مراجعة ليلة الامتحان.. وبنك أسئلة لأهم المسائل المتوقعة

روشتة تفوق.. لن يخرج عنها الامتحان.. أعدها خبراء في صناعة الأوائل

أسرة الرياضيات باللغة الإنجليزية

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1- Choose the correct answer from those given:

[1] The distance between the point (3 , 4) and the origin point =…… length unit.

(a) 3 (b) 4 (c) 5 (d) 7

[2] If $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{1}{2}$, then the slope of $\overleftrightarrow{CD}$ = ……

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 2 (d) – 2

[3] If $\tan(x + 10°) = \sqrt{3}$ where x is an acute angle, then m (∠ x) = ….°

(a) 0 (b) 20 (c) 35 (d) 50

[4] If Δ XYZ is right-angled at Z, XY = 25 cm, YZ = 7 cm and XZ = 24 cm, then sin X + sin Y = ….

(a) $\frac{31}{25}$ (b) $\frac{17}{25}$ (c) 2 (d) 1

[5] If $\sin x = \frac{1}{2}$ and x is an acute angle, then $\sin 2x$ = …….

(a) 1 (b) $\frac{1}{4}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{3}}$

[6] If x and y are two measures of two complementary angles, where $x : y$ = 1 : 2, then $\sin x + \cos y$ =...

(a) 90° (b) 60° (c) 30° (d) 1

[7] If $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{2}{3}$, then the slope of $\overleftrightarrow{CD}$ = ……

(a) $\frac{-2}{3}$ (b) 5 (c) – 6 (d) $\frac{2}{3}$

[8] If (4 , – 3) is the midpoint of $\overleftrightarrow{AB}$ such that A(3 , – 4), then the coordinates of B are ……..

(a) (5 , 2) (b) (5 , – 2)

(c) (2 , 5) (d) (0 , 2)

[9] The equation of the straight line whose slope is 1 and passes through the origin point is ……

(a) $x = 1$ (b) $y = 1$

(c) $y = x$ (d) $y = -x$

[10] tan 45° sin 30° = ……

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{2}{3}$ (d) $\frac{1}{4}$

[11] In Δ ABC, if m (∠ B) = 90°, then sin A + cos C = ……

(a) 2 sin A (b) 2 sin C

(c) 2 sin B (d) 2 cos A

[12] The equation of the straight line which passes through the point (2 , – 3) and parallel to x- axis is …

(a) $x = -2$ (b) $y = -3$

(c) $x = 2$ (d) $y = 3$

[13] The points (– 3 , 0) , (0 , 3) and (3 , 0) are vertices of ……

(a) a scalene triangle.

(b) an equilateral triangle.

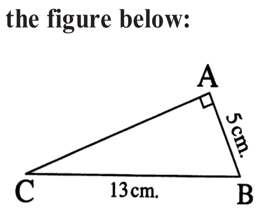
(c) an obtuse-angled triangle.

(d) a right-angled triangle and isosceles.

[14] The slope of the straight line which is parallel to the x-axis is …

(a) – 1 (b) 0 (c) 1 (d) undefined.

[15] In the figure below:



ABC is a triangle in which m (∠ A) = 90°, AB = 5 cm and BC = 13 cm, then tan B = …..

(a) $\frac{5}{13}$ (b) 2.4 (c) $\frac{13}{5}$ (d) $\frac{25}{13}$

[16] A circle its centre is the origin and its radius length is 2 length unit, which of the following points belongs to the circle?

(a) (1 ,2) (b) (−2 , 1)

(c) ($\sqrt{3}$, 1) (d) ($\sqrt{2}$, 1)

[17] If (x_1 , y_1) and B(x_2 , y_2), then AB = ……….

(a) $x_1 x_2 + y_1 y_2$

(b) $\sqrt{x_1 x_2 + y_1 y_2}$

(c) $(x_2 - x_1 , y_2 - y_1)$

(d) $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

[18] $2 \tan 45 - \frac{1}{\cos 60°}$ = …..

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

[19] For any acute angle X, tan X = …

(a) $\frac{\cos X}{\sin X}$ (b) sin X cos X

(c) $\frac{\sin X}{\cos X}$ (d) sin X + cos X

[20] If the straight line whose equation $x + 3y - 6 = 0$ is perpendicular to the straight line whose equation $a x - 3y + 7 = 0$, then a = ……

(a) 2 (b) 9 (c) – 9 (d) – 2

Answers

[1] c	[2] d	[3] d	[4] a
[5] c	[6] d	[7] d	[8] b
[9] c	[10] a	[11] a	[12] b
[13] d	[14] b	[15] b	[16] c
[17] d	[18] a	[19] c	[20] b

2- If A (3 , 1), B (1 , 2) and C (5 , 4), prove that BC = 2AB.

