and c are in continued proportion

$\therefore \frac{a}{b}=\frac{b}{c}=m$

$\therefore \textcircled{b}=cm, \textcircled{a}=cm^2$

(1) $\therefore \frac{a-b}{a}=\frac{cm^2-cm}{cm^2}=\frac{cm(m-1)}{cm^2}=\frac{m-1}{m}$

$\therefore \frac{a-c}{a+b}=\frac{cm^2-c}{cm^2+cm}=\frac{c(m^2-1)}{cm(m+1)}$

$=\frac{c(m-1)(m+1)}{cm(m+1)}=\frac{m-1}{m}$

From (1) and (2): $\frac{a-b}{a}=\frac{a-c}{a+b}$

(2) $\therefore ab-c^2=cm^2 \times cm-c^2=c^2m^3-c^2$

$=c^2(m^3-1)$

$\therefore (b-c)(a+b+c)$

$=(cm-c)(cm^2+cm+c)$

$=c(m-1) \times c(m^2+m+1)$

$=c^2(m-1)(m^2+m+1)$

$=c^2(m^3-1)$

From (1) and (2) we deduce that:

$ab-c^2=(b-c)(a+b+c)$

[11] If $a^2+4b^2=4ab$, prove that: $a \propto b$

Answer

$\therefore a^2+4b^2=4ab \therefore a^2-4ab+4b^2=0$

$\therefore (a-2b)^2=0 \therefore a-2b=0$

$\therefore a=2b \therefore a \propto b$

[12] The following table shows the distribution of ages of 20 persons in years:

The age	15	20	22	23	25	30	Total
No. of persons	2	3	5	5	1	4	20

Find the standard deviation of the ages.

Answer

The age (x)	No. of persons (k)	$x \times k$
15	2	30
20	3	60
22	5	110
23	5	115
25	1	25
30	4	120
Total	20	460

The mean (\bar{x}) = $\frac{\sum(x \times k)}{\sum k}=\frac{460}{20}=23$ years.

x	k	$x-\bar{x}$	$(x-\bar{x})^2$	$(x-\bar{x})^2 \times k$
15	2	-8	64	128
20	3	-3	9	27
22	5	-1	1	5
23	5	0	0	0
25	1	2	4	4
30	4	7	49	196
Total	20			360

$\sigma=\sqrt{\frac{\sum(x-\bar{x})^2 \times k}{\sum k}}=\sqrt{\frac{360}{20}}=\sqrt{18} \approx 4.24$ years.

[13] If $\frac{21x-y}{7x-z}=\frac{y}{z}$, prove that $y \propto z$.

Answer

$\therefore \frac{21x-y}{7x-z}=\frac{y}{z} \therefore 21xz-z^2-yz=7xy-yz$

$\therefore 21xz=7xy \therefore 3z=y \therefore y \propto z$

[14] $(x-2, 3)=(5, 3y+1)$, then find the value of x and y.

Answer

$\therefore (x-2, 3)=(5, 3y+1)$

$\therefore x-2=5 \therefore x=7$

$3y+1=3 \therefore 3y=2 \therefore y=\frac{2}{3}$

[15] If a, b, and c are in continued proportion, prove that:

$\frac{4a^2-3b^2}{4b^2-3c^2}=\frac{a}{c}$

Answer

Let $\frac{a}{b}=\frac{b}{c}=m \therefore b=cm$ and $a=cm^2$

$\therefore \frac{4a^2-3b^2}{4b^2-3c^2}=\frac{4(cm^2)^2-3(cm)^2}{4(cm)^2-3c^2}$

$=\frac{4c^2m^4-3c^2m^2}{4c^2m^2-3c^2}=\frac{c^2m^2(4m^2-3)}{c^2(4m^2-3)}=m^2$ (1)

\there