Answer

AB = $\sqrt{(1-3)^2 + (2-1)^2}$

= $\sqrt{4+1} = \sqrt{5}$ length unit.

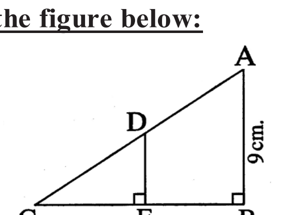
BC = $\sqrt{(5-1)^2 + (4-2)^2}$

= $\sqrt{16+4} = \sqrt{20}$

= $2\sqrt{5}$ length unit.

∴ BC = 2AB

3- In the figure below:



ABC is a right-angled triangle at B in which:

AB = 9 cm., D ∈ $\overline{AC}$, E ∈ $\overline{BC}$ where $\overline{DE} \perp \overline{BC}$ and 4 DE = 3 EC.

Calculate: The area of Δ ABC.

Answer

∵ 4 DE = 3 EC ∴ $\frac{DE}{EC} = \frac{3}{4}$

∵ $\tan C = \frac{DE}{EC} = \frac{3}{4}$, $\tan C = \frac{AB}{BC}$

∴ $\frac{AB}{BC} = \frac{3}{4}$ ∴ $\frac{9}{BC} = \frac{3}{4}$

∴ BC = 12 cm.

∴ The area of Δ ABC = $\frac{1}{2} \times 12 \times 9$

= 54 cm²

4- If ABCD is a rectangle, where A(1 , 1), B(1 , 4), C(5 , 4) and D(5 , 1), then find its surface area.

Answer

∵ AB = $\sqrt{(1-1)^2 + (4-1)^2} = \sqrt{9}$

= 3 length unit

BC = $\sqrt{(5-1)^2 + (4-4)^2} = \sqrt{16}$

= 4 length unit

∴ The area of the rectangle ABCD

= 3 × 4 = 12 square unit

5- Without using the calculator find the numerical value of the expression:

cos 60° sin 30° – sin 60° cos 30°

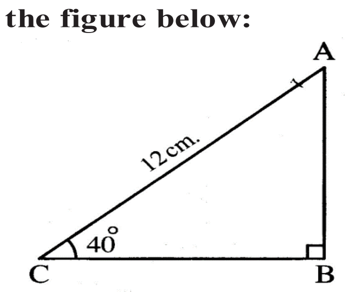
Answer

cos 60° sin 30° – sin 60° cos 30°

$= \frac{1}{2} \times \frac{1}{2} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} - \frac{3}{4}$

$= -\frac{1}{2}$

6- In the figure below:



m (∠ C) = 40°

and AC = 12 cm.

Find the surface area of :

Δ ABC to the nearest cm².

Answer

∵ m (∠ B) = 90° ∴ $\sin C = \frac{AB}{AC}$

∴ AB = AC sin 40° ≃ 7.7 cm.

∴ (BC)² = (12)² – (7.7)² = 84.71

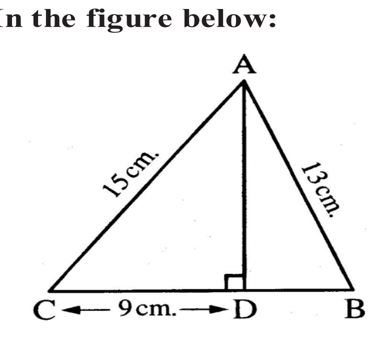
∴ BC = $\sqrt{84.71}$ ≃ 9.2 cm.

∴ The area of Δ ABC

$= \frac{1}{2} \times 7.7 \times 9.2$

= 35.42 cm² ≃ 35 cm²

7- In the figure below:



$\overline{AD} \perp \overline{BC}$, AB = 13 cm. , AC = 15 cm. , CD = 9 cm.

Find in the simplest form the value of :

$\frac{\tan(\angle CAD) + \tan(\angle BAD)}{\tan(\angle CAD) - \tan(\angle BAD)}$

Answer

In Δ ADC :

(AD)² = (15)² – (9)² = 144

∴ AD = $\sqrt{144}$ = 12 cm.

In Δ ADB :

(BD)² = (13)² – (12)² = 25

∴ BD = $\sqrt{25}$ = 5 cm.

∴ $\frac{\tan(\angle CAD) + \tan(\angle BAD)}{\tan(\angle CAD) - \tan(\angle BAD)}$

$= \frac{\frac{9}{12} + \frac{5}{12}}{\frac{9}{12} - \frac{5}{12}} = \frac{\frac{14}{12}}{\frac{4}{12}} = \frac{7}{2}$

8- Find the value of A where A is the measure of an acute angle if:

$\cos A \times \tan A = \frac{1}{2}$

Answer

∵ $\cos A \times \tan A = \frac{1}{2}$

∴ $\cos A \times \frac{\sin A}{\cos A} = \frac{1}{2}$

∴ $\sin A = \frac{1}{2}$ ∴ A = 30°

9- A straight line, its slope is $\frac{1}{2}$ and intersects a positive part from y-axis with length of two units, find:

(1) the equation of this straight line.

(2) its intersection point with y-axis.

Answer

(1) ∵ The slope of the straight line = $\frac{1}{2}$

and it intercepts a part of length of 2 units from the positive part of the y-axis

∴ The equation of the straight line is

$y = \frac{1}{2}x + 2$

(2) put $x = 0$

∴ $y = \frac{1}{2} \times 0 + 2$ ∴ y = 2

∴ The coordinates of intersection point with y-axis is : (0 , 2)

10- Without using the calculator, prove each of the following:

[a] cos 60° = 2 cos² 30° – 1

[b] cos 60° = cos² 30° – sin² 30°

Answers

[a] The left side = cos 60° = $\frac{1}{2}$

The right side = 2 cos² 30° – 1

$= 2\left(\frac{\sqrt{3}}{2}\right)^2 - 1 = 2 \times \frac{3}{4} - 1 = \frac{1}{2}$

∴ The two sides are equal.

[b] The left side = cos 60° = $\frac{1}{2}$

The right side = cos² 30° – sin² 30°

$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

∴ The two sides are equal.

11- Prove that: The triangle whose vertices are A(3 , 2), B(−4 , 1) and C(2 , −1) is right-angled, then find its area.

Answer

∵ AB = $\sqrt{(3+4)^2 + (2-1)^2}$

= $\sqrt{49+1} = \sqrt{50}$ length unit.

BC = $\sqrt{(-4-2)^2 + (1+1)^2}$

= $\sqrt{36+4} = \sqrt{40}$ length unit.

AC = $\sqrt{(3-2)^2 + (2+1)^2}$

= $\sqrt{1+9} = \sqrt{10}$ length unit.

∵ (AC)² + (BC)² = 10 + 40 = 50,

(AB)² = 50

∴ (AC)² + (BC)² = (AB)²

∴ The triangle is right-angles at C.

∴ The area of the triangle ABC

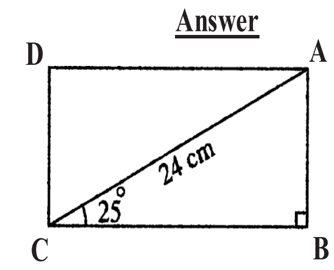
$= \frac{1}{2}$ AC × BC $= \frac{1}{2} \times \sqrt{10} \times 2\sqrt{10}$

= 10 square unit.

12- ABCD is a rectangle whose diagonal length AC = 24 cm and m (∠ ACB) = 25°.

Find: the length of $\overline{BC}$

Answer



∵ ABCD is a rectangle.

∴ m (∠ B) = 90°

In Δ ABC:

∴ cos (∠ ACB) = $\frac{BC}{AC}$

∴ cos 25° = $\frac{BC}{24}$

∴ BC = 24 cos 25° ≃ 21.8 cm.

13- Prove that:

The straight line passing through the two points (2 , 5) and (6 , 9) is parallel to the straight line which makes an angle of measure 45° with the positive direction of the x-axis.

Answer

∵ $m_1 = \frac{9-5}{6-2} = 1$, $m_2 = \tan 45° = 1$

∴ $m_1 = m_2$

∴ The two straight lines are parallel.

14- Prove that:

The point A (−2 , 5), B (3 , 3) and C (−4 , 2) are non-collinear and if D (−9 , 4), then prove that the figure ABCD is a parallelogram.

Answer

AB = $\sqrt{(3+2)^2 + (3-5)^2}$

= $\sqrt{25+4} = \sqrt{29}$ length unit.

BC = $\sqrt{(-4-3)^2 + (2-3)^2}$

= $\sqrt{49+1} = \sqrt{50}$ length unit.

CA = $\sqrt{(-2+4)^2 + (5-2)^2}$

= $\sqrt{4+9} = \sqrt{13}$ length unit.

∵ BC is the greatest distance,

BC < AB + CA

∴ A , B and C are non-collinear point.

∵ AD = $\sqrt{(-2+9)^2 + (5-4)^2}$

= $\sqrt{49+1} = \sqrt{50}$ length unit.

CD = $\sqrt{(-9+4)^2 + (4-2)^2}$

= $\sqrt{25+4} = \sqrt{29}$ length unit.

∴ AB = CD and BC = AD

∴ ABCD is a parallelogram